

Interaction of short-crested random waves and large-scale currents

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ABSTRACT

Methods are outlined for determining the transformations to directional wave spectra induced by large-scale currents. The problems considered are those where waves move from quiescent water on to a current, or from one current region to another. Situations involving wave generation on currents are not discussed. The principle of wave action conservation is used to relate the wave energy densities in the two regions, and an equilibrium range constraint is applied to the high frequency tail of the transformed spectrum in instances where wave action is not conserved and energy is dissipated by wave breaking. Examples are presented which highlight how current-induced wave refraction and energy dissipation may have important consequences for the transformed spectrum.

NOTATION

A^*, B^*	numerical constants associated with the equilibrium range of the surface elevation spectrum for waves on a current ($A^* = 2B^*$)
b	distance between adjacent wave orthogonals
C_a	absolute wave celerity
C_{ga}	absolute wave group velocity
C'_{ga}	component of the absolute wave group velocity in the direction of the wave orthogonal ($= \partial\omega_a/\partial k$)
C_{gr}	relative wave group velocity
C_r	relative wave celerity
d	water depth
E	wave energy density
e	distance between adjacent wave rays
$G_{ER}(\theta)$	equilibrium range directional spreading function
g	acceleration due to gravity
H	wave height
H_s	significant wave height
k	wave number ($= 2\pi/L$)

L	wavelength
$S_{\eta\eta}(\)$	spectral density of surface elevation; a function of the parameters contained within the brackets
$S_{\eta\eta\text{ER}}(\)$	limiting value of $S_{\eta\eta}(\)$ associated with the equilibrium range of the surface elevation spectrum
$S_{\eta\eta\text{WA}}(\)$	spectral density of surface elevation predicted on the assumption that wave action is conserved
T_a	absolute wave period
T_r	relative wave period
T_z	mean zero-upcrossing period
U	current velocity
α	angle between wave front and current direction
β	angle between two vectors for evaluation of their scalar product
θ	direction of a component wave, measured relative to the predominant wave direction
π	$4.0 \tan^{-1}(1.0)$
ρ	water density
$\sigma_{\eta\text{WA}}^2$	total variance of surface elevation predicted on the assumption that wave action is conserved
ω_a	absolute wave angular frequency
ω_r	relative wave angular frequency
∇	spatial gradient operator, $\partial/\partial x + \partial/\partial y$, x and y being Cartesian co-ordinates in the horizontal plane

Subscripts (other than those defined above):

\sim_1	value in region 1 (containing incident waves)
\sim_2	value in region 2 (containing transformed waves)
\simeq	a vector quantity (i.e. \underline{r} is the vector quantity r)

INTRODUCTION

A satisfactory estimation of the hydrodynamic forces experienced by the submerged elements of offshore structures depends upon an adequate description of the water-particle motions. These motions are usually predicted with the aid of a wave theory based upon a number of simplifying assumptions. One possible assumption is that the water is quiescent — that the only water motions are those induced by the waves. However, in reality, ocean waves rarely propagate on quiescent water.

Waves may combine with currents in a variety of ways. For example, the wind may blow over water already in motion under tidal forces. If the wind is opposing the tidal flow then the waves tend to be higher than if it follows the

flow. This effect has been studied in the laboratory by Francis and Dudgeon (1967). Another possibility is that waves generated on quiescent water propagate from the generation area on to a current some distance away. This possibility is the one considered in the present study.

The scope of the investigation is limited to large-scale currents which vary slowly in space and time. It is these currents which are most relevant to the design of offshore structures. Large-scale currents have horizontal variations which may be regarded as negligible within a wavelength whilst time variations in the current are negligible within a wave period (see Peregrine, 1976).

It has also been necessary, at present, to limit the study to the consideration of currents which are essentially uniform with depth. Such an idealisation may be appropriate for tidal flows but is unlikely to be wholly satisfactory for situations in which there is strong shearing of the water surface by the wind. Nevertheless, even for the latter case, calculations involving a vertically-uniform current will provide some insight into the likely behaviour of waves encountering the real flow conditions.

