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Bispectra of Wind-waves and Wave-wave Interaction*

Norihisa IMASATO** and Hideaki KUNISHI**

Abstract: Bispectra of wind-waves in wind tunnels were calculated in order to understand the characteristics of the nonlinear wave-wave interaction in actual wind-wave field. It is shown that the nonlinearity in wind-waves increases in magnitude with the development of wind-waves and that the characteristics of nonlinearity in wind-waves in the early stage of development differ from those in the late stage. It is shown that the bispectra are classified into five types (I~V), and that the bispectral type changes from the type I to the type V as the wind-waves develop from the stage of the "initial-wavelets" to that of the "sea-waves". The relations between frequencies of the component waves interacting each other are discussed in each bispectral type.

1. Introduction

The nonlinear wave-wave interaction in wind-waves has been discussed theoretically some investigators (see KINSMAN, 1965). Using the observed power spectra of wind-waves, some investigators (BARNETT, 1968; HASSELMANN *et al.*, 1973; and others) tried to estimate the actual features of wind-wave interaction. Recently, bispectrum has been used in order to understand the nonlinear interaction in wind-waves (HASSELMANN *et al.*, 1963; KAKINUMA *et al.*, 1968). IMASATO (1976b) demonstrated a few examples of the bispectrum of wind-waves in wind tunnels in order to prove his model on the development of wind-waves and suggested that the wave-wave interaction of a wave spectral peak with other frequencies plays an important role in development of wind-waves.

In this paper, we will illustrate some characteristics of the nonlinearity in the wind-wave field, using bispectra of wind-waves in the wind tunnels. Wave data used in this paper were obtained by KUNISHI (1963) and KUNISHI and IMASATO (1966), and the development process of these wind-waves has been already discussed by KUNISHI (1963) and IMASATO (1976a, b). Definitions on the Run number and the development state of wind-waves (*e.g.* "sea-waves", "initial-wavelets" *etc.*) are same as those in the papers of IMASATO (1976a, b).

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** Geophysical Institute, Faculty of Science, Kyoto University, Kitashirakawa Oiwake-cho, Sakyo-ku, Kyoto 606, Japan

2. Bispectra of wind-waves in wind tunnels

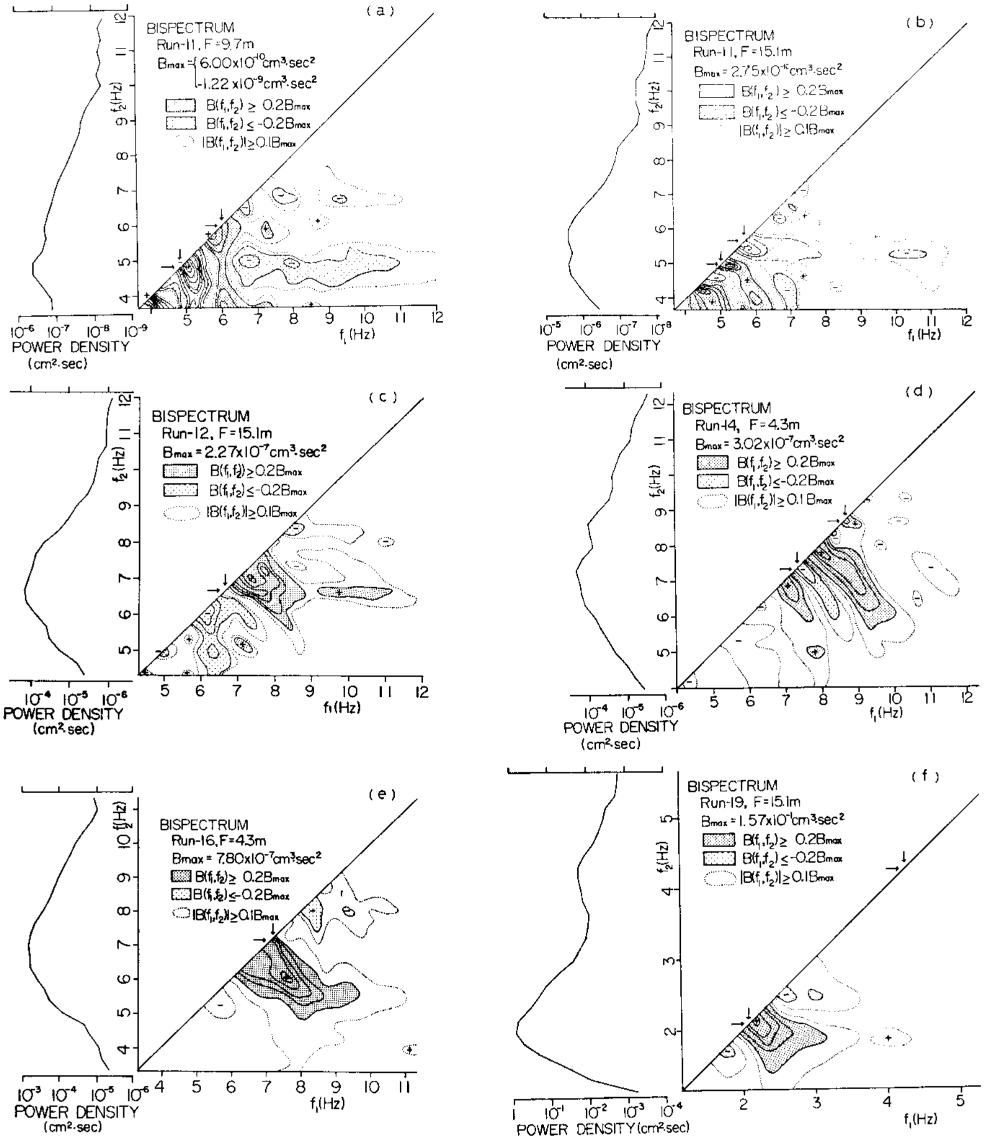
Bispectrum $B(\omega_1, \omega_2)$ is defined as the Fourier transform of an auto-bicorrelation function $R(\tau_1, \tau_2)$ of wind-waves $\eta(t)$ (TUKEY, 1963), *i.e.*

