Estimates of underwater sound (and infrasound) produced by nonlinearly interacting ocean waves

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Estimates of sound (and infrasound) spectral levels are given using a theoretical model of acoustic production based on quadratic interactions of oppositely traveling ocean waves, and incorporating present day surface wave spectral models. It is shown that reasonable agreement with measurements exists for frequenices less than 10 Hz but predictions are 10–15 dB low and have the wrong spectral shape at higher frequencies.

Subject Classification: [43]30.40; [43]20.15; [43]30.70; [43]28.65.

INTRODUCTION

One of the properties of underwater ambient noise is that its intensity is wind dependent (Piggott,[1] Crouch and Burt,[2] and Urick[3]). Several physical mechanisms have been proposed and explored which transfer energy from the wind field into the underwater acoustic field, for example, turbulent pressure fluctuations in the atmospheric boundary layer (Isakovich and Kuryanov[4]), oscillating bubbles and impacting water droplets (Wenz[5]), nonlinearly interacting surface waves (Brekhovskikh,[6] Kuo,[7] Marsh,[8] and Harper and Simpkins[9]).

Our concern in the present paper is with the last mechanism and our primary contribution will be to make ambient noise predictions by combining present day models of the surface wave field with second-order acoustic production theory. Specifically, our models are as follows: Phillips, both gravity and capillary regions behaving as (wave number)$^{-4}$ but with different levels; Pierson–Stacy, same as Phillips but with smooth connecting regions and a somewhat different overall level; Toba, based on the high-frequency surface wave measurements of Mitsuyasu and Honda[10] and not displaying any extended (wave number)$^{-4}$ region. In all models we use a directional distribution similar to Tyler *et al.*[11] and Mitsuyasu *et al.*[12]; namely, $\cos^q(\theta/2)$, where θ is measured from the wind vector and q is effectively constant for our frequency range.

Our main conclusion is that this (second-order) mechanism fails to account for the underwater noise field above 10 Hz but could well provide the necessary energy below 10 Hz. In order to make stronger conclusions and more detailed predictions it is necessary to model bottom losses with more precision than the present state of knowledge warrants.

I. GENERAL THEORY

This particular theoretical model has been explored at length in the literature and so we shall merely provide enough detail to define our notation and provide the highlights of the description. Brekhovskikh[6] gives a straightforward account of the theoretical model, Hasselmann[13] gives a general, full description with emphasis on statistical features of microseism generation, and Longuet-Higgins[14] provides a fundamental analytical treatment that is primarily deterministic and again oriented mainly toward microseisms. Harper and Simpkins[9] have treated the problem deterministically by using matched asymptotic expansions. They assume that the surface wave is a finite-amplitude standing wave and they determine not only the second-order, second-harmonic radiated acoustic field but the fourth-order second- and fourth-harmonic fields as well. In the analysis by Kuo[7] the model is put in terms of radiation accompanying the modulation of capillary waves by long surface waves, and the primary emphasis is on showing consistency with general acoustic observations and predicting the surface roughness in terms of acoustic measurements.

A. Equations of motion and perturbation expansions

For the dynamical equations, we have expressions for conservation of momentum, conservation of mass, and the equation of state[15]:

$$\rho\, \partial\vec{u}/\partial t + \rho(\vec{u}\cdot\nabla)\vec{u} = -\nabla p + \rho\vec{g}, \tag{1}$$

$$\partial\rho/\partial t + \rho\nabla\cdot\vec{u} + \vec{u}\cdot\nabla\rho = 0, \tag{2}$$

and

$$p = c^2\rho, \tag{3}$$

where ρ is density, $\vec{u}$ is fluid velocity, p is pressure, $\vec{g}$ is acceleration of gravity, and c is the speed of sound, assumed to be uniform. In our coordinate system we take z positive downwards, and we shall incorporate the following boundary conditions[16]:

$$\partial\zeta/\partial t = w - \vec{u}\cdot\nabla_H\zeta, \tag{4}$$

$$p = P_a + \mu\nabla_H^2\zeta/(1+|\nabla_H\zeta|^2)^{3/2}, \tag{5}$$

both to be applied at the free surface $z=\zeta(x,y,t)$, and the remaining statement that energy is bounded as $z\to+\infty$. In the above, μ is the dynamical surface tension, P_a is the atmospheric surface pressure (which we shall assume to be constant), w is the z component of $\vec{u}$, and ∇_H is the horizontal gradient operator with components $[(\partial/\partial x), (\partial/\partial y)]$.

We now make a perturbation expansion in all dependent variables as follows:

$$p = p_0 + \epsilon p_1 + \epsilon^2 p_2 + \cdots,$$

$$\rho = \rho_0 + \epsilon\rho_1 + \epsilon^2\rho_2 + \cdots,$$

$$\vec{u} = \epsilon\vec{u}_1 + \epsilon^2\vec{u}_2 + \cdots,$$

$$\zeta = \epsilon\zeta_1 + \epsilon^2\zeta_2 + \cdots,$$

$$P_a = \epsilon P_1, \quad (6)$$

and we expand the boundary conditions (4) and (5) about $z=0$ by means of a Taylor series. In Eq. (6), p_0 and ρ_0 refer to hydrostatic quantities, and ϵ is used only as an ordering parameter. We are leaving the order of magnitude and the dimensionality in each term. In fact, we could have omitted ϵ entirely and just insisted that all nth-order quantities satisfy differential equations in which any inhomogeneous terms are composed of products of lower-order quantities and such that the sum of the subscripts in each term is n.

