# Some Characteristics of the Development Process of the Wind-wave Spectrum\*

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Abstract: The development process of wind-waves of which spectral peak distributes from 0.6 cps to 9.3 cps will be discussed on the basis of the wind tunnel experiments and of the field observations performed at Lake Biwa. The characteristics of power and slope spectra are here presented. The development process of these wind-waves is characterized by three stages; i.e. "initial-wavelets", "transition stage" and "sea-waves". In the wind tunnel experiments, the transition from the stage of the "initial-wavelets" to the "transition stage" occurs when the wave spectral peak arrives at the line  $6.40 \times 10^{-4} k^{-2} \text{ cm}^2 \cdot \text{sec}$  (where k is wave number) or when the slope spectral density at the frequency  $f_{\text{max}}$  becomes larger than  $6.40 \times 10^{-4}$  sec. In the stage of sea-waves, the component wave of a wave-spectral peak is steepest in the component waves. And the wave spectral peak develops along the line  $1.02 \times 10^2 f^{-6}$  cm<sup>2</sup>·sec (where f is the frequency corresponding to the wave number k) untill it reaches the line  $33.3 f^{-4} \text{ cm}^2 \cdot \text{sec}$ , and thereafter develops along the latter line, which indicates the constant density of slope spectrum. It is suggested that the nonlinearity of wind-waves must become stronger as wind-waves develop. The effective momentum flux  $\tau_{ws}$  from the air flow to wind-waves in this stage is evaluated to be about  $4 \sim 9 \%$  of the total stress  $\tau_0$ .

#### 1. Introduction

A large number of observations and experiments have been performed by many investigators in order to solve the fascinating phenomenon of water surface being undulated by wind blowing over the surface. Their studies have brought much information on this phenomenon, and several excellent theories on the generation and development of the wind-waves have been proposed by several investigators (KINSMAN, 1965).

KUNISHI (1963) and KUNISHI and IMASATO (1966) discussed the development process of wind-waves in wind tunnels by using the mean properties of wind-waves under the wind in the range from 1.5 to 34 m·sec<sup>-1</sup>. KUNISHI (1963) showed that the development of wind-waves can be classified in three stages; the initial-tremor, the initial-wavelets and the sea-waves.

In order to compair the development process of wind-waves in a field with that of windwaves in the wind tunnels, we carried out field observations at Lake Biwa. Power and slope spectra of wind-waves are calculated using the data obtained by KUNISHI (1963) and KUNISHI and IMASATO (1966) and also using the data obtained from the field observations at Lake Biwa. Spectra  $\tau_w(f)$  of momentum flux from the air flow are estimated using the energy equation for a component wave of wind-wave spectrum. Spectral density of  $\tau_w(f)$  indicates the effective momentum flux which the component wave of the frequency f needs to receive from the wind in order to develop from an energy level to another.

It is shown that the development process of wind-waves and of the effective momentum flux can be traced by three stages; *i.e.* the initial-wavelets, the transition stage and the sea-waves. Some discussions are given about some characteristics of the power spectra of wind-waves in these stages, *e.g.* the equilibrium state in a wind-wave spectrum, the overshoot phenomenon in the wave spectral peak and the nonlinearity of the wind-wave field. The first problem of the equilibrium state in a wave

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spectrum was introduced by PHILLIPS (1958) from dimension analysis. According to him, the wave spectrum has an equilibrium range which is proportional to  $f^{-5}$ . This  $f^{-5}$  law has been supported to hold in the frequency range from 0.2 cps to 2.1 cps from the field observations (PHILLIPS, 1966) and in the frequency range from 2.5 cps to 40 cps from the wind tunnel experiments (PLATE, CHANG and HIDY. 1969, and others). The second problem of the overshoot phenomenon has been reported by

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Fig. 1. The map of the south basin of Lake Biwa. The observation site is shown by a double circle.

BARNETT and WILKENSON (1967), BARNETT and SUTHERLAND (1968) and others. The third problem of the nonlinear wave-wave interaction has been studied by PHILLIPS (1960), HASSELMANN (1962, 1963), BARNETT (1968) and others. According to the present study, the spectrum  $\tau_w(f)$  of sea-waves has a frequency ragion where the spectral density is negative. If our assumption that this phenomenon is

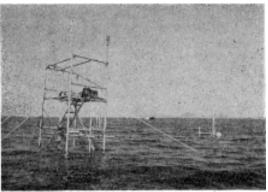


Photo 1. The observation tower at Lake Biwa.