Longuet-Higgins and Stewart (1961) were the first to deal correctly with the interaction between water waves and currents. They introduced the concept of "radiation stress" and showed the existence of energy transfer between waves and currents. Bretherton and Garrett (1968) later drew attention to a quantity which they called "wave action". Wave action is important in the study of waves on currents as, unlike wave energy, it is conserved in the absence of wave generation or dissipation. This approach is adopted in the present study.

In more recent years, the interaction of short-crested waves and currents has received attention from, amongst others, Tayfun et al. (1976), Mathiesen (1984), Brink-Kjaer (1985) and Treloar (1986). The present investigation extends this work by the introduction of an equilibrium range constraint (see Hedges, 1981). This concept has received little attention in the literature relating to the interaction of short-crested waves and currents. However, the implications of neglecting to apply such a constraint may be of considerable importance.

ABSOLUTE AND RELATIVE WAVE FREQUENCIES

Before developing the theory to describe the transformation of a general three-dimensional wave field on encountering a current, the absolute and relative wave frequencies need to be defined. These quantities are well-known (see Hedges, 1987), and only a brief outline is given here.

Initially, consider a train of two-dimensional regular waves of wavelength L and height H travelling on a steady, horizontally and vertically uniform current. The current velocity is U in the direction of wave propagation. Viewing the waves in a stationary frame of reference containing the wave orthog-

onal, Fig. 1 (a), they have a celerity C_a and a period T_a . However, if the waves are viewed in a frame of reference moving along the wave orthogonal at velocity U , Fig. 1 (b), they appear to be travelling on “still” water with celerity C_r and period T_r .

Hence

$$C_a = C_r + U \tag{1}$$

or

$$\frac{L}{T_a} = \frac{L}{T_r} + U \tag{2}$$

Alternatively

$$\omega_a = \omega_r + kU \tag{3}$$

where ω_a is the absolute wave angular frequency ($= 2\pi/T_a$); ω_r is the relative wave angular frequency ($= 2\pi/T_r$); and k is the wave number ($= 2\pi/L$).

In the moving frame of reference, the waves seem to be propagating on “still” water and all the usual equations describing wave motion are valid. Thus, according to linear wave theory:

$$\omega_r^2 = gk \tanh kd \tag{4}$$

in which g is the acceleration due to gravity and d is the water depth.

Substituting for ω_r in Eq. (3) gives:

$$\omega_a = \pm (gk \tanh kd)^{1/2} + kU \tag{5}$$

which is the dispersion relationship for small amplitude, two-dimensional waves on a current. Given d , U and ω_a , Eq. (5) may be solved for k . Equation (3), or (4), then yields ω_r . Note that, in general, there is no unique solution to Eq. (5). A discussion of the full solution of the dispersion relationship for waves on a current is provided by Peregrine (1976).

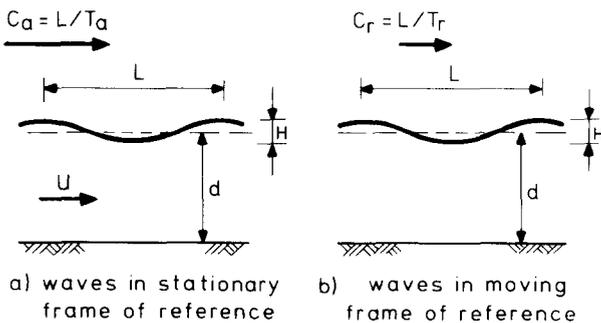


Fig. 1. Waves on a uniform current.