On substituting Eq. (6) in Eqs. (1)–(5) and setting to zero each group of equations of different order, we find that the usual hydrostatic equations are given at zero'th order and the linear radiation equations are given at first order. The latter are

$$\nabla^2 p_1 = \frac{1}{c^2}\frac{\partial^2 p_1}{\partial t^2} \quad \text{throughout the fluid}, \quad (7)$$

$$p_1 = P_a + \mu\nabla_H^2\zeta_1 + \rho_A g\zeta_1, \quad z=0, \quad (8)$$

$$\partial\zeta_1/\partial t = w_1 \quad \text{at } z=0, \quad (9)$$

and boundedness as $z \to +\infty$. Here, $g = |\vec{g}|$, we have omitted $(g/c^2)(\partial p_1/\partial z)$ compared to $\nabla^2 p_1$ and we have labeled the surface density ρ_A. With a similar omission, the second-order equations become

$$\nabla^2 p_2 - \frac{1}{c^2}\frac{\partial^2 p_2}{\partial t^2} = -\nabla\cdot\{\rho_0\vec{u}_1\cdot\nabla\vec{u}_1 + \vec{u}_1\nabla\cdot(\rho_0\vec{u}_1)\}, \quad (10)$$

$$p_2 = \mu\nabla_H^2\zeta_2 - \rho_A g\zeta_2 - \rho_A g\zeta_1^2/2c^2 - \zeta_1\partial p_1/\partial z, \quad z=0, \quad (11)$$

$$\partial\zeta_2/\partial t = w_2 - \vec{u}_1\cdot\nabla_H\zeta_1 + \zeta_1\partial w_1/\partial z, \quad z=0, \quad (12)$$

boundedness as $z \to +\infty$. (13)

We now identify first order variables with the prescribed *surface* wave field, i.e., $c^2=\infty$ in Eqs. (7)–(9), and solve Eqs. (10)–(13) for the *acousticlike* part of p_2, i.e., $c^2 \neq \infty$ in Eqs. (10)–(13). With the recognition that $\nabla\times\vec{u}_1 \approx 0$ and that the phase velocity of the surface waves is very much less than c we may reduce the right hand side of Eq. (10) to zero by redefining p_2 and ζ_2:

$$p_2' = p_2 + \tfrac{1}{2}\rho_0(\vec{u}_1\cdot\vec{u}_1), \quad (14)$$

$$\partial\zeta_2'/\partial t = \partial\zeta_2/\partial t + \nabla_H\cdot(\vec{u}\zeta_1). \quad (15)$$

This gives us

$$\nabla^2 p_2' = \frac{1}{c^2}\frac{\partial^2 p_2'}{\partial t^2}, \quad \text{throughout the fluid}, \quad (16)$$

$$p_2' = \mu\nabla_H^2\zeta_2' - \rho_A g\zeta_2' + \tfrac{1}{2}\rho_A(\vec{u}_1\cdot\vec{u}_1) - \zeta_1\partial p_1/\partial z + \rho_A g^2\zeta_1^2/2c^2 \quad \text{at } z=0, \quad (17)$$

$$\partial\zeta_2'/\partial t = w_2, \quad \text{at } z=0, \quad (18)$$

and boundedness as $z \to +\infty$. (19)

We can recognize Eqs. (16)–(19) as specifying linear acoustic radiation of energy from the plane $z=0$, with a source given by the "external pressure field"

$$P_{2a} = \tfrac{1}{2}\rho_A(\vec{u}_1\cdot\vec{u}_1) + \rho_A\zeta_1\partial^2\zeta_1/\partial t^2. \quad (20)$$

This last equation is obtained by comparing Eqs. (17) and (8) and noting that the set (7)–(9) also specifies ensonification of the body of the fluid by the acousticlike part of P_1 (with $c^2 \neq \infty$). In gathering the three last terms in Eq. (17) we have omitted the third term because it is negligibly small, and we have replaced $-\partial p_1/\partial z$ with $+\rho_0\partial w_1/\partial t$, which at $z=0$, is $\rho_A\partial^2\zeta_1/\partial t^2$. In the body of the fluid $\vec{u}_1$ is exponentially small, thus $p_2' \approx p_2$. In using Eq. (20) we must include only those parts of the right-hand side that have "acousticlike" properties, i.e., low wave number and high frequency.

B. Renormalization of expansion scheme

If we assign typical parameters to our surface wave field, in particular, wave number k, frequency ω, and amplitude a, all associated with the peak in the energy spectrum, we find that $p_2/p_1 \sim ak$ and thus our perturbation scheme should provide useful results if the rms surface slope is not too large and if the acoustic energy is being produced by surface wave components near the peak in the spectrum. In principle we may extend the range of usefulness of our results to include radiation from components well removed from the peak by using a slightly different expansion system. From Eq. (17) we see that the second order forcing terms are of order $\zeta_1\partial p_1/\partial z$. By continuing the expansion to the next order, we find that the third-order forcing term is of order $\zeta_1^2\partial^2 p_1/\partial z^2$. This is equivalent to $k_s\zeta_1^2\partial p_1/\partial z$, and, if we examine the wave-number–frequency Fourier transform of this term we indeed find an acousticlike part resulting from the convolution of ζ_L with $k_s\zeta_1(\partial p_1/\partial z)_2$. Here ζ_L is the low-frequency, long wavelength part of ζ_1, k_s is the surface-wave wave number producing the acoustic field, and $(\zeta_1\partial p_1/\partial z)_2$ is the second-order forcing term, i.e., the high frequency, low wave-number part of the product of ζ_1 and $\partial p_1/\partial z$. Thus, the acousticlike third-order terms are of order $\zeta_L k_s$ times the second-order terms, and, since k_s generally refers to the high-frequency part of the surface wave spectrum, $\zeta_L k_s$ may be much greater than 1. To reduce the magnitude of these terms it is merely necessary to expand the surface boundary conditions (4) and (5) about $z=\zeta_L$ instead of $z=0$. This may be done consistently by splitting ζ_1 into two parts, $\bar{\zeta}_1$ and $\hat{\zeta}_1$, where $\bar{\zeta}_1$ pertains to low frequencies and $\hat{\rho}$ pertains to high frequencies, i.e., those frequencies directly responsible for the production of the acoustic energy being considered. Then we have $\zeta_1 = \bar{\zeta}_1 + \hat{\zeta}_1$ and the expansions are performed about $z = \epsilon\bar{\zeta}_1$. We find that the only change in our solution occurs in Eq. (20), which becomes

$$P_{2a} = \tfrac{1}{2}\rho_A(\vec{u}_1\cdot\vec{u}_1) + \rho_A\hat{\zeta}_1\partial^2\hat{\zeta}_1/\partial t^2, \quad (21)$$

and P_{2a}, as an "external pressure," is applied at $z=\epsilon\bar{\zeta}_1$. We also find that the ratio of the third-order terms to the second-order terms is now of order $\hat{\zeta}_1 k_s$, so we must split ζ_1 apart in such a way that $\hat{\zeta}_1 k_s$ is minimized. This will usually mean that $\bar{\zeta}_1$ pertains to all wave components with $k < k_s$.