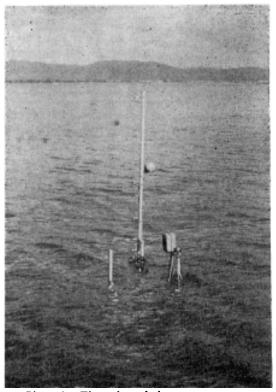


Photo 2. The pole and the cup-anemometers.

related with the nonlinear wave-wave interaction of sea-waves is reasonable, nonlinearity of sea-waves must become stronger as sea-waves develop.

#### 2. Field observations at Lake Biwa

Field observations were carried out at the south basin of Lake Biwa in 1968 and 1969. The location of an observation tower shown in Fig. 1 by a circle was about 100 m offshore, and the mean water depth around the tower was about 2.4 m. This observation tower, shown in Photo 1, consists of three portable aluminium scaffolds 1.7 m in width, 2.25 m in length and 2.0 m in height. A pole, which is shown in Photo 2, was set about 5 m away from the tower in order to prevent the effects of the tower in vertical mean wind profiles.

The mean wind vertical profiles were measured by seven small anemometers, which have three cups of 2.0 cm in diameter. They were set on the pole at the height of 10, 20, 35, 65, 110, 230 and 430 cm, respectively above the mean water surface. Wind-waves were measured by capacitance-type wave gauges at the tower. Observed data of wind-waves and wind are tabulated in Table 1.

## 3. Development of wind-wave spectrum at Lake Biwa

Power spectra of wind-waves are calculated by the well known method of BLACKMAN and TUKEY (1958). Values of the wave spectral density  $S_{\text{max}}$  and the frequency  $f_{\text{max}}$  of a spectral peak are shown in Table 1. They are obtained from the wave data which were measured

Table 1.  $f_{\text{max}}$ ,  $S_{\text{max}}$ , and  $\phi_{\text{max}}$  and several characteristic variables of the wind field obtained from the field observations at Lake Biwa.

Run No	Date		me m	$f_{ m max} \ ( m cps)$	$S_{\text{max}}$ (cm <sup>2</sup> ·sec)	$\phi_{ ext{max}}$ (sec.)	z <sub>0</sub> (mm)	$U_*$ (cm·sec <sup>-1</sup> )	$W_{10} \pmod{\mathrm{m \cdot sec^{-1}}}$	γ2	F (km)
IL-01	1968 June 9	19	50	0.50	02.70	×10 <sup>-3</sup>				×10 <sup>-8</sup>	
02	1908 Julie 9	13	55	0.50 0.40	93.79 227.7	$9.57 \ 9.33 \ $	0, 45	26.0	6.50	1.60	4.7
03			59	0.40	293.7	$\frac{9.33}{12.03}$	0.40	20.0	0.50	1.00	4. /
04		14	37	0.40	36.64	7.70	0.51	16.2	4.01	1.64	4.9
05		1.1	43	0.55	96.06	14.30	0.24	18.2	4.84	1.42	4.9
06			52	0.55	107.8	16.06	0.86	22.7	5, 31	1,83	4.9
07	June 10	9	53	0.58	30.74	5.77	0.30	23.9	6, 21	1.48	4,9
08	•	-	59	0.53	46, 51	6. 15	0.19	23. 4	6.37	1.35	4.9
09		12	07	0.45	60.50	4,07	0.16	27.2	7.52	1.31	4.0
10			17	0,45	67.75	4.56)					
11			21	0.45	75.83	5. 10	0.21	27.4	7.38	1.38	4.7
12	1969 June 6	11	34	0.67	10.03	3. 21	0.11	22.3	6.36	1.22	6.8
13			48	0.60	19.85	4.17)			0.00	1 00	
14			52	0.60	22.87	4.81	0.19	25.7	6.96	1.36	6.8
15	June 8	6	53	0.60	24.79	5.21)	0.15	14.0	4.10	1 00	4.0
16			56	0.60	28.86	6.07}	0.15	14.9	4.16	1.29	4.9
IS-02	1969 June 4	11	52	1.20	1.003	3.37	0.09	10.9	3, 83	0.81	0.20
03		12	04	1.00	2.038	3.31	0.25	19.8	5.60	1.25	0.20
04	June 5	10	30	3.30	0.00527	0.986	1.37	5.40	1.20	2.03	0.20
05			35	2.50	0.02316	1.85	0.65	5,94	1.43	1,73	0.20
06			48	1.80	0.1910	3.24	0.082	6.84	2.00	1.17	0.20
07			57	1.50	0.5113	4.20	0.088	8.43	2.45	1, 18	0.20
08		11	03	1.30	0.7422	3.44	0.089	10.02	2,91	1.19	0.20
09			29	1.20	0.9294	3.13	0.030	8.21	2.61	0.99	0.20
10			59	1.20	1.202	4.04	0.0073	7.20	2.54	0.80	0.20
11 12	June 7	6	13 23	2.30 2.00	0, 0542 0, 2306	2. 44) 5. 96)	2.55	17.04	3.84	2.34	0.18

