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EXTREME VALUE STATISTICS FOR WAVE RUN-UP ON A NATURAL BEACH

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ABSTRACT

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Statistics of wave run-up maxima have been calculated for 149 35-minutes data runs from a natural beach. During the experiment incident wave height varied from 0.4 to 4.0 m, incident wave period from 6 to 16 s, and beach face slope from 0.07 to 0.20. Four extreme statistics were calculated; the maximum run-up height during each run, the 2% exceedence level of shoreline elevation, the 2% exceedence height for individual run-up peaks, and the 2% exceedence level for swash height as determined by the zero-upcrossing method. These statistics were best parameterized when normalized by the incident significant wave height and plotted against the Iribarren number, $\xi = \beta/(H/L_0)^{1/2}$. The swash data (with set-up removed) showed less scatter than total run-up (with set-up included). For Iribarren number greater than 1.5 the run-up was dominated by the incident frequencies, for lower Iribarren number longer period motions dominated the swash. A reasonable value of wave steepness for a fully developed storm sea is 0.025 so that a storm Iribarren number can be estimated as 6.3 times the beach slope. Using this and an offshore design wave height, the included graphs may provide guidance in determining a design run-up height.

INTRODUCTION

The study of wave run-up on artificial and natural beaches has been the topic of much past interest. To a great extent this interest has been dictated by economics; the cost of a structure such as a dyke will rise approximately as the square of the design run-up height. Uncertainties in our understanding of run-up and particularly extreme values of run-up are directly reflected in more expensive structures.

Figure 1 is a definition diagram of the many measures of run-up and swash. $\eta(t)$ is the time series of shoreline elevation (defined as the instantaneous elevation of the intersection of the water surface and the beach face, expressed relative to SWL, the still water level measured at offshore tide gauge. $\bar{\eta}$, the time mean of shoreline elevation, is just the set-up, and the swash, $\eta'(t)$ is defined as the fluctuations of shoreline elevation about

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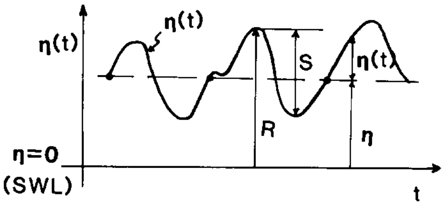

Fig. 1. Definition diagram of run-up variables.

the set-up, $\eta'(t) = \eta(t) - \bar{\eta}$. The term run-up, symbol R, will refer to a local maximum of shoreline elevation. The symbol S is the swash range between consecutive zero up-crossings of $\eta(t)$.

The statistics of these various measures have been examined in a number of studies, most of which have been carried out on laboratory beaches at reduced scales. One of the most notable papers is by Hunt, 1959. He summarizes lab data on the run-up, R, finding that the non-dimensional run-up (run-up normalized by H, the incident significant wave height) was best described by:

$$\frac{R}{H} = 2.3 \frac{\tan\beta}{(H/T^2)^{1/2}} \tag{1}$$

Battjes (1974) synthesized data relating to a number of surf zone processes and pointed out the central importance of the variable combination on the right-hand side of eq. (1). He proposed a dimensionless surf similarity parameter, to be called the Iribarren number, given as:

$$\xi = \frac{\beta}{(H/L_0)^{1/2}} \tag{2}$$

L_0 is the deep water wavelength ($= gT^2/2\pi$, where T is the wave period and g is the acceleration due to gravity) and β is the beach slope in radians (actually an excellent approximation to $\tan\beta$ for all naturally occurring slopes). Using the Iribarren number, Hunt's equation becomes:

$$R/H = 1.0\xi \tag{3}$$

The dynamics of shoreline elevation have been best understood by examining its two components, set-up and swash, separately. The theoretical understanding of set-up was provided by Longuet-Higgins and Stewart (1964) who introduced the concept of radiation stress. Using this, Bowen et al. (1968) showed theoretically and with data from the laboratory that the non-dimensional set-up at the shoreline is a function only of γ, the ratio of breaking wave height to water depth. They also showed data representing γ as only a function of ξ, so:

$$\bar{\eta}/H_0 = f(\gamma) = f'(\xi) \tag{4}$$

The Iribarren number has also been applied to monochromatic swash in the laboratory, where it has been found that for Iribarren numbers down to some minimum (reported in the literature to be in the range 1.0—1.8) the incident waves will not break. Below the minimum, breaking occurs to reduce wave height such that the minimum value of Iribarren number is maintained on the shoreface (Moreas, 1970; Battjes, 1974; Guza and Bowen, 1976; Van Dorn, 1978). For broad-band incident waves a spectrum of swash will occur with energy transfer to both higher frequencies (Huntley et al., 1977; Guza and Thornton, 1982) and lower frequencies (Gallagher, 1971; Suhayda, 1974; Huntley et al., 1981; Symonds et al., 1982). Galvin, 1972, applied a surf similarity parameter (which can be directly converted to the Iribarren number) to a number of surf zone problems including the onset of breaking, breaker type and the number of waves in the surf zone.