It may also be noted that the perturbation scheme we have chosen gives rise to resonant interactions between surface waves. Thus second- and third-order terms in the surface wave field will grow to become first order. By a simple modification of the perturbation method we may circumvent this problem. We find no change in the (second-order) acoustic radiation results if we include all orders of the surface wave field with the first order terms, except those that possess significant values in the frequency-wave-number region associated with acoustic propagation. Thus ζ_1 becomes virtually the total surface wave amplitude.

C. Determination of acoustic spectra

We now solve Eqs. (16), (17), and (19) by a Fourier transform. We satisfy (16) and (19) automatically with

$$p_2' = \iiint_{-\infty}^{+\infty} Q(\omega, \vec{k}) \exp[i\omega t - i\vec{k}\cdot\vec{x} - i(\mathrm{sgn}\omega) \times (\omega^2/c^2 - k_H^2)^{1/2} z]\, d\vec{k}\, d\omega, \tag{22}$$

where $\vec{k}$ and $\vec{x}$ are horizontal vectors only, $k_H^2 = \vec{k}\cdot\vec{k}$, and the negative imaginary root is taken if $\omega^2/c^2 < k_H^2$. For simplicity we have chosen the case in which all waves propagate towards $+z$; we shall briefly discuss the effect of a bottom in a later section. The form of Q is given by the surface boundary condition, Eq. (17), modified by the previous discussion and including only the acousticlike terms:

$$\iiint_{-\infty}^{+\infty} Q(\omega, \vec{k}) \exp[i\omega t - i\vec{k}\cdot\vec{x} - i(\mathrm{sgn}\omega)(\omega^2/c^2 - k_H^2)^{1/2} \times \zeta_1(\vec{x}, t)]\, d\vec{k}\, d\omega = P_{2a}(\vec{x}, t). \tag{23}$$

We wish to discuss acoustic *spectral* levels and so we must incorporate stochastic features in Eqs. (22) and (23). We will assume that $\bar{\zeta}_1$, p_2', and $\hat{\zeta}_1$ are all Gaussian processes exhibiting wide sense stationarity, then, forming the covariance of P_{2a} from Eq. (23), we obtain (see Appendix)

$$\iiint S(\omega, \vec{k}) \exp[-k_3^2\sigma^2(1 - R(\vec{\Delta}, \tau)) - i\omega\tau + i\vec{k}\cdot\vec{\Delta}]\, d\omega\, d\vec{k} = U(\vec{\Delta}, \tau), \tag{24}$$

where $S(\omega, \vec{k})$ is the power spectrum of p_2', $k_3^2 = \omega^2/c^2 - k_H^2$, σ^2 is the variance of $\bar{\zeta}_1$, $R(\vec{\Delta}, \tau) = \langle \bar{\zeta}_1(\vec{x}, t)\, \bar{\zeta}_1(\vec{x} + \vec{\Delta}, t + \tau)\rangle/\sigma^2$ is the autocorrelation of $\bar{\zeta}_1$, U is the autocovariance of P_{2a}, and the integration is over all ω, $\vec{k}$ such that $k_3^2 \geq 0$. This equation has an immediate solution if we set $\bar{\zeta}_1 = 0$ (i.e., $\sigma^2 = 0$):

$$S(\omega, \vec{k}) = \begin{cases} F(\omega, \vec{k}), & \omega^2 \geq c^2 k_H^2, \\ 0, & \omega^2 < c^2 k_H^2, \end{cases} \tag{25}$$

where $F(\omega, \vec{k})$ is the Fourier transform of U, i.e., the power spectrum of the P_{2a} process. In general, $\sigma^2 \neq 0$, and we are left with the difficult task of solving Eq. (24) for $S(\omega, \vec{k})$. We do not intend to pursue this general case, except for some later discussion.[17] Our further treatment of the acoustic generation problem will be based on Eq. (25), and it will be shown later that our treatment does not suffer from this restriction.

If we define the power spectrum of the $\hat{\zeta}_1$ process to be $\Phi(\omega, \vec{k})$, it can be shown that

$$F(\omega, \vec{k}) \simeq \rho_A^2 \iiint_{-\infty}^{+\infty} \omega_1 \omega^3 \Phi(\omega, \vec{k}_1)\, \Phi(\omega - \omega_1, \vec{k} - \vec{k}_1)\, d\omega_1\, d\vec{k}_1. \tag{26}$$

This is obtained by transforming Eq. (21), forming the power spectrum, and converting the fourth-order statistical moment into products of second-order moments. From the known properties of surface waves, $\Phi(\omega, \vec{k})$ is very narrow in frequency if $\vec{k}$ is specified, and, for frequencies above a few hertz, the directional properties are separable from the wave number magnitude properties. Thus

$$\Phi(\omega, \vec{k}_1) = \tfrac{1}{2} X(k_1)\, G(\theta)\, [\delta(\omega - \omega_f) + \delta(\omega + \omega_f)], \tag{27}$$

where

$$\omega_f = (g k_1 + T k_1^3)^{1/2}, \tag{28}$$

$k_1 = |\vec{k}_1|$, θ is the direction of propagation of the surface waves, and T is the kinematic surface tension. We have defined $X(k_1)$ and $G(\theta)$ so that

$$\langle \hat{\zeta}_1^2 \rangle = \iint_{-\infty}^{+\infty} X(k_1)\, G(\theta)\, d\vec{k}_1, \tag{29}$$

$$= \int_0^\infty k_1 X(k_1)\, dk_1, \tag{30}$$

and

$$\int_0^{2\pi} G(\theta)\, d\theta = 1. \tag{31}$$

With these definitions, we find

$$F(\omega, \vec{k}) = \tfrac{1}{8}\rho_A^2 \left[\omega^4 \left(\frac{k_1 X^2(k_1)}{(\partial\omega_f/\partial k_1)}\right)\right] \int_0^{2\pi} G(\theta)\, G(\theta + \pi)\, d\theta. \tag{32}$$

The expression in the braces { } is to be evaluated at $\omega_f = \tfrac{1}{2}|\omega|$.

