Waves on the Arctic Ocean

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Abstract. A continuous vertical oscillation of the ice of the Arctic Ocean has been observed at four U. S. drifting research stations. In the period range of 15 to 60 sec, the oscillations were detected with seismometers and gravity meters, and the displacement amplitudes were about $\frac{1}{2}$ mm at 30 sec. Amplitudes increase roughly as the square of the period. These oscillations are, in part at least, generated by wind action. In one case, oscillations in this period range have been identified as propagating waves. Theoretical dispersion curves for propagating flexural-gravity waves are presented. Oscillations in the 10- to 100-min period range were detected with a tide recorder operating at a grounded ice station, T-3, on the continental shelf, 130 km northwest of Point Barrow, Alaska. Displacement amplitudes were about 1 cm at a period of 30 min and were roughly proportional to period. The Arctic Ocean wave spectrum contrasts with that of other oceans. The spectral peak due to sea and swell is absent, and the present data indicate a monotonic increase in displacement amplitude as the periods range from about 0.1 to 60 sec in deep water or from about 0.1 sec to 100 min on the continental shelf.

Introduction. The ice cover of the Arctic Ocean is an exceptional boundary between air and ocean which has certain effects on wave motion. To the visual observer, it is evident that the sea and swell of open oceans are absent on the ice-covered Arctic Ocean. However, recent studies have shown that ice floes and ice islands are in nearly continuous oscillation, with amplitudes so small as to be detectable only with instruments. These oscillations are of particular interest for their bearing on the understanding of ocean wave generation and propagation in general.

Early observations of natural ice vibrations were reported by *Crary*, *Oliver*, and *Cotell* [1952], who made a series of landings on ice floes in the Beaufort Sea for geophysical studies. Using a gravity meter as a seismometer, they observed oscillations with a maximum vertical amplitude of about 0.05 cm in the period range of 5 to 40 sec. Later, an extensive series of gravity meter readings were made at Fletcher's ice island (T-3), a 60-m-thick section broken from an ice shelf [*Crary*, 1956; *Bushnell*, 1959]. On this drifting ice island similar oscillations were noted that had a maximum vertical motion in the 35- to 45-sec period range of 0.02 to 0.03 cm and a seasonal variation in both amplitude and period. The largest amplitudes occurred during the winter months, but the longest predominant periods occurred in the summer.

The present paper reports new investigations of natural vibrations at four floating ice stations: Alpha, Charlie, Fletcher's ice island, and Arlis II. The oscillations discussed here occur when the ocean is almost entirely covered with ice. Recording instruments used were a gravity meter, a long-period seismometer, and a tide gage. It is of interest that these three instruments were intended, respectively, for the study of gravity, earthquakes, and tides in the Arctic Ocean and that they yielded data on ice oscillations as an auxiliary result. Discussed here are (1) the spectral composition of this motion, (2) the relationship between amplitude and wind speed, and (3) the theory of long waves in a floating ice layer.

Waves with 15- to 60-sec periods. During 1957 and 1958, observations of ice motion were made with a gravity meter at drifting station Alpha, a floating ice research site maintained by the United States Air Force in cooperation with the United States National Committee of the International Geophysical Year. The drift track of the station is shown in Figure 1. Water depths are about 2 to 3 km along this track.

Lamont Geological Observatory Contribution 547.



Fig. 1. Drift tracks of stations Alpha and Charlie in the Arctic Ocean.



Fig. 2. Wave records taken with gravity meter at station Alpha in 1957. Dots indicate readings.

The Frost gravity meter, C-1-15, used at station Alpha, was nearly identical with the North American meter used at Fletcher's ice island. A horizontal boom is supported by a zerolength spring and enclosed in a double case with two thermostatically controlled sections. The boom is compensated for atmospheric pressure changes, and its position, magnified by an optical lever, is viewed through an ocular. The practice was to read the boom position at 3-sec intervals for a period of 4 to 5 min. The sampling schedule was chosen to span nearly all the visually observable periods, so that the problem of 'aliasing' is small. The amplitudes were frequently great enough to cause the boom to strike the stops. It was possible to read the



Fig. 3. Wave records taken with a gravity meter at station Alpha in 1958. Dots indicate readings.