A number of investigations have studied the effect of a spectrum of incident waves on the extremes of run-up through the use of probability distributions (Saville, 1962; Van Oorschot and d'Angremond, 1968; Battjes, 1971; Ahrens, 1983). The earlier studies would generally invoke the principle of equivalency, defined by Battjes (1971), as the assumption that the distribution of run-ups of an irregular wave train can be found by assigning to each individual wave the run-up value of a periodic wave train of corresponding height and period. This assumption met with some success in explaining early laboratory data and, qualitatively, one set of field data from a dike of the IJssellake in the Netherlands (Battjes, 1971). While not strictly an assumption of linearity, the principle of equivalency seems to be essentially dependent on linearity. Thus its success seems surprising given the strong nonlinear nature of the surf zone and the apparent sensitivity of monochromatic, laboratory run-up data on plane smooth slopes to incident nonlinearities (Ahrens, 1983). Interestingly, remarkably linear behavior has also been observed in the surf zone for wave kinematics (Guza and Thornton, 1980) and phase velocity (Thornton and Guza, 1982).

Few extensive datasets from the field exist to show the applicability of these theoretical and laboratory ideas to natural beaches. Wright and Short (1983) summarize field data taken in a variety of environments and show the importance of a surf similarity parameter like the Iribarren number (they actually use a slightly different version). Guza and Thornton (1981, 1982) present extensive datasets regarding set-up and swash on a natural beach taken during the National Sediment Transport Study (NSTS). While they show no dependence on the Iribarren number, their data encompasses only a small range of ξ, about 0.25—0.5. No reference is made to extreme value statistics from these studies.

The most extensive field dataset to date is that of Holman and Sallenger, 1985. It presents set-up and swash data from 154 data runs under a variety of incident wave conditions and beach slopes. The data span an Iribarren number from 0.5 to 4.0 and indicate the strong dependence of nearshore dynamics on this parameter. Like the data of Guza and Thornton (1981, 1982), only the significant statistics were presented with no reference to extreme value statistics. It is the intention of this paper to extend the analysis of their dataset to look specifically at the extreme value statistics. As in Holman and Sallenger (1985), the extreme values of run-up are also found to depend on the Iribarren number.

In the following section the field experiment is described followed by the analysis techniques. The data are then presented in a variety of formats to show the utility of different statistical measures. Finally the results are discussed along with ideas on relevant future research.

THE FIELD EXPERIMENT

In October 1982 a large field experiment was carried out at the CERC Field Research Facility (FRF) at Duck, North Carolina. The FRF is located in the middle of a 100-km stretch of barrier islands with the only topographic perturbation in the longshore being the pier itself which extends 560 m offshore and causes some interruption of the natural contours (Miller et al., 1983). The average beach face slope was approximately 1:10 although this value varied significantly through the experiment and on several occasions (when the shoreline was very rhythmic) in the longshore. A bar system was present approximately 50 m offshore although the position and amplitude of the bar varied in response to storms. Bar morphology varied from linear to crescentic. A typical example of the three-dimensional morphology is shown in Fig. 2.

Run-up data were collected using longshore-looking time-lapse photography from super-8 movie cameras mounted on scaffolding on the pier, approximately 13 m above mean sea level. Large markers were placed in pairs, spaced 10 m in the cross-shore direction, every 50 m down the beach for 300 m on either side of the pier. Additional single markers were placed at odd multiples of 25 m. The markers served as a reference for the beach profile grid and provided scale for the film images.

A data run usually consisted of running two movie cameras synchronously, one pointed to the north and one to the south. A frame was shot every second for a total run length of 35 min, or 2100 frames. Slight differences in the camera speed were corrected by carefully timing the length of each run, counting the number of frames taken, and calculating the average Δt. Laboratory studies have shown no noticeable drift in this number through a 35-min period. Spectral analyses showed the energy to be down by at least two and a half orders of magnitude by the Nyquest frequency, indicating that sampling at 1.0-s intervals was adequate.

Digitization of the film data for any of the longshore locations is accomplished with a computer-assisted digitization scheme described in Holman and Guza (1984). Replicate digitizations by different operators, performed on a number of films, showed the standard deviation on set-up and significant swash height measurements presented here to be approxi-

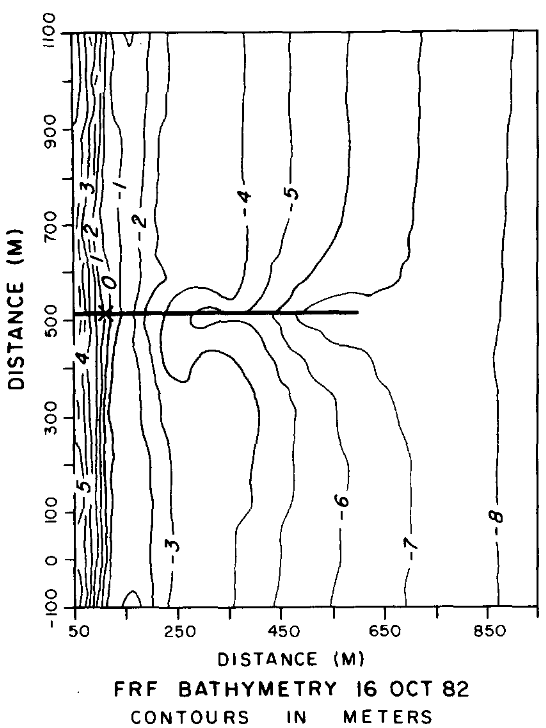

Fig. 2. Typical FRF bathymetry for the field experiment period. The heavy line represents the pier, the X the camera locations. Run-up data were digitized up to 300 m away from the pier on either side. (Bathymetry data from CERC, 1982).

mately 10%. Intercalibration of the film technique with the dual resistance wire run-up sensor on a low-slope beach showed some systematic differences in measured means and standard deviations, with the film technique registering a slightly higher mean, and a 35% larger standard deviation (83% larger variance) than the wire sensor (Holman and Guza, 1984). This is partly related to the sensitivity of the wire sensor to the height of the wire above the beach, and partly to the subjective interpretation of rundown of the films.