WAVE ACTION CONSERVATION

When waves propagate from quiescent water on to a current, their absolute period remains constant but the wave amplitude is changed as energy is transferred between the waves and the current. In the general three-dimensional situation, wave refraction also occurs. Changes in wave amplitude induced by a current may be determined using the principle of wave action conservation (Bretherton and Garrett, 1968). Wave action is defined as E/ω_r , where E , the wave energy density, is given by:

$$E = \rho g H^2 / 8 \tag{6}$$

in which ρ is the water density. Provided that there is no energy dissipation or wave generation, wave action is conserved and the governing equation may be written:

$$\frac{\partial}{\partial t} \left(\frac{E}{\omega_r} \right) + \nabla [(\underline{U} + \underline{C}_{gr}) \frac{E}{\omega_r}] = 0 \tag{7}$$

where \underline{U} is the current vector, \underline{C}_{gr} is the vector representing the wave group velocity relative to the current, and ∇ is the spatial gradient operator. From linear wave theory, the magnitude of the relative wave group velocity is:

$$C_{gr} (= \partial\omega_r / \partial k) = \frac{\omega_r}{2k} \left[1 + \frac{2kd}{\sinh 2kd} \right] \tag{8}$$

Before proceeding further, it is important to clearly define the various co-existing flow directions (see Fig. 2). Firstly, streamlines are defined as running in the direction of the current velocity vector. Secondly, wave orthogonals give the direction of wave travel — they are normal to the wave fronts. Finally, wave rays run in the direction of the absolute group velocity, \underline{C}_{ga} , given by:

$$\underline{C}_{ga} = \underline{U} + \underline{C}_{gr} \tag{9}$$

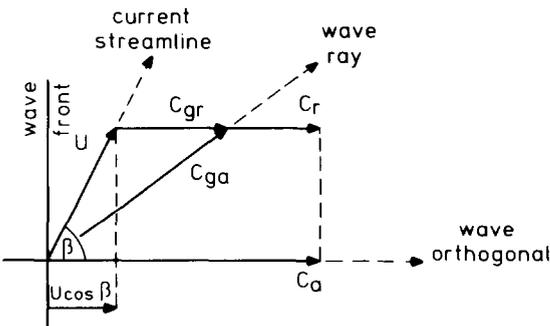


Fig. 2. A streamline, a wave ray and a wave orthogonal.

(Note that streamlines, wave orthogonal and wave rays are all in the same plane if the current is directly following or directly opposing the waves.)

Jonsson and Skovgaard (1978) developed a method for determining the changes in wave direction, wavelength and wave height as regular waves cross a shearing current. They applied Snell's Law to evaluate the degree of refraction induced by the current (see Fig. 3), giving:

$$\frac{k_2}{k_1} = \frac{L_1}{L_2} = \frac{\sin \alpha_1}{\sin \alpha_2} \quad (10)$$

in which α is the angle between the wave fronts and the current direction. Subscript "1" refers to a value in region 1 containing the incident waves and subscript "2" refers to a value in region 2. Note that for incident waves on quiescent water, $U_1 = 0$.

Generalising Eq. (3):

$$\omega_a = \omega_r + \underline{k} \cdot \underline{U} \quad (11)$$

in which \underline{k} is the wave number vector. $\underline{k} \cdot \underline{U}$ is the scalar product of vectors \underline{k} and \underline{U} (i.e. $\underline{k} \cdot \underline{U} = kU \cos \beta$ where β is the angle between \underline{k} and \underline{U}). Thus $\underline{k} \cdot \underline{U}$ represents the wave number multiplied by the magnitude of the current velocity component in the wave direction (see Fig. 2). Together with Eq. (4), Eqs. (10) and (11) can be used to find the changes in k , ω_r and the orthogonal direction as waves pass from one current region to another. If the incident waves are on quiescent water, then $\omega_{r1} = \omega_a$ and the procedure is simplified accordingly.

Although the shearing current changes the wave ray spacing (see Fig. 4), wave action is conserved between wave rays in the absence of wave generation or dissipation. Thus, for steady-state conditions, Eq. (7) gives:

$$\frac{E_1}{\omega_{r1}} C_{ga1} e_1 = \frac{E_2}{\omega_{r2}} C_{ga2} e_2 \quad (12)$$

in which e is the distance between adjacent wave rays.

