Inverse Estimation of Sea Surface Drag Coefficient Based on Waves Observed Away from Strong Wind Region

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ABSTRACT

A wave prediction model including data assimilation method was improved to estimate the appropriate sea surface drag coefficient in high speed wind. In the present study, the accuracy of the estimated coefficients was evaluated for several observation stations that changes the distance from the strong wind area. As a result, it was clarified to be able to deduce drag coefficient if waves were propagated from strong wind region even if the strong wind had not been generated in the observation station.

KEY WORDS: Inverse estimation; sea surface drag coefficient; adjoint method; WAM; identical twin experiment; wave model.

INTRODUCTION

The third generation wave models, particularly WAM, SWAN and Wave Watch III, are widely used in many countries. These models describe wave status more accurately than the previous models, and have been applied to various practical applications. In the wave models however, the sea surface drag coefficient, an important factor of the energy transfer process from winds to waves, is generally described as a linear function of wind speed such as Wu (1980) or Mitsuyasu & Honda (1982) as shown in Fig.1.

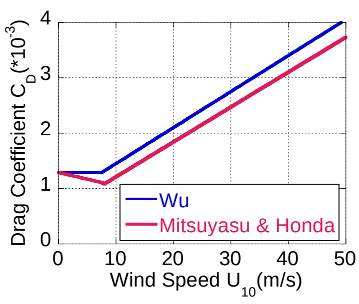

Fig.1 Drag coefficients proposed by Wu (1980) and Mitsuyasu & Honda (1982)

These functions were deduced from the observational and the experimental data under wind conditions slower than 25m/s at the fastest and were extrapolated to faster wind speed conditions. Accordingly, accuracy of the computed wave height under strong wind conditions seems to be unreliable. Actually, the recent report of an aerodynamic observation suggested that the sea surface drag coefficient declines when wind speed exceeds about 30m/s (Zhang et al, 2006; Powell et al, 2003; Andreas, 2003).

To investigate this fact, theoretical approach for clarifying the mechanism of the energy transfer under strong wind is difficult due to complicated physical processes such as wave breaking and spray generated by the strong wind. As an alternative method in such case, inverse estimation method that presumes the internal structure through model is considered to be effective (Hashimoto et al, 2004; Hashimoto et al, 2007). For the purpose of clarifying the optimum value of the sea surface drag coefficients in high wind speed, the wave prediction model including data assimilation method was improved to deduce the sea surface drag coefficients as its control variables.

The validity of the new model was already examined through identical twin experiments that use the wave data observed in strong wind condition. However, it seems difficult to carry out wave observation that aims at strong wind condition. On the other hand, the sea surface drag coefficient in high wind speed may be inversely estimated by assimilating the wave data propagated from strong wind area even if the strong wind had not been generated in the wave observation station.

In this paper, the accuracy of the estimated coefficients was evaluated for several observation stations that changes the distance from the strong wind area. As a result, it was clarified that deduced drag coefficient is accurate if waves were propagated from strong wind region even if the strong wind had not been generated in the observation station. It is also confirmed that the accuracy of the sea surface drag coefficient tends to decrease with the increase of the distance between wave observation station and typhoon track.

INVERSE ESTIMATION METHOD

Sea Surface Drag Coefficients

First of all, the expression of the energy transfer term in WAM Cycle4 was modified by replacing C_d with Mitsuyasu & Honda's equation expressed by Eq.1 from Janssen's method (Janssen, 1991).

$$C_D = (1-1.89\times U_{10}\times 10^{-2})\times 1.28\times 10^{-3} \qquad U_{10}\le 8\text{m/s}$$
$$C_D = (1+1.078\times U_{10}\times 10^{-1})\times 5.81\times 10^{-4} \qquad U_{10}\ge 8\text{m/s} \tag{1}$$

After that, in order to deduce the sea surface drag coefficient C_d as an arbitrary function with respect to wind speed, it was assumed as a piecewise-constant function over the wide wind speed range as shown in Fig.2. In this paper, range of wind speed is defined from 0 to 50 m/s and the number of the unknown parameter is fixed as 50 (1m/s interval).