$$R(\tau_1, \tau_2)=\overline{\eta(t)\,\eta(t+\tau_1)\,\eta(t+\tau_2)} \qquad (1)$$

$$B(\omega_1, \omega_2)=\frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} R(\tau_1, \tau_2) \times \exp\left[i(\omega_1\tau_1+\omega_2\tau_2)\right] d\tau_1 d\tau_2 \qquad (2)$$

where τ_1 and τ_2 are the lag time and the over bar indicates the time mean, and ω_1 and ω_2 are the angular frequencies.

Figs. 1a-g show some examples of bispectra of wind-waves in the wind-tunnels. Solid contour curves are drawn by every 0.2 B_{max}, where B_{max} indicates the bispectral density of a dominant bispectral peak. Dotted contour curves indicate $|B(f_1,f_2)|=0.1\,B_{max}$, where $f=\omega/2\pi$. In these figures, power spectra are also shown along the frequency coordinate f_2. In early stage of development of wind-waves (*e.g.* in the stage of "initial-wavelets"), bispectrum is rather complicated in the distribution pattern and the significant bispectral densities spread wide over the frequency area. It is distinguished that the fairly dominant bispectral ridges strech toward the lower and the higher frequencies from the point (f_{max}, f_{max}) (f_{max} is the frequency of the power spectral peak). On the other hand, in the late stage of the development of wind-waves, *i.e.* in the stage of "sea-waves", bispectrum



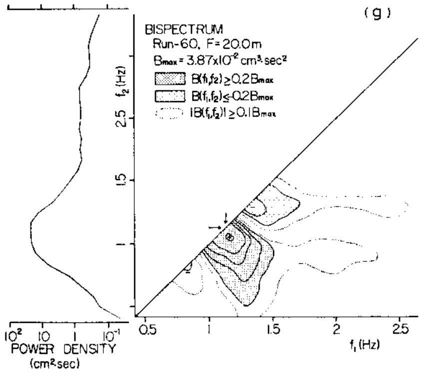


Fig. 1. Bispectrum of wind-waves in wind tunnels (right side) and power spectrum (left side). (a) Run 11, Fetch 9.7 m, (b) Run 11, Fetch 15.1 m, (c) Run 12, Fetch 15.1 m, (d) Run 14, Fetch 4.3 m, (e) Run 16, Fetch 4.3 m, (f) Run 19, Fetch 15.1 m and (g) Run 60, Fetch 20.0 m. Wind-waves in (a), (b) and (c) belong to the stage of "initial-wavelets", those in (d) and (e) to the "transition stage" and those in (f) and (g) to the stage of "sea-waves".

becomes fairly simple in the distribution pattern. The significant bispectral densities concentrate around the point (f_{max}, f_{max}) to make a dominant bispectral peak, and significant bispectral ridges strech toward the lower and the higher frequencies. These bispectra indicate that a dominant power spectral peak nonlinearly interacts with the higher and the lower frequencies.