To obtain the frequency spectrum of pressure we must integrate $S(\omega, \vec{k})$ over all $\vec{k}$, i.e., integrate $F(\omega, \vec{k})$ over all $\vec{k}$ such that $k_H^2 \leq \omega^2/c^2$. From Eq. (32) we see that F is independent of $\vec{k}$, but we must recognize that only downward propagating waves are allowed (as yet there is no bottom), therefore with S_p as the acoustic frequency spectrum,

$$S_p(\omega) = \rho_A^2 \frac{\pi\omega^6}{4c^2} \left(\frac{k_1 X^2(k_1)}{(\partial\omega_f/\partial k_1)}\right)_{|\omega|/2} \int_0^{2\pi} G(\theta)\, G(\theta + \pi)\, d\theta. \tag{33}$$

It should be noted that this is in agreement with the final expression given by Hasselmann[13] [his Eq. (2.15)], and of the same form as that given by Brekhovskikh[6] [his Eq. (53); although his is basically one-half[18] of ours due to a different method of relating the fourth- and second-order statistical moments, and his displays an insignificant difference in the effect of surface tension]. Also, as shown by Brekhovskikh, our theoretical model [Eq. (32)] corresponds to dipole radiation from a plane surface, with the dipole axis oriented vertically.

II. PREDICTIONS USING SPECIFIC SURFACE WAVE MODELS

Recent measurements of the directional surface-wave spectrum indicate that a good estimate for $G(\theta)$ is

$$G(\theta)=\frac{\cos^q[\frac{1}{2}(\theta-\theta_0)]}{[2^{q+1}/\Gamma(q+1)]\Gamma^2(\frac{1}{2}(q+1))}, \quad (34)$$

where θ_0 is the wind vector angle and q is a nondimensionalized function of frequency. Tyler *et al.*[11] find that $q \leq 1$ for frequencies greater than $g/(12.5\pi u_*)$ where u_* is the friction velocity characterizing the wind profile. If we use a representative value of 10^{-3} for the wind stress coefficient, we can relate u_* to U_{10}, the wind speed at 10 m height, (see, e.g., Ref. 16, p. 144), and we find

$$q \leq 1 \text{ for } f \gtrsim 2.5\, g/(\pi U_{10}). \quad (35)$$

Mitsuyasu *et al.*[12] find a similar behavior for $G(\theta)$, and their results produce

$$q \leq 1 \text{ for } f \gtrsim g/(\pi U_{10}). \quad (36)$$

Equations (35) and (36) indicate that we may take $q \leq 1$ for all wind speeds greater than 10 knots if we restrict our attention to frequencies greater than approximately 1 Hz. In fact, we shall take $q=1$, and we shall use the frequency range $f \geq 0.5$ Hz (this corresponds to *acoustic* frequencies greater than 1 Hz). Even if q varies between 0 and 2 our predicted values of S_p will vary by only ± 1.5 dB and we consider this to be insignificant.

The directional term in S_p can be integrated for general q to give

$$\int_0^{2\pi} G(\theta)G(\theta+\pi)\,d\theta=\frac{\pi^{-1/2}2^{-q-1}\Gamma(\frac{1}{2}q+1)}{\Gamma[\frac{1}{2}(q+1)]}; \quad (37)$$

this provides us with a value of $\frac{1}{8}$ at $q=1$.

For the frequency distribution of the surface-wave spectrum, i.e., $X(k)$, we shall use three forms:

I. Phillips[19]:

$$X=\begin{cases}0.46\;10^{-2}\,k^{-4}, & k<\sqrt{g/T},\\ 1.5\;10^{-2}\,k^{-4}, & k>\sqrt{g/T},\end{cases} \quad (38)$$

II. Toba[12,20]:

$$X=\frac{0.01u_*}{(g+Tk^2)^{1/2}}\left\{\frac{g+3Tk^2}{g+Tk^2}\right\}k^{-7/2}, \quad (39)$$

III. Pierson-Stacy[21]:

$$X=\begin{cases}Bk^{-4}, & k<k_1,\\ Bk^{-7/2}k_1^{-1/2}, & k_1<k<k_2,\\ BD(u_*)k^{p-4}k_3^{-p}, & k_2<k<k_3,\\ BD(u_*)k^{-4}, & k_3<k<k_\nu,\\ Ek^{-10}, & k>k_\nu,\end{cases} \quad (40)$$

where, in III, $B=4.05\;10^{-3}$, $p=2.39\log_{10}(12D(u_*)/u_*)$, $k_1=51.7/u_*^2$, $k_2=0.359$, $k_3=0.942$, $D(u_*)=(1.274+0.0268u_*+6.03\;10^{-5}u_*^2)^2$, $k_\nu=2.09u_*^{1/2}/D^{1/6}$, $E=0.33u_*^3$, and all units are cgs. We have simplified all spectra by ignoring the low-frequency rolloff.

To convert u_* into wind speed, we use the following equations

$$U_{10}=25\,u_*[1-0.2\ln(u_*/u_0)],$$
$$u_0=0.50\text{ m/sec}. \quad (41)$$

These are obtained by assuming Charnock's constant is 0.0156 (Ref. 11, Sec. 8).

It should be noted that we will use the three forms of X for surface wave frequencies up to 1500 Hz. This is an order of magnitude beyond the present range of measurements. Mitsuyasu and Honda[10] provide measurements up to 100 Hz and their spectral levels show no tendency to deviate from the Toba form even at 100 Hz. We are perhaps justified, therefore, in extending their results to 300 Hz, i.e., an acoustic frequency of 600 Hz. On the other hand, the viscous cutoff specified in the Pierson-Stacy form begins at one-sixth of this frequency (for $U_{10}=30$ knots). We think this may be an excessive restriction and so we have shown the Pierson-Stacy predictions with and without a viscous cutoff region.

Predicted values of S_p, at $U_{10}=30$ knots and for each of the three forms of X, are shown in Fig. 1 along with two sets of experimental data. The high-frequency data is obtained from Crouch and Burt[2] and is the *wind-dependent* part of the acoustic spectral measurements originally published by Perrone.[22] The measurement site (near Bermuda) had an ocean depth of about 4400 m and the (omnidirectional) hydrophone was suspended approximately 120 m above the bottom. Recordings were made over a 30-day period (January). The low frequency data is obtained from Perrone[23] and in contrast to the previous case, it is the *total* spectral level: no separation has been made into wind-dependent and non-wind-dependent parts. The measurement site in this case (Grand Banks) has an ocean depth of about 1100 m and the hydrophone was bottom mounted. Recordings were made for eight days (July 1972).