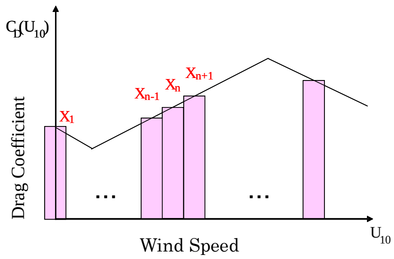

Fig.2 Drag coefficients assumed as a piecewise-constant function

Adjoint Method

The adjoint method, one of the data assimilation methods, is considered to be a kind of remote sensing that utilizes observation data to connect the model with the reality when we try to reproduce natural phenomena by a model. The adjoint method can presume the optimum value of unknown parameter including nonlinearity by using the maximum likelihood estimation method. Hersbach (1998) applied the adjoint method to the WAM and developed the ADWAM to get a better prediction by correcting several model parameters in the WAM. In the ADWAM, the most suitable parameters are automatically estimated from initial values by minimizing the cost function $J(x)$ composed of the sum of the error margin of estimation and observed values, expressed by Eq.2

$$J(x) = \sum_{t=0}^{T}\frac{1}{2}(H_t(X)-y_t)^{\mathrm{T}}\mathbf{R}_t^{-1}(H_t(X)-y_t) \tag{2}$$

where X is the vector of model parameters, and y_t is the vectors of observations, H_t is the matrix of the operator that converts the model state X into y_t, $\mathbf{R}_t$ is the covariance matrix of the observation errors.

In order to obtain the optimum value of X, the minimization of the cost function must be performed. Generally, analytical approach is difficult for the minimization. Instead, a method of descent is usually applied, which requires the following descendent value of the cost function ($g = \partial J/\partial x$).

$$g = \mathrm{H}_t^{\mathrm{T}}(X)\mathbf{R}_t^{-1}\sum_{t=0}^{T}(H_t(X)-y_t) \tag{3}$$

To compute g, the transpose of the operator matrix, $\mathbf{H}^{\mathrm{T}}$, has to be computed. This matrix corresponds to the adjoint operator $\mathbf{H}^*$ of the tangent liner operator of $\mathbf{H}$.

In the actual computation of Eq.3, it is directly computed through the adjoint run with the adjoint model code. For constructing the adjoint code of WAM, we used AMC (Adjoint Model Compiler, Giering, 1995). For the minimization of Eq.2, a descent method with Quasi-Newton Method was used.

The procedure of data assimilation with adjoint WAM is almost the same as that of Hersbach (1998). For normal applications of wave hindcasting, the WAM code is used and the energy balance equation is integrated in the forward direction of time t (forward run), while for data assimilation, the ADWAM code is used and the equation is integrated in the reverse direction of time t (adjoint run) to obtain the information with respect to the control parameters to be modified for data assimilation. When nonlinearities are included in the forward run, the nonlinearities at each integration time step have to be stored. To avoid the storage problem in the computations for realistic applications, a 'check-point' method has been implemented same as WAM Cycle 5 (Hersbach, 1998).

A Priori Condition

In this new adjoint WAM, the larger the assumed division number is, the more difficult and unstable the inverse estimation of parameters become, since the division number of wind speed in the assumed piecewise-constant function is equal to the number of unknown parameters in the model. This is namely an ill-conditional inverse problem and the minimization of the cost function composed of only the observation errors leads to unstable and unreliable computation. To solve this problem, a priori condition that the sea surface drag coefficient is continuous and smooth between the adjoining pieces of wind speeds was added as a background error term to the cost function in the adjoint model of WAM, as expressed by Eq.4.

$$J(x) = W\sum_{n=1}^{N}(x_n - x_{n-1})^{\mathrm{T}}\mathrm{B}_t^{-1}(x_n - x_{n-1}) + \sum_{t=0}^{T}(H_t(X)-y_t)^{\mathrm{T}}\mathbf{R}_t^{-1}(H_t(X)-y_t) \tag{4}$$

where x_n is the model parameters, B_t is the covariance matrix of the background errors, and W is a weighting coefficient between the observation error and the background error.

The most reasonable and suitable parameters are inversely estimated by minimizing the cost function assumed as the summation of observation error (the difference between observed wave data and the hindcasted wave data) and background error (the degree of satisfaction of the a priori condition). In this paper, the weighting coefficient W is assumed to be constant as 10^4.