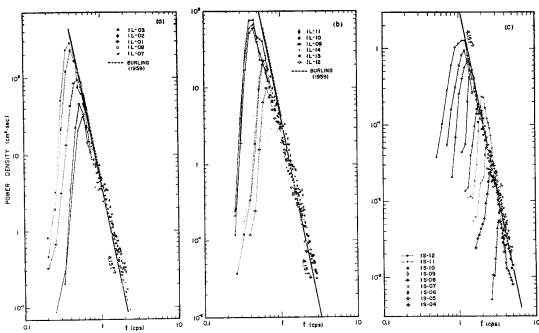


Fig. 2. Wave spectra obtained at long fetches (a, b) and at short fetches (c) in Lake Biwa.

when wind field was developing with times  $S_{\rm max}$  is distributed from  $0.005\,{\rm cm^2\cdot sec}$  to 230 cm<sup>2</sup>·sec, and also  $f_{\rm max}$  from 0.4 cps to 3.33 cps. The wind-wave spectrum with a peak in the high frequency of 3.33 cps has acarcely been discussed from field observations (PHILLIPS, 1966).

Some examples of the wind-wave spectrum are shown in Fig. 2: (a) and (b) show the spectra at long fetch (4~6 km), and (c) shows those at short fetch (0.1~1 km). In Fig. 2(c), two series of wind-wave spectra are included; the wave spectra from IS-04 to IS-08 in Table 1 belong to one series and the wave spectra IS-11 and 12 belong to another. Wind-waves in these two series were generated on a calm water surface and developed by the wind which began suddenly to blow from the lake shore. The high frequency part of a wave spectrum lies on the line of  $f^{-5}$  as shown by a solid line in this figure. From Fig. 2, it is found that the high frequency part of most of the wave spectra from the observations at Lake Biwa follows the  $f^{-5}$  law, which is proposed by PHILLIPS (1958).

The high frequency parts of twenty-seven wave spectra, which are listed up in Table 1,

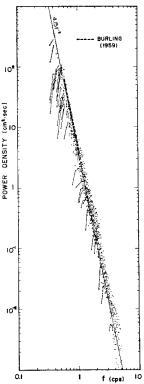


Fig. 3. The line  $4.15 f^{-5}$  which was obtained from the high frequency parts of twenty seven wave spectra shown in Table 1.

are plotted in Fig. 3. The constant  $\alpha$  of the  $f^{-5}$  law is given from Fig. 3 to be  $4.15 \, \mathrm{cm^2 \cdot sec^{-4}}$  in the range of the frequency from  $0.4 \, \mathrm{cps}$  to  $5.0 \, \mathrm{cps}$ . Therefore, spectral density of the high frequency part in a wind-wave spectrum is expressed as follows:

$$S(f)=4.15f^{-5}$$
  $(f>f_{max}),$  (1)

in the region of the frequency from 0.4 cps to 5.0 cps. This present result agrees well with BURLING's observed result (BURLING, 1959) where the coefficient  $\alpha$  is given to be 4.25 cm<sup>2</sup>· sec<sup>-4</sup> in the range of the frequency from 0.6 cps to 1.8 cps. BURLING's result has been accepted in the range of the frequency from 0.2 cps to 2.1 cps by the field observations (PHILLIPS, 1966).