Beach surveys were carried out using the FRF Zeiss Elta-2 electronic total station system. This gives profile data, corrected to mean sea level, with an accuracy of better than 0.5 cm over the area of filming. These data were used to transform the raw cross-slope run-up data to a vertical signal. All data presented in this paper will be in terms of the vertical component of run-up. The profile data were also used to define a foreshore beach slope, β, as the mean slope over the 5 m width of beach surrounding the mean sea level at the time of the run. Profile data were collected at least every two days and up to twice per day when the profiles were changing rapidly.

Incident wave data were available from two sources. A waverider buoy positioned 3 km offshore in 20 m depth gave deep water data. Data from intermediate depth were available from a Baylor gauge located at the end of the FRF pier in 6 m depth (the depth immediately surrounding the gauge is approximately 8 m due to scour around the pier pilings). Results using both values will be given, with the subscripts 0 and s used to indicate data from the waverider and Baylor gauges respectively. Incident significant wave height is calculated as $H = 4\sigma$, where σ is the standard deviation of a twenty-minute time series. Incident period is the period associated with the peak energy in the spectrum. Tide data is provided by a NOAA tide gauge attached to the end of the pier. Raw tide gauge data, consisting of spot measurements of sea surface elevation every six minutes, showed a standard deviation of 0.04 m during storms. Mean sea level was estimated from the average of the 6 consecutive measurements corresponding to the data run. The tide gauge was outside the surf zone for all but the largest storms.

OBSERVATIONS

Data were collected over a three-week period in October, 1982. Two storms occurred during the experiment, with deep water significant wave heights ranging from 0.4 to 4.0 m, periods from 6 to 16 s, and foreshore slope variations of a factor of 3. In short, data were collected over a wide portion of the relevent parameter space. A summary of incident wave statistics for the duration of the experiment is shown in Fig. 3.

Sixty-one films have been digitized, most at two longshore locations, 100 and 150 m from the camera. Some films, where longshore variability has been apparent, have been analyzed intensively, with up to 9 ranges being digitized. A total of 149 run-up time series are discussed in this paper. After digitization of a run-up time series all data were transformed to the vertical component and the tide removed. The set-up, $\bar{\eta}$, was calculated as the mean elevation of the run-up time series above mean water level at the tide gauge location and the significant swash height, η_s, as $4\sigma_\eta$ where σ_η is the standard deviation of the shoreline elevation time series.

Ideally extreme statistics of some parameter are found by fitting a known frequency distribution such as Rayleigh distribution (Van Oorschot and d'Angremond, 1968) or Weibull distribution (Ahrens, 1983) to the set of observed values of that parameter. The tails of the distribution are then described by an analytical function so that extreme statistics at any recurrence rate can quickly be calculated. For the present dataset, each run consisted of approximately 150—200 swashes. When these were fit to a distribution it was felt that the small number of data points was insufficient to select the appropriate frequency distribution, especially given the sensitivity of the final "rare event" wave statistics on the details of the tail of the chosen distribution. Thus it was decided not to attempt to match the data

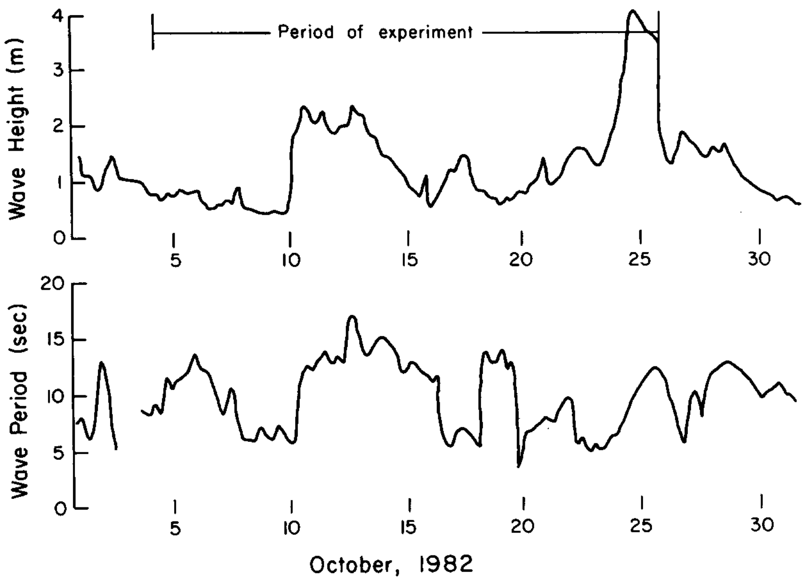

Fig. 3. Incident significant wave height and period for the duration of the experiment as measured by an offshore waverider in 20 m water depth (CERC, 1982). Two major storms occurred, 10—13 and 24—26 October.

to any known frequency distribution but to simply calculate particular values from the observed frequency distribution for each dataset.