NUMERICAL EXPERIMENTS

Experiment Method

Accuracy of the inverse estimation was examined by confirming that unknown drag coefficients are corrected to the vicinity of the target value from initial value by assimilating the time series of computed significant wave height. Fig.3 shows the flow of the experiment.
First, the target drag coefficients were assumed as a function of wind speed. In this study, Eq.5 is assumed as a target function which declines in high wind speed faster than 30m/s.

$$C_D = (1-1.89\times U_{10}\times 10^{-2})\times 1.28\times 10^{-3} \qquad U_{10}\le 8\text{m/s}$$
$$C_D = (1+1.078\times U_{10}\times 10^{-1})\times 5.81\times 10^{-4} \qquad 30\text{m/s}\ge U_{10}\ge 8\text{m/s} \tag{5}$$
$$C_D = (7.5-1.078\times U_{10}\times 10^{-1})\times 5.81\times 10^{-4} \qquad 50\text{m/s}\ge U_{10}\ge 30\text{m/s}$$

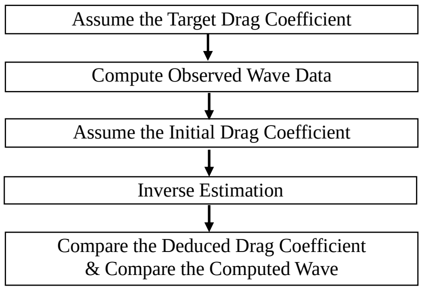

Fig.3 Flow of experiment

Then, the time series of significant wave heights were computed with the target function. These time series data were used as the wave observation data in the numerical experiments. Next, the unknown parameters were inversely estimated from initial values by assimilating the wave observation data. In this study, Eq.1 was assumed as the initial function and iterated assimilation process 25 times at each case. Finally, the estimated parameters were compared with the target values.

Fig.4 compares the time series of computed wave height, where the initial values, the target values and the deduced values are expressed by the broken line, the solid line and white circles respectively. Although the wave height computed with the initial parameter overestimate the peak wave height compared with the observed (target) data, the wave height computed with the inversely deduced parameter shows good agreement with the observed (target) data.

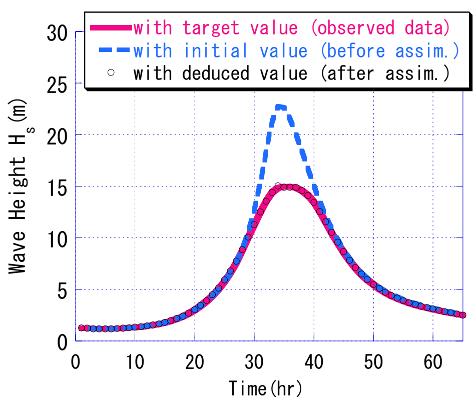

Fig.4 Comparison of the time series data

Simulation Condition

The condition of the numerical simulation is that a typhoon passes through the area from the south toward the north as seen in Fig.5. The size of computational area is 10 degrees for all sides and grid interval is 0.5 degree. The computation for sea surface winds was carried out using typhoon model. To reproduce strong wind condition faster than 30m/s, the condition of the typhoon was assumed as the central atmospheric pressure of 850hPa, the maximum wind speed radius of 100km and migration velocity of 50km/h. In addition, for the purpose of examine the possibility of inverse estimation with propagated wave data, northern part of the area, latitude of 30.5 degree to 35 degree, was assumed as the calm (wind speed of 0m/s) area. An example of the wind field (t=24h) is also shown in Fig.5. In this study as shown in fig.5, four observation stations (d2, d3, d4, d5) in the calm area that changes the distance from the wind area and seven observation stations (a, b, c, d, e, f, g) in wind area that changes the distance from the typhoon track (1.5 degree interval) were examined. As the examples, the time series of wind speed at “a”, “b”, “c” and “d” are shown in Fig.6. As seen in the figures, the maximum wind speed exceeds 40m/s at the station ”d” while the maximum wind speed at the station “a” is about 20m/s.

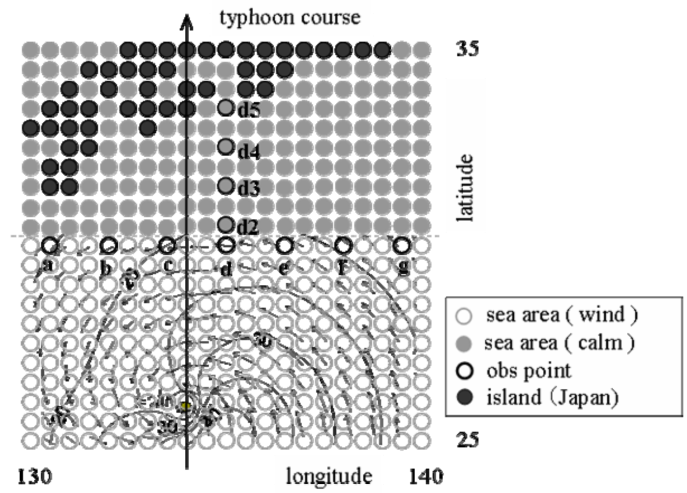

Fig.5 Example of wind field (t=24h)

