A Universal Form for Shoreline Run-Up Spectra?

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Time series of shoreline run-up on two natural beaches have been measured by using a time-lapse camera. Spectra of these time series and two other run-up spectra measured by Suhayda (1972) suggest that for the frequency band over which incident waves are large enough to break a universal 'saturation' form for the vertical run-up spectrum occurs, with energy density $E(f) = [\hat{\epsilon}_c{}^s g\beta^2/(2\pi f)^2]^2$, where g is the gravitational acceleration, β is the beach slope, and f is the frequency (in hertz). Parameter $\hat{\epsilon}_c{}^s(\Delta f)^{1/2}$ is a universal nondimensional constant, found to have a value of about 1, where Δf is the bandwidth over which incident waves are large enough to break in the surf zone. This result is discussed in relation to previous laboratory experiments and theories, based on monochromatic waves, which suggest the existence of a limiting amplitude for standing waves formed by reflection at the shoreline. This limiting amplitude is related to a critical parameter $\epsilon_c{}^s$ by $a = \epsilon_c{}^s g\beta^2/(2\pi f)^2$. A possible interpretation of $\hat{\epsilon}_c{}^s(\Delta f)^{1/2}$ in terms of $\epsilon_c{}^s$ is given based on percentage exceedances of the critical downslope acceleration $g\beta^2$. In this interpretation we have assumed a Gaussian distribution for run-up acceleration. This assumption cannot be tested directly, but the observed distribution functions for run-up elevation suggest that it may need to be modified. Departures from the universal spectrum at higher and lower frequencies are briefly discussed.

BACKGROUND

Since the work of *Miche* [1944, 1951] and *Iribarren and Nogales* [1949] it has become clear that the nondimensional parameter

$$a\omega^2/g\beta^2 = \epsilon \tag{1}$$

is of particular importance in describing conditions within a surf zone on a plane slope [*Bowen et al.*, 1968; *Battjes*, 1974, 1975]. In (1), a is some measure of wave amplitude, ω is the wave angular frequency, g is the acceleration due to gravity, and β is the beach slope in radians (assumed small, so that tan $\beta \simeq \sin\beta \simeq \beta$). The wave amplitude a is taken by different authors to be either the wave amplitude at the shoreline, a_s, or the amplitude of the wave traveling toward the shore either in deep water, a_∞, or 'at the toe of the slope,' a_T, assuming that the plane shoreface slope levels to a constant depth d at some distance offshore. Note that when the shoreward traveling wave is partially reflected at the shoreline, the reflected wave interferes with the incident wave to create a standing wave component; if the wave is totally reflected, a pure standing wave is set up with an amplitude at a deepwater antinode of $2a_\infty$. Inserting these forms of a into (1) gives ϵ^s, ϵ^∞, and ϵ^T, respectively. The theory of small-amplitude standing waves on a plane slope shows that $a_s/a_\infty = (2\pi/\beta)^{1/2}$ [*Miche*, 1944; *Stoker*, 1957, p. 73], and we can probably assume that $a_T \approx a_\infty$ for the laboratory experiments using a_T, since $a_T = a_\infty$ to within 10% when $d/L_0 > 0.03$, where L_0 is the deepwater wavelength. Thus we have $\epsilon^\infty \simeq \epsilon^T \simeq \epsilon^s(\beta/2\pi)^{1/2}$.

At present there is no agreement as to whether ϵ^s or ϵ^∞ (ϵ^T) is the more significant parameter. The theoretical work of *Carrier and Greenspan* [1958] shows that ϵ^s is of controlling influence in the exact solution of the nonlinear sloping bottom shallow water equations; *Guza and Davis* [1974] and *Guza and Bowen* [1975] find ϵ^s as the nonlinearity parameter in the perturbation analysis of incident and edge wave interactions in the nearshore zone. On the other hand, *Battjes* [1974, 1975] defines a parameter ζ, where

$$\zeta = \tan\beta/(2a_T/L_0)^{1/2} = (\pi/\epsilon^T)^{1/2}$$

and shows that for laboratory beaches, ϵ^T can be used to define the onset of wave breaking, the range of occurrence of different breaker types, the phase difference across the surf zone, breaker height to depth ratios, set-up, run-up, run-down, and reflection and absorption of wave energy.