TABLE 1. Wind Speeds at Station Alpha

Hour AST	Date	Wind Speed, m/sec
1330 1630 1930 1330 1630 1630 1630 0100 0400 0400 0400 0400 0400 0400	7/24/57 7/24/57 7/25/57 7/25/57 7/26/57 7/26/57 7/26/57 7/19/58 7/19/58 7/19/58 7/20/58 7/20/58 7/20/58 7/20/58 7/21/58 7/21/58	2 1 3 2 3 3 4 $1/2$ 1 2 1 $1/2$ 2 $1/2$ 2 $1/2$ 2 2 $1/2$ 3

instrument only during periods of calm or of light winds when maximum ice amplitudes were about 0.04 cm in the 30-sec period range. This contrasts with conditions on Fletcher's ice island, where the amplitudes were rarely sufficient to cause the boom to strike the stops [Crary and Goldstein, 1957].

Six typical records from Alpha are shown in Figures 2 and 3, and the contemporaneous wind data are contained in Table 1. Numerical Fourier analyses of these records were made with an IBM 650 computer. In preparation for analysis, linear drift was removed from the records with a ramp function, and the ends of the record were



Fig. 4. Fourier spectra of records in Figures 2 and 3 plotted in terms of displacement. Log-log scale.

chosen as the points of first and last zero crossings. The uncorrected Fourier spectra are plotted in Figure 4, but it is necessary to examine these results in light of the instrument response. They were corrected for the displacement re-



Fig. 5. Calculated response curves for Frost gravity meter. Log-log scale.

sponse of the instrument (Fig. 5). This response curve was calculated on the basis of the 17-sec natural period and 0.70 critical damping of the gravity meter. The corrected Fourier spectra (Fig. 6) are plotted from periods of 6 sec out to periods equal to the record length (240 or 300 sec), but the central period range from about 15 to 60 sec is the most reliable. The reliability of the spectral amplitudes at the shortest periods is limited by the sampling interval, and at the longest periods by the record length. A general increase of displacement amplitude with increasing period is the most outstanding feature of all records. This increase in displacement amplitude corresponds to nearly constant acceleration. Superimposed on the general amplitude increase is a fine structure which does not show sufficient correspondence between records for a definite interpretation. It should be noted that station Alpha was situated on different ice floes in the years 1957 and 1958. In 1957 the floe was about $1\frac{1}{2}$ km wide and 3 km long, and in 1958, about 1×1 km. The thickness was approximately 3 m both years.



Fig. 6. Fourier spectra of records in Figures 2 and 3 plotted in terms of displacement. Dashed line is constant acceleration. Log-log scale.

2480



Fig. 7. Records taken with two gravity meters at ice island Arlis II. The upper records were taken simultaneously at the same location. The lower records were taken simultaneously with a 400-m instrument separation. Readings at 5-sec intervals are indicated with dots.

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Fig. 8. Comparison of wind speed and peak-to-peak trace amplitude for long-period seismometer at station Charlie. Seismometer natural period is 10 sec for left diagram and 40 sec for right diagram.



Fig. 9. Comparison of wind speed and wave amplitude at Fletcher's ice island. Peak-to-peak boom displacement in terms of optical scale divisions.

Oscillations similar to these were also recorded at ice island Arlis II, a floating research station operated by the Arctic Research Laboratory under support of the Office of Naval Research. The ice island dimensions were about $3\frac{1}{2}$ × 31/2 km and 20 m in thickness. Arlis II was at 74°33'N, 162°33'W, at the time in 1500-m water depth. Two gravity meters, a World-Wide and a Worden model, were read simultaneously at two locations which were separated by an interval of 400 m (Fig. 7). The meters were about 300 m from the island edge, and the waves must have traversed only a small portion of island ice. Most of the records of peaks and troughs can be correlated over this distance, but they exhibit a phase difference between the locations. When the two meters are read simultaneously at the same location. no phase differences are observed. We are clearly observing progressive waves in this case. A Fourier analysis was made of the records in Figure 7, and the phase velocities between the two meters were determined. Velocities are 37.3 m/sec at a period of 40 sec, 30.5 m/sec at 36 sec, and 24.7 m/sec at 27.7 sec (Fig. 13). Since only two instruments were used, this is an apparent phase velocity and is an upper limit of the true phase velocity. Before the measurements were made, winds had been blowing in the direction of the line between the two instruments, from the Worden toward the World-Wide meter, and the velocity measured may be close to the true phase velocity in this case.

Wave amplitude and wind speed. A longperiod seismometer was operated at station Charlie, a drifting ice research station which traveled along the track in Figure 1 during 1959 and early 1960 [Cromie, 1961]. The station was located on an ice floe which originally measured 7×10 km and 3 m in thickness, but the area was considerably reduced during the drift. The U. S. Air Force supported the station as a contribution to the International Geophysical Cooperation-1959.