Four particular measures of extreme swash were used. η_{max} is the maximum of the shoreline elevation time series for the 35-minute period of the data run. η_2 is the 2% exceedence statistic; the shoreline elevation which only 2% of the data exceeded. η_2 is based on the entire 2100 points in each record. The time series were then separated into individual waves and the statistics of the associated wave heights examined. Individual waves were defined in two ways. The first is by the zero-upcrossing method where an individual swash occurs in the time between successive upward crossings of the shoreline elevation time series through the mean level for the record (as opposed to zero crossings through the still water level which intermingles the problems of set-up and swash statistics). The associated swash height is the range of run-up for each individual swash (Fig. 1). Alternately, the individual swashes could be defined as a local maximum or peak in η, with the associated heights for each run-up being the elevation above still water level of each peak. S_2 is just the 2% exceedence level for swash height as defined by the zero upcrossing method, and R_2 is the 2% exceedence for run-up peaks. In defining the individual run-ups using either of the above two methods, a count was kept of the number of run-ups. Dividing this into the 35-min length of the record gave a representative run-up period denoted T_z and T_p for the zero upcrossing and the local peak definitions of swashes respectively. All notation is summarized in appendix 1.

In the following analysis, incident wave height could be represented by either deep water significant wave height, H_0, or 6 m depth significant wave height, H_s. Plots were produced using both of these statistics, however, the two sets of plots were sufficiently similar that only those using H_s (chosen as more representative of the wave input to the surf zone) are presented. Regression statistics for both are listed in Table 1 and confirm the similarity of the statistical measures.

Figures 4a, b, c and d plot the four above-mentioned extreme run-up statistics against the incident significant wave height, H_s. The data show a great deal of scatter in all cases and do not indicate any one measure to be particularly better than any other. The concentration of points at certain values of H_0 is partly due to digitizations of multiple longshore locations from the same film. Interestingly, the trend of the data shows a positive y-intercept in each plot. The cause of this will become apparent when the data are expressed in terms of the Iribarren number, itself a function of H_s. The trends and intercepts for each of the plots and for all subsequent plots are listed in Table 1a for plots using H_s as a measure of incident wave height and Table 1b for those using H_0.

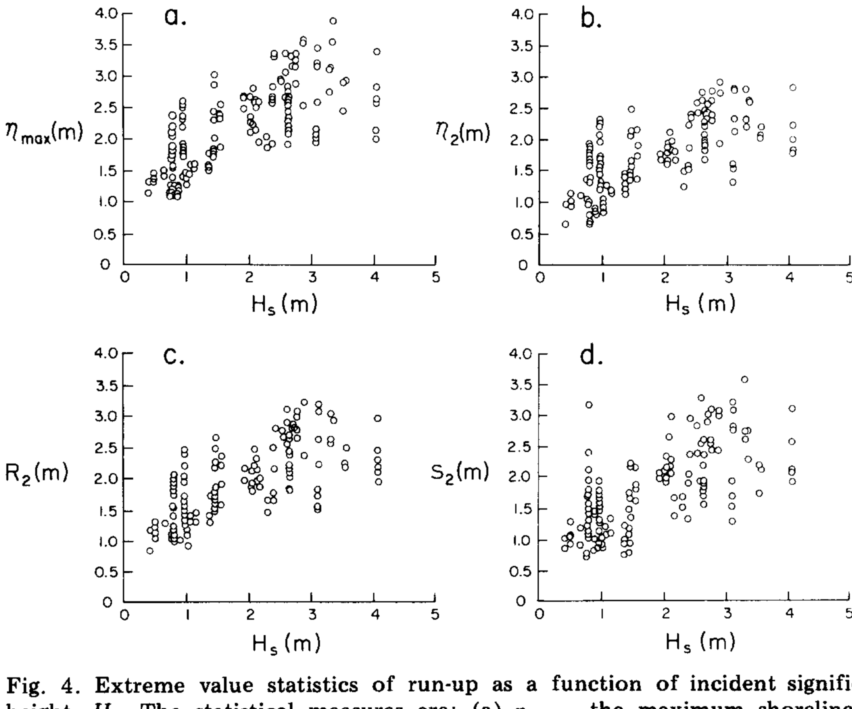

Fig. 4. Extreme value statistics of run-up as a function of incident significant wave height, H_s. The statistical measures are: (a) η_{max}, the maximum shoreline elevation height for each run; (b) η_2, the 2% exceedence height of run-up; (c) R_2, the 2% exceedence height for the run-up heights associated with individual run-up peaks; and (d) S_2, the 2% exceedence height for swash range for the individual swashes as defined by the zero-upcrossing method.

