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Wave setup and setdown generated by obliquely incident waves

Tai-Wen Hsu [a,*], John R.-C. Hsu [b,1], Wen-Kai Weng [c], Swun-Kwang Wang [d], Shan-Hwei Ou [e]

[a] *Department of Hydraulic and Ocean Engineering, National Cheng Kung University, Tainan 701, Taiwan*
[b] *Department of Marine Environment and Engineering, National Sun Yat-sen University; ASIA-PACIFIC Ocean Research Center, Kaohsiung 804, Taiwan*
[c] *Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung 202, Taiwan*
[d] *Department of Safety, Health and Environmental Engineering, Chung Hwa College of Medical Technology, Rende 717, Taiwan*
[e] *Department of Environmental Engineering and Sciences, Tajen University, Pingtung 907, Taiwan*

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Abstract

An analytical theory is developed for the wave setup and setdown induced by obliquely incident waves on an impermeable swell-built beach profile. The wave setup and setdown are found to decrease as wave obliquity increases. The incorporation of wave obliquity in wave setup and setdown formulation offers the physical reality in engineering applications. The general solutions presented in this paper yield the limiting case of normal wave incidence and the result is consistent with the classical theories published. The present theory is primarily applicable to the spilling and plunging breaker across the surf zone, within which wave amplitude is assumed to be linearly related to the local water depth. Experiments were conducted in a large-scale wave basin to compare with theoretical results and especially to investigate the applicability of this assumption to the case of obliquely incident waves. The dimensionless setup versus the distance offshore within the surf zone is found to depend on wave breaking angle and the shape of the beach profile; and it has a non-zero value at the original shoreline position. This implies that the original shoreline will advance landwards, and that the extent of this movement can be related to wave angle at breaking and the beach profile under consideration. The results of the present theory are in good agreement with experimental data and field measurements available.

Keywords: Wave setup and setdown; Radiation stress; Obliquely incident waves; Beach profile; Shoreline advance

1. Introduction

The phenomena of wave-induced setdown and setup have attracted the attention of many coastal scientists since the 1960s, and almost all the theoretical outputs so far have focused on the scenario of normal wave incidence. When shoaling waves reach the nearshore and eventually break on a beach, wave height and momentum flux reduce. Wave breaking produces not only a change in momentum flux across the surf zone, but also a compensating force on the water column. This force is then balanced by an increase in the mean water surface above the still water level (SWL) of the sea, which is termed "wave setup" with a gradient to balance the momentum flux induced by the breaking waves. Outside the breaker line, where waves have undergone rapid transformations in height and energy due to shoaling, a depression in the mean sea level is found to balance the excess momentum flux resulting in wave setdown prior to breaking.

The fundamental theory of Longuet-Higgins and Stewart (1963, 1964) on wave motions in the nearshore have shown the wave-induced excess momentum flux (or the onshore component of the radiation stress) is responsible for the wave setup and setdown on a sloping beach. It means that the changes in the onshore component of the radiation stresses are balanced successively by wave setup and setdown as wave transformation takes place during shoaling, breaking, across the surf zone and finally arriving at the shoreline. The concept of radiation stress was verified by several researchers in the laboratory. For example, Saville (1961) was the first to conduct measurements

* Corresponding author. Tel.: +886 6 2372315; fax: +886 6 2741463.
E-mail address: twhsu@mail.ncku.edu.tw (T.-W. Hsu).
[1] Honorary Research Fellow, School of Civil and Resource Engineering, University of Western Australia, Crawley, WA 6009, Australia.

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of wave setup and setdown in the laboratory. Later, Bowen et al. (1968), Bowen (1969) and Van Dorn (1976) also carried out detailed measurements on wave setup and setdown in laboratory condition. Their results confirmed the validity of Longuet–Higgin's theory for the wave-induced radiation stresses. They also found wave setdown commenced from offshore and reached its largest depression at the breaking point, followed by a rapid rise of setup shoreward across the surf zone. All the laboratory experiments so far performed were on beach in uniform slope and normal wave incidence.

While investigating beach profile variations in field condition, Dean (1977) derived an expression for equilibrium beach profile, based on a concept of mean energy conservation within the surf zone. His results not only showed a good agreement with the empirical relationship between water depth and its distance offshore, but also confirmed that uniform beach profile is rare in nature. In the mean time, wave setup and setdown were observed in the field (Guza and Thornton, 1981; Holman and Sallenger, 1985; Hanslow and Nielsen, 1993). Among them, Guza and Thornton (1981) measured wave setup on beaches in southern California. They found that the maximum setup, $\overline{\eta}_{\max}$, above the SWL at the shoreline can be expressed as

$$\overline{\eta}_{\max} = 0.17H_{1/3} \tag{1}$$

in which $H_{1/3}$ is a significant wave height in deep water. From field measurements on a steep beach at the field research facility (FRF) in Duck, North Carolina, Holman and Sallenger (1985) obtained an empirical regression equation for the maximum wave setup as a function of wave steepness and beach slope,

$$\overline{\eta}_{\max}/H_0 = 0.45\xi_0 \tag{2}$$

in which H_0 is the wave height at deep water and $\xi_0 = \tan\beta/\sqrt{H_0/L_0}$ is the Iribarren number, where $\tan\beta$ is the local beach slope in the vicinity of the breaker line, and L_0 is the wavelength at deep water. Hanslow and Nielsen (1993) have shown that the slope of wave setup surface increases within the swash zone on several natural beaches in Australia. They have confirmed that Eq. (2) is appropriate for steep beaches, and the discrepancy between the prediction from the equation and the result of field measurements increases on dissipative beaches in gentle slopes. They also discussed the effects of tides on the maximum setup at the shoreline. On the other hand, based on the concept of radiation stress McDougal and Hudspeth (1983) derived analytical solutions for the wave setup and setdown induced by normal incident waves on three different types of beach profile. An analytical model was developed by Ruggiero and McDougal (2001) to predict the time- and depth-averaged cross-shore and longshore sediment transport on the planar beach backed by a seawall. The incident and reflected waves and the total water depth including wave setup are able to be determined in the model.

Using continuity and momentum equations, numerical models have been developed to compute the wave-induced setup and setdown, as well as the variation in mean water level and nearshore currents (e.g. Kawahara and Kashiyama, 1984; Nishimura et al., 1985; Horikawa, 1988; Péchon et al., 1997; DHI, 1998; Hsu et al., 2000). However, the task to calculate nearshore current distribution is rather complicated, because the characteristics of the wave field (including the variations in mean sea level) have to be solved first. Comparing with the procedure used in many numerical models, theoretical analysis applies the concept of radiation stresses in the calculation of wave setup and setdown, thus making the task relatively straightforward. Numerical model also applied the radiation stress concept to calculate wave setup, setdown and current field. Due to some assumptions made in the theoretical formulation, analytical solutions are generally suitable for simple situations.

Despite the advance described above, theories and data are not available for the analyses of wave setup and setdown induced by oblique breaking waves. Moreover, although the analytical theory of Longuet-Higgins and Stewart (1963, 1964) on wave setup and setdown has been well received and later verified in several laboratory and field studies, some limitations still remain due to assumptions in the original formulations. For example, firstly, the shortcoming may arise from the use of linear wave theory (LWT) to estimate local wave height $H=\kappa(h+\overline{\eta})$ within the surf zone and corresponding radiation stresses in the nearshore, where H is the local wave height, κ an empirical coefficient, h the mean local depth, and $\overline{\eta}$ the setup. Secondly, the existing theory is based on two-dimensional wave transformation nearshore and across the surf zone with normal wave incidence. On natural beaches, however, incident waves break obliquely and produce an onshore radiation stress components S_{xx} and S_{yx} associated with the wave setup and setdown, where the x-axis is the direction perpendicular to a straight shoreline with positive offshore, and the y-axis is parallel to the straight shoreline.

Because wave conditions in a laboratory flume are different from that on a natural beach, where they are mostly with oblique incidence and wave spreading is as important as wave mean direction, it would be worthwhile to investigate the effect of wave oblique incidence on wave setup and setdown. In general, theoretical formulation provides a better alternative for understanding the wave setup and setdown in the surf zone than any complex numerical method. The quantity of wave setup and setdown can be computed by explicit formulae upon the input of known incident wave conditions and other physical parameters. Results obtained from theoretical work can also be used for the comparison and validation with numerical models.

The main purpose of this paper is to establish a theory for the calculation of wave setup and setdown induced by an obliquely incident wave. A large-scale experiment was also conducted in a wave basin to compare with theoretical results and particularly to examine the validity of the spilling breaker assumption in which the envelope of the total breaker amplitude is linearly proportional to the local water depth. Comparison is also included with experimental data provided by Hamliton and Ebersole (2001) and Svendsen et al. (2003) to investigate the assumptions used in the case of obliquely incident waves. Mathematical equations for setup and setdown across the surf zone are derived on the basis of the water depth at the breaking point and parameters related to wave breaker angle and beach profile. Theoretical results calculated from the present theory are compared favorably with laboratory data and field measurements available from which the suitability of the theory is established.