The seismometer, a Sprengnether long-period vertical instrument, was placed directly on the ice surface and enclosed in a protecting shelter. Seismometer output was filtered to remove high frequencies, electronically amplified, and recorded with a pen and ink oscillograph. The response of the instrument was peaked near the natural period, and variation in period is small on the records. Local wind speed and maximum peak-to-peak trace displacement are plotted in Figure 8 for two different intervals which included storms. The natural period of the seismometer was adjusted to 10 sec in one case and 40 sec in the other. Correlation between wind and oscillation amplitude is notable only during periods when winds exceed 10 or 12 m/sec. This is evidently a threshold wind speed which is necessary for efficient excitation of the oscillations. It is also notable that surface waves from the earthquake on August 18, 1959, at Hebgen Lake, Montana, were recorded above the natural ice background with this instrument. Gravity meter recordings at Fletcher's ice island in 1960 also show a similar type of correlation between wind speed and wave amplitude. At this time the island was drifting between 72°11'N, 154°02'W, and 71°43'N, 157°27'W. The maximum boom displacement found in a



Fig. 10. Tide records at T-3 from 0800 AST 5/27/61 to 0800 AST 5/28/61 (top) and 0800 AST 5/16/61 to 0755 AST 5/17/61 (bottom). Amplitude scale in centimeters.

5-min interval versus local wind speed is shown in Figure 9. Again the correlation is most notable for higher wind speeds.

For high wind speeds, maximum amplitudes of ice oscillation show no marked tendency to lead wind speed maxima, indicating that the waves are generated at a relatively close distance. Only slight correlations are found between light local winds and the oscillations which accompany them. It is possible that the oscillations during periods of calm or light wind are generated in distant storm centers where wind speeds exceed the threshold value and then propagate to other areas of the ocean.

 TABLE 2. Wind Speeds at Fletcher's Ice Island

 (T-3)

Hour	v	Wind Speed,
AST	Date	m/sec
0750	5/16/61	8
1050	5/16/61	10
1350	5/16/61	8
1650	5/16/61	11
1950	5/16/61	7
0750	5/17/61	8
0750	5/27/61	8
1050	5/27/61	10
1350	5/27/61	9
1650	5/27/61	10
1950	5/27/61	9

Waves with 10- to 100-min periods. Waves in a much longer period range were also observed at Fletcher's ice island, which became grounded in 1960 on the continental shelf at 71°55'N, 160°20'W, about 130 km northwest of Point Barrow, Alaska (Fig. 1). A tide gage was operated on 2-m-thick pack ice in Colby Bay, adjacent to the island. A cable was anchored to the bottom with a large weight in 37 m of water. The other end of the cable was led over the pulley of the tide gage and a small weight suspended from it. The records thus represent changes in ice level with respect to the ocean bottom. The chart drum was clockdriven at a rate of 8 inches per 24 hours. Trace displacement was equal to actual changes in ice level. The short-period resolutions of the instrument were limited to about 5 min by the chart speed and trace width.

Three general groups of water-level changes are revealed in the tide records (Fig. 10; wind data, Table 2). The tidal effects are the most evident, having a maximum amplitude of 18 cm at spring tide. Irregular water-level changes extending over several days are associated with atmospheric pressure and wind changes. This effect is not noticeable on the daily records in the illustration. Finally, there are 'short-period' waves ranging in period from 5 min to several hours and appearing as small fluctuations su-



Fig. 11. Fourier analyses of four T-3 tide records. Waves of tidal period have been removed. Linear scale in angular velocity and in displacement.

2484



Fig. 12. Group and phase velocity dispersion for long waves in 3-m ice layer floating on water 3000 m deep. Semilog scale.

perimposed on the tides and on the long-period water-level changes. *Beal* [1957] also observed these three types of oscillations with a tide gage operated in a similar manner close to shore near Point Barrow. The advantage of the ice island installation is its location far from the influence of the irregular shoreline. The fact that tides were well recorded at the ice island gives evidence that the ice was responding faithfully to water-level changes. No differential movements between ice and water level were noted at the recording site.