TABLE 1a

Regression coefficients for run-up statistics using incident wave height from 6 m depth, H_s

Variables	Slope, m	Δm	Intercept, b	Δb
η_{max} vs H_s	0.53	±0.04	1.25	±0.08
η_2 vs H_s	0.42	±0.04	0.98	±0.07
R_2 vs H_s	0.47	±0.04	1.09	±0.07
S_2 vs H_s	0.50	±0.04	0.88	±0.09
η_{max} vs η_s	0.94	±0.05	0.66	±0.09
η_2 vs η_s	0.77	±0.05	0.48	±0.08
R_2 vs η_s	0.86	±0.05	0.53	±0.08
S_2 vs η_s	1.11	±0.03	0.03	±0.05
$\hat{\eta}_{max}$ vs ξ_s	0.90	±0.06	0.21	±0.09
$\tilde{\eta}$ vs ξ_s	0.69	±0.05	0.18	±0.08
R_2 vs ξ_s	0.78	±0.05	0.20	±0.08
$\hat{S}_2$ vs ξ_s	0.83	±0.05	0.78	±0.07
$\tilde{\eta}_{max}$ vs ξ_s	0.65	±0.05	−0.01	±0.07
$\tilde{\eta}_2$ vs ξ_s	0.45	±0.02	−0.04	±0.04
$\tilde{R}_2$ vs ξ_s	0.53	±0.03	−0.02	±0.05

TABLE 1b

Regression coefficients for run-up statistics using incident wave height from depth water, H_0

Variables	Slope, m	Δm	Intercept, b	Δb
η_{max} vs H_0	0.52	±0.05	1.37	±0.08
η_2 vs H_0	0.40	±0.04	1.09	±0.08
R_2 vs H_0	0.45	±0.04	1.21	±0.08
S_2 vs H_0	0.46	±0.05	1.05	±0.10
η_{max} vs η_s	0.94	±0.05	0.66	±0.09
η_2 vs η_s	0.77	±0.05	0.48	±0.08
R_2 vs η_s	0.86	±0.05	0.53	±0.08
S_2 vs η_s	1.11	±0.03	0.03	±0.05
$\hat{\eta}_{max}$ vs ξ_0	0.96	±0.07	0.22	±0.11
$\hat{\eta}_2$ vs ξ_0	0.75	±0.07	0.18	±0.10
R_2 vs ξ_0	0.83	±0.06	0.20	±0.10
$\hat{S}_2$ vs ξ_0	0.85	±0.05	0.06	±0.07
$\tilde{\eta}_{max}$ vs ξ_0	0.67	±0.05	0.02	±0.07
$\tilde{\eta}_2$ vs ξ_0	0.46	±0.03	−0.03	±0.04
$\tilde{R}_2$ vs ξ_0	0.55	±0.03	0.00	±0.05

In an effort to express the extreme value statistics in terms of a more familiar statistic, the four measures were plotted against the significant swash height, η_s. These plots are shown in Figs. 5a, b, c and d and the

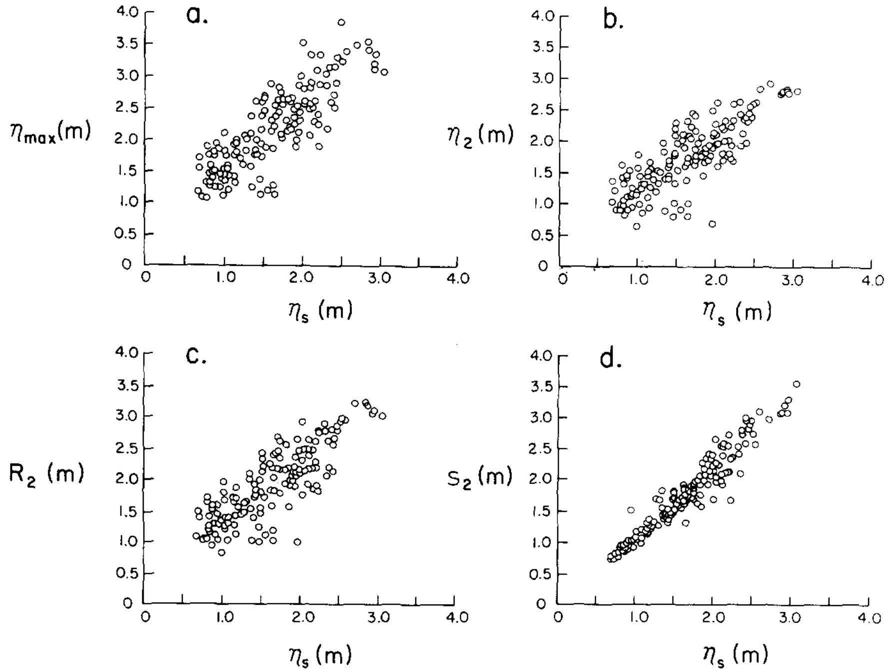

Fig. 5. Extreme value statistics of run-up as a function of the significant swash height, η_s. η_s is defined as four times the standard deviation of the shoreline elevation time series, so the plots are an indication of the spread of the frequency distribution. Individual measures are as defined in Fig. 4 and in the text.

regression coefficients are listed in Table 1. S_2 is best parameterized by this form, showing an intercept of 0.03 and a trend 1.11. The other statistical measures are more scattered and all have a positive intercept. It is expected that these intercepts are related to set-up. Later analysis shows that the data are better described when set-up is removed.