Of particular interest here is the dependence on ϵ of the onset of breaking of incident waves and the amplitude of standing waves at the shoreline once breaking is occurring. A variety of laboratory experiments and theoretical arguments has confirmed that breaking occurs when ϵ exceeds some critical value ϵ_c. *Iribarren and Nogales* [1949] suggested that the incident waves break when their amplitude exceeds the mean undisturbed depth in the one-quarter wavelength adjacent to the waterline and obtained a value $\epsilon_c{}^T \approx 0.6$, in good agreement with their laboratory experiments. On the other hand, *Miche* [1944] solved the linear small-amplitude wave equations for a plane sloping beach and found that the critical condition should be applied to ϵ^s, not to ϵ^∞ or ϵ^T. His critical condition $\epsilon_c{}^s = 2$ corresponds to the condition that the water surface at the point of greatest run-up is just tangential to the beach slope. *Munk and Wimbush* [1969] argued that breaking sets in once the downslope acceleration reaches $g\beta$ and calculated from this a value $\epsilon_c{}^s = 1$. *Carrier and Greenspan* [1958] used the nonlinear shallow water equations and found a limiting condition for nonbreaking waves corresponding to a wave with a vertical face at the lowest point of run-down; their value of $\epsilon_c{}^s$ was similar to that of Munk and Wimbush.

It is of interest to ask what happens when the critical conditions are exceeded and the waves break on the beach. *Miche* [1951] assumes that the reflected wave amplitude at the shoreline remains equal to the maximum amplitude possible for nonbreaking waves of the same period on the same beach slope and that the energy corresponding to the wave amplitude greater than the critical amplitude is dissipated by breaking through the surf zone. In other words, this excess energy corresponds to the progressive wave component of the incident wave, which, though contributing to the mean set-up of the water level inside the surf zone, is assumed to have no periodic motion at the shoreline [*Bowen et al.*, 1968]. In terms of ϵ we therefore expect that for a given beach slope ϵ^s will increase linearly with ϵ^∞ or ϵ^T (with slope $a_s/a_\infty = (2\pi/\beta)^{1/2}$) until ϵ^s reaches the critical value $\epsilon_c{}^s$; further increase in ϵ^∞ or ϵ^T

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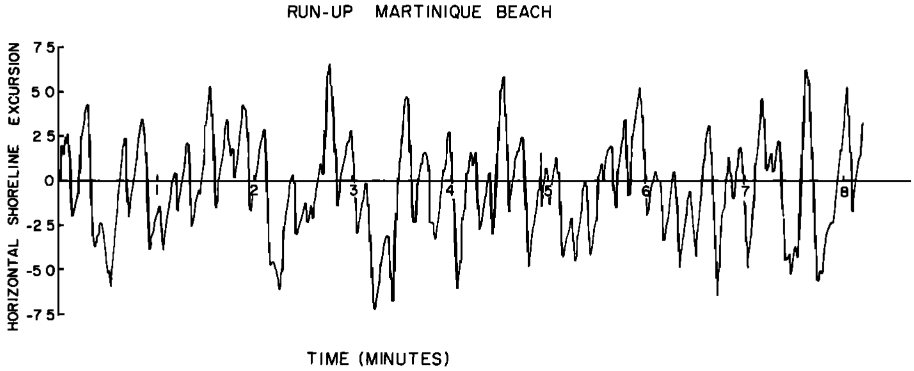

Fig. 1. An example of the run-up variation measured on beach 1. The horizontal shoreline excursion is in meters.

then leaves ϵ^s unchanged at ϵ_c^s, which thus acts as a saturation level for ϵ^s.

Guza and Bowen [1976] have investigated Miche's hypothesis by making extensive measurements of run-up for monochromatic waves on a large laboratory beach with slopes in the range 0.04–0.12. Their data show that although breaking begins when $\epsilon_c^s \approx 1$, as was suggested by *Carrier and Greenspan* [1958], the amplitude at the shoreline continues to increase, reaching a saturation value of around 3.