From Fig. 3, the  $f^{-5}$  law is considered to universally hold to the extent to the high frequency of 5.0 cps, *i.e.*, the spectral peak develops along this line, and a high frequency part of wave spectrum lies on this line. Nevertheless it must be emphasized here that some spectra whose peaks rise from the  $4.15f^{-5}$  line have been observed at Lake Biwa. The spectrum of IS-12 shown by a dotted line in Fig. 2(c) is one of the examples.

### Development of wind-waves in the wind tunnels

Power spectra of fetch-limited wind-waves are calculated from wave data of the two wind tunnel experiments. One of the experiments was performed by KUNISHI (1963), and is designated as the experiment Ex-K in this article. The other was performed by KUNISHI and IMASATO (1966) and is designated as the experiment Ex-KI. The wind tunnel used in the former experiment is 21 m in length, 0.5 m in width and 0.5 m in depth, and that used in the latter experiment is 40 m in length, 0.8 m in width and 1.2 m in depth. The range of the wind speed is from 1.5 to 11.0 m·sec-1 in the experiment Ex-K, and from 8 to 34 m·sec-1 in the experiment Ex-KI. The data used in the spectrum analysis in this article have been obtained at the fetches of 4.3, 9.7 and 15.1 m, respectively, in the experiment Ex-K and at the fetches of 13.6 and 20.0 m, respectively, in the experiment Ex-KI.

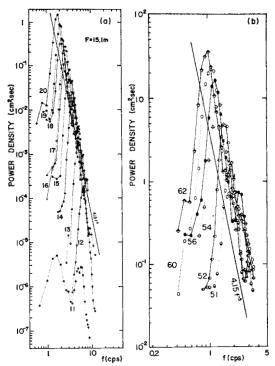


Fig. 4. Development of wind-waves at a short fetch in the wind tunnels; (a) is obtained from the experiment Ex-K (F=15.1 m), and (b) from the experiment Ex-KI (F=20.0 m).

Fig. 4 shows the development of wind-waves with the wind speed: (a) indicates the development of the wind-wave spectrum at the fetch of 15.1 m in the experiment Ex-K, and (b) at the fetch of 20.0 m in the experiment Ex-KI<sub>•</sub>

The wave spectral densities  $S_{max}$  of the spectral peaks at the fetch 4.3, 9.7 and 15.1 m. respectively, in the experiment Ex-K and at the fetch 13.9 and 20.0 m, respectively, in the experiment Ex-KI are plotted against the spectral peak frequency  $f_{\text{max}}$  in Fig. 5. When wind begins to blow over a calm water surface in a wind tunnel, a wave spectral peak develops with wind speed, and the frequency  $f_{\text{max}}$  of the spectral peak also increases, i.e. the wave height of a dominant wave increases but its wave length decreases with wind speed. This region in the development process of wave spectrum is called the stage of "initial-wavelets." When  $S_{\text{max}}$  goes over the line  $6.40 \times$  $10^{-4}k^{-2}$  or  $1.62 \times 10^{-5}$   $C(f)^{-2}f^{-2}$  cm<sup>2</sup>·sec (C(f) is 26 Imasato

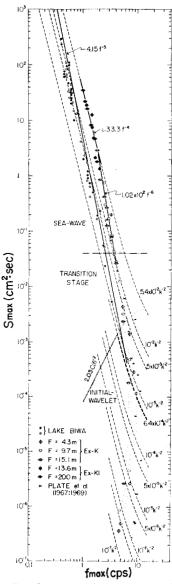


Fig. 5. Development of wave spectral peaks. The solid line shows the line 4.15 f<sup>-5</sup> obtained at Lake Biwa, and the broken curves show the several wave spectra given by Eq. (4). Data of the wave spectral peaks, obtained at Lake Biwa and shown by the small circles in the figure, shown in Table 1.

the phase velocity of wave given as  $C(f) = (g/k + kT/\rho_w)^{1/2}$  and T is the surface tension of water of density  $\rho_w$ ),  $f_{\text{max}}$  begins to decrease, and  $S_{\text{max}}$  increases with wind speed and becomes gradually independent of fetch. At f =

 $4.0 \, \mathrm{cps}$ ,  $S_{\mathrm{max}}$  becomes independent of fetch, and continues to grow along the line  $1.02 \times 10^2 f^{-6} \, \mathrm{cm^2 \cdot sec}$ . This is called the stage of "seawaves", and the transient stage from "initial-wavelets" to "sea-waves" is called the "transition stage".