Given the observed dependence of both previous laboratory and field data on the Iribarren number, ξ, scatter is expected in the data when it is plotted dimensionally as in Fig. 4. Two data runs with the same incident wave height could easily have different Iribarren numbers, hence different run-up statistics. Thus all run-up statistics were normalized by the incident wave height and plotted against the Iribarren number in Fig. 6. The symbol ^ is used to denote the non-dimensional variable, e.g. $\hat{R}_2 = R_2/H$. The data in Fig. 6, while still showing some scatter, show a very definite trend; the non-dimensional run-up is clearly a function of the Iribarren number for a natural beach. Again, S_2 shows the least scatter of the four statistical measures, possibly because it, by definition, excludes the phenomenon of set-up. The data tend to be more scattered at high Iribarren numbers, corresponding to narrow surf zone conditions. During the periods of high Iribarren number during the experiment the beach was observed to become

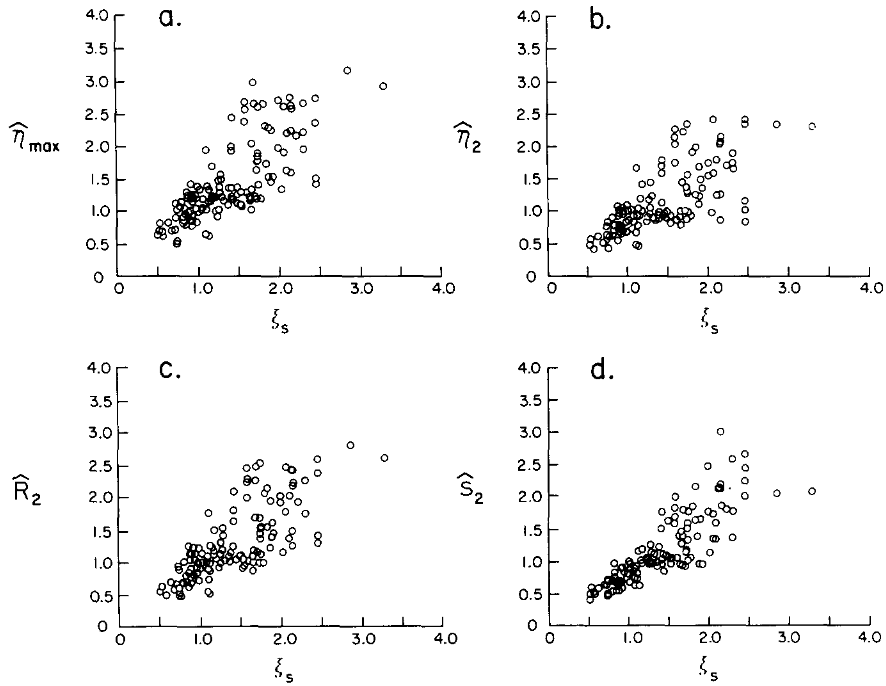

Fig. 6. Extreme value run-up statistics normalized by the incident significant wave height, plotted as a function of the Iribarren number, ξ_s. The individual statistics are ordered in the four plots as described in Fig. 4.

quite three-dimensional with a strong longshore variability in foreshore beach slope and run-up statistics. On the other hand, low Iribarren number data were generated during the two storms when the beach was observed to become linear and run-up statistics became more uniform. The Iribarren number dependence explains the positive y-intercept in the dimensional data of Fig. 4. In general, lower H is associated with higher Iribarren number, hence a higher nondimensional run-up.

The three statistical measures η_{max}, η_2, and R_2 contain contributions from both the swash and the set-up. Given the reduced scatter of S_2, the measure with no contribution from set-up, and given that set-up and swash act according to different dynamics, it is clearly of interest to remove the set-up contribution from these measures and examine the extreme value statistics of the swash component only. The symbol $\sim$ is used to denote the run-up measures with the set-up removed and nondimensionalized by incident wave height, e.g. $\tilde{R}_2 = (R_2 - \bar{\eta})/H$. The swash data are plotted in Figs. 7a, b and c. The removal of set-up does not change S_2, plotted in Fig. 6d. The extreme swash statistics data are very well behaved in this format. The scatter is small, particularly at lower Iribarren numbers (corresponding to storms which are the times of greatest interest). The central