The assumption of a constant ϵ^s for breaking waves is also consistent with the empirical formulae for run-up and run-down. *Hunt*'s [1959] formula for vertical run-up Z_{up} for breaking waves on a smooth slope is

$$Z_{up} = (2aL_0)^{1/2}\beta$$

where a is the incident wave amplitude and L_0 is the deepwater wavelength. Measurements of run-down by Battjes and Roos [see *Battjes,* 1975] suggest that run-down is given by

$$Z_{down} \simeq [1 - 0.4(L_0/2a)^{1/2}\beta]Z_{up}$$

The shoreline amplitude $a_s = \frac{1}{2}(Z_{up} - Z_{down})$ and, when calculated from these empirical formulae, corresponds to a constant $\epsilon^s = 1.26$.

Battjes [1974, 1975] uses the extensive laboratory measurements of reflection coefficient made by *Moraes* [1970] to see if it is possible to distinguish between the assumptions of a critical ϵ^s or a critical ϵ^∞ to determine the maximum standing wave amplitudes in breaking wave regimes. He concludes that the data agree somewhat better with the assumption of a critical value of ϵ^∞, but the result is inconclusive, especially for beach slopes less than 0.3.

The available laboratory and theoretical evidence therefore tends to suggest that the appropriate parameter for shoreline run-up is ϵ^s rather than ϵ^∞.

We have conducted field experiments to investigate the run-up which results from a broad spectrum of incident wave frequencies rather than the essentially monochromatic incident wave conditions previously studied. The observed run-up spectra suggest that a universal spectrum may exist for run-up at a shoreline due to breaking incident waves.

FIELD DATA

We have obtained run-up spectra from two beaches in Nova Scotia. Beach 1 is at Martinique, on the Atlantic coast, 50 km northeast of Halifax, with a slope of 0.07 in the vicinity of the measurements; beach 2 is at Inverness, on the Gulf of Saint Lawrence coast of Cape Breton Island, with a foreshore slope of 0.12. Run-up was measured by a time-lapse movie camera taking one frame every half second. The camera was mounted approximately at mean shoreline, aimed alongshore. Within its field of view were a set of vertical posts placed at 10-m intervals along a range line perpendicular to the shoreline, providing a horizontal scale for the shoreline excursions. After the film had been processed, it was replayed frame by frame, and the instantaneous shoreline position along the range line was recorded for each frame. The films were of approximately 20-min duration and thus provided a discrete time series of shoreline position consisting of approximately 2400 points.

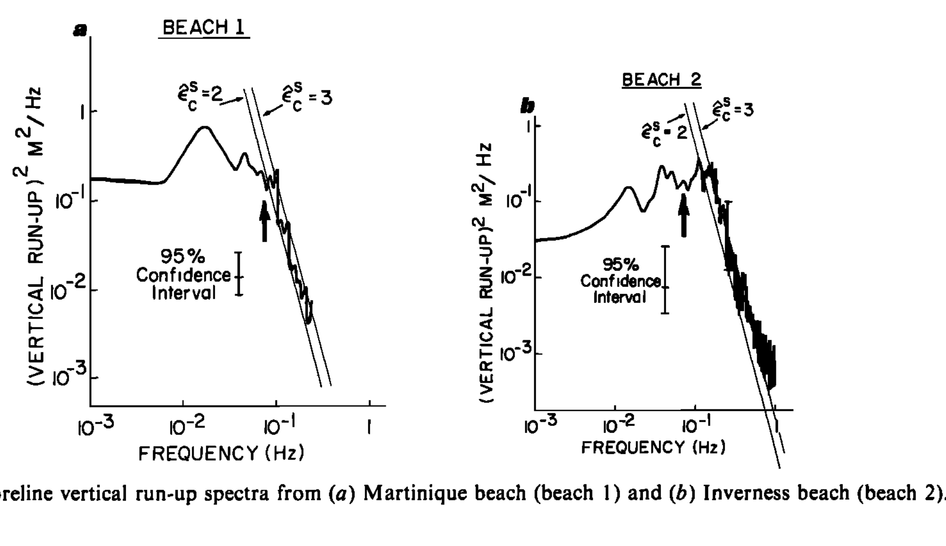

Fig. 2. Shoreline vertical run-up spectra from (*a*) Martinique beach (beach 1) and (*b*) Inverness beach (beach 2).