As a measure of the order of nonlinearity of the wind-wave field, we will adopt the bispectral density B_{max} of the dominant bispectral peak and the relative area A which is defined as A_{sig}/A_{all}, where A_{sig} is the area of $|B(f_1,f_2)| \geq 0.2\, B_{max}$ and A_{all} is the area of the triangle frequency domain where the bispectrum is defined. Fig. 2 illustrates the relation between B_{max} and $S^{3/2}_{max}$, where S_{max} is the spectral density of the dominant power spectral peak. Relation between B_{max} and $S^{3/2}_{max}$ in the region $S^{3/2}_{max}<10^{-9}$ seems to differ from that in the region $S^{3/2}_{max}>10^{-9}$, and the two solid lines in Fig. 2 are the fittest ones to the data. Fig. 3 illustrates the relation between the relative area A and the bispectral density B_{max}. It is clearly seen in the figure that the area A is very wide in the early stage of the development of wind-waves. These differences in B_{max} and A seem to correspond to the stage of the development of wind-waves such as "initial-wavelets" and "sea-waves".* In the "initial-wavelets", the order of nonlinearity is small but the significant interaction occures wide over the frequency area. As the wind-waves develop, the area A becomes rapidly narrower and the order of nonlinearity becomes gradually larger. On the other hand, in the stage of the "sea-waves", the former becomes gradually narrower and the latter becomes rapidly larger. Anyway, we can conclude that the results in these two figures indicate that the order of nonlinearity and the area of the significant bispectral densities of

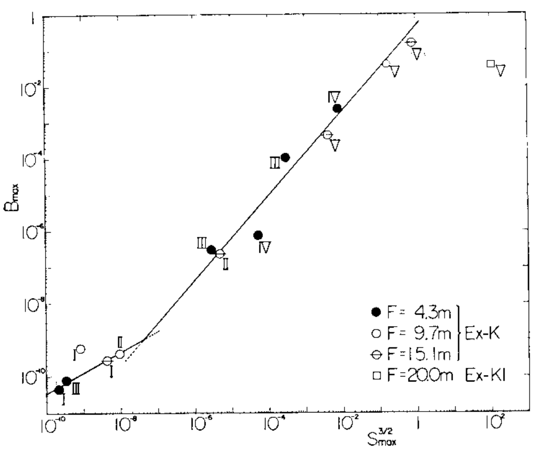


Fig. 2. B_{max} versus $S^{3/2}_{max}$. Bispectral type is shown by Roman numerals I～V.

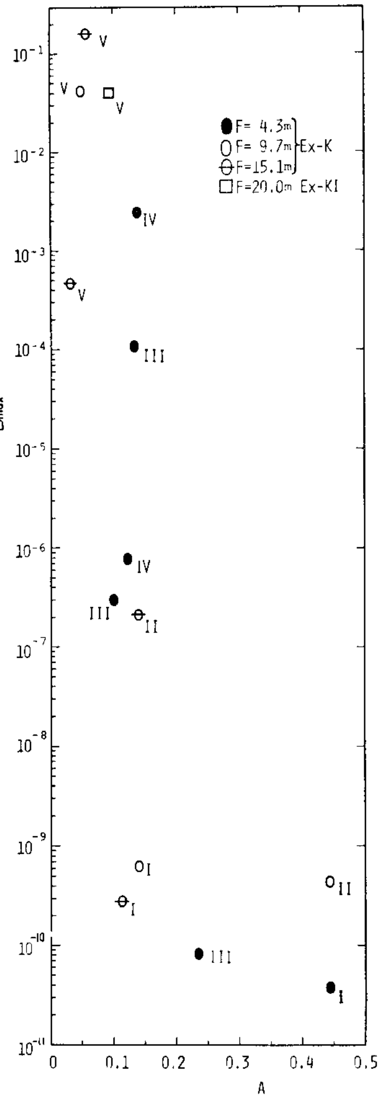


Fig. 3. B_{max} versus the relative area A. Bispectral type is shown by Roman numerals I～V.