The 'short-period' waves are of interest in this study; the tides and long fluctuations will be the subject of a later publication. A Fourier analysis of four daily tide records was made. The records were first converted to digital form with an instrument built for this purpose by Paul Pomeroy of Lamont Geological Observatory. The record is placed on a drum, and when the drum is set in rotation the record trace is followed manually with a lens and target which are free to move laterally. The position of the target is digitally encoded at periodic intervals by an encoder which operates on an IBM card punch. A series of three digit numbers proportional to trace amplitude are thus punched on IBM cards for further analvsis. In each case a 24-hour record was analyzed with a drum speed and punching interval which

correspond to sampling at 52- or 53-sec intervals in real time. The remaining analysis was performed by Professor Yasuo Satô, using a program he devised for the IBM 7090 computer. The tides were removed from the digitized data by making a least squares fit of tidal components to the record, then synthesizing a tidal curve from these components. Five tidal components were used: M_{*} (principal lunar, semidiurnal), S₂ (principal solar, semidiurnal), O_1 (principal lunar, diurnal), S_1 (solar, diurnal), N_2 (larger lunar elliptic, semidiurnal). The synthesized tide curve was then subtracted from the original data, leaving a residual curve. A Fourier integral analysis was then made of the residuals, and the results are plotted in Figure 11. The periods between 10 and 100 min are the most reliable section of these spectra. The most outstanding feature of the spectra is again a general increase of displacement amplitude with period in all cases. Superimposed on the general increase in this range is a fine structure which shows only a slight correspondence between records. Some of the increase with period may be due to tidal contamination, but this is not considered to be important since the tides have been largely removed. Also, waves with periods of up to 1 hour are clearly evident on the records.

The origin of these long waves is not yet well understood. This long-period wave background is present even on records made during almost complete calm. Such long-period waves on open continental shelves have been identified with seiches and with edge waves by various authors [Marmer, 1951; Munk, Snodgrass, and Carrier, 1956]. Seiches should be identifiable as peaks of corresponding period on the various records but are not evident in these spectra. It is possible that these very long waves originate in response to atmospheric pressure changes and then propagate from the source as edge waves or gravity waves; however, there is no assurance that this is so.

Waves with periods of several minutes, unlike the longer-period waves, are evidently generated by local wind action. Only tide records from periods of calm or light wind were Fourier analyzed, since storms produced irregular water level changes which would be difficult to eliminate. But it is evident from a visual examination of tide records made in storm periods that



Fig. 13. Phase velocity dispersion for long waves in ice of various thicknesses floating on water 3000 m deep. Dots indicate apparent phase velocities measured at Arlis II. Semilog scale.

periods of the order of several minutes are enhanced by strong local wind.

Theoretical flexural-gravity waves. These observations of oscillations on the Arctic Ocean lead to the study of waves on a floating ice layer. Progressive waves are treated here, since records from Arlis II suggest that they may be the most important type. Long waves in a floating ice sheet under steady-state, plane-wave conditions were investigated by Ewing and Crary [1934]. They assumed an infinite, perfectly elastic ice sheet floating on a body of water having a plane, rigid bottom, and they included the effect of gravity in their calculations. For waves which are extremely long in comparison with ice thickness, their results reduce to the case of gravity waves on water. For short waves, the results apply to flexural vibrations and are valid down to a frequency of 20 cps when the ice sheet is 3 m thick [Hunkins, 1960]. These two wave types exhibit contrasting behavior. Deep water gravity waves are normally dispersed with a prograde orbital motion; flexural waves are inversely dispersed with an orbital motion which is retrograde at the ice surface and prograde at the ice-water interface.

The transition region between flexural and gravity waves was examined numerically using the period equation of Ewing and Crary which, for infinitely deep water, may be written in the form

$$G\lambda^4 - D(c^2/\alpha)\lambda^3 - \rho c^2\lambda^2 + F = 0$$

where

- λ = wave length.
- c = phase velocity.
- ρ = ice density = 0.90 g/cm³.
- ρ_1 = water density = 1.025 g/cm³.
- h = ice thickness = 3 m.
- g =acceleration due to gravity = 983 cm/sec².

$$G = g \rho_1 / 4 \pi^2 h$$

- $D = \rho_1/2\pi h.$
- $F = \frac{1}{3}\pi^2 h^2 \rho V_{p^2}.$
- $\alpha = (1 c^2 / V_w^2)^{1/2}.$
- V_w = sound velocity in water = 1440 m/sec.
- $V_p =$ longitudinal plate velocity of ice = 2800 m/sec.

This is a quartic equation in λ , and it was solved in terms of λ for various assigned values of c. The other parameters were given the constant values listed above, which are representative of the Arctic Ocean [Hunkins, 1960].