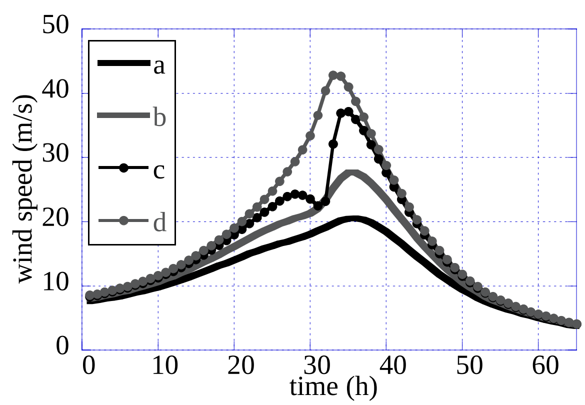

Fig.6 Time series of wind speed

Fig.7 compares the time series of wave height computed with the target value at the stations in the calm area and the station d in wind area. As seen in figure, the maximum wave height at the observation station becomes small by dissipation as the distance from wind area becomes large. In this study, experiments were carried out with these time series wave data as observation data. The number of the observation is 65 (1 hour interval)

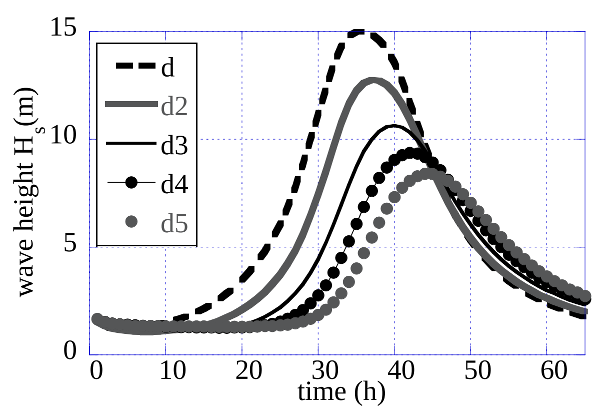

Fig.7 Comparison of the time series of wave height

Characteristics of the Deduced Values

Fig.8 compares the deduced value with the target value where the computations were carried out for several observation stations at the calm area that changed the distance from the wind area. Because the maximum wind speed generated in this typhoon is approximately 45m/s, the values in the wind speed higher than 45m/s do not corrected to the target values but change according to a priori condition. At the station “d” where the maximum wind speed exceeds 40m/s, the values in the wind speed of higher than 30m/s (where the initial values were intentionally separated from the target values) were corrected to the vicinity of the target values from the initial values and were almost agree with the target values. Moreover, the deduced values estimated by the inversion method in the wind speed of less than 40m/s were accurate enough at all the other stations in calm area. From these results, it can be said that the deduced drag coefficient is accurate if waves were propagated from strong wind region even if the strong wind had not been generated in the observation station. Also it can be said that the dissipation due to the propagation is not so critical to the inverse estimation accuracy.