* Dr. Y. NAGATA suggests, in his kind personal communications, that these differences may not be associated with the stage of the development of wind-waves but with the power spectral form of wind-waves. The study on his suggestion will be done in near future.

wind-wave field become larger and larger as the wind-waves develop.

3. Wave-wave interaction

From Figs. 1a-g, it is found that the bispectra of the wind-waves in wind tunnels can be classified into five types; *i.e.* the types I, II, III, IV and V. The bispectrum of the type I is illustrated in Figs. 1a and b, that of the type II in Fig. 1c, that of the type III in Fig. 1d, that of the type IV in Fig. 1e and that of the type V in Figs. 1f and g. In these figures, it is seen that a bispectrum has some fairly dominant bispectral ridges. Therefore, we may schematically represent a bispectral ridge with a line in order to discuss the basic characteristics of the wave-wave interaction. Schematic bispectra of these five types are shown in Figs. 4a-e, where the significant bispectral ridges are given by solid lines, *i.e.* a thick line indicates the relatively large bispectral density and a thin line does the relatively small one. It must be mentioned that the schematic bispectrum in Fig. 4b is only an example of the type II.

Fig. 4a shows that, in the bispectrum in the type I, a dominant wave with the frequency f_{max} interacts nonlinearly with the component waves with the lower or higher frequencies. Therefore, $f_1=$const. or $f_2=$const.

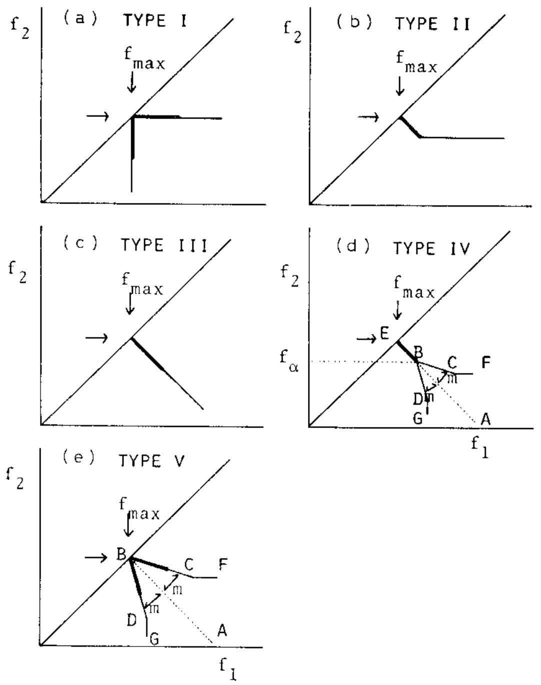


Fig. 4. Schematic line bispectrum in the five types I~V. Thick lines indicate the relatively large bispectral density. Arrows indicate the power spectral peak.

In the case of a line bispectrum in the type III (Fig. 4c), we have the relation $f_1+f_2=$const. (*e.g.* $2f_{max}$). In the actual bispectrum in this type (Fig. 2), significant nonlinear interaction concentrates around the point (f_{max}, f_{max}). Combining the type I with the type III, we can obtaine the line bispectrum in the type II as is shown in Fig. 4b. Therefore, the bispectrum in the type II can be understood to be in the transition stage from the type I to the type III. Therefore, the three figures in the left hand side in Fig. 4 illustrate the basic types of the wind-wave bispectrum.

In the case of a bispectrum in the type IV, the relation in the frequencies interacting each other in the line bispectrum $\overline{EB}$ is the same as that in the type III, and the relation in the frequencies in the line $\overline{CF}$ is the same as that in the type I. The relation in the line $\overline{BC}$ is given by Eq. 3,

$$f_2+f_1\tan\left(\frac{\pi}{4}-m\right)=2f_{max}+f_{1\alpha}\left[\tan\left(\frac{\pi}{4}-m\right)-1\right] \quad (3)$$

where $f_{1\alpha}$ is the abcissa of the point B, and the positive m is the angle ABC in radian. On the other hand, Eq. 3 with negative m (the angle ABD) gives the relation for the line $\overline{BD}$. As the point B approaches toward the point E along the line $\overline{BE}$ in Fig. 4d, the interaction type IV changes to the type V. Therefore, the relation between f_1 and f_2 for the line $\overline{BC}$ in the line bispectrum in the type V in Fig. 4e is easily obtained from Eq. 3 by putting $f_{1\alpha}=f_{max}$, *i.e.*

$$f_1\tan\left(\frac{\pi}{4}-m\right)+f_2=f_{max}\left[1+\tan\left(\frac{\pi}{4}-m\right)\right] \quad (4)$$