Figure 1 shows an example of the shoreline variation measured on beach 1. For each beach, profiles along lines normal to the shoreline were measured within a few hours of the filming, and from these profiles, approximate plane beach slopes for the swash zone were found. By using the beach slopes, shoreline excursions along the range lines were converted to vertical run-up variations, and spectra of the resulting time series were computed from the fast Fourier transform technique and a Tukey smoothing filter. The spectra in Figure 2 are plotted on a log-log scale.

Two additional run-up spectra have been obtained by *Suhayda* [1972, 1974], who used a digital run-up meter, and these are shown in Figure 3. Beach 3 (Figure 3*a*) is Scripps beach, La Jolla, California, and has a complex beach profile, with a beach face slope of 0.065 and a slope over most of the surf zone of 0.02. Beach 4 (Figure 3b) is El Moreno beach, Baja, California, with a slope in the surf zone of 0.13.

It is clear from Figures 2 and 3 that for each spectrum there is a frequency band over which the energy density spectrum $E(f)$ takes the form

$$E(f) = \alpha f^{-4} \qquad (2)$$

where α is constant for each beach. On beach 1 this frequency band is approximately 0.09–0.25 Hz; on beach 2, approximately 0.15–0.5 Hz; on beach 3, approximately 0.08–0.30 Hz; and on beach 4, approximately 0.25–0.70 Hz; in each case the band covers the predominant incident wind-wave band.

Confirmation that the f^{-4} saturation frequency band corresponds to the frequency band of breaking waves at least on beach 3 can be found by comparing the run-up spectrum with an offshore spectrum. In Figure 3*a* the dashed line is an elevation spectrum measured by Suhayda about 150 m offshore. Throughout the f^{-4} frequency band the offshore amplitude is larger than the shoreline amplitude, confirming that breaking was occurring in this band.

At frequencies greater than the f^{-4} band the spectra in Figures 2*b* and 3 show more energy than predicted. Digitization noise from the run-up films and the electronic run-up meter is probably too small to account for the high-frequency noise. Percolation of the swash, on the other hand, would result in apparently rapid changes of shoreline position, and it is possible that this contributes most to a broad high-frequency spread of spectral energy.

COMPARISON WITH MONOCHROMATIC RUN-UP

It is not obvious how this result, with a spectrum of incident waves causing the run-up, can be related to the laboratory data, with a monochromatic incident wave train.

The existence of the monochromatic limiting parameter ϵ_c^s suggests perhaps that we might write α in the form

$$\alpha = [\hat{\epsilon}_c^s g\beta^2/(2\pi)^2]^2 \qquad (3)$$

where $\hat{\epsilon}_c^s$ is in units per $Hz^{1/2}$. Table 1 summarizes the results of fitting this form of α to the f^{-4} range of each spectrum to obtain values of $\hat{\epsilon}_c^s$.

How is $\hat{\epsilon}_c^s$ related to its monochromatic equivalent ϵ_c^s? This question involves consideration of the possible meaning of the bandwidth component in $\hat{\epsilon}_c^s$. The form of the run-up elevation spectrum (2) implies that the run-up acceleration spectrum has constant amplitude (i.e., is white). Hence for finite accelerations, (2) must be valid only in a finite bandwidth Δf. From (3) the magnitude of the acceleration spectrum is $[\hat{\epsilon}_c^s g\beta^2]^2$, so the rms acceleration is $\hat{\epsilon}_c^s g\beta^2(\Delta f)^{1/2}$, and the mean acceleration amplitude is $(2\Delta f)^{1/2}\hat{\epsilon}_c^s g\beta^2$.