2. Theoretical derivations

Radiation stresses induced by shoaling waves reach their maxima close to the breaking point on a beach. Consequently, they cause the deviation in water level, either lower or higher, relative to the SWL. Now let's consider the case of a shoaling wave approaching obliquely toward a straight shoreline with parallel offshore bathymetry, as seen in Fig. 1, where the wave ray is angled θ to the direction normal to the shoreline. The Cartesian coordinate system is used and the alongshore variations of all variables in the y direction are vanished, i.e. $\partial\overline{\eta}/\partial y=0$, $\partial S_{xy}/\partial y=0$, where $\overline{\eta}$ is the fluctuation of the mean sea level and S_{xy} is the onshore radiation stress of the longshore component. The initial water depth $h=h(x)$ anywhere within the domain of interest (shown in Fig. 1) is related to the SWL.

Now consider the simplified momentum equation for the wave setup and setdown in the x-direction,

$$-\rho g(h+\overline{\eta})\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}x}-\frac{\mathrm{d}S_{xx}}{\mathrm{d}x}-\overline{\tau_x^s}+\overline{\tau_x^b}=0 \tag{3}$$

in which $\overline{\tau_x^s}$ and $\overline{\tau_x^b}$ are the frictional stresses at the free surface and the bottom, respectively. In the case of oblique wave incidence, the radiation stress term S_{xx} in the x-direction may be given by

$$\begin{aligned}S_{xx}&=E\left[\left(2n-\frac{1}{2}\right)\cos^2\theta+\left(n-\frac{1}{2}\right)\sin^2\theta\right]\\&=\frac{1}{2}\rho ga^2\left[\left(2n-\frac{1}{2}\right)\cos^2\theta+\left(n-\frac{1}{2}\right)\sin^2\theta\right]\end{aligned} \tag{4}$$

In Eq. (4), parameter $n=C_g/C=(1/2+kh/\sinh 2kh)$ is the ratio of group velocity to wave celerity of the shoaling wave; k is the wavenumber; E is the wave energy flux; and a is the wave amplitude. The value of $n=1$ may be taken in shallow water nearshore where wave setup and setdown occur.

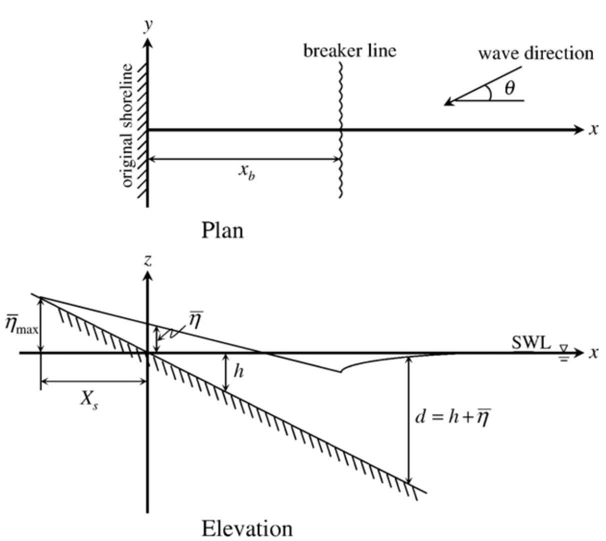

Fig. 1. Schematic diagram showing the Cartesian coordinates system used in the present study of wave setup and setdown induced by an obliquely incident wave.

Surface roller effects were ignored in the present approach. Basically, inside the surf zone the effect of a roller is added to the short wave averaged parameters. According to Svendsen et al. (2003), the inclusion of these terms can lead to an improvement of the theoretical results. To eliminate the uncertainty, we therefore omit the influence of roller for simplicity in the calculation of the radiation stresses that the waves are in shallow water.

For spilling and plunging breakers across the surf zone in which the amplitude of the broken wave may assume a linear relationship with the total water depth, such as

$$a=\frac{\kappa}{2}d=\frac{\kappa}{2}(h+\overline{\eta}) \tag{5}$$

Using the criteria that the particle velocity at the wave crest equals the phase velocity, the theoretical derived critical value of κ is given in the range of 0.73–1.03, in which the 0.78 value determined by McCowan (1894) is most commonly cited. However, laboratory measured values of the critical κ for both solitary and periodic waves as reviewed by Kishi and Saeki (1966) and Weggel (1972) have shown that it depends on both the beach slope and the wave steepness. For a given wave steepness, the greater the beach slope the higher the value of κ at breaking point. Kamisky and Kraus (1993) derived the empirical formula in a review of 17 data sets obtained by various investigators in laboratory experiments.

$$\kappa=1.20\xi_0^{0.27} \tag{6}$$

For the case of obliquely incident waves it is plausible to assume that the value of κ is approximated by

$$\kappa=a_1\xi_\theta^{a_2} \tag{7}$$

where a_1 and a_2 are empirical coefficients to be determined by experimental data, ξ_θ is the modified Iribarren number defined as

$$\xi_\theta=\xi_0\cos\theta_0=\frac{\tan\beta}{\sqrt{H_0/L_0}}\cos\theta_0 \tag{8}$$

here θ_0 is the incident wave angle in deep water. The number ξ_θ has been used by Losado et al. (1986) to predict longshore current velocity at the breaker line on a beach with oblique incidence. A theoretical analysis by Hsu (1998) also shows that the geometric characteristic of a storm–beach profile is governed by the modified Iribarren number. In this paper, the experimental study will be conducted to confirm the applicability of Eqs. (5) and (8).

Substituting Eqs. (4) and (5) into Eq. (3), and upon omitting the frictional terms at the free surface and the bottom, respectively, renders

$$\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}x}=-\frac{\kappa^2}{4}\left(\frac{1}{2}+\cos^2\theta\right)\frac{\mathrm{d}(h+\overline{\eta})}{\mathrm{d}x}+\frac{\kappa^2(h+\overline{\eta})}{8}\frac{\mathrm{d}\sin^2\theta}{\mathrm{d}x} \tag{9}$$

As the variation in wave angle due to refraction prior to and after breaking is small, it may be reasonable, at least for practical application, to further assume $\cos\theta\approx\cos\theta_b$ (Longuet-

Higgins, 1970a,b). On a straight and parallel beach, Snell's law gives

$$\frac{\sin\theta}{\sin\theta_b} = \frac{C}{C_b} \approx \frac{\sqrt{gh}}{\sqrt{gh_b}} = \sqrt{\frac{h}{h_b}} \tag{10}$$

in which the assumption of $\overline{\eta} \ll h$ is used in the surf zone. Substituting Eq. (10) into Eq. (9) yields

$$\frac{d\overline{\eta}}{dx} = -\frac{\kappa^2}{4}\left(\frac{3}{2} - \sin^2\theta_b\right)\frac{d(h+\overline{\eta})}{dx} + \frac{\kappa^2 \sin^2\theta_b}{8h_b} h \frac{dh}{dx} \tag{11}$$

Integrating Eq. (11) with respect to x and considering the range from 0 to x_b gives an expression for the fluctuation in mean water level within the surf zone, that is

$$\overline{\eta} = -\frac{3\kappa^2 - 2\kappa^2\sin^2\theta_b}{8 + 3\kappa^2 - 2\kappa^2\sin^2\theta_b} h + \frac{\kappa^2\sin^2\theta_b}{2h_b(8 + 3\kappa^2 - 2\kappa^2\sin^2\theta_b)} h^2 + C_1 \;, x \le x_b \tag{12}$$

In Eq. (12), C_1 is a constant of integration, which can be obtained from satisfying the continuity condition of wave setup and setdown at the point of wave breaking. For the case of normal wave incidence, i.e. $\theta_b = \theta_0 = 0°$, Eq. (12) reduces to

$$\overline{\eta} = -\frac{3\kappa^2}{8 + 3\kappa^2} h + C_1 \tag{13}$$

This equation is identical to one of the relation given by McDougal and Hudspeth (1983).

Now, Bernoulli equation is applied to the free surface to calculate wave setdown beyond the breaker line, as is done by Longuet-Higgins and Stewart (1964) or McDougal and Hudspeth (1983), such that

$$-\frac{1}{g}\frac{\partial\phi}{\partial t} + \frac{1}{2g}(u^2 + w^2) + \frac{p}{\rho g} + \eta = C_b \;,\; z = 0 \tag{14}$$

in which ϕ is the velocity potential of the waves, g is the acceleration due to gravity, p is the wave pressure at the free surface, C_b is a Bernoulli constant, and u and w are the water-particle velocities in the x- and z-directions, respectively. In the linear wave theory (LWT), the velocity potential of an oblique propagating wave may be taken as,

$$\phi = \frac{ag}{\omega}\frac{\cosh k(h+z)}{\cosh kh}\sin(kx\cos\theta + ky\sin\theta - \omega t) \tag{15}$$

where ω is the angular frequency of the waves.