The stage of initial-wavelets in the present classification from the wave spectrum analysis corresponds to the initial-tremor and the earlier stage of the initial-wavelets in the classification of KUNISHI (1963). And also the stage of transition stage and the stage of sea-waves in the present classification corresponds to the latter stage of the initial-wavelets and the stage of sea-waves, respectively. The difference cannot be found between the wave spectra of the initial-tremor and those of the earlier stage of the initial-wavelets. Development process of the spectral peaks from the experiments (simbol × in Fig. 5) by PLATE and HIDY (1967) and PLATE, CHANG and HIDY (1969) agrees very well with our present results shown in Fig. 5. PLATE et al. (1969) classified the development process of wind-wave spectrum in three stages. From comparison between their spectral peaks and ours in Fig. 5, it is found that the transition from the second stage to the third stage in their definition corresponds to the transition from the stage of initial-wavelets to the transition stage in our present definition, and that this occurs when a spectral peak arrives at the line  $6.40 \times 10^{-4}k^{-2}$  or  $1.62 \times 10^{-5}C(f)^{-2}f^{-2}$ . It is very interesting to note that this line in the region of the frequency from 5 cps to about 12 cps corresponds to the line  $4.15f^{-5}$  obtained from our field observations at Lake Biwa.

It is not easy to identify the development stage of the wave spectra obtained from the field observations at Lake Biwa. If we give attention to the fact that they seem to develop independent of fetch, they may be considered to belong to the stage of sea-waves. From the results of the wave spectral density and the slope spectral density (which will be discussed in the following section) at the frequency  $f_{\text{max}}$  of a wave spectral peak shown in Fig. 5, we regard the wave spectra as belonging to the stage of sea-waves when the dominant wave spectral peak has power density larger than

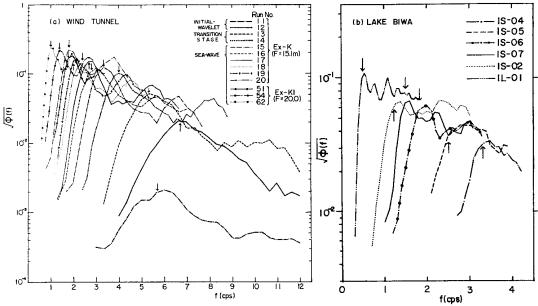


Fig. 6. Slope spectra of wind-waves: (a) is obtained from the wind tunnel experiments Ex-K and Ex-KI, and (b) from the field observations at Lake Biwa. Arrows indicate the frequency  $f_{max}$  of the wave spectral peak.

 $4 \times 10^{-2}$  cm<sup>2</sup>·sec.

A spectral peak of the wave spectrum in the stage of sea-waves in the wind tunnel rises high from the  $f^{-5}$  line given by Eq. (1), and therefore spectral form is very sharp as shown in Fig. 4, and is different from a spectral form found in the field observations at Lake Biwa. The spectral density of the high frequency part is not proportional to  $f^{-5}$ , but is proportional to  $f^{-7}$  or  $f^{-8}$  as reported by HAMADA et al. (1963), MITSUYASU (1968), and other investigators. As shown in Fig. 2(c), overshooted spectra were also obtained in the field observations at Lake Biwa and they have the similar form to that of the wave spectra in the wind tunnel experiments. These overshooted spectra indicate that the  $f^{-5}$  law is not always universal in the development of wind-waves. According to PHILLIPS (1958), the  $f^{-5}$  law means that the component waves in the high frequency part of the wind-wave spectrum are in the equilibrium state which is maintained by the breaking of steep waves. If the breaking occures at the steepest component waves, it must occure at the spectral peak as shown in the section 5. Breaking of wind-waves was observed in the case of the sea-waves in the experiment Ex-KI, but it was not observed\* in the wind-waves in the transition stage in the experiment Ex-K. It seems that these facts have put into chaos the physical meaning of the  $f^{-5}$  law.