The equation was solved on an IBM 650 computer with a program for quartic equations which yields all four roots, real or complex. For values of c greater than a certain minimum



Fig. 14. Phase velocity dispersion for long waves in 3-m ice layer floating on water 3000 m and 40 m deep. Semilog scale.

value, the equation yields two real roots corresponding to the two types of wave motion, and the results are plotted in Figure 12 in terms of period, T. At very long wavelengths the assumption of infinite water depth causes the wave velocity to approach infinity. To correct this, the long-wave portion of the curve was calculated from the gravity wave formula,

$$c = \left(rac{g\lambda}{2\pi} anh rac{2\pi h}{\lambda}
ight)^{1/2}$$

and added later.

The short-period flexural waves and the long-period gravity wave branches meet at a point of minimum phase velocity, c = 22 m/sec and T = 12 sec. Group velocity was derived graphically and has a minimum at c = 15 m/sec and T = 16 sec. The velocity minimum in this case is analogous to that for surface waves on water where a minimum occurs between the capillary and gravity wave branches. The ice layer and the layer of surface tension both act to produce inverse dispersion at short wavelengths. It is to be noted that the phase-velocity minimum in this type of dispersion corresponds to the minimum wind speed capable of producing simple resonant coupling between the atmosphere and the ocean.

Some of the constant parameters in the previous calculation may vary under natural conditions, particularly ice thickness and water depth. Changes in ice thickness alter the flexural wave branch of the dispersion curve, the velocity minimum occurring at progressively higher velocities and longer periods as ice thickness increases (Fig. 13). Changes in water depth alter the gravity wave branch if ice thickness remains constant (Fig. 14). Dispersion in water 40 m deep is compared with the 3000-m depth of Figure 12. The velocity minimum virtually disappears in shallow water, and the flexural wave branch joins the shallow water gravity wave branch of constant velocity.

Standing waves on individual ice floes or ice islands may also occur. They should be identifiable as a series of sharp spectral peaks corresponding to the various modes. Although such a series is not evident in the present data, mention may be made of two possible types of standing waves. The simplest type is the bobbing of ice under the influence of gravity and buoyancy alone. The period of the oscillation, T, is given by

$$T = 2\pi (h\rho/g\rho_1)^{1/2}$$

and, using the previous constants, the value is 3.25 sec for ice 3 m thick and 14.55 for ice 60 m thick. The damping effect of the water will modify these periods under actual conditions and the system may be overdamped. Flexural standing waves on an ice floe or ice island may exist in analogy with the flexural vibrations of a free elastic plate.

Discussion. Observations with various recording instruments on different ice floes and ice islands in the Arctic Ocean all indicate that the ice oscillates almost continually. Some properties of these oscillations have been determined. On the ice floes of station Alpha, the displacement amplitudes of these oscillations generally increase with period in the range of 15 to 60 sec. The amplitudes increase from a fraction of a millimeter to several millimeters, roughly in proportion to the square of the period. On the ice island Arlis II, oscillations in this period range have been identified as propagating waves. Although only apparent phase velocities were determined, the values show fair agreement with the gravity wave branch of the theoretical flexural gravity-wave dispersion. These oscillations are at least partially generated by wind action. The fact that the local wind speed must reach a threshold value of 10 or 12 m/sec before large amplitudes are developed suggests



Fig. 15. Generalized spectrum of wave amplitudes on the Arctic Ocean. Crosshatching indicates observed period ranges with approximate limits of variation under average conditions. Dashed lines are interpolations. Log-log scale.

that some form of resonant coupling may be the method of excitation.

There is evidence that the general amplitudeperiod relationship found for 15- to 60-sec waves extends to very short periods. Observations of vertical oscillations of pack ice in the period range of 5 to 200 cps with exploration seismograph systems indicate that the general background is about the same as that on land. According to *Brune and Oliver* [1959], average displacement amplitudes of background motion on land are about 2 m μ at 5 cps and decrease with increasing frequency.

Another group of oscillations has been recorded in the period range of 10 to 100 min, but so far they have been observed only on the continental shelf in the Arctic Ocean. Amplitudes are in the centimeter range and increase slightly with period.

The Arctic Ocean spectra differ from those of other oceans. On an open ocean the typical wave spectrum is distinguished by an amplitude peak in the period range of 5 to 20 sec. This is the region of sea and swell which propagates as gravity waves [Roll, 1957]. On the ice-covered Arctic Ocean amplitudes in this range are small; sea and swell are effectively prohibited by the ice layer. A tentative diagram of the typical Arctic Ocean spectrum based on the results presented here is sketched in Figure 15. Large gaps exist and the known sections need to be verified at many more locations, but the broad features of a pattern can be seen.

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