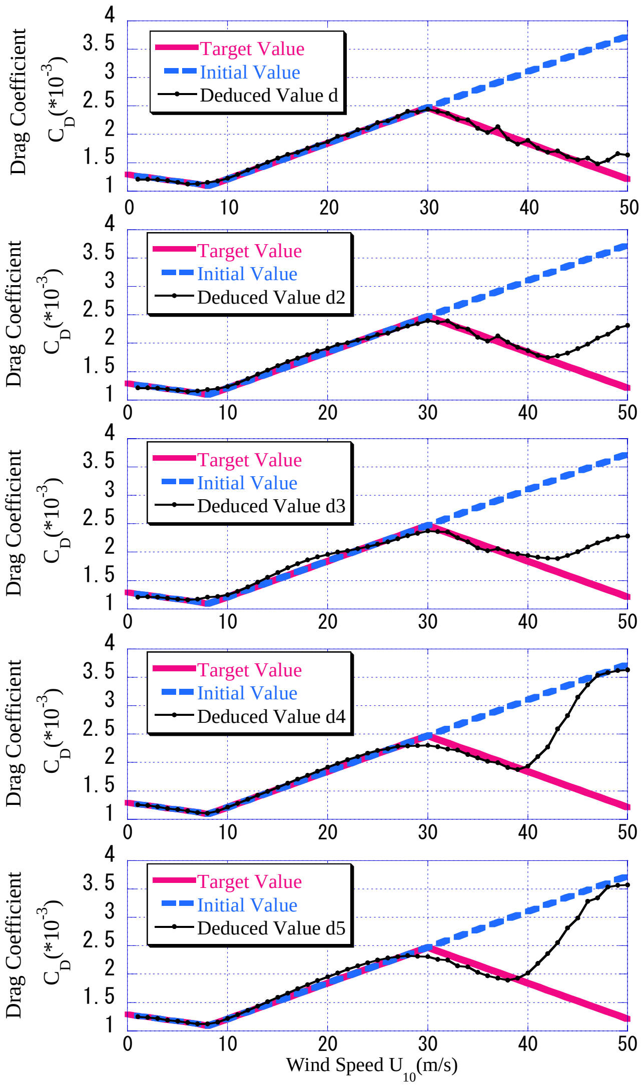

Fig.8 Deduced drag coefficients where distance from wind area is different

Fig.9 in the next page compares the deduced value with the target value where the computations were carried out for several observation stations that changed the distance from the typhoon track. The left side panels assumes the initial value as Eq.1 and right side panels assumed the initial values as constant value given by 1*10^{-3}. The arrow shown in each panel indicates the maximum wind speed generated in each observation station. In all stations, the deduced values were quite accurate in the wind speed range slower than maximum wind speed at each station. In addition, the deduced values have a tendency to decline in high speed wind range. However, the accuracy of the sea surface drag coefficient tends to decrease with the increase of the distance between wave observation station and typhoon track. Especially, the accuracy of the inverse estimation at the stations “a”, “b”, “f”, and “g” are insufficient. It may be due to the influence of winds or waves from several directions during the propagation.

The lower panel in Fig.10 compares the computed wave heights at the station “d” where the examination was carried out for the station “a” as the observation station while the upper panel compares the wave heights at the station “a” for the same case. Because the deduced values at the station “a” are not accurate, the wave heights computed with the deduced value in lower panel are not agree with those of the target values. On the other hand, the wave heights computed with the deduced values at the station “a” are agree with the wave heights computed with the target value. Namely, it can be said that the target value was not estimated among two or more solutions that satisfy cost function since observation data was insufficient. In addition to the significant wave height as the observation data, the use of the wave period and the wave direction may be effective to improve the accuracy of the inverse estimation in such case.

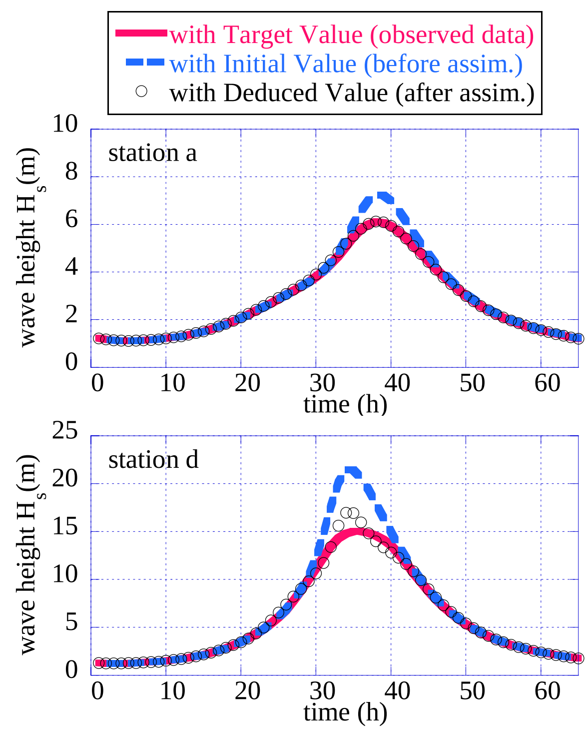

Fig.10 Comparison of computed wave heights at the station “a” and “d” where the examination was carried out for the case of the station “a” as the observation station.