For short-crested random waves on a current, the wave energy density, E , is related to the surface elevation spectral density, $S_{\eta\eta}(\omega_a, \theta, U)$, by:

$$E = \rho g S_{\eta\eta}(\omega_a, \theta, U) d\omega_a d\theta \quad (13)$$

where θ is the direction of a spectral component (here measured relative to the predominant direction of the incident waves). Thus, Eq. (12) may be rewritten:

$$\frac{S_{\eta\eta 2}(\omega_a, \theta_2, U_2)}{S_{\eta\eta 1}(\omega_a, \theta_1, U_1)} = \frac{\omega_{r2}}{\omega_{r1}} \cdot \frac{C_{ga1}}{C_{ga2}} \cdot \frac{e_1}{e_2} \cdot \frac{d\theta_1}{d\theta_2} \quad (14)$$

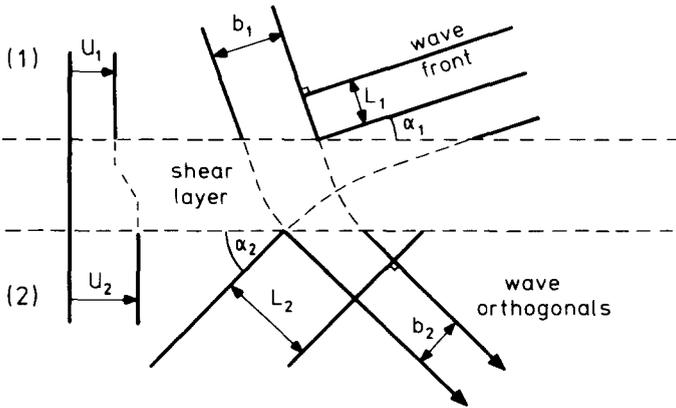


Fig. 3. Change in wave direction across a shearing current.

Note that the wave directions in the two regions, θ_1 and θ_2 , will not generally be the same owing to refraction at the boundary between the regions. This implies that a component of the transformed spectrum will have been produced by a component in the incident spectrum of the same frequency (as ω_a remains constant across the boundary) but in a different direction.

Variations in θ are related to the behaviour of the wave orthogonals, rather than the wave rays, and it may be shown (Longuet-Higgins, 1957) that:

$$\frac{d\theta_1}{d\theta_2} = \frac{b_2 k_2}{b_1 k_1} \tag{15}$$

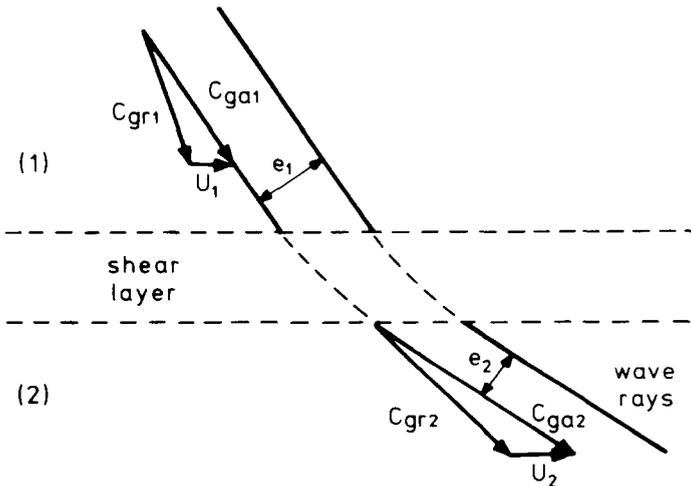


Fig. 4. Change in wave ray direction across a shearing current.

where b is the distance between adjacent wave orthogonals. From geometric considerations (see Fig. 3):

$$\frac{b_2}{b_1} = \frac{\cos \alpha_2}{\cos \alpha_1} \quad (16)$$

Furthermore, since ω_a remains constant across the current boundary:

$$\frac{k_2}{k_1} = \frac{C_{a1}}{C_{a2}} \quad (17)$$

Equation (14) may now be re-written with the aid of Eqs. (15) and (17) as follows:

$$\frac{S_{\eta 2}(\omega_a, \theta_2, U_2)}{S_{\eta 1}(\omega_a, \theta_1, U_1)} = \frac{\omega_{r2}}{\omega_{r1}} \cdot \frac{C_{ga1}}{C_{ga2}} \cdot \frac{e_1}{b_1} \cdot \frac{b_2}{e_2} \cdot \frac{C_{a1}}{C_{a2}} \quad (18)$$