In order to solve Eq. (14), we first expand $\phi(\eta)$ in Eq. (15) in a Taylor series with respect to the SWL, and combine the result with the linear kinematic free surface boundary condition (KFSBC), i.e. $\partial\phi/\partial z = \partial\eta/\partial t$ at $z = 0$. Again in LWT, because the variation in the water surface elevation $\eta = a\cos(kx\cos\theta + ky\sin\theta - \omega t)$ is a simply periodic function in time with angular frequency ω, so that the mean rate of η over one wave period and the rate of the velocity potential can be linked in the form

$$\overline{\frac{\partial\phi(\eta)}{\partial t}} \approx \overline{\frac{\partial\phi(0)}{\partial t}} + \overline{\eta\frac{\partial^2\eta(0)}{\partial t^2}} = \overline{\frac{\partial\phi(0)}{\partial t}} - \overline{\omega^2\eta^2} = -\frac{\omega^2 a^2}{2} \tag{16}$$

in which $\phi(\eta)$ is the velocity potential expanded to a second-order on the SWL. Since the velocity potential on the SWL is periodic in time, its average is zero in Eq. (16). Again using LWT and Eq. (15), the time-average over one wave period for the squared terms of horizontal and vertical velocity components u and w on the SWL in Eq. (14) can be expressed as

$$\overline{\left[\frac{\partial\phi(0)}{\partial x}\right]^2} = \overline{u^2(0)} = \frac{1}{2}\left[a\omega\frac{\cosh kh}{\sinh kh}\right]^2 \cos^2\theta \tag{17}$$

$$\overline{\left[\frac{\partial\phi(0)}{\partial z}\right]^2} = \overline{w^2(0)} = \frac{1}{2}a^2\omega^2 \tag{18}$$

By taking the time-average over one wave period for Eq. (14), together with $p = 0$ at the free surface, and substituting Eqs. (16), (17) and (18) into Eq. (14), yield

$$\frac{1}{2g}\left[\frac{a^2\omega^2}{2}(\coth^2 kh\cos^2\theta - 1)\right] + \overline{\eta} = C_b \tag{19}$$

Upon satisfying the far field boundary condition at deep water as $kh \to \infty$, where wave setdown does not exist, i.e. $\overline{\eta} = 0$, Eq. (19) produces an expression for the Bernoulli constant C_b as

$$C_b = \frac{a^2\omega^2}{4g}(\cos^2\theta - 1) \tag{20}$$

Substituting Eq. (20) into Eq. (19) and using the dispersion relation of LWT, wave setdown beyond the breaker line becomes

$$\overline{\eta} = -\frac{a^2 k}{2\sinh 2kh}\cos^2\theta \;;\; x > x_b \tag{21}$$

In Eq. (21), wave refraction is taken into account so that the setdown $\overline{\eta}$ depends on the wave approaching wave angle θ. Using the Green formula of shallow water approximation, i.e. $ah^{1/4} = a_b h_b^{1/4} = \kappa h_b^{5/4}/2$, and the approximation of $\sinh 2kh = 2kh$ for depth beyond the breaker line for $x > x_b$, the variation in mean water level is thus related to water depth, such that

$$\overline{\eta} = -\frac{\kappa^2 h_b^{5/2}}{16h^{3/2}}\cos^2\theta \;;\; x > x_b \tag{22}$$

The present theory assumes shallow water, small angle of wave propagation in the surf zone for spilling and plunging breaker. From equating the mean water level variation within the surf zone, Eq. (12), and that beyond the breaker line, Eq. (22), at the breaker line, $x = x_b$ and $h = h_b$, a general expression for the wave setup induced by an obliquely incident wave within the surf zone is

$$\overline{\eta} = \frac{\kappa^2 \sin^2\theta_b}{2h_b(8+3\kappa^2-2\kappa^2\sin^2\theta_b)}(h^2-h_b^2) - \frac{3\kappa^2-2\kappa^2\sin^2\theta_b}{8+3\kappa^2-2\kappa^2\sin^2\theta_b}(h-h_b) - \frac{\kappa^2(1-\sin^2\theta_b)}{16}h_b\ ;\ x\leq x_b \tag{23}$$

Combining Eqs. (22) with (23) and normalizing the resultant form using the water depth at the breaker gives the wave setup and setdown within and beyond the surf zone, respectively, i.e.

$$\frac{\overline{\eta}}{h_b} = \begin{cases} \dfrac{\kappa^2\sin^2\theta_b}{2(8+3\kappa^2-2\kappa^2\sin^2\theta_b)}\left[\left(\dfrac{h}{h_b}\right)^2-1\right] - \dfrac{3\kappa^2-2\kappa^2\sin^2\theta_b}{8+3\kappa^2-2\kappa^2\sin^2\theta_b}\left(\dfrac{h}{h_b}-1\right) \\ -\dfrac{\kappa^2}{16}(1-\sin^2\theta_b) & ;x\leq x_b \\ -\dfrac{\kappa^2}{16}\left(\dfrac{h_b}{h}\right)^{3/2}\left(1-\dfrac{h}{h_b}\sin^2\theta_b\right) & ;x>x_b \end{cases} \tag{24}$$

For the case of normal wave incidence, $\theta_b=\theta_0=0°$, Eq. (24) leads to the same expression given by McDougal and Hudspeth (1983). We notice that Eq. (24) reflects properly the influence of the incident wave angle on wave setup and setdown for an impermeable beach with parallel straight bottom.

3. Wave setup and setdown on a beach face

3.1. Wave setup and setdown for generic beach profiles

Following the approach outlined in McDougal and Hudspeth (1983), the new general expressions for wave setdown and setup derived in this paper are applied to three different types of beach profile, e.g., concave-down, planar and concave-up in the form of $h=A\,x^{3/2}$, $h=Ax$ and $h=A\,x^{2/3}$, respectively. Substitution of these profile expressions into Eq. (24) results in dimensionless wave setup and setdown distribution as a function of offshore distance, that is

3.1.1. Concave-down beach profile

$$\frac{\overline{\eta}}{h_b} = \begin{cases} \dfrac{\kappa^2\sin^2\theta_b}{2(8+3\kappa^2-2\kappa^2\sin^2\theta_b)}(X^3-1) - \dfrac{3\kappa^2-2\kappa^2\sin^2\theta_b}{8+3\kappa^2-2\kappa^2\sin^2\theta_b}\left(X^{3/2}-1\right) \\ -\dfrac{\kappa^2}{16}(1-\sin^2\theta_b) & ;X\leq 1 \\ -\dfrac{\kappa^2}{16}X^{9/4}\left(1-X^{3/2}\sin^2\theta_b\right) & ;X>1 \end{cases} \tag{25a}$$

3.1.2. Planar beach profile

$$\frac{\overline{\eta}}{h_b} = \begin{cases} \dfrac{\kappa^2\sin^2\theta_b}{2(8+3\kappa^2-2\kappa^2\sin^2\theta_b)}(X^2-1) - \dfrac{3\kappa^2-2\kappa^2\sin^2\theta_b}{8+3\kappa^2-2\kappa^2\sin^2\theta_b}(X-1) \\ -\dfrac{\kappa^2}{16}(1-\sin^2\theta_b) & ;X\leq 1 \\ -\dfrac{\kappa^2}{16}X^{3/2}(1-X\sin^2\theta_b) & ;X>1 \end{cases} \tag{25b}$$

3.1.3. Concave-up beach profile

$$\frac{\overline{\eta}}{h_b} = \begin{cases} \dfrac{\kappa^2\sin^2\theta_b}{2(8+3\kappa^2-2\kappa^2\sin^2\theta_b)}(X^{4/3}-1) - \dfrac{3\kappa^2-2\kappa^2\sin^2\theta_b}{8+3\kappa^2-2\kappa^2\sin^2\theta_b}\left(X^{2/3}-1\right) \\ -\dfrac{\kappa^2}{16}(1-\sin^2\theta_b) & ;X\leq 1 \\ -\dfrac{\kappa^2}{16}X\left(1-X^{2/3}\sin^2\theta_b\right) & ;X>1 \end{cases} \tag{25c}$$

in which $X=x/x_b$ is a non-dimensional offshore distance normalized against the width of the surf zone. Therefore, Eqs. (25a)–(25c) are applicable to monochromatic waves approaching obliquely to an impermeable beach in concave-down, planar and concave-up beach profile, respectively.