#### 5. Slope spectrum of wind-wave field

The slope spectrum  $\Phi(f)$  is writen as follows:

$$\Phi(f) = k^2 \cdot S(f). \tag{2}$$

Fig. 6 shows the change of the steepness of the wave field by using the root  $\{\Phi(f)\}^{1/2}$ ; (a) is obtained from the wind tunnel experiments and (b) from the field observations at Lake Biwa. It is seen from this figure that the component wave of the wave spectral peak has the largest value of the slope spectral density in the component waves of the wave spectrum. In Fig. 6(a), the slope spectra shown by the thin curves (six spectra from Run-15 to Run-20 of Ex-K and three spectra of Ex-KI) belong to the stage of sea-waves, and the component waves of spectral peaks in this stage have the

<sup>\*</sup> Personal communication with Dr. KUNISHI.

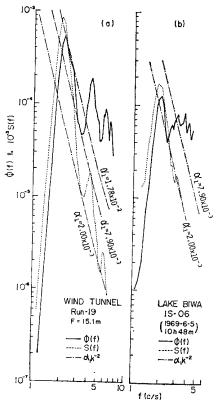


Fig. 7. Relation between the slope spectrum (solid curve) and the wave (power) spectrum (broken curves) of wind-waves; (a) is obtained from the wind tunnel experiment and (b) from the field observations at Lake Biwa. The chain curves show the wave spectra given by Eq. (4) with several constant values of  $\alpha_1$ .

almost same value in the slope spectral density. On the other hand, the slope spectral density at the frequency  $f_{\max}$  in the stage of initial-wavelets in the wind tunnel and the slope spectral density from the field observations at Lake Biwa increase with the development of the wind-waves.

If all component waves of the wind-wave spectrum have the same density  $\alpha_1$  in the slope spectral density  $\Phi(f)$ , the wind-wave spectrum is proportional to  $k^{-2}$  as follows,

$$S(f) = \alpha_1 k^{-2}. \tag{3}$$

Therefore, it is proportional to  $f^{-4}$  in the frequency range of gravity waves, and to  $f^{-4/3}$ 

in the frequency range (f>10 cps) of capillary waves, *i.e.* 

$$S(f) = \begin{cases} \alpha_2 f^{-4} & \text{(for gravity waves),} \\ \alpha_3 f^{-4/3} & \text{(for capillary waves).} \end{cases}$$
 (4-1)

The observed wave spectrum S(f), the wave spectrum given by Eq. (3) and the slope spectrum  $\Phi(f)$  are shown in Fig. 7; (a) is an example obtained from the wind tunnel experiments and (b) is that obtained from the field observations at Lake Biwa. These wave spectra belong to the stage of sea-waves. In this figure, wave spectra given by Eq. (3) are plotted by the broken curves for the different values of the constant  $\alpha_1$ . It is clearly seen from Fig. 7 that the slope spectral density of the high frequency part in the wind-wave spectrum is not constant but decreases with the frequency.

Figure 5 shows that the wave spectral peaks in the late stage of sea-waves develop along the line  $33.3f^{-4}$ . It indicates that the wave field in this stage has the constant and ultimate value in the slope spectral density, and that this state seems to be attained by breaking of the component waves around the wave spectral peak by the wind over 20 m·sec<sup>-1</sup>. According to TOBA (1973), wave spectral peak develops along the line  $f^{-4}$  if the friction velocity  $U_*$  of wind is constant. But, in the case of the fetchlimited wind-waves in a wind tunnel, the friction velocity  $U_*$  varies with the fetch and therefore the spectral peak of wind-waves in the stage of sea-waves develops along the line  $f^{-6}$  presented in the section 4. And the development of the spectral peak along the line  $f^{-4}$  can be attained only when the wind-waves develop. keeping its steepness constant even if the friction velocity  $U_*$  increases. In the wind tunnel experiment, this state is attained by breaking of the component waves around the wave spectral peak.