Now, defining C'_{ga} to be the component of the absolute wave group velocity vector in the direction of the wave orthogonal (see Fig. 2) so that:

$$C'_{ga} (= \partial\omega_a / \partial k) = U \cos \beta + C_{gr} \quad (19)$$

it may be shown that

$$\frac{e}{b} = \frac{C'_{ga}}{C_{ga}} \quad (20)$$

Thus, finally:

$$\frac{S_{\eta 2}(\omega_a, \theta_2, U_2)}{S_{\eta 1}(\omega_a, \theta_1, U_1)} = \frac{\omega_{r2}}{\omega_{r1}} \cdot \frac{C'_{ga1}}{C'_{ga2}} \cdot \frac{C_{a1}}{C_{a2}} \quad (21)$$

The above derivation of this result is for the simple case of current shear shown in Figs. 3 and 4. But as Mathiesen (1984) demonstrates, Eq. (21) can be used more generally to account for changes in both water depth and current.

In the special case of incident waves travelling on quiescent water ($U_1 = 0$), the above formulation is slightly simplified as $\omega_{r1} = \omega_a$ and $C'_{ga1} = C_{ga1} = C_{gr1}$. If, in addition, the incident waves are long-crested and encounter either a directly following or a directly opposing current so that there is no refraction, then the spectral densities in the current and quiescent water regions are related to the wave energy densities by:

$$\frac{S_{\eta 2}(\omega_a, U_2)}{S_{\eta 1}(\omega_a)} = \frac{\rho g S_{\eta 2}(\omega_a, U_2) d\omega_a}{\rho g S_{\eta 1}(\omega_a) d\omega_a} = \frac{E_2}{E_1} \quad (22)$$

For two-dimensional, steady-state conditions, Eq. (7) becomes:

$$\frac{\partial}{\partial x} [(U + C_{gr}) \frac{E}{\omega_r}] = 0 \quad (23)$$

so that for waves travelling from quiescent water on to a directly following or directly opposing current:

$$\frac{E_1 C_{gr1}}{\omega_a} = \frac{E_2 (U_2 + C_{gr2})}{\omega_{r2}} \quad (24)$$

or

$$\frac{E_2}{E_1} = \frac{\omega_{r2}}{\omega_a} \frac{C_{gr1}}{(U_2 + C_{gr2})} \quad (25)$$

Combining Eqs. (22) and (25) gives the two-dimensional, wave-current interaction model described by Hedges (1981) and supported by the experimental investigation of Hedges et al. (1985).

Note that if $C'_{ga2} = 0$ then Eq. (21) predicts an infinite spectral density in region 2. This situation arises when $C_{gr2} = -U_2 \cos \beta_2$ (see Eq. 19) and the wave energy cannot propagate on to the current; wave breaking then occurs at the current boundary. Only spectral components having $C'_{ga2} > 0$ can cross from region 1 to region 2.

Even when waves do advance into region 2, any increase in spectral density will be limited by the breaking process. Such considerations have received little attention in previous work on the interactions of three-dimensional wave fields with currents and are now discussed below.

THE EQUILIBRIUM RANGE CONSTRAINT

The previous section describes how the principle of wave action conservation may be used to obtain estimates of the spectral density when waves are transformed on encountering currents. A decrease in spectral density is predicted for some conditions, an increase for others. However, wave growth at a particular frequency and in a given direction cannot continue without limit: wave breaking will be induced. As in the absence of currents, there exists a range of frequencies, the equilibrium or saturation range, over which the spectrum becomes saturated.

A general approach to calculating the equilibrium range spectrum, taken as the upper limit to spectral densities, has been given by Kitaigordskii et al. (1975). Their method is based upon earlier work by Phillips (1958) for waves on quiescent water. More recently, Hedges et al. (1985) have produced experimental evidence supporting the application of an equilibrium range constraint in the case of long-crested random waves encountering opposing currents.