For frequencies greater than about 10 Hz it is apparent that the theoretical estimates are all much too

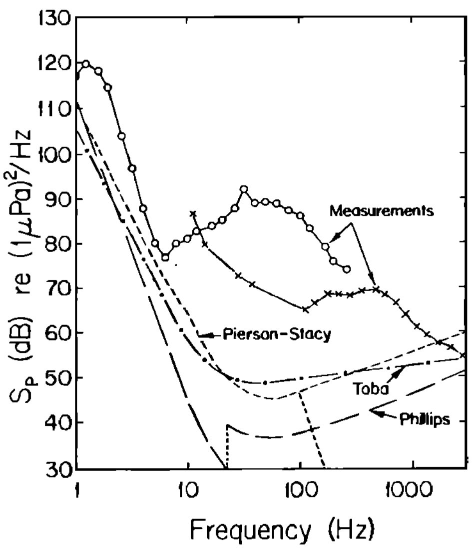

FIG. 1. Comparison of predicted acoustic spectra with measurements. The high-frequency measurements (crosses) are the wind-dependent part only (Crouch and Burt[2]); the low frequency measurements (circles) are the total acoustic field (Perrone[23]). Pierson-Stacy (dashed line) is shown with and without a viscous cutoff. All curves are for a wind speed U_{10} of 30 knots.

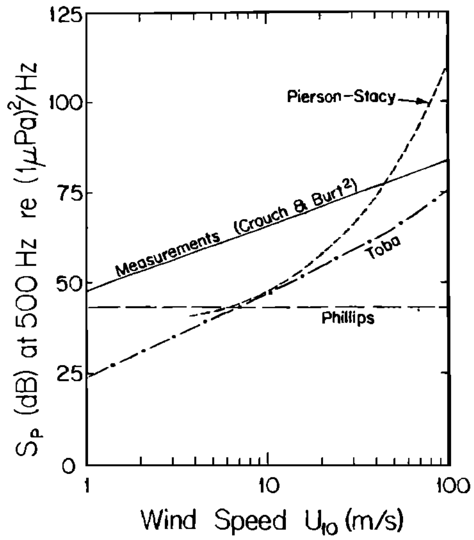

FIG. 2. Wind speed dependence of S_p at 500 Hz.

low, and they predict S_p ultimately increasing with frequency whereas the measurements have the opposite trend. For low frequencies (1–10 Hz), on the other hand, the frequency dependence of the theoretical estimates is quite similar to the measured data although the predicted levels are still somewhat low (10–15 dB).

A comparison of wind-speed dependence at high frequency is given in Fig. 2. The solid line represents the experimental data and is obtained by averaging the best-fit parameters for 446 and 562 Hz as given by Crouch and Burt.[2] Its equation is $S_p = 47.4 + 18.2 \log U_{10}$. (For the theoretical estimate based on Pierson–Stacy, we have omitted the viscous cutoff region, i.e., arbitrarily shifted k_ν from >6.5 and >31.9 rad/cm.) Except for predictions using Phillips's form, the dependence for winds less than about 80 knots is predicted reasonably well.

In Fig. 3 the low-frequency measurements for all wind speeds up to 40 knots are depicted along with the comparable Pierson–Stacy predictions. Both measurement and prediction exhibit similar variations with wind speed, although the frequency bands with largest variations do not coincide. [For the 7.5 knot boundary on the Pierson–Stacy data we have muliplied X by the factor $\exp(-0.74 g^2/U_{19.5}^4 k^2)$ because surface-wave frequencies radiating 1-Hz acoustic energy are near the peak in the surface wave spectrum. The effect of this is the slight curvature near 1 Hz. Our previous arguments suggest that in this region we should also reduce the directional contribution to S_p given by Eq. (37). This is indicated in Fig. 2 by the dotted line.]

III. DISCUSSION

There are two further aspects of the acoustical ocean environment which we will now consider in more detail: (a) the effect of multiple reflections between ocean surface and bottom (along with volume absorption at very high frequencies), and (b) the influence of low-frequency roughness at the free surface, i.e., $\sigma^2 \neq 0$ in Eq. (24).

A. Multiple reflections

We shall make estimates by using a very simplistic multiple reflection model of the atmosphere–ocean–bottom system: horizontal air–water interface with unity reflection coefficient, horizontal ocean bottom with an energy reflection coefficient γ (the energy reflected upwards is γ times the incident energy), straight rays, incoherent addition of different multiple reflections, and a simple exponential attenuation factor α to allow for volume absorption. As a result we find we must multiply our previous estimates of $S_p(\omega)$ by a factor $L(\omega)$, given by[24]

$$L(\omega) = 2\int_0^{\pi/2} \sin\theta\cos\theta \left[\frac{\exp[-z\alpha(\omega)/\cos\theta] + \gamma(\theta,\omega)\exp[-(2H-z)\alpha(\omega)/\cos\theta]}{1-\gamma(\theta,\omega)\exp[-2H\alpha(\omega)/\cos\theta]}\right] d\theta, \tag{42}$$

where z is the hydrophone depth, H is the ocean depth, and θ is the propagation angle measured from the vertical (90°—grazing angle). We have (somewhat arbitrarily) chosen the following form for $\gamma(\theta, \omega)$:

$$10\log_{10}\gamma(\theta,\omega) = -\{8450 + 589.5\,[\log_{10}(\omega/2\pi)]^4\}^{1/4} + 8.588 \tag{43}$$

independent of θ, where the right-hand side has been determined from data[25] given in Urick.[26,27] For $\alpha(\omega)$ we have used the results for the North Atlantic presented by Mellen and Browning,[28]

$$\alpha = 0.003 + 0.1f^2/(1+f^2) + 0.01f^2, \tag{44}$$

where f is in kHz and α is dB/kyd. $L(\omega)$ is shown in Fig. 4 (solid line) along with estimates for $\gamma = 0$ and $\gamma = 1$ (dotted and dashed lines, respectively). To correspond to the experiment conditions, we have used $z = H$ for all estimates,[29] and $H = 1100$ m for low frequencies, $H = 4400$ m for high frequencies.