3.2. Shoreline advancement landward

Once within the surf zone ($X\leq 1$), wave setup commences and its magnitude increases with the distance toward the original shoreline. The total water depth anywhere within the surf zone is the sum of the SWL and the wave setup, i.e. $d=h+\overline{\eta}$. For a beach profile in generic form, $h=Ax^m$, Eqs. (25a)–(25c) gives the total water depth, that is

$$\frac{d}{h_b} = X^m + \frac{\kappa^2\sin^2\theta_b}{2(8+3\kappa^2-2\kappa^2\sin^2\theta_b)}(X^{2m}-1) - \frac{3\kappa^2-2\kappa^2\sin^2\theta_b}{8+3\kappa^2-2\kappa^2\sin^2\theta_b}(X^m-1) - \frac{\kappa^2}{16}(1-\sin^2\theta_b) \tag{26}$$

Consequently, the actual dimensionless mean shoreline position is located at the point where the total depth d is zero, and this new distance X_s can be obtained from Eq. (26), such that

$$X_s = \begin{cases} -\left[\dfrac{\kappa^2(40-3\kappa^2)}{128}\right]^{1/m} & ;\theta_b=0 \\ -\{[-32+(1024-80\kappa^4\sin^2\theta_b+6\kappa^6\sin^2\theta_b+64\kappa^4\sin^4\theta_b \\ \quad -10\kappa^6\sin^4\theta_b+4\kappa^6\sin^6\theta_b)^{1/2}]/(4\kappa^2\sin^2\theta_b)\}^{1/m} & ;\text{otherwise} \end{cases} \tag{27}$$

For the limiting case of normal wave incidence on a plane beach, i.e. $\theta_b=0°$ and $m=1$, Eq. (27) renders the dimensionless shoreline distance,

$$X_s = -\frac{\kappa^2(40-3\kappa^2)}{128} \tag{28}$$

Again, Eq. (28) is consistent with the expression given by McDougal (1993).

3.3. Wave setup at original shoreline position

The total wave setup at the intersection of the SWL and the beach profile (i.e. the original shoreline) may be estimated from Eqs. (25a)–(25c) upon setting $X=0$, i.e.

$$\frac{[\overline{\eta}]_{X=0}}{h_b} = \frac{\kappa^2(40-3\kappa^2-32\sin^2\theta_b+5\kappa^2\sin^2\theta_b-2\kappa^2\sin^4\theta_b)}{16(8+3\kappa^2\sin^2\theta_b)} \tag{29}$$

Eq. (29) may be used to calculate the dimensionless increase of wave setup at the original shoreline for various wave obliquities, including the case for $\theta_b=0°$ in McDougal (1993).

Eq. (27) may be used to estimate the maximum wave setup, $\overline{\eta}_{max}$, from Eq. (23), occurring at $X=X_s$, such that

$$\frac{\overline{\eta}_{max}}{h_b}=\begin{cases}\dfrac{320\kappa^2+96\kappa^4-9\kappa^6}{128(8+3\kappa^2)} & ;\ \theta_b=0\\ [32-(1024-80\kappa^4\sin^2\theta_b+6\kappa^6\sin^2\theta_b+64\kappa^4\sin^4\theta_b & \\ -10\kappa^6\sin^4\theta_b+4\kappa^6\sin^6\theta_b)^{1/2}]/(4\kappa^2\sin^2\theta_b) & ;\ \text{otherwise}\end{cases} \tag{30}$$

In order to improve the applicability of wave setup estimation, Rattanapitikon and Shibayama (2000) have suggested an alternative form to the wave breaker height originally given by Komar and Gaughan (1972). By taking bottom slope into account and with 574 cases of experimental data, they derive a new expression for breaking wave height H_b as a function of beach face slope and wave steepness.

For inclinedly incident waves, it is noted that the effect of the wave angle must be incorporated into the breaking criterion. In some applications it is desirable to calculate wave breaker heights from their deep water parameters going through the entire shoaling transformations. Such a formulation was derived by LéMéhauté and Wang (1980) and their formula is given by

$$\frac{H_b}{H_0}=0.76(\cos\theta_b)^{\frac{1}{7}}(\tan\beta)^{\frac{1}{7}}K_R^{0.75}\left(\frac{H_0}{L_0}\right)^{-1/4} \tag{31}$$

in which K_R is the refraction coefficient. For the simple case of a straight shoreline with parallel contours, Snell's law is utilized and K_R is computed by

$$K_R=\left(\frac{1-\sin^2\theta_0}{1-\tanh^2 kh_b\sin^2\theta_0}\right)^{1/4} \tag{32}$$

Again, using the linear relationship of wave height and water depth at breaking point, $H_b=\kappa h_b$, and the dispersion relation, we can derive the maximum wave setup from Eq. (30) in which the deep water wave steepness H_0/L_0, the local beach slope tan β and the breaking wave angle θ_b are incorporated in a general form,

$$\frac{\overline{\eta}_{max}}{H_0}=\begin{cases}0.76(\tan\beta)^{\frac{1}{7}}\left(\dfrac{H_0}{L_0}\right)^{-1/4}\times\dfrac{320\kappa+96\kappa^3-9\kappa^5}{128(8+3\kappa^2)} & ;\ \theta_b=0\\ 0.76(\cos\theta_b)^{\frac{1}{7}}(\tan\beta)^{\frac{1}{7}}K_R^{0.75}\left(\dfrac{H_0}{L_0}\right)^{-1/4} & \\ \times[32-(1024-80\kappa^4\sin^2\theta_b+6\kappa^6\sin^2\theta_b+64\kappa^4\sin^4\theta_b & \\ \quad -10\kappa^6\sin^4\theta_b+4\kappa^6\sin^6\theta_b)^{1/2}]/(4\kappa^3\sin^2\theta_b) & ;\ \text{otherwise}\end{cases} \tag{33}$$

In such a way, Eq. (33) gives the maximum wave setup at the original shoreline as a function of deep water wave steepness, beach face slope, wave obliquity and value κ is a function of the modified Iribarren number governing the wave amplitude and the local depth within the surf zone.

4. Experiments and verifications

A large-scale experiment was conducted in a wave basin to verify the crucial assumption of Eq. (5) in which the wave height is assumed to be proportional to the total water depth inside the surf zone for obliquely incident waves. On the other hand, the wave breaking criterion used in normally incident waves may not be applicable to the case of various obliquities. The main objective of the laboratory experiment is therefore to use the measured data and all published data to give a comprehensive review of the simple approach inside the surf zone.

The layout of the facility in the National Taiwan Ocean University (NTOU), Taiwan, is shown in Fig. 2. The experiments were conducted in a three-dimensional wave basin of 50 m cross-shore, 50 m alongshore and 1 m deep. The test arrangement for both normally and inclinedly incident waves are varied by tuning the connected wave boards. The wave profiles, setup and setdown were driven by long crested waves, which are generated by a snake-type wavemaker. The beach was carefully constructed by concrete with straight and parallel contours having a beach slope of 1:10. There are totally 21 capacitance-type wave gauges were placed in a cross-shore array to measure the wave height as well as water surface elevation. All the experimental data were measured at the center of the beach. The hydrodynamic experiments were first conducted to verify the facility's capability to generate the desired wave conditions. Three comprehensive test series were performed, i.e. the incident wave angles $\theta_0=0°$, 15° and 30°; the incident wave heights $H_0=4.5$ cm, 6.5 cm and 8.5 cm; the wave periods $T=1.0$ s, 1.25 s and 1.5 s; and the mean water depth $h=50$ cm in deep water. In the test, the incident wave direction is fixed and three relative wave conditions ($H_0=4.5$ cm, $T=1.0$ s; $H_0=6.5$ cm, $T=1.25$ s; $H_0=8.5$ cm, $T=1.5$ s) were produced by the snake-type wave generator. According to Battjes' (1974) classification of the breaker-type, the Iribarren number ξ_0 falls in the range of $0.58<\xi_0<0.64$, and that corresponds to the plunging breaker.

The coordinate system in the wave basin is the same with the definition shown in Fig. 1. The position axis is directed offshore and measured relative to the upper edge of the beach slope. Wave guides were used in both lateral boundaries to guide waves propagating from deep water to shallow water smoothly in incident obliquities. Wave setup and setdown were obtained by using FFT (Fast Fourier Transform) from the measured water surface elevation in which the SWL has been subtracted in the analysis.

To examine the validity of the present theory, comparisons are necessary between the results of theoretical calculation and the data from laboratory experiments and field observations that have been published by various researchers. It is perceived that uncertainties may result due to the linear assumptions in deriving Eqs. (25a)–(25c) for the wave setup and setdown. For example, LWT is used to estimate the radiation stress, even though applications in most cases are for moderate to large waves in shallow water where LWT may not be fully adequate. Another likely limitation is due to the assumption of a linear relation between the wave height and the local water depth inside the surf zone, Eq. (5), which may only be applicable to spilling and plunging breaker for normally incident waves.