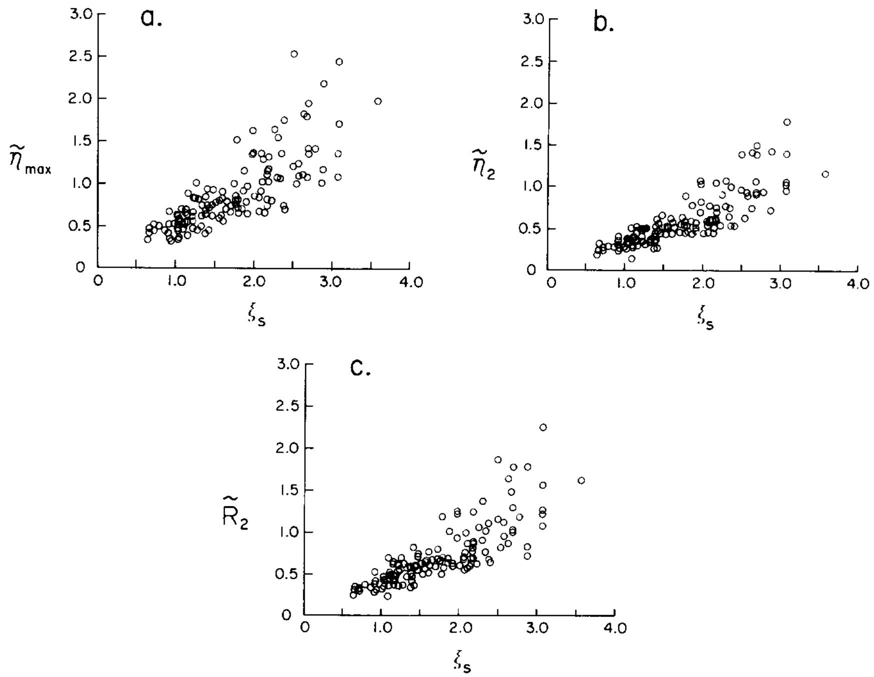

Fig. 7. Extreme swash statistics, normalized by incident significant wave height, as a function of the Iribarren number, ξ_s. The swash statistics were calculated by subtracting the set-up from the corresponding run-up statistical measure. The individual plots are as described in Fig. 4. There is no plot for S_2 since that measure had already excluded the set-up.

importance of the Iribarren number to surf zone and swash processes as suggested by Battjes (1974) is again confirmed.

DISCUSSION

For coastal engineering applications the run-up measure of most interest is the total run-up or extreme statistics associated with the total run-up. The engineer must design according to an elevation above still water level. For this purpose the distinction of the two components of run-up, set-up and swash, is perhaps not necessary. However, from the point of view of trying to understand wave processes, the distinction is quite real. To model the total run-up theoretically the equations are split into equations dealing with the mean (set-up), and variations about that mean (the swash). It is of interest that the data on swash, the more poorly understood of the two processes, is the better behaved. Figure 8 shows the set-up data (from Holman and Sallenger, 1985) that was used to produce Fig. 7. Even though set-up is theoretically well understood, the field data are very scattered. Bowen et al., 1968, show from laboratory data that the set-up gradient across the surf zone is well behaved except very near the shoreface where it rises to approach the beach face slope asymptotically. They explain this behaviour in terms of the small standing wave component of the incident wave field. Unfortunately, this renders measurements of the set-up sensitive to the height of the measuring device above the shore face, a problem noted by Holman and Guza (1984) in their intercomparison of two techniques of measuring set-up. It is apparent that to achieve a good understanding of set-up on natural beaches and to allow intercomparison of measurements taken using different techniques, a detailed study of the behaviour of run-up in the immediate vicinity of the shoreface must be undertaken.

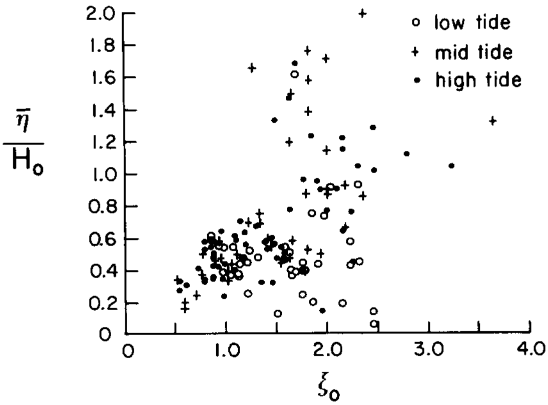

Fig. 8. Set-up data, normalized by the incident significant wave height, as a function of the Iribarren number, ξ_s.

The run-up and swash statistics presented here are monochromatic representations of spectral phenomena. In fact, the spectral characteristics of the swash have been the topic of considerable interest, particularly the presence of energy at low frequencies (Huntley, 1976; Sasaki et al., 1976; Wright et al., 1979; Huntley et al., 1981; Holman, 1981; Mase and Iwagaki, 1984 for example). A representative period of the swash, T_z, was calculated using the zero-upcrossing technique. This was normalized by the incident wave period (period associated with the peak energy in the offshore wave spectrum) and plotted against the Iribarren number (Fig. 9). The plot shows that for ξ greater than about 1.5 the swash is dominated by the incident wave frequency band. However, for lower Iribarren numbers the swash periods become progressively longer as lower-frequency wave motions become increasingly important. Since storms are usually associated with low Iribarren numbers, it is clear that further research is needed on the nature and dynamics of these long-period motions.

The data presented here show the importance of the Iribarren number to run-up kinematics. Unfortunately, for erodeable beaches, the Iribarren

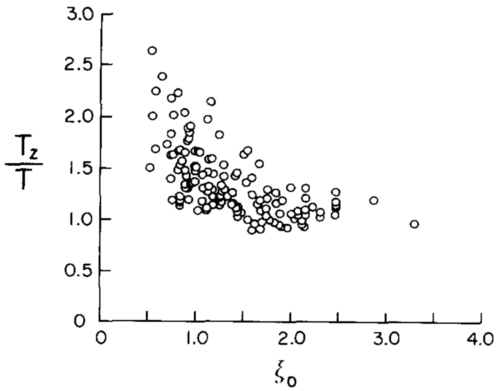

Fig. 9. Average swash period, normalized by the incident wave period (period corresponding to the energy maximum in the offshore wave spectrum) as a function of the Iribarren number.

number cannot be determined a priori. The beach slope is itself a function of the incident wave characteristics. Ideally we should be able to express that function in terms of external variables (presumably including sediment as well as offshore wave variables) but that problem has been attempted many times and is known to be complicated.