Fig. 8 shows the wind-wave spectra in the stage of initial-wavelets and several examples of spectra S(f) (chain curves) given by Eq. (3). Fig. 8(a) shows three wave spectra in Run-12, and (b) those in Run-11 of the experiment Ex-K. It is very interesting to show that the wave spectra of initial-wavelets in the early stage have the very wide frequency ranges of respec-

tive constant values in the slope spectral density, which are shown by the thick solid curves in the figure. This range is situated at the higher frequency than the wave spectral peak, and it becomes narrow with the development of the wind wave spectrum and vanishes at the late stage of the development. A wave spectrum in the first stage of initial-wavelets has two dominant peaks as shown in Fig. 8. At first, the spectral peak with the lower frequency has a larger value in wave spectral density than that with the higher frequency. As they develop with the fetch, both of them grow toward the higher frequency, and the wave spectral peak with the higher frequency becomes to have a large value in the wave spectral density than that with the lower frequency. The latter vanishes and the former becomes the dominant peak in the wave spectrum and grows toward the lower frequency until the wind-wave spectrum becomes similar in form to that of the spectrum in the stage of sea-waves.

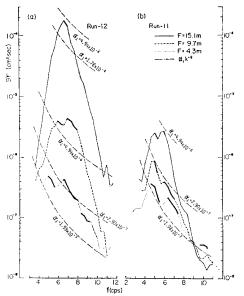


Fig. 8. Wave spectra in the stage of initial-wavelets. The parts of the wave spectrum with a constant density in the slope spectrum are shown by the thick solid curve. The chain curves show the wave spectra given by Eq. (4) with the different values of  $\alpha_1$ .

#### 6. Effective momentum flux to wind-waves

If wind-waves dissipate the energy due only to the kinematic viscosity of water  $\nu_w$ , momentum flux  $\tau_w(f)$  from the air flow to a component wave with the frequency f is given by Eq. (5) in the case of the field observations as follows,

$$\tau_w(f) = \frac{\rho_w g}{2} \left\{ \frac{\partial}{\partial t} \cdot \frac{S(f)}{C(f)} + 4\nu_w k^2 \frac{S(f)}{C(f)} \right\} \quad (5)$$

and by Eq. (6) in the case of the wind tunnel experiments as follows,

$$\tau_w(f) = \frac{\rho_w g}{2} \left\{ \frac{1}{2} \cdot \frac{\partial S(f)}{\partial x} + 4\nu_w k^2 \frac{S(f)}{C(f)} \right\} \quad (6)$$

where  $\rho_w$  is the density of water, g the acceleration of gravity, C(f) the phase velocity of the wave of the frequency f, and the x-axis is chosen for the direction of wave propagation. Therefore, effective momentum flux  $\tau_{ws}$  from the air flow to the wave field is defined as follows,

$$\tau_{ws} = \int_0^\infty J \cdot \tau_w(f) \, df \,, \tag{7}$$

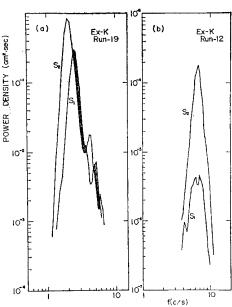


Fig. 9. Schematic diagram of development of wind-waves: (a) is in the stage of sea-waves and (b) in the stage of initial-wavelets. The wave spectrum  $S_0(f)$  is estimated from the observed spectra  $S_1(f)$  and  $S_2(f)$ .

where J is unity in the frequency range where  $\tau_w(f)$  is positive, and is zero in the frequency range where  $\tau_w(f)$  is negative.  $\tau_{ws}$  indicates the necessary amount of momentum flux for a wind-wave spectrum to develop from  $S_2$  to  $S_1$ which are schematically shown in Fig. 9. Evaluation of the ratio  $\tau_{ws}/\tau_0$   $(\tau_0 = \rho_a U_*^2)$  has been made by STEWART (1961), KORVIN-KROUKOVSKY (1965), and TAIRA (1972) from field observations and by HAMADA et al. (1966) and IMASATO and KUNISHI (1971) from wind tunnel experiments. From their results,  $\tau_{ws}$ seems to be from 2 to 20 % of  $\tau_0$ . Values of  $\tau_{ws}/\tau_0$  from the present study are plotted in Fig. 10 against the friction velocity  $U_*$ . In the stage of initial-wavelets of the present experiment,  $\tau_{ws}/\tau_0$  increases rapidly with  $U_*$ , depending on fetch. And in the stage of sea-waves, it has the almost constant value, which is 0.041 at  $F=7.1\,\mathrm{m}$  (the middle point between  $4.5\,\mathrm{m}$ and 9.7 m), and 0.093 at F=12.4 m (the middle point between 9.7 m and 15.1 m).  $\tau_{ws}/\tau_0$  from the experiment Ex-KI decreases with  $U_*$ , because in this range of  $U_*$ ,  $au_{ws}$  can not increase largely as the results of the breaking of waves by a strong wind over 20 m·sec<sup>-1</sup>. The ratio  $\tau_{ws}/\tau_0$  from the observations at Lake Biwa is