Kitaigordskii et al. (1975) showed that the directional frequency equilibrium range spectrum, $S_{\eta\eta ER}(\omega_a, \theta, U)$, may be written:

$$S_{\eta\eta ER}(\omega_a, \theta, U) = \sum_{i=1}^N \left[\frac{B^* k^{-3} G_{ER}(\theta)}{\partial \omega_a / \partial k} \right]_{k=k_i} \tag{26}$$

where B^* is a non-dimensional constant, $G_{ER}(\theta)$ is a spreading function describing the angular distribution of wave component energy within the equilibrium range, and $k=k_i$ are the N roots of the dispersion relationship. For the conditions considered here, waves are crossing a boundary from one current region to another (or moving from quiescent water on to a current) and there will be only one appropriate solution for k for specified values of d , U , ω_a and θ . If this were not the case then the following development would be modified as a result of the need to continue to apply the summation indicated in Eq. (26).

In the present case, Eq. (26) becomes simply:

$$S_{\eta\eta ER}(\omega_a, \theta, U) = \frac{B^* k^{-3}}{C'_{ga}} G_{ER}(\theta) \tag{27}$$

where C'_{ga} is defined by Eq. (19).

Information on the form of $G_{ER}(\theta)$ is scarce, although some possibilities are discussed by Kitaigordskii et al. (1975). In the absence of currents, a deep-water spectrum is often assumed to be symmetrical about the predominant wave direction. However, this assumption is not necessarily appropriate when waves are refracted on encountering a current. Whatever the form of the spreading function, it must satisfy:

$$\int_{\theta_n - \pi/2}^{\theta_n + \pi/2} G_{ER}(\theta) d\theta = 1 \tag{28}$$

where θ_n is the direction of the normal to the boundary between region 1 and region 2 (here measured relative to the predominant direction of the incident waves). Implicit in Eq. (28) is the assumption that the transformed wave field contains only components which have propagated into region 2 from region 1.

In the absence of any better information, a possible form for $G_{ER}(\theta)$ is now proposed.

The actual limiting spectral densities associated with the equilibrium range are likely to depend to some degree upon the shape which the transformed spectrum attempts to adopt as a result of the interaction between the waves and the current. Thus, in this study, the distribution of surface elevation variance with θ has been predicted for region 2 on the basis of wave action conservation (Eq. 21), and then spectral densities for each direction have had a

limit assigned to them which is in proportion to the unlimited value associated with that direction. Thus, if $S_{\eta\eta_{WA}}(\omega_a, \theta, U)$ is the spectral density predicted on the assumption that wave action is conserved, and $\sigma_{\eta_{WA}}^2$ is the associated total variance of surface elevation:

$$\sigma_{\eta_{WA}}^2 = \int_{\theta_n - \pi/2}^{\theta_n + \pi/2} \int_0^\infty S_{\eta\eta_{WA}}(\omega_a, \theta, U) d\omega_a d\theta \tag{29}$$

then the equilibrium range spreading function is given by:

$$G_{ER}(\theta) = \frac{1}{\sigma_{\eta_{WA}}^2} \int_0^\infty S_{\eta\eta_{WA}}(\omega_a, \theta, U) d\omega_a \tag{30}$$

Note that in the case of short-crested waves encountering a current flowing in, or directly against, their predominant direction, the equilibrium range constraint provided by Eqs. (27), (29) and (30) reduces to the relationship for long-crested waves (see below) as the spread of wave directions is narrowed. This is in accordance with expectations. Note, also, that the results above are for a general water depth.

For the special case when long-crested waves meet an opposing or a following current, $G_{ER}(\theta)$ may be omitted from Eq. (27). If deep-water conditions also exist then:

$$\omega_r^2 = gk \tag{31}$$

and

$$C'_{ga} = U + \frac{g}{2\omega_r} \tag{32}$$

Equation (27) then becomes:

$$S_{\eta\eta_{ER}}(\omega_a, U) = 2B^* g^2 \omega_r^{-5} [1 + (2U\omega_r/g)]^{-1} \tag{33}$$

in which $\omega_r = \omega_a - kU$ (Eq. 3). Equation (33) agrees with that given by Hedges (1981) and used by Hedges et al. (1985), except that they replaced constant $2B^*$ by another factor, A^* . (For a discussion of the value of constant B^* for conditions when $U \approx 0$, see Phillips (1977).)