For structures such as a dike, the beach slope is known so that the run-up statistics can be determined from only the offshore wave data. We would like to know the worst possible run-up levels that can be experienced by a structure with a certain slope. To calculate this we would need to know about the joint probability distribution of offshore wave height and wave steepness since the wave steepness enters into the Iribarren number and the wave height into the nondimensional run-up. These data may not be available for all sites. However we can make a simplification if we assume that for the worst storms the sea will be well developed and the offshore wave steepness will be limited by breaking. The value of this limiting steepness for stochastic waves is unknown to the author but some guidance can be found from theory and from the data. Michell (1893) showed that the theoretical value of maximum wave steepness for deep water deterministic waves is 1/7. For stochastic waves the value will undoubtedly be smaller. Fortunately, the data collected during this experiment included a major storm, considered qualitatively to have a recurrence rate of from two to five years. The storm caused significant coastal erosion including the loss of twelve houses from the town of Kittihawk. Incident wave data from the offshore waverider showed the incident wave steepness during this major storm to be approximately 0.025, a number found also by Battjes (1970) to be typical of storms. If we assume this value to be representative of destructive storms, then the associated storm Iribarren number will just be:

$$\xi_{0\,\mathrm{storm}} = 6.3\,\beta \tag{5}$$

Using this value of Iribarren number the extreme run-up statistics can be found in a number of ways. The simplest would be to use the regression coefficients from Table 1. For example, the 2% exceedence level for swash height determined by zero-upcrossings would be:

$$R_2 = (5.2\,\beta + 0.2)\,H_s \tag{6}$$

an equation that will give results quite similar to the Dutch formula $R_2 \simeq 8 \tan \beta$ (Wassing, 1957). Equation (6) could then be used directly to find the design swash height given an offshore design wave height. Alternately, a greater safety factor could be achieved by examining the plots and finding a line which just exceeded all the data, as opposed to the linear regression which lies below half of the points. Details of the application of the data to particular projects are left to the reader.

The four measures presented here give different degrees of scatter. However, no judgement will be made of which is better. The utility of each clearly depends on the application and so comparisons are again left to the reader.

Finally, a recent paper by Mase and Iwagaki (1984) gives η_{max} data for the incidence of random waves on shallow slopes in a wave flume. Using surface elevation maximum data based on run lengths of 650—900 incident wave periods, they found the best formulation to be:

$$\hat{\eta}_{max} = 2.3\,\xi_0^{0.77} \tag{7}$$

The values given by eq. (7) are substantially larger than those presented in this dataset from a natural beach. The field data presented in Fig. 6 are also too scattered to legitimize anything but a linear fit.

CONCLUSIONS

Four measures of extreme run-up height have been investigated using an extensive dataset from a natural beach. The data are best parameterized in terms of the Iribarren number, $\xi = \beta/(H/L_0)^{1/2}$. Statistics on total run-up (with set-up included) are considerably more scattered than with the set-up removed (just swash). This is largely due to the scatter in the set-up data and indicates the need for further study of set-up.

For Iribarren numbers greater than 1.5 the shoreline elevation time series are dominated by incident wave frequencies. For lower Iribarren numbers the swash become progressively lower frequency.

The Iribarren number for an erodeable beach under storm attack cannot be calculated a priori since the beach slope is unknown (although good estimates can be made from experience). For a structure of known slope a reasonable estimate of the storm Iribarren number can be made by:

$\xi_{0\ \mathrm{storm}} = 6.3\ \beta$

This can be used as a tool, combined with the data plots, to predict the extreme value statistics for nondimensional run-up. True run-up can then be found for any design offshore wave height.

It should be noted that while the data presented are felt to be of high quality, the relationships derived in this paper are based on one location. The author and Oregon State University are in no way responsible for liabilities arising from the use of the relationships in design or construction of coastal structures.

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APPENDIX 1 — NOTATION

The following symbols were used in this paper:

b	y-intercept of regressions
g	acceleration due to gravity
H_0	incident significant wave height in 20 m depth
H_s	incident significant wave height in 6 m depth
L_0	deep water wave length ($gT^2/2\pi$)
m	slope of regression lines
R	height of wave run-up crests above still water level
R_2	2% exceedence height defined from run-up peak heights
S_2	2% exceedence height of swash range defined from zero-upcrossings
T	incident wave period associated with peak spectral energy
T_p	average swash period defined by counting swash peaks
T_z	average swash period defined by counting zero-upcrossings
$\hat{}$	variable has been normalized by H
$\tilde{}$	set-up has been subtracted, then variable has been normalized by H
s	as subscript to indicate that incident wave data was from 6 m depth
0	as subscript to indicate incident wave data was from 20 m depth

β	beach face slope
γ	empirical ratio of breaking wave height to depth
η	shoreline elevation time series
$\bar{\eta}$	set-up
η_2	2% exceedence height of shoreline elevation
η'	swash time series, $\eta' = \eta - \bar{\eta}$
η_s	significant swash height
ξ	Iribarren number
σ	standard deviation of incident wave time series
σ_η	standard deviation of run-up time series

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