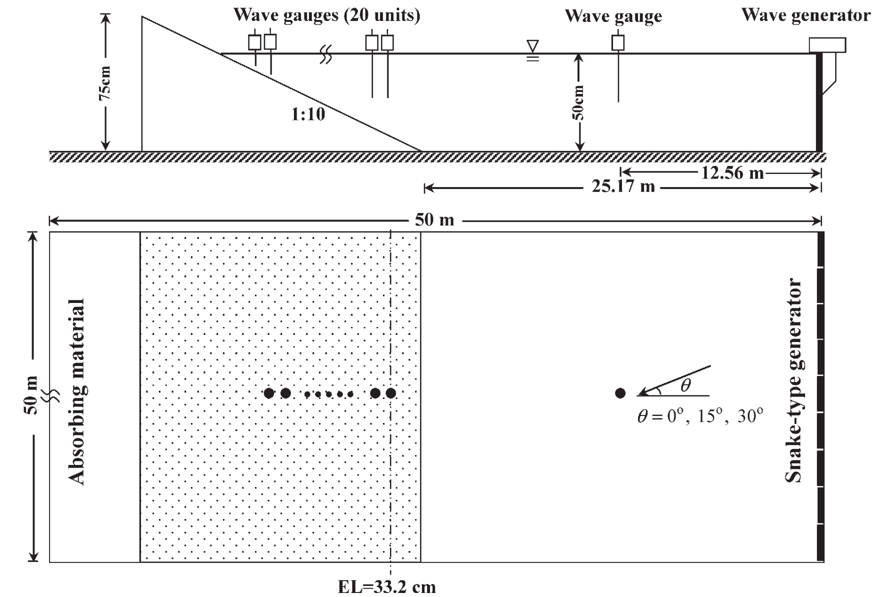

Fig. 2. Plan view of the large-scale facility in a three-dimensional wave basin.

On dissipative beaches of low slope where a continuous surf zone of breaking waves and bores always exists, it is commonly assumed that there is a constant ratio of $\kappa = H/h$ across the surf zone, having values in the range of 0.78–1.3, as established by the initial wave breaking. However, field measurements of Thornton and Guza (1982, 1983) at Torrey Pines Beach, U.S.A., has found that the root-mean-square wave heights H_{rms} within the inner surf zone are essentially dependent on the local water depths and the ratio $\kappa = H_{rms}/h$ approaches the saturation value of $\kappa = 0.84$. From the laboratory experiments of Horikawa and Kuo (1966) indicates that the value of κ approximately approaches on the order of 0.78 which has been determined by McCowan (1894).

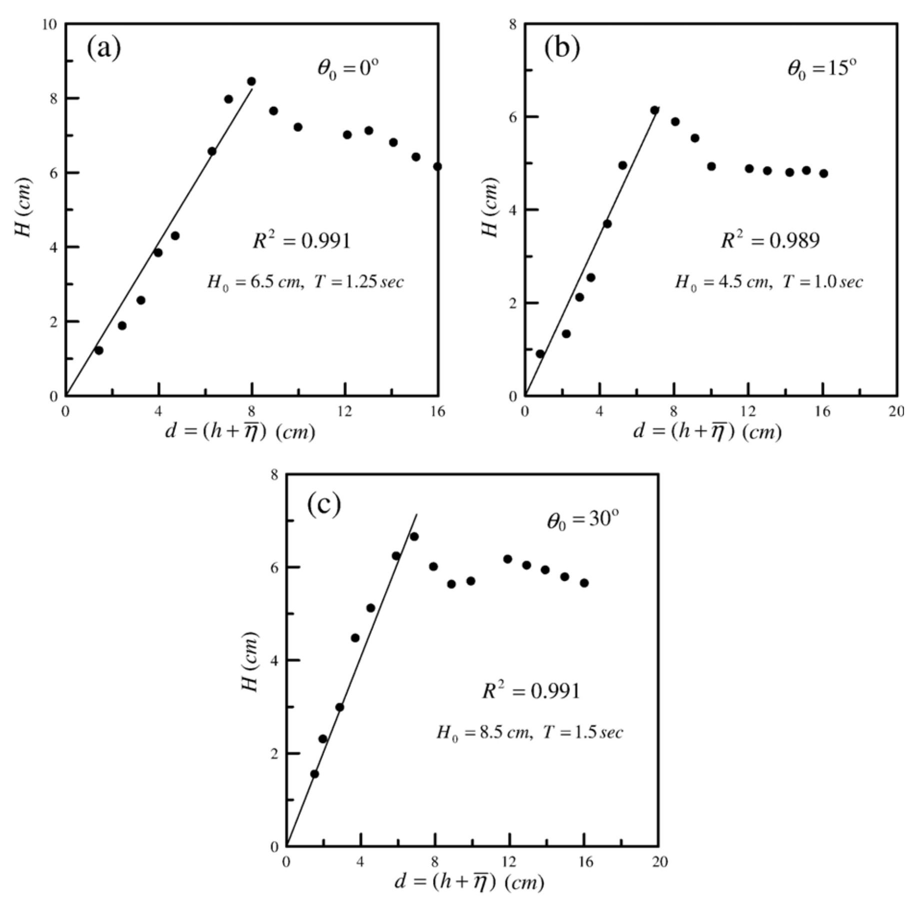

Fig. 3. Wave height distribution inside and beyond the surf zone for different incident wave angles.

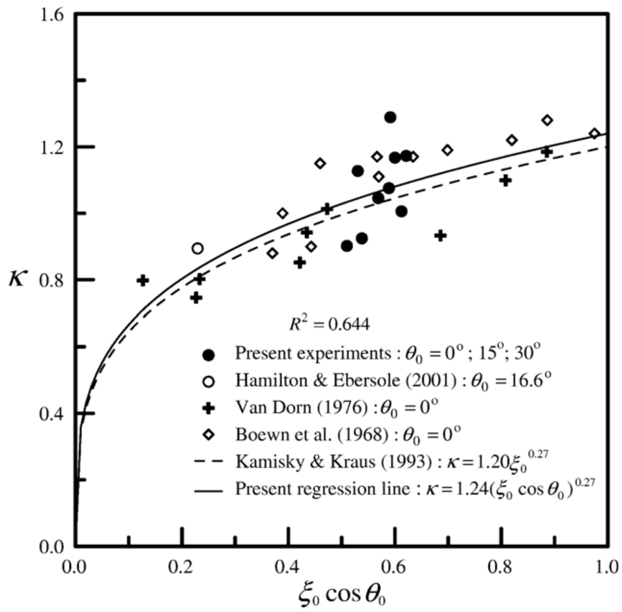

Fig. 4. The κ value related to the modified Iribarren number $\xi_0 \cos\theta_0$.

Some typical wave height variation of the present experimental results were plotted in Fig. 3. The regression line was obtained based on measured data. It is interesting to note that the linear relationship between wave height and local water depth still holds for obliquely incident waves, while the value of κ is varied with different wave conditions.

Referring Eq. (5), it has been found by Weggel (1972) that the critical ratio for wave breaking varies considerably in both laboratory and field observations. As aforementioned in Section 2, the greater the beach slope the higher value of κ at breaking point for a given wave steepness. Kamisky and Kraus (1993) proposed an empirical formula of Eq. (6) in which the κ value depends on the deep water form of the Iribarren number. In the present study, we extended Eq. (6) to the case of wave breaking and dissipation for obliquely incident waves. The ratio κ is assumed to be related to the modified Iribarren number as given in Eq. (7) which includes the influence of beach slope, wave steepness and incident wave angle. Eq. (7) is plotted in Fig. 4 along with the field and laboratory wave breaking data. It can be seen that the relationship fairly agrees with the measurements. This leads to a regression line

$$\kappa = 1.24(\xi_0 \cos\theta_0)^{0.27} \tag{34}$$

This equation provides the best agreement of the κ value with existing laboratory data. From Fig. 4, it is also interesting to note that a larger incident wave angle would produce a higher values of κ for a given wave condition and beach face slope. The relationship of Eq. (7) appears to be a little higher than that of Eq. (6) as the case of normal incidence, $\theta_0 = 0°$. From this analysis, it is concluded that the linear relationship of wave height and local water depth is still applicable to the case of obliquely incident waves, but the value of the proportionality κ is modified by the modified Iribaren number in practical applications.

Despite classical theory for the wave setup and setdown was proposed as early as in the 1960s, results of laboratory experiments were not generally available for obliquely incident waves. In Fig. 5, experimental results of wave setup and setdown obtained by Bowen et al. (1968), Van Dorn (1976), Hamlition and Ebersole (2001) and the present laboratory measurements for both normally and inclinedly incident waves are compared with theoretical results calculated using the present theory. Good agreement can be found with these laboratory measurements. Both the theoretical results and laboratory data show that wave setdown increase from offshore almost linearly to its maximum depression at the breaking point, then followed by a rapid rise in the mean water level of setup shoreward inside the surf zone. The pattern of the wave setup in Fig. 5 indicates the uniform gradient $(\partial\overline{\eta}/\partial x)$ is less than the beach face slope. Both experiments present similar pattern of the dimensionless wave setup and setdown for the mean water level beyond and across the surf zone.