For monochromatic waves the critical run-up acceleration amplitude is now $a\omega^2 = \epsilon_c^s g\beta^2$. Hence $(2\Delta f)^{1/2}\hat{\epsilon}_c^s$ is in some sense a spectral analog of ϵ_c^s. *Munk and Wimbush* [1969] argued that for monochromatic incident waves the downslope acceleration of the run-up $a\omega^2/\beta$ could not exceed the downslope acceleration due to gravity $g\beta$; they therefore suggested $\epsilon_c^s = 1$. On this basis the larger values of ϵ_c^s discussed in the opening section of this paper may be interpreted as the factor by which the amplitude of sinusoidal vertical run-up acceleration exceeds the Munk and Wimbush limiting value $g\beta^2$. Similarly, $(2\Delta f)^{1/2}\hat{\epsilon}_c^s$ represents the factor by which the mean amplitude of vertical acceleration in a spectrum exceeds $g\beta^2$.

It seems reasonable to take Δf as the bandwidth over which wave breaking is occurring. Referring again to the data for beach 3 we can obtain a rough estimate of this bandwidth from the relative size of the offshore spectrum, i.e., $\Delta f = 0.32$ Hz. With this value we find that $\hat{\epsilon}_c^s(\Delta f)^{1/2} \simeq 1$. In fact, although the upper frequency limit is not sharply defined, approximately the same result is obtained for each of the observed spectra when Δf is taken as the bandwidth of the f^{-4} run-up elevation spectrum (Table 1). Hence the mean acceleration amplitude is approximately equal to $(2)^{1/2}g\beta^2$.

The observed value of $\hat{\epsilon}_c^s(\Delta f)^{1/2} = 1.22 \pm 0.35$ is reasonably constant and shows no clear trend with changing beach slope, although both beach slope and bandwidth vary by as much as a factor of 2. The observed run-up on these natural beaches therefore suggests the existence of a universal form for the run-up spectrum:

$$E(f) = [\hat{\epsilon}_c^s g\beta^2/(2\pi f)^2]^2 \qquad (4)$$

where $\hat{\epsilon}_c^s(\Delta f)^{1/2}$ is a universal constant of about 1.0.

It is also clear from these spectra, particularly on beach 3,

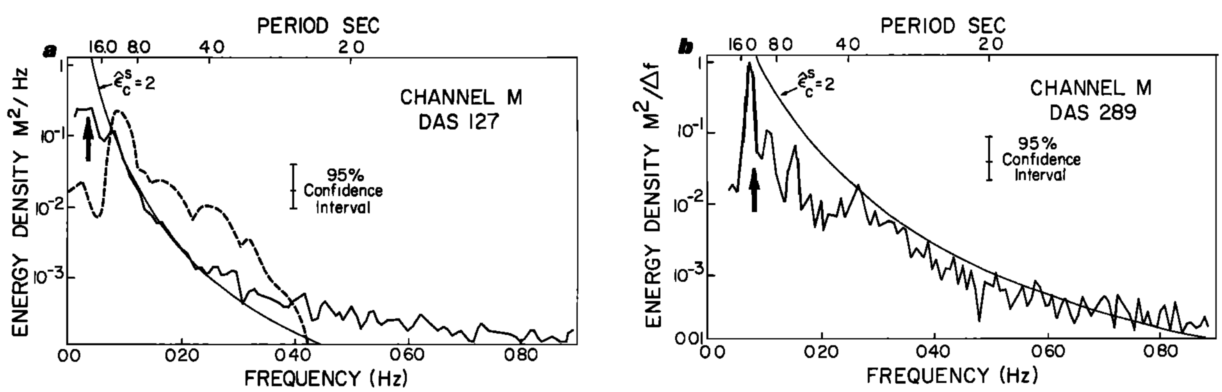

Fig. 3. Shoreline vertical run-up spectra from (*a*) Scripps beach (beach 3) and (*b*) El Moreno beach (beach 4).

