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Discussion

# Discussion of “A new predictive formula for inception of regular wave breaking” by Yu Liu, Xiaojing Niu and Xiping Yu. Volume 58, Issue 9, September 2011, pp. 877–889. DOI: 10.1016/j.coastaleng.2011.05.004

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ABSTRACT

The authors have provided an interesting review of existing methods for predicting the onset of wave breaking. They have then proposed a new formula which requires the evaluation of the breaking wave celerity, $C_b$. This discussion aims to explore further the issue of predicting $C_b$, relating the authors' present method, based on Hedges' (1976) modification of Airy wave theory, to later work.

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## 1. Applying the free-surface boundary conditions in linear wave theory

Hedges' (1976) empirical modification to the Airy equation for the celerity, $C$, of water waves may be written as:

$$C = \sqrt{\frac{gL}{2\pi}\tanh\frac{2\pi}{L}(h+Z)} \tag{1}$$

Here, $g$ = gravitational acceleration, $L$ = wavelength, $h$ = water depth, and $Z$ = the elevation above mean-water-level at which the dynamic and kinematic free surface boundary conditions are applied during derivation of linear wave theory. These boundary conditions are applied at mean-water-level (i.e. at $Z = 0$) in Airy theory. However, by substituting the wave height, $H$, for $Z$:

$$C = \sqrt{\frac{gL}{2\pi}\tanh\frac{2\pi}{L}(h+H)}. \tag{2}$$

Eq. (2) reduces to the Airy expression as $H$ reduces to zero:

$$C = \sqrt{\frac{gL}{2\pi}\tanh\frac{2\pi h}{L}}. \tag{3}$$

It also reduces to the Airy result for deep water, regardless of the value of $H$:

$$C = \sqrt{\frac{gL}{2\pi}} \tag{4}$$

while matching the solitary wave expression for shallow water:

$$C = \sqrt{g(h+H)}. \tag{5}$$

Furthermore, Eq. (2) mimics the results for cnoidal waves in conditions in which cnoidal theory is most valid, i.e. when $HL