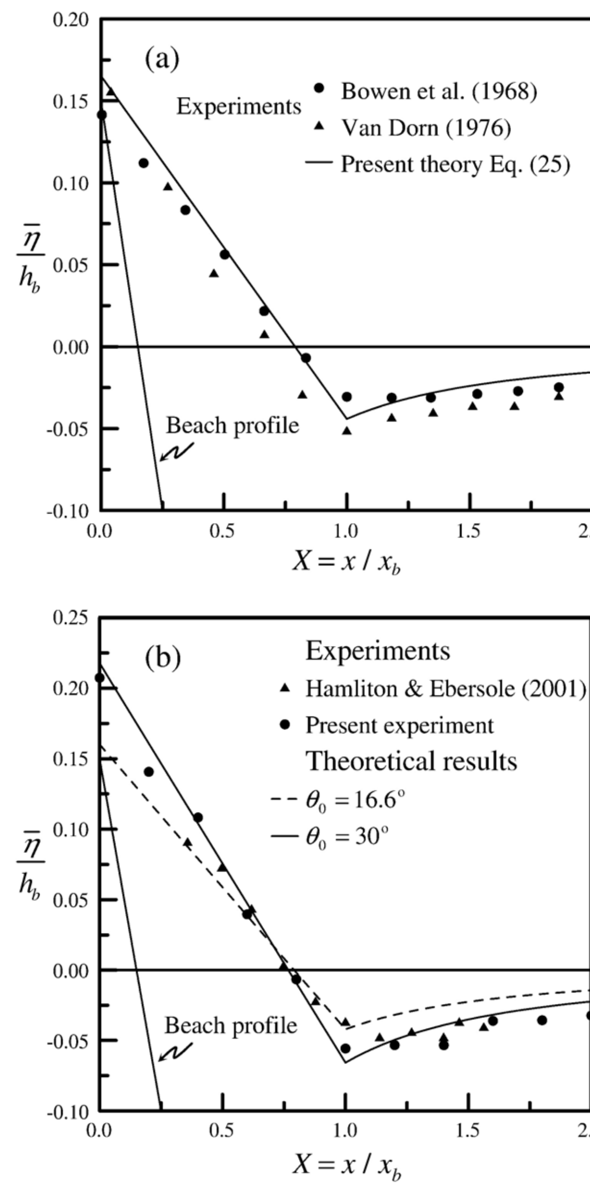

Fig. 5. Comparisons on wave setup and setdown between the present theory and laboratory data. (a) normally incident waves; (b) obliquely incident waves.

Field measurements of wave setup at high tide on Torrey Pines Beach, California, with average slope 1:50 were obtained by Guza and Thornton (1981). Holman and Sallenger (1985) also conducted a series of field observations on a steep beach of 1:10 at the FRF in Duck, North Carolina. The maximum wave setup $\overline{\eta}_{\max}$ at the new shoreline was recorded as high as 1.6 m and the variations in offshore wave heights ranging from 0.4 to 4.0 m. They found that a direct correlation between the maximum setup and the significant wave height as presented in Eq. (1) was highly scattered, but, the scatter is greatly reduced if the $\overline{\eta}_{\max}$ divided by deep water wave height H_0, is related to the deep water form of the Iribarren number of Eq. (2). The line fitted to the data at mid-tide is presented in Fig. 6(a). Notably, the statistical regression of Eq. (2) has forced the line to pass the original point due to data limitation. The angles of breaking waves were estimated by Snell's law as given by Eq. (10). The theoretical curve of the maximum setup at the position of shoreline, based on Eq. (33) for the same wave and beach data and the present laboratory measured shown in Fig. 6(a) is demonstrated in Fig. 6(b). This figure indicates that the maximum setup depends on local beach slope, wave steepness and breaking wave angle having a similar property like Eq. (2). Hence, this theoretical relationship may be extended to field applications to evaluate the maximum setup induced by obliquely incident waves on swell-built beach profiles. The data used to regression analysis of Eq. (2) were all from field measurements at mid-tide. The Iribarren number covered a range from 0.5 to 3.3. The applications of Holman and Sallenger's (1985) empirical formula should take this limitation into consideration. The present theoretical development provides a more convenient assessment of variations in wave setup across the surf zone. From Fig. 6(b), it is also noted that the present theory is applicable to the spilling and plunging breaker for $0.5<\xi_0<3.3$.

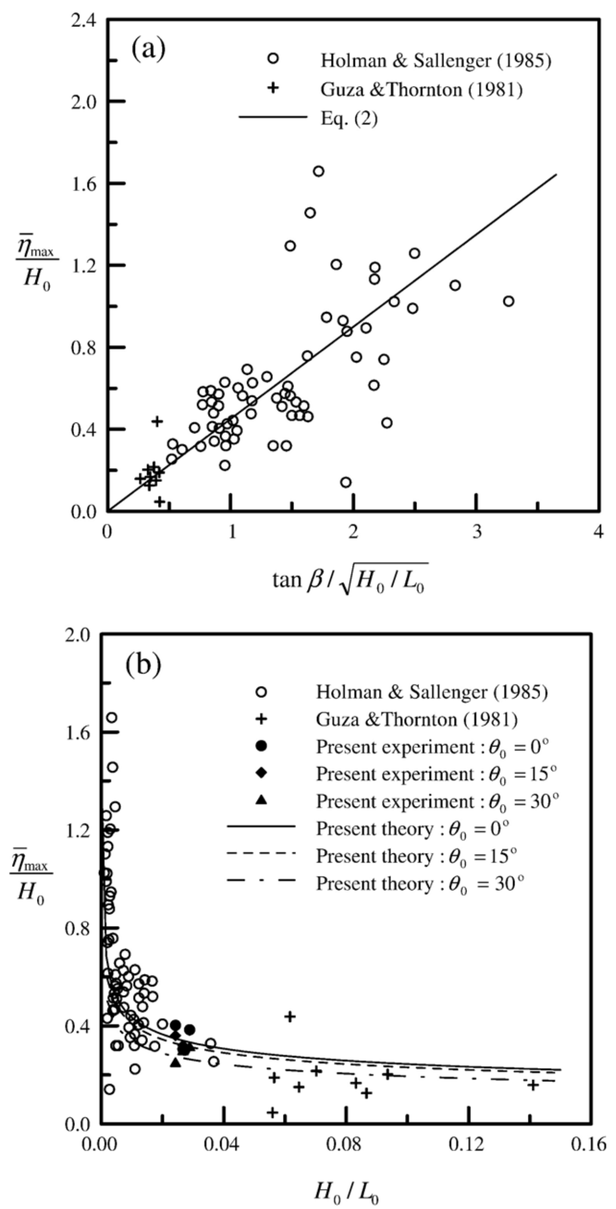

Fig. 6. Comparison on the maximum wave setup between the present theory and measured data. (a) The regression analysis of the empirical formula given by Eq. (2) obtained by Holman and Sallenger (1985); (b) the present theory of Eq. (33) for $\theta_b=0°$, 15° and 30°.

5. Results and discussions

It is necessary to examine whether Eqs. (25a)–(25c) is applicable for most wave conditions in natural environment. As pointed out earlier by Longuet-Higgins (1970a,b) that the assumption of $\cos\theta\approx\cos\theta_b$ is satisfied only if longshore current velocity is smaller than the water-particle velocity within the oblique incident waves. In this case, θ_b should be less than 24°, which is the norm in almost field conditions. Moreover, Komar (1998) also pointed out that measured longshore current velocities agree well with theoretical predictions, for θ_b up to 45°. Judging from the validation using field data given previously, therefore, it may be stated that the general form for wave setup and setdown, Eqs. (25a)–(25c), would be applicable for most wave conditions in field applications.

5.1. Effects of beach profile and wave obliquity on setup and setdown

The general expression for the wave setup and setdown of Eqs. (25a)–(25c), is based on the assumption of a linear relationship between the amplitude of water surface elevation and local water depth within the surf zone, therefore it would only be appropriate for a beach with foreshore depth increases monotonically with distance offshore rather than with a significant bar profile. In order to examine the applicability of Eqs. (25a)–(25c) for a beach profile rather than a uniform slope, the resulting wave setup and setdown for three different types of beach profile (concave-up, planar and concave-down) subjected to a number of wave obliquities ($\theta_b=0°$, 15°, 30° and 45°) are to be compared. These are shown collectively in Fig. 7. By taking $\kappa=0.78$ in Eqs. (25a)–(25c), Fig. 7 indicates that wave setdown commences offshore and reaches a maximum at breaker line ($X=1$); and the value decreases with the increase in wave obliquity θ_b at breaker line. The maximum wave setdown occurs for $\theta_b=45°$ is less than that for $\theta_b=0°$. The maximum setdown only changes the total water depth up to 4% at the breaker line in the case of normal incidence. Therefore,