CHANGES IN SPECTRA: TWO EXAMPLES

The changes in spectral form when long-crested random waves are transformed by following and opposing currents have been discussed by Hedges (1981), Burrows and Hedges (1985) and Hedges et al. (1985). Generally, following currents lead to a reduction of spectral density over all frequencies while adverse currents lead to an increase in spectral density (provided that

the waves can cross on to the current — see earlier discussion of Eq. 21). However, increases in spectral density are limited by wave breaking as parts of the spectrum become saturated. The situation is further complicated in short-crested seas by wave refraction, as the following examples illustrate. In each of these examples, it is assumed that spectral densities in the current region are given by Eq. (21) provided that values do not exceed the equilibrium range limit given by Eq. (27); otherwise Eq. (27) is assumed to apply with $G_{ER}(\theta)$ determined from Eqs. (29) and (30).

Figure 5 shows the typical changes in the surface elevation spectral density as short-crested waves encounter a current opposed to their predominant direction. The current velocity is 0.5 m/s and the water depth is 120 m. The wave condition on quiescent water is modelled using a Jonswap spectrum

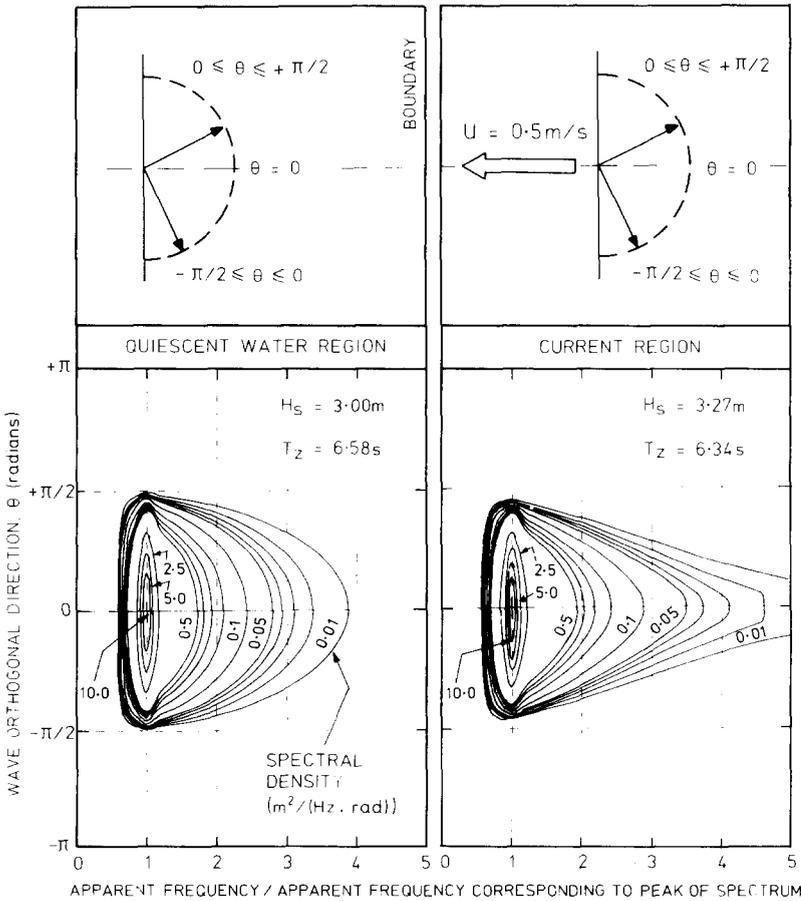


Fig. 5. Changes in surface elevation spectral density as short-crested random waves encounter an adverse current.

with the significant wave height, H_s , equal to 3.00 m, the zero-upcrossing period, T_z , equal to 6.58 s and with the peak at a value of ω_a of 0.767 rad/s. The corresponding peak enhancement factor is approximately 4 (see Isherwood, 1987). A cosine-squared spreading function has been used to distribute the wave energy about the predominant wave direction on quiescent water. This predominant direction is at right angles to the current boundary.