B. Low-frequency air–water roughness

The second aspect we referred to earlier was the effect of a rough air–sea interface. Intuitively, we may expect the roughness merely to reorient the direction of energy propagation and not to affect its total level summed over all directions. Indeed, this is approximately the case, as can be seen by appropriate manipulations of Eq. (24). If we first take the inverse (three-dimensional) Fourier transform of both sides, and then integrate over all wave number space, we obtain

$$\frac{1}{2\pi}\iiint_{-\infty}^{+\infty} S(\omega,\vec{k})\,d\omega\,d\vec{k}\int_{-\infty}^{+\infty}\exp\{i\tau(\omega-\omega') - k_3^2\sigma^2[1-R(0,\tau)]\}\,d\tau = \frac{2\pi\omega'^2}{c^2}F(\omega',0), \tag{45}$$

where the right-hand side has been limited to only the acoustic part of the spectrum, i.e., $|\vec{k}|^2 \le \omega'^2/c^2$, and we have used the fact that F is independent of $\vec{k}$. We

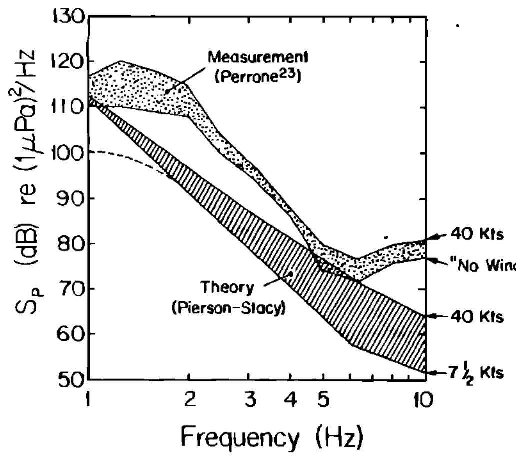

FIG. 3. Comparison of low frequency ambient noise for a range of wind speeds.

may approximate $R(0, \tau)$ by the parabolic form $R = 1 - \frac{1}{2}\omega_p^2\tau$, where ω_p is a frequency near the peak in the surface wave spectrum. For small τ, this form is a good approximation; for large τ, a consideration of the behavior of the exponent in (45) shows, after some analysis, that the form of R is unimportant. By expanding the Gaussian part of the τ-integrand in a power series in τ^2, we find that (46) can be written as

$$\iint_{-\infty}^{+\infty} S(\omega', k)\left\{1 + \frac{\omega_p^2\sigma^2}{c^2} - \frac{2\omega_p^2\sigma^2}{c^2}\left(\frac{\omega'}{S}\frac{\partial S}{\partial\omega}\right) + \frac{\omega_p^2\sigma^2}{2c^2}\frac{\omega'^2}{S}\frac{\partial^2 S}{\partial\omega^2} + \cdots\right\} dk = \frac{2\pi\omega'^2}{c^2} F(\omega', 0). \qquad (46)$$

Since the parameter $(\omega_p\sigma/c)^2$ is independent of $\vec{k}$ and is approximately 10^{-6} for a 30-knot wind, and less for weaker winds, we see that our original assumption of $\sigma^2 = 0$ has not jeopardized our results. (In treating the bottom, $R = 1$ for all (significant) τ and again this aspect of roughness is unimportant. The effect of roughness on γ is quite separate.)

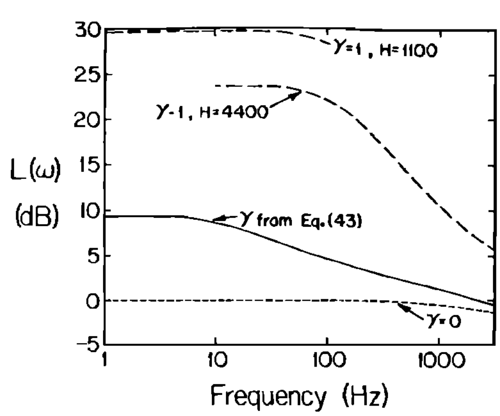

FIG. 4. Bottom factor $L(\omega)$. The solid line is for bottom losses given by Eq. (43). The lines labeled $\gamma = 1$ represent no bottom absorption (perfect reflection), the line labeled $\gamma = 0$ represents complete bottom absorption (no reflection). Volume absorption is calculated using Eq. (44). For all curves two ocean depths were used, 1100 m for 1–100 Hz, and 4400 m for 10–3000 Hz (corresponding to the experimental conditions).

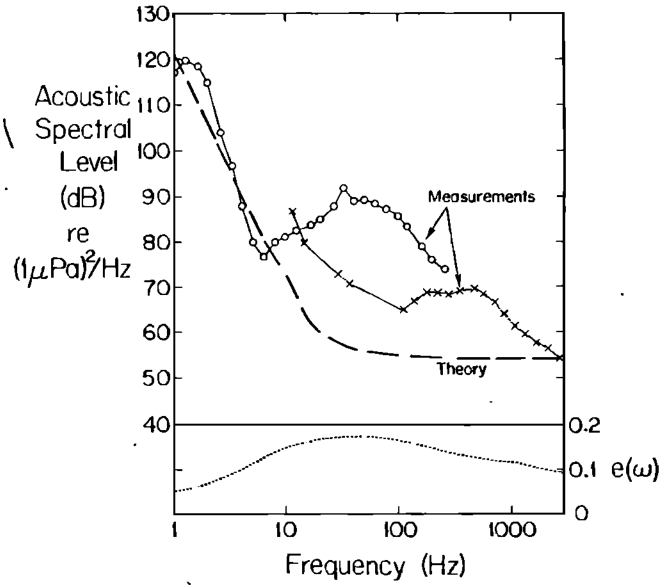

FIG. 5. Acoustic spectral level for $U_{10} = 30$ knots including nonzero bottom loss and volume absorption. The theoretical curve (broken line) uses the Pierson–Stacy form for less than 20 Hz, and the Toba form otherwise. Measurements (solid lines with symbols) are as in Fig. 1. Also shown (dotted line) is the perturbation expansion convergence factor $e(\omega)$ as defined in Eq. (47). It uses the right-hand vertical axis.