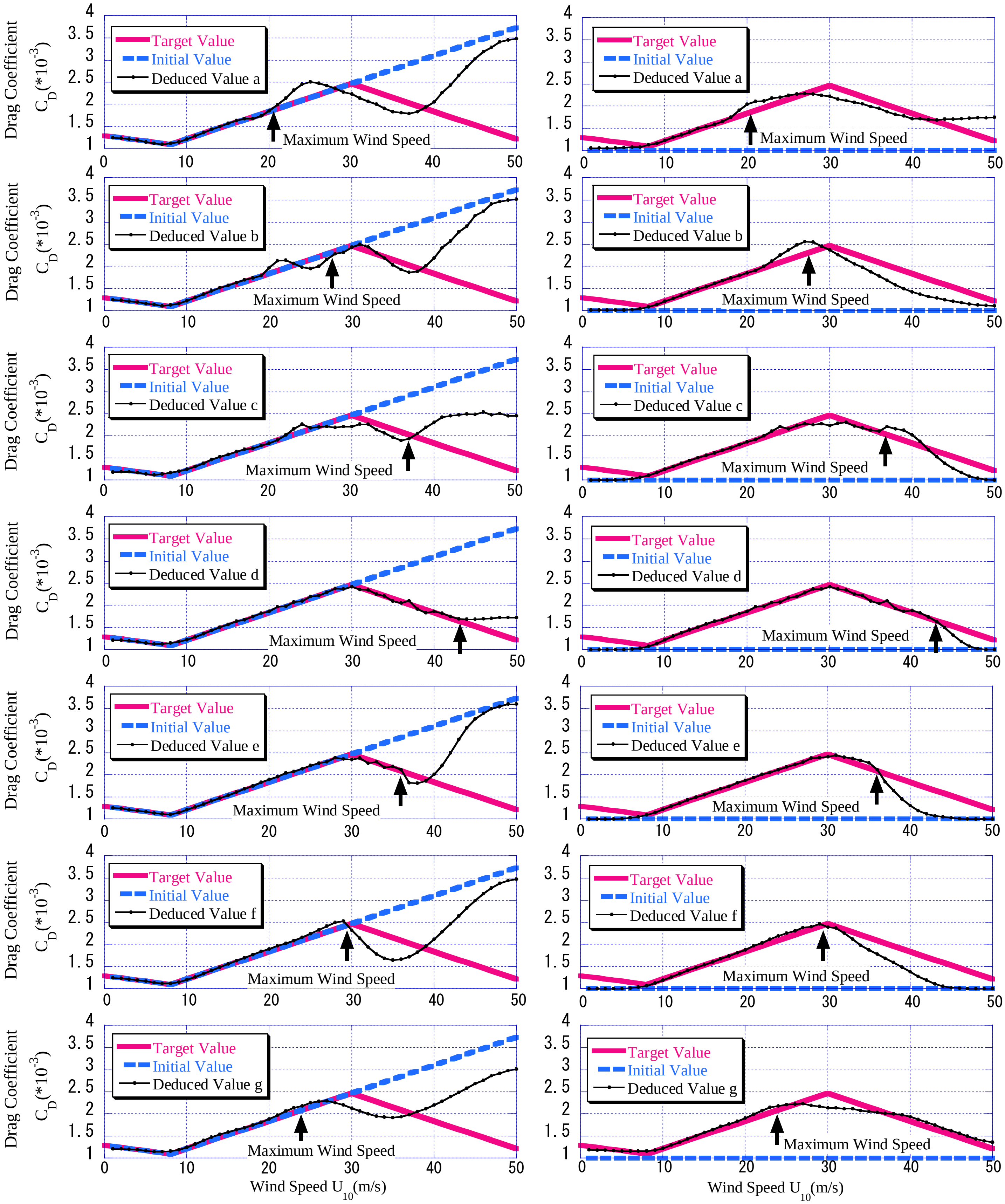

Fig.9 Deduced drag coefficients where distance from typhoon track is different

CONCLUSIONS

In this study, the accuracy of the deduced parameter was evaluated for several observation stations that changes the distance from the strong wind area. As results, it can be said that the deduced drag coefficient is accurate if waves were propagated from strong wind region even if the strong wind had not been generated in the observation station. It is also confirmed that the accuracy of the sea surface drag coefficient tends to decrease with the increase of the distance between wave observation station and typhoon track. From these results, it can be said that the dissipation due to the propagation is not so critical to the inverse estimation accuracy.

The wave prediction accuracy in severe sea conditions may be improved if the drag coefficients are clarified by applying the method to the actual wave data measured under severe sea conditions. We will therefore try to apply the proposed method to the actual wave observation data.

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