TABLE 1. Approximate Incident Wave Conditions

	Beach	Foreshore Slope	Height, m	Period, s	$\hat{\epsilon}_c^s$, Hz$^{-1/2}$	Δf, Hz	$\hat{\epsilon}_c^s(\Delta f)^{1/2}$
1.	Martinique	0.07	1.3	9	2.5	0.17	1.03
2.	Inverness	0.12	1.0	6	3.0	0.36	1.8
3.	Scripps	0.065	1.0	10	2.0	0.2	0.89
						0.32*	1.13
4.	El Moreno	0.13	0.3	3†	2.0	0.40	1.26
			0.05	12‡			

*Bandwidth of breaking waves based on offshore wave spectrum. All other bandwidths in this column are the range of the f^{-4} spectrum slope.
†Indicates value due to wind waves.
‡Indicates value due to swell.

that the appropriate beach slope in (4) is that at the shoreline itself. On beach 3 the foreshore slope of 0.065 extends offshore only approximately as far as the first zero crossing of a standing wave of longest period in the saturation range. This suggests that the limiting amplitude results from interaction of the waves with the beach slope within a quarter of a wavelength of the shoreline. This is consistent with the theoretical criteria for limiting amplitudes, which are concerned with conditions at the shoreline, and is also consistent with the full solution for standing waves on a beach, where conditions very close to the shoreline are found to control the complete solution.

For the spectral case we might also ask what proportion of the time the downslope acceleration exceeds the value $g\beta^2$. Since $\hat{\epsilon}_c^s(\Delta f)^{1/2} \simeq 1$, the standard deviation of the run-up acceleration distribution function is just $g\beta^2$, so we need to know what proportion of the distribution function lies outside 1 standard deviation on the downslope acceleration side. If we assume a Gaussian distribution of acceleration, then this proportion is approximately 16%. For comparison, in the monochromatic case the proportion lying outside a downslope acceleration of $g\beta^2$ is given by $[0.5 - (\sin^{-1} k)/\pi]$, where $k = 1/\epsilon_c^s$.

Thus the monochromatic parameter corresponding to the Gaussian value of 16% is $\epsilon_c^s = 1.14$. In fact, the standard deviation of the observed $\hat{\epsilon}_c^s(\Delta f)^{1/2}$ values puts ϵ_c^s in the range 1.09–1.46. This is in good agreement with the lower values found in laboratory experiments, for example that of *Battjes* [1974, 1975], where $\epsilon_c^s \simeq 1.26$, but not with the $\epsilon_c^s \simeq 3$ value found by *Guza and Bowen* [1976].

Unfortunately, the presence of high-frequency digitization noise and percolation effects in the observed run-up records

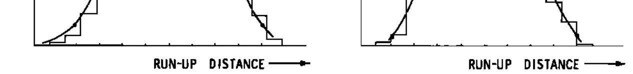

Fig. 4. Run-up distribution histograms for beaches 1 and 2. The smooth line is a Gaussian function with the same mean and variance values as the observed run-up.

prevent a satisfactory direct calculation of the acceleration distribution functions. Nevertheless, the distribution functions for run-up elevation on beaches 1 and 2 have been plotted and are shown in Figure 4, compared to Gaussian curves with the same variance. Both functions have insignificant skewness ($\simeq$0.2 standard deviations from zero) but have kurtosis values of 2.41 (beach 1) and 2.39 (beach 2). These kurtosis values are significantly different from the value of 3.00 expected for a Gaussian process (the observed values are about 6 standard deviations from the mean, based on an expected standard deviation of kurtosis for a Gaussian process). These low values of kurtosis suggest that the run-up tends toward a regular sawtooth or triangular variation (with an expected kurtosis value of 1.8), and this seems consistent with the observed run-up motion (Figure 1). However, it is interesting that water currents close to the shoreline and in surf, while showing a tendency toward a sawtooth variation, also have kurtosis values greater than normal [*Huntley and Bowen,* 1975]. Clearly, the low values of run-up kurtosis are likely to result in an acceleration distribution function which is rather different from normal, perhaps changing significantly the proportions calculated above.