IV. CONCLUSIONS

Our final "best" estimate for the acoustic spectral level at a wind speed of 30 knots is given in Fig. 5. Here we have used the Pierson–Stacy form for frequencies less than 20 Hz and the Toba form for higher frequencies.[30] We have also incorporated $L(\omega)$. It is apparent that our estimates are in reasonable agreement with the data for frequencies less than 10 Hz and not otherwise.

We offer one further speculation: that the main weakness of our theoretical model is its failure to include higher-order nonlinearities. We expect these to be large where the surface waves are steepest, i.e., where waves are breaking, and thus we may expect their inclusion to model, at least partially, the acoustic effects concomitant with whitecaps and spray. (We do not expect our perturbation scheme to be capable of including all the acoustic effects of breaking waves: we have expanded the surface conditions in Taylor series' and so are restricted to single-valued surfaces only. On the other hand, we have used surface wave spectra which are based on measurements made, presumably, in the presence of breaking waves.) An estimate of the relative importance of the higher order terms as a function of acoustic frequency ω was shown to be given by the convergence factor $k_s\hat{\zeta}_1$, where k_s is the surface-wave wave number producing ω, and $\hat{\zeta}_1$ is the amplitude of the surface wave field including all wave components with $k \geq k_s$. Numerical values may be assigned to this parameter by allowing $\hat{\zeta}_1$ to be calculated from X, i.e., allowing $\hat{\zeta}_1$ to be the rms amplitude. Defining $e(\omega) = k_s(\hat{\zeta}_1)_{rms}$, we have

$$e(\omega) = k_s\left[\int_{k_s}^{\infty} k X(k)\, dk\right]^{1/2}. \qquad (47)$$

Using the combined Pierson–Stacy and Toba form for X, and using $U_{10} = 30$ knots, we obtain values for e as illustrated in Fig. 5 (dotted line, right hand vertical axis). Smaller values of e result for weaker winds. It is perhaps noteworthy that the largest values of e, i.e., the slowest (estimated) convergence rates in our perturbation expansions, exist in the frequency range where the second-order theory deviates most from the measurements. It is tempting (although admittedly premature) to conclude that our correspondence would be improved if all orders were included.

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APPENDIX

With $U(\vec{\Delta}, \tau)$ as the autocovariance of P_{2a}, and $k_3 = \operatorname{sgn}\omega(\omega^2/c^2 - k_H^2)^{1/2}$,

$$\int \langle Q(\omega, \vec{k})\, Q^*(\omega', \vec{k}') \exp[-ik_3\bar{\zeta}_1(\vec{x}, t) + ik_3'\bar{\zeta}_1(\vec{x}+\vec{\Delta}, t+\tau)]\rangle \times \exp[i\omega t - i\vec{k}\cdot\vec{x} - i\omega'(t+\tau) + i\vec{k}'\cdot(\vec{x}+\vec{\Delta})]\, d\vec{k}\, d\omega\, d\vec{k}'\, d\omega' = U(\vec{\Delta}, \tau)\,. \tag{A1}$$

The right-hand side is wide sense stationary, as is $\bar{\zeta}_1$, therefore all terms in $\vec{x}$ and t must disappear from the left-hand side. This is accomplished with

$$\langle Q(\omega, \vec{k})\, Q^*(\omega', \vec{k}') \exp[-ik_3\bar{\zeta}_1 + ik_3'\bar{\zeta}_1(\vec{x}+\vec{\Delta}, t+\tau)]\rangle \equiv F_M(\omega, \vec{k}, \vec{\Delta}, \tau)\, \delta(\omega - \omega')\, \delta(\vec{k} - \vec{k}')\,, \tag{A2}$$

where F_M is to be determined. We then have

$$\int F_M(\omega, \vec{k}, \vec{\Delta}, \tau) \exp(-i\omega\tau + i\vec{k}\cdot\vec{\Delta})\, d\omega\, d\vec{k} = U\,. \tag{A3}$$

In Eq. (A2) we may replace the Q's by inverse transforms of p_2':

$$F_M\delta(\omega - \omega')\,\delta(\vec{k} - \vec{k}') = \frac{1}{64\pi^6}\int \langle p_2'(\vec{x}, z, t)\, p_2'(\vec{x}', z, t') \times \exp\{-ik_3[\bar{\zeta}_1(\vec{x}'', t'') - z] + ik_3'[\bar{\zeta}_1(\vec{x}''+\vec{\Delta}, t''+\tau) - z]\}\rangle \times \exp(-i\omega t + i\omega' t' + i\vec{k}\cdot\vec{x} - i\vec{k}'\cdot\vec{x}')\, d\vec{x}'\, d\vec{x}\, dt\, dt'\,. \tag{A4}$$

We now restrict our definition of p_2' to only the acoustic part, i.e., real k_3 only. This provides no loss of generality and ensures that F_M is determined only in terms of the acoustic spectrum.

Measurements indicate that both $\bar{\zeta}_1$ and p_2' are approximately Gaussian (Phillips[31] and Urick[32]). We shall make the further assumption that they are jointly Gaussian. The acoustic pressure p_2' is produced by Fourier components in the surface wave field that are separated from the components defining $\bar{\zeta}_1$; therefore, since ζ_1 is assumed to be wide sense stationary (and thus its components are mutually orthogonal) we may assume that p_2' and $\bar{\zeta}_1$ are orthogonal, i.e., $\langle p_2'(\vec{x}, z, t) \times \bar{\zeta}_1(\vec{x}'', t'')\rangle = 0$. This allows us to write the ensemble average in the integrand in Eq. (A4) as the product of the autocovariance of p_2' and the ensemble average of the exponential term. The former is dependent only on $\vec{x} - \vec{x}'$, z, $t - t'$ and thus all eight integrals may be performed: four produce the same delta functions that appear on the left-hand side, and the remaining four serve to define the power spectrum, S, of the p_2' process. Finally, by the Gaussian character of $\bar{\zeta}_1$, $\langle\exp\{ik_3[\bar{\zeta}_1(\vec{x}'', t'') - \bar{\zeta}_1(\vec{x}''+\vec{\Delta}, t''+\tau)]\}\rangle$ becomes $\exp\{-k_3^2\sigma^2[1 - R(\vec{\Delta}, \tau)]\}$ where σ^2 is the variance of $\bar{\zeta}_1$ and R is its autocorrelation function (see also Medwin and Hagy[33]). We finally have

$$F_M(\omega, \vec{k}, \vec{\Delta}, \tau) = S(\omega, \vec{k}) \exp\{-k_3^2\sigma^2[1 - R(\vec{\Delta}, \tau)]\}\,. \tag{A5}$$

Substituting this in Eq. (A3) and noting the restriction to real k_3 produces Eq. (24).