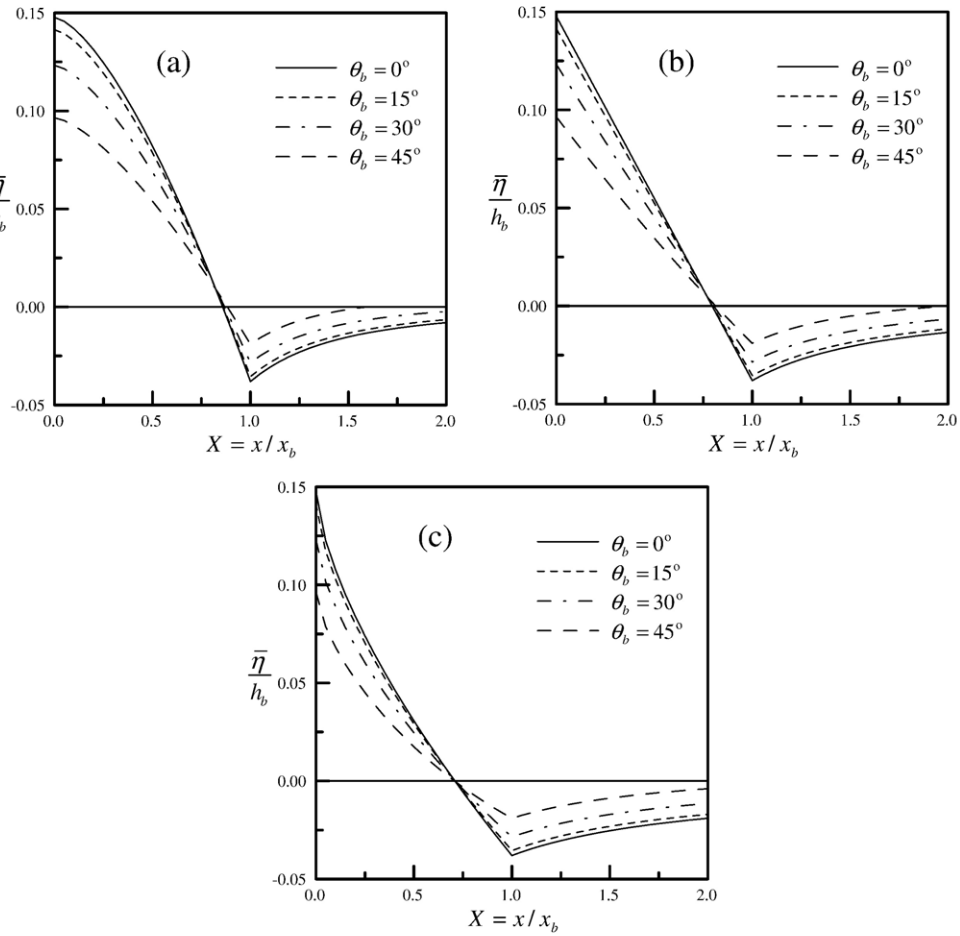

Fig. 7. Dimensionless wave setup and setdown induced by obliquely incident waves versus distance offshore, for (a) concave-down beach profile $h = Ax^{3/2}$; (b) planar beach profile $h = Ax$ and (c) concave-up beach profile $h = Ax^{2/3}$.

according to McDougal and Hudspeth (1983), it seems reasonable to omit the influence of setdown in engineering application in most field situations.

In Fig. 7, the outlines of the theoretical curves for wave setup seem to conform to the geometrical shape of its own beach profile. In the general expression for a beach profile with depth increases monotonically offshore as $h/h_b = X^m$, where m is an arbitrary constant, Eqs. (25a)–(25c) implies the contribution of the first term $(h/h_b)^2 = X^{2m}$ to the total setup is a smaller quantity than that of the second term $h/h_b = X^m$. By omitting the second-order term of $(h/h_b)^2$, the dimensionless setup in Eqs. (25a)–(25c) can be approximated by

$$\frac{\bar{\eta}}{h_b} = -\frac{3\kappa^2 - 2\kappa^2 \sin^2\theta_b}{8 + 3\kappa^2 - 2\kappa^2 \sin^2\theta_b}\left(\frac{h}{h_b} - 1\right) - \frac{\kappa^2}{16}(1 - \sin^2\theta_b);\ X \le 1 \tag{35a}$$

or

$$\frac{\bar{\eta}}{h_b} = -\frac{3\kappa^2 - 2\kappa^2 \sin^2\theta_b}{8 + 3\kappa^2 - 2\kappa^2 \sin^2\theta_b}(X^m - 1) - \frac{\kappa^2}{16}(1 - \sin^2\theta_b);\ X \le 1 \tag{35b}$$

Eqs. (35a) and (35b) envisages that the dimensionless wave setup is a linear function of the dimensionless water depth which represents the geometry of each individual beach profile for $X \le 1$, i.e. within the surf zone. Interestingly, similar reflection to the profile geometry is not found for the dimensionless wave setdown versus depth, as seen in Fig. 7.

It would be worthwhile to compare the effect of beach profile on the resulting wave setup and setdown for a particular wave obliquity θ_b measured at the breaker line. The results of such calculations using Eqs. (25a)–(25c) are illustrated in Fig. 8, for $\theta_b = 0°$, 15°, 30° and 45°, respectively. It is obvious that wave setup is an important component to the variation in the total water depth near the shoreline. Among the three beach profiles under consideration, the concave-down profile has the greatest change in mean sea level near the shoreline and the smallest setdown offshore. The extent of the setup seems to be a reflection of the geometry of the beach profile and it decreases as wave obliquity increases.

5.2. New shoreline position X_s

Again, for the typical value of $\kappa = 0.78$ and the three beach profiles examined earlier (concave-down, planar and concave-up), the new shoreline positions X_s against breaking wave obliquity θ_b are shown in Fig. 9. For these three types of beach profile, radiation stress decreases as wave obliquity increases,

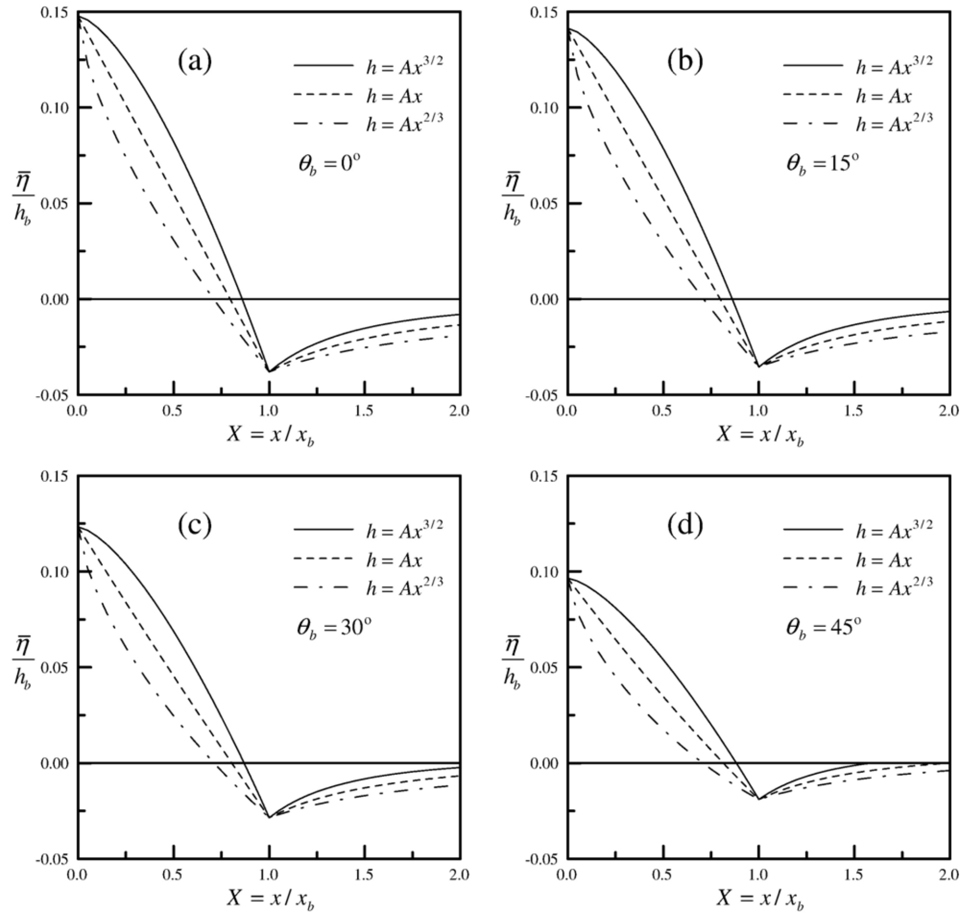

Fig. 8. Dimensionless wave setup and setdown versus distance offshore for three different types of beach profile.

resulting in the decrease in wave setup and advancement of shoreline position landwards. From Eq. (27), the maximum percentages of the expansion in surf zone width due to wave setup occur for the case of $\theta_b = 0°$ are estimated at 8%, 18% and 32% for concave-down, planar and concave-up beach profile, respectively. As also seen in Fig. 9, a concave-down beach profile produces the largest shoreline advancement than the other two, due to increase in radiation stress in the former.

The approximate new shoreline position could be obtained from Eqs. (35a) and (35b), without considering the second-order term $(h/h_b)^2$. For a beach profile in the generic form $h = Ax^m$, Eq. (35b) yields the approximate shoreline position,

$$X_s = -\left(\frac{40\kappa^2 - 3\kappa^4 - 24\kappa^2\sin^2\theta_b + 5\kappa^4\sin^2\theta_b - 2\kappa^4\sin^4\theta_b}{128}\right)^{1/m} \tag{36}$$

For a normally incident wave over a plane beach, i.e. $\theta_b = 0°$ and $m = 1$, Eq. (36) readily reduces to Eq. (28).