LOW-FREQUENCY RUN-UP

At frequencies below the f^{-4} band the run-up energy is below the predicted saturation level, suggesting that the waves are pure standing waves. For Scripps beach (beach 3) and El Moreno beach (beach 4), *Suhayda* [1972, 1974] showed that the ratios of the offshore amplitude to the run-up amplitude for most of these low-frequency waves (Figure 3) are consistent with the ratios predicted for small-amplitude standing waves, which have an elevation proportional to the zero-order Bessel function $J_0(X)$ where $X = [4\omega^2 x/g\beta]^{1/2}$ and x is the distance offshore. *Guza* [1974] shows that for beach 4 the energy at some of the low frequencies considered by Suhayda is also consistent with trapped edge wave energy of mode zero or 1. Trapped edge waves could also account for the low-frequency energy on beach 3. For the spectrum from beach 1 the large low-frequency peak at about 1.6×10^{-2} Hz can only be explained in terms of an edge wave and not in terms of a standing wave of Bessel function form. Simultaneous measurements of longshore and onshore/offshore water velocities at a point offshore show that this energy is due to a mode 2 edge wave trapped to the shoreline by refraction and trapped in the longshore direction by the ends of the bay containing the beach (R. Holman, private communication, 1976). It is clear from this observation that, at least on beach 1, a major part of the low-frequency energy is due to edge waves which may be created by nonlinear interactions between the higher-frequency standing waves [*Guza and Davis*, 1974; *Guza and Bowen*, 1975]. To what extent this is true of the other beaches is not known, since in the absence of velocity measurements or elevation measurements well offshore it is not possible to distinguish between high-mode edge waves and reflected incident waves. Nevertheless, if in general the low-frequency energy is generated by nonlinear interactions of the higher-frequency standing waves, the constant amplitude of the latter waves in a breaking regime implies a forcing of low-frequency motion which is independent of incident wave height. The influence of incident wave height will then be confined to determining the viscous dissipation of these low-frequency waves by varying the width of the surf zone.

In passing, it is worth noting that these spectra do not show a peak at the frequencies predicted by *Emery and Gale* [1951]. They observed the swash on a large number of North American beaches and concluded from visual obseravtions that the motion had a long period variation, whose period T, in the range 10–40 s, was a function only of beach slope and not of incident wave energy or period; their data suggest that $T \propto \beta^{-1/2}$. In Figures 2 and 3 an arrow marks the frequencies predicted by their relationship. Only on beach 4 is there any suggestion that the arrow corresponds to a well-defined peak; replays of the run-up films for beaches 1 and 2, shown to determine whether an observer with a stopwatch picked out the predicted period by some form of low-frequency band averaging, also failed to produce results in agreement with the predictions. In principle, it is possible to find a peak frequency approximately dependent on $\beta^{-1/2}$ and essentially independent of incident wave energy if the run-up spectrum increases rapidly with increasing frequency until the saturation curve is reached (the $\beta^{-1/2}$ dependence is exact if the low-frequency energy increases as ω^{+4}), but with the possible exception of beach 4 there is no indication in the observed spectra that such a rapid rise of energy occurs.

CONCLUSIONS

The field data, from beaches with slopes ranging from 0.065 to 0.13, suggest that for breaking waves a universal saturation spectrum of run-up variation occurs with energy density

$$E(f) = [\hat{\epsilon}_c{}^s g\beta^2/(2\pi f)^2]^2$$

where $\hat{\epsilon}_c{}^s(\Delta f)^{1/2} \approx 1.0$ and Δf is the bandwidth over which the offshore spectrum contains waves large enough to break at the shoreline.

A possible interpretation of $\hat{\epsilon}_c{}^s(\Delta f)^{1/2}$ in terms of the monochromatic limiting parameter $\epsilon_c{}^s$ is given, based on percentage exceedances of the critical downslope acceleration $g\beta^2$. This interpretation depends upon the assumption of a Gaussian distribution of run-up acceleration. This assumption cannot be directly tested from the data, but the distribution of the elevation suggests that the assumption may need to be modified.

The problem of relating monochromatic results to naturally occurring spectral cases is a general one for which no satisfactory solution exists, although several authors have suggested approaches to the problem [e.g., *Longuet-Higgins*, 1969]. The form of the nondimensional spectral parameter for run-up proposed in this paper should remain valid in the monochromatic case, where $\Delta f \rightarrow 0$ and the spectrum tends to a Dirac δ function at the monochromatic frequency.

The existence of this run-up saturation regime may have some interesting implications for nearshore processes. In particular, the fact that shoreline beach slope is used in the parameterization indicates that wave energy at the shoreline is primarily determined by conditions at the shoreline itself and not significantly by conditions at the break point or through the surf zone.

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