When the waves meet the current, the shape of the spectrum is changed considerably, with spectral densities generally being increased. The value of the constant B^* used to establish the limiting values associated with the equilibrium range of the spectrum has been taken as 0.025. Note that in this particular example, whilst the value of H_s is increased as the waves move on to the current, the value of T_z is slightly reduced.

Figure 6 shows the same quiescent water wave conditions as in the previous

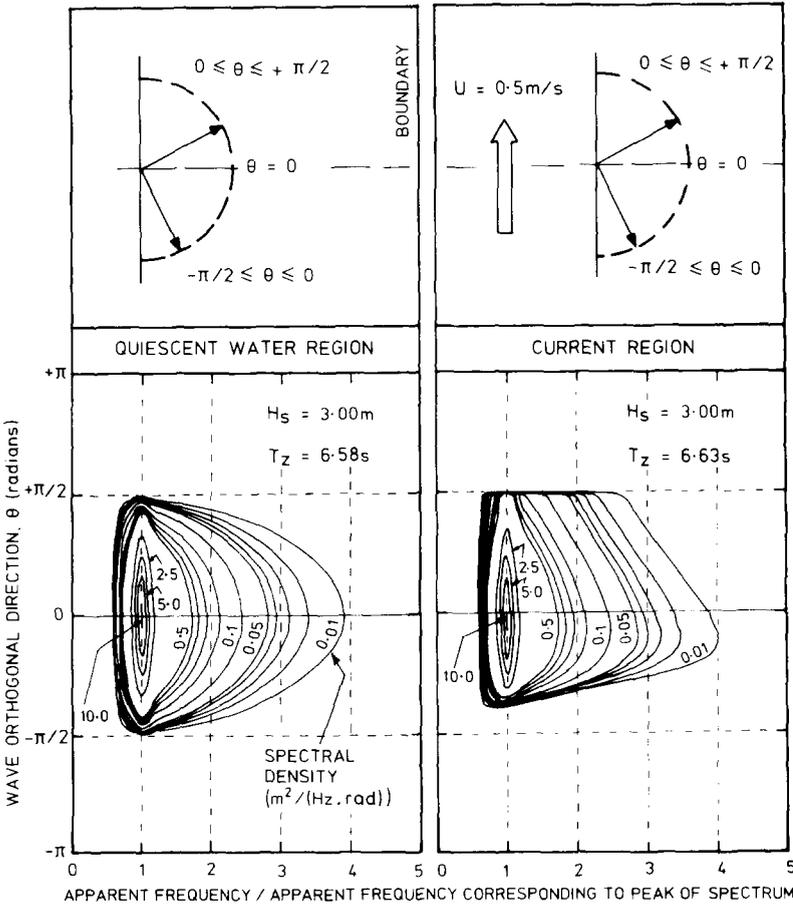


Fig. 6. Changes in surface elevation spectral density as short-crested random waves encounter a shearing current.

example, but in this case the waves encounter a current of 0.5 m/s travelling at right angles to the predominant incident wave direction. The effects of refraction distort the spectral form so that it is no longer symmetrical about the predominant wave direction once the waves move on to the current. Note, however, that despite the obvious changes to the directional distribution of wave energy, the value of H_s remains unchanged in this particular example. T_z is slightly increased as the waves move on to the current. Again, the value of B^* has been taken to be 0.025.

CONCLUDING REMARKS

A theoretical model has been developed to describe the interaction of short-crested random waves with large-scale currents. The transformed spectral densities are predicted using the principle of wave action conservation and due allowance is made for wave refraction. An equilibrium range constraint is then applied to account for the limit to the growth of spectral densities due to energy dissipation. The examples presented show the marked changes in spectral form which may be induced by currents. Whilst it is common practice in the offshore industry to adopt a parametric spreading function which is symmetrical about the mean wave direction, this study has shown that strong asymmetry may exist as a result of current action. The ability of the present theory to predict this asymmetry requires confirmation by field measurements or physical model tests.

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