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[1] C. L. Piggott, J. Acoust. Soc. Am. **36**, 2152–2163 (1965).

[2] W. W. Crouch and P. J. Burt, J. Acoust. Soc. Am. **51**, 1066–1072 (1972).

[3] R. J. Urick, *Principles of Underwater Sound* (McGraw–Hill, New York, 1975), 2nd ed., p. 186.

[4] M. A. Isakovich and B. F. Kur'yanov, Sov. Phys.—Acoustics **16**, 49–58 (1970).

[5] G. M. Wenz, J. Acoust. Soc. Am. **34**, 1936–1956 (1962).

[6] L. M. Brekhovskikh, Izv. Atmos. Ocean. Phys. **2**(9), 582–587 (1966).

[7] E. Y. T. Kuo, J. Acoust. Soc. Am. **43**, 1017–1024 (1968).

[8] H. W. Marsh, J. Acoust. Soc. Am. **35**, 409–410 (1963).

[9] E. Y. Harper and P. G. Simpkins, J. Sound Vib. **37**(2), 185–193 (1974).

[10] H. Mitsuyasu and T. Honda, J. Oceanogr. Soc. Jpn. **30**, 185–198 (1974).

[11] G. L. Tyler, C. C. Teague, R. H. Stewart, A. M. Peterson, W. H. Munk, and J. W. Joy, Deep-Sea Res. **21**, 989–1016 (1974).

[12] H. Mitsuyasu, F. Tasai, T. Suhara, S. Mizuno, M. Ohkusu, T. Honda, and K. Rikiishi, J. Phys. Oceanogr. **5**, 750–760 (1975).

[13] K. Hasselmann, Rev. Geophys. **1**, 177–210 (1963).

[14] M. S. Longuet-Higgins, Philos. Trans. R. Soc. London, Ser. A **243**, 1–35 (1950).

[15] G. K. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge University, London, 1974), Sec. 3.6. In our equations we have ignored viscosity. Volume attenuation is included in a later section.

[16] O. M. Phillips, *The Dynamics of the Upper Ocean* (Cambridge University. London, 1969), Eqs. (3.1.3), (3.1.6), and following (3.1.11).

[17] For further research on the effect of sea surface roughness see, for example, H. Medwin *et al.*, J. Acoust. Soc. Am. **54**, 99–109 (1973); F. M. Labianca and E. Y. Harper, J. Acoust. Soc. Am. **59**, 799–812 (1976); and R. L. Swarts and C. J. Eggen, J. Acoust. Soc. Am. **59**, 846–851 (1976).

[18] We also believe his definition of the surface displacement spectrum [his Eq. (38)] leads to an extra factor of one-half multiplying the right hand side of his Eq. (54). This has apparently been erroneously omitted, and, if we insert it (twice, since the spectrum appears squared), and allow for the previously mentioned factor of one-half, we obtain satisfactory agreement in our expressions.

[19] Reference 16, p. 120, Table 4.2.

[20] Y. Toba, "The Science Reports of the Tôhoku University, Series 5," Geophys. **22** (2), 61–73 (1974).

[21] W. J. Pierson, Jr., and R. A. Stacy, NASA Report CR-2247, National Technical Information Service, Springfield, VA 22151 (1973).

[22] A. J. Perrone, J. Acoust. Soc. Am. **46**, 762–770 (1969).

[23] A. J. Perrone, J. Acoust. Soc. Am. **55**, 754–758 (1974).

[24]This expression can be obtained by summing the infinite geometric series that expresses the total energy received at the hydrophone along a given elevation angle. The first term in the series is the contribution from the surface along a direct path, i.e., no reflections; the second is the contribution that has suffered one surface and one bottom reflection; the third, two surface and two bottom reflections, etc. At each bottom reflection the energy is reduced an amount γ, and along each path energy is attenuated at a spatial rate α. Equation (42) then results by integrating this sum over all relevant elevation angles after multiplying by the dipole radiation pattern, $\sin\theta$, and the Jacobian for circular symmetry, $\cos\theta$.

[25]We have also set $\gamma = -1$ dB at 1 Hz.

[26]Reference 3, p. 131, Fig. 5.30.

[27]See also H. E. Morris, J. Acoust. Soc. Am. 48, 1198–1202 (1970), and H. P. Bucker, J. A. Whitney, G. S. Yee, and R. R. Gardner, J. Acoust. Soc. Am. 37, 1037–1051 (1965).

[28]R. H. Mellen and D. G. Browning, J. Acoust. Soc. Am. 59, 700–702 (1976). We have included the "scatter" loss term 0.003 dB/kyd mainly in the spirit of completeness. We believe that long-range ducted propagation may be of importance to infrasonic noise levels, and that the "scatter" loss mechanism is not fully understood yet. It seems, therefore, somewhat more conservative to include this term at present. As it turns out, it makes virtually no difference to our final "best estimate" and thus its presence or absence is largely immaterial to the present problem.

[29]A full normal-mode treatment indicates that in this case $L(\omega)$ should be doubled for $\gamma \approx 1$. This is because the reflected and incident radiation are approximately equal and are coherent at the reflecting boundary. In view of the facts that the bottom is rough, $\gamma \neq 1$, and $z \neq H$ for all frequencies for the measured data, we have not included this increase in $L(\omega)$.

[30]This corresponds closely to a revised version of the Pierson-Stacy form; see W. J. Pierson, Jr., NASA Report CR-2646, Prepared by City Univ. of New York, New York, for Goddard Space Flight Center (1976).

[31]Reference 16, pp. 154–157, Sec. 4.10.

[32]Reference 3, pp. 199–200, Sec. 7.7.

[33]H. Medwin and J. D. Hagy, Jr., J. Acoust. Soc. Am. 51, 1083–1090 (1972).