Mean value of m in the bispectra in the type V is ± 0.419 radian, and therefore, Eq. 4 becomes Eqs. 5.1 and 5.2,

$$0.384f_1+f_2=1.38f_{max} \quad (\text{for } \overline{BC}) \quad (5.1)$$

$$2.36f_1+f_2=3.36f_{max} \quad (\text{for } \overline{BD}) \quad (5.2)$$

From the above discussions, we understand that the bispectrum of the type IV is in a transition stage from the type III to the type V.

The actual bispectrum of wind-waves does not have the schematic form of a line but has the bispectral ridges as well illustrated in Figs. 1a-g. Therefore, combinations of interacting waves will be very complicated. Nevertheless, it must be emphasized that the most significant interaction can be represeted by the line bispectra shown in Figs. 4a-e.

We discussed the bispectral types and the basic relations between the frequencies of the component waves interacting each other. In Figs. 2 and 3, where the types of bispectrum are denoted in Roman numerals (I∼V), it is shown that bispectral type changes from I to V as the wind-waves develop. In this paper, we could not clearify why a wind-wave field at some condition chooses some one of bispectral types, and this problem will be left in future to be studied.

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References

BARNETT, T.P. (1968): On the generation, dissipation and prediction of ocean wind waves. J. Geophys. Res., **73**, 513-529.

HASSELMANN, K., W. MUNK and D. MACDONALD (1963): Bispectra of ocean waves. Proc. Symposium on Time Series Analysis, John Wieley and Sons, Inc., New York, pp. 125-139.

HASSELMANN, K., T.P. BARNETT, E. BOUNS, H. CARLSON, D.E. CARTWRIGHT, K. ENKE, J.A. EWING, H. GIENAPP, D.E. HASSELMANN, P. KRUSEMAN, A. MEERBURG, P. MÜLLER, D.J. OLBERS, K. RICHTER, W. SELL and H. WALDEN (1973): Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). Ergänzungsheft zur Dt. Hydro. Z., Reihe A, **12**, 1-95.

IMASATO, N. (1976a): Some characteristics on the development process of the wind-wave spectrum. J. Oceanogr. Soc., Japan, **32**, 21-32.

IMASATO. N. (1976b): The mechanism of the development of wind-wave spectra. J. Oceanogr. Soc., Japan, **32**, 253-266.

KAKINUMA, T., A. ISHIDA and T. MONJI (1968): Analysis of nonlinearity of coastal ocean waves. Proc. 15th Conf. on Coastal Engn. in Japan, pp. 73-79 (in Japanese).

KINSMAN, B. (1965): Wind Waves. Prentice Hall, Inc., Englewood Cliffs, New Jersey, pp. 584-638.

KUNISHI, H. (1963): An experimental study on the generation and growth of wind waves. Bull., Disast. Prev. Res. Inst., Kyoto Univ., **61**, 1-41.

KUNISHI, H. and N. IMASATO (1966): On the growth of wind waves in high-speed wind flume. Annuals, Disast. Prev. Res. Inst., Kyoto Univ., **9**, 667-676 (in Japanese).

TUKEY, J.W. (1963): What can data analysis and statistics offer today? *In*, Ocean Wave Spectra, Prentice Hall, Inc., Englewood Cliffs, New Jersey, pp. 347-351.

風波のバイスペクトルと相互作用

今 里 哲 久*, 國 司 秀 明*

要旨: 風洞水槽実験の風波の資料を用いて，現実の風波の相互作用の特性を調べるためにバイスペクトルを計算した．その結果，風波の発達の初期と後期とでは，非線形性の様子が異なっていること，また，風波が発達するにつれてその非線形性は強くなることが分かった．風波のバイスペクトルは五つの型（基本的には三つ）に分類することができて，風波が発達するにつれて，タイプはⅠからⅤへと変って行く．相互作用を行なっている成分波の周波数間の関係を各々のタイプについて示した．

* 京都大学理学部地球物理学教室
〒606 京都市左京区北白川追分町