Therefore, the new shoreline position can be obtained from Eq. (27) or Eq. (36) for exact and approximate solution, respectively. Taking $\kappa = 0.78$ and breaking wave obliquity θ_b, the percentage increases of the surf zone width for a concave-down, planar or concave-up beach profile are tabulated in Table 1. In the three profiles investigated, both Eqs. (27) and (36), indicate surf zone width decreases as wave obliquity increases. Comparison between the results calculated from Eqs. (27) and (36) reveals that the percentage differences are all less than 2% for the conditions examined. This implies that the

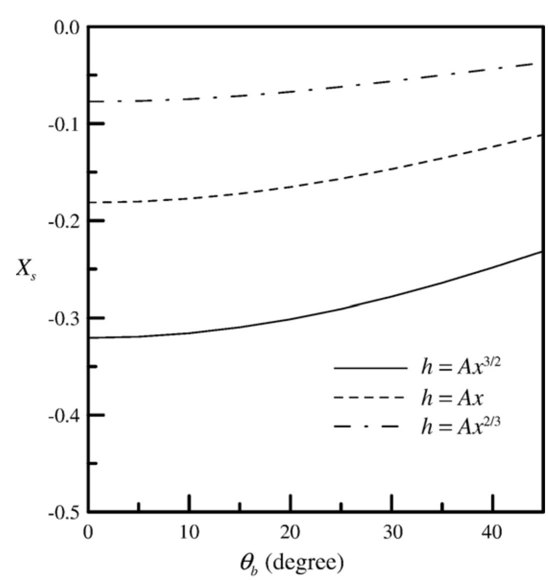

Fig. 9. New shoreline position versus breaking wave angle for three different types of beach profile.

Table 1
Percentage of the increases in the surf zone width due to wave setup

Wave obliquity θ_b	Beach profile					
	$h=Ax^{3/2}$		$h=Ax$		$h=Ax^{2/3}$	
	Eq. (27)	Eq. (36)	Eq. (27)	Eq. (36)	Eq. (27)	Eq. (36)
0°	32.05	32.05	18.14	18.14	7.73	7.73
15°	30.96	31.26	17.23	17.48	7.15	7.31
30°	27.84	29.00	14.69	15.62	5.63	6.17
45°	23.15	25.69	11.14	13.02	3.72	4.70

contribution from the second-order term of $(h/h_b)^2$ is very small and could be omitted in practical applications. However, Table 1 also indicates that discrepancy in the percentage width increases as incident wave angle increases.

5.3. Increase in total depth at original shoreline

Eq. (29) can be used to calculate the wave setup at the original shoreline. The results calculated are converted into percentage increase relative to the depth at the breaker, as shown in Table 2. Approximate values obtained from Eqs. (35a) and (35b) are presented in the parentheses in the same table. It is worth noting that the percentages in the wave setup at the original shoreline position decrease as wave obliquity increases. Eqs. (35a) and (35b) can also be used to estimate the approximate values for maximum wave setup over a wide range of wave obliquity, including the limiting case of $\theta_b=0°$. The estimated maximum wave setup at the new shoreline position for $\theta_b=0°$, 15°, 30° and 45°, are also given in Table 2. Again, the larger an incident wave angle is, the smaller percentage of the maximum setup becomes. From Table 2, the maximum wave setup is over 10% relative to the breaker depth, indicating that the maximum wave setup is a significant component to the total water depth within the proximity of the shoreline on an impermeable beach. Based on the present analysis, it may be concluded that maximum wave setup relative to breaker depth is independent of the beach profile and its value decreases as wave obliquity increases.

Finally, Eq. (33) can be used to calculate maximum wave setup at the original shoreline with respect to deep water steepness H_0/L_0. The results calculated for two typical bottom slopes, tan β=0.01 and 0.15, are illustrated in Fig. 10. The effect of beach slope and wave steepness on relative maximum wave setup is evident. This figure also reveals that the dimensionless maximum setup decreases with the increase in wave steepness and wave obliquity for a particular beach slope. Notably the relationship between $\bar{\eta}_{max}/H_0$ and H_0/L_0 is not linear.

Table 2
Percentage of the increases in total depth at shoreline positions estimated by Eq. (29)

Wave setup	Wave obliquity at breaker θ_b			
	0°	15°	30°	45°
$[\bar{\eta}]_{X=0}/h_b$ (original shoreline)	14.77	14.14	12.32	9.65
	(14.77)	(14.35)	(13.12)	(11.30)
$\bar{\eta}_{max}/h_b$ (new shoreline)	18.14	17.23	14.69	11.14
	(18.14)	(17.48)	(15.62)	(13.02)

Note: the values in the parentheses are calculated using an approximate formula, Eqs. (35a) and (35b).

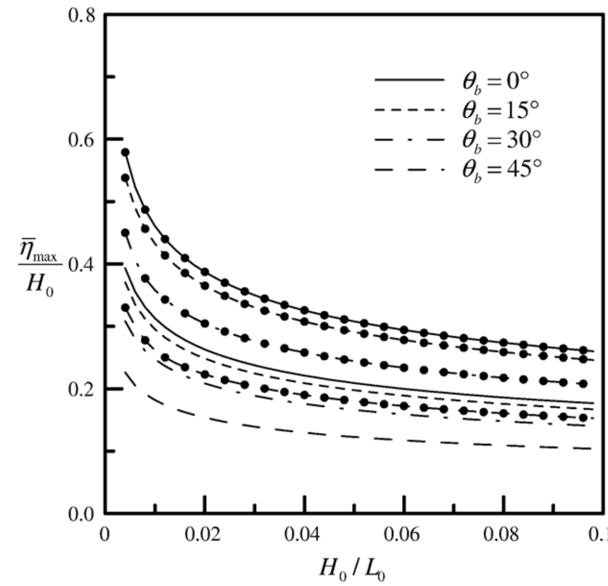

Fig. 10. The normalized maximum setup at the new shoreline position versus deep water wave steepness. Lines without dots for tan β=0.01 and those with dots for tan β=0.15.

6. Concluding remarks

Based on the analysis presented in this paper on the wave setup and setdown across the surf zone induced by an obliquely incident wave, the following remarks may be made.

1. On the basis of a linear relationship between the wave amplitude and water depth within the surf zone, the new mathematical expressions derived for wave setup and setdown in this paper are only appropriate for a beach profile with its depth increasing monotonically offshore. In addition, the target of the present theory is primarily for a spilling and plunging breaker beyond and across the surf zone. Consequently, these equations are recommended for a swell-built beach profile rather than a storm profile with a significant bar feature.
2. Experiments were performed in a large-scale wave basin to examine the linear relationship between wave height and local water depth $H=\kappa h$ employed in deriving the theory. Experimental data of obliquely incident waves varying from $\theta_0=0°$ to 30° are used to confirm the crucial assumption. Experimental results show that the linear assumption is still applicable to the case of inclinedly incident waves, but the value of κ is modified as a function of the modified Iribarren number. Wave breaking criterion for obliquely incident waves is different from the case of normally incident waves. In this paper, LéMéhauté and Wang's (1980) formula was implemented in the theoretical formulation, in which influence parameters of beach slope, deep water wave steepness and breaking angles are included.
3. Although the equations for the wave-induced setup and setdown reported in this paper are derived under a similar

limitation of $\cos\theta \approx \cos\theta_b$ in the nearshore (Longuet-Higgins, 1970a,b), its applicability may be extended to most field conditions for $\theta_b < 24°$, and even up to 45°, as explained in the context of field measurements reported in Komar (1998).

4. Regardless of the seemingly large variations in incident wave obliquity, wave setdown produced by shoaling waves beyond the surf zone reaches its maximum at the breaker line. Wave setup then commences across the surf zone and finally attains to a maximum at the shoreline on an impermeable slope.
5. The outline of a non-dimensional setdown distribution beyond the surf zone versus the distance offshore does not reflect the geometric shape of the beach profile on which the wave propagates. Beyond the breaker line, the effects of the second-order term $(h/h_b)^2$ on setup are small compared with the first-order term (h/h_b). On the contrary, the outline of the non-dimensional wave setup distribution within the surf zone is conformable to the geometry of the beach profile of either concave-down, planar or concave-up due to the fact that the radiation stress in shallow water is highly dependent on water depth.
6. Affected by wave setup on an impermeable beach profile, the total water depth at the original shoreline position has a non-zero value, as obtained in the procedure of mathematical derivation. The increase in water depth and the landwards "advancement" of the original shoreline can be estimated using the equation derived in this paper. The maximum setup relative to the depth at the breaker does not vary with beach profile and its value decreases as wave obliquity increases.
7. The relationship between the maximum wave setup $\overline{\eta}_{\max}/H_0$ at the shoreline and wave steepness H_0/L_0 is derived through wave breaking criterion provided by LéMéhauté and Wang (1980), the trend agrees fairly well with the field data and empirical formula given by Holman and Sallenger (1985). On the same beach slope, the value of $\overline{\eta}_{\max}/H_0$ decreases as H_0/L_0 and θ_b increase. A large value of $\overline{\eta}_{\max}/H_0$ is to be expected on a steep slope.

Acknowledgments

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