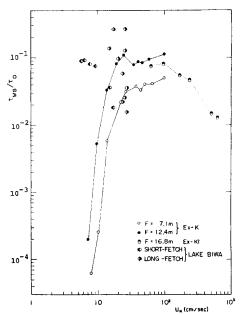


Fig. 10. The ratio  $\tau_{ws}/\tau_0$  versus the friction velocity  $U_*$ .

scattered around the values from the wind tunnel experiments and the mean value is 0.081. Those results of  $\tau_{ws}/\tau_0$  in the stage of sea-waves agree well with the previous results of other investigators.

 $\tau_{ws}$  in this article excludes the negatiqe parts of the spectrum  $\tau_w(f)$ , which comes from the negative  $\partial S(f)/\partial x$ , i.e. from the shaded part of the wave spectrum which is schematically shown in Fig. 9. If the component waves linearly receive the momentum from the air flow and linearly dissipate it,  $\tau_{ws}$  indicates the momentum flux which wind-waves need to receive from the air flow. However, because dissipation of wave energy due to the viscosity is taken into account in the estimation of  $\tau_w(f)$ , a negative part of  $\tau_w(f)$  must be transported to other component waves or must be used to produce the turbulence in water. Therefore,  $\tau_{NWS}$  defined in Eq. (8)

$$\tau_{NWS} = \int_0^\infty G \cdot \tau_w(f) \, df, \qquad (8)$$

can be considered as a measure of nonlinearity of wind-waves. The relation between  $\tau_{ws}$  and

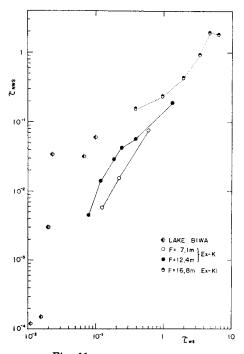


Fig. 11.  $\tau_{ws}$  versus  $\tau_{NWS}$ .

 $\tau_{NWS}$  is shown in Fig. 11, which shows that  $\tau_{NWS}$  indreasrs with  $\tau_{ws}$  depending on the fetch. It indicates that the nonlinearity in the wave spectrum becomes stronger with development of the wind-waves.

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### 風波スペクトルの発達過程の特徴

#### 今 里 哲 久\*

要旨: 風洞水槽実験とびわ湖での観測結果をもとにして,風波の発達過程を検討し,風波のパワースペクトルS(f) とスロープスペクトル $\phi(f)$  の特徴について述べる.風波の発達過程は,"initial-wavelets (初期波)","transition stage","sea-waves (風浪)" の 3 段階に分けられる.水槽実験では,initial-wavelets から transition stage への移行は,スペクトルピークが  $6.40 \times 10^{-4}$ 

 $k^2 \, \mathrm{cm}^2 \cdot \mathrm{sec}$  (k: 波数)の線に達するか,または  $\theta_{\mathrm{max}}$  が  $6.40 \times 10^{-4} \, \mathrm{sec}$  より大きくなった時に生ずる.風浪域ではスペクトルピークの成分波が最も急峻であり,ピークは  $1.02 \times 10^2 \, f^{-6} \, \mathrm{cm}^2 \cdot \mathrm{sec}$  に沿って発達し, $33.3 \, f^{-4} \mathrm{cm}^2 \cdot \mathrm{sec}$  に達した後は,この線に沿って発達する.この  $f^{-4}$  の線は  $\theta(f)$  が一定値となることを意味している・風波の非線形性が風波の発達と共に強くなることを示唆すると共に,風から波への運動量フラックスは全応力の  $4 \sim 9\%$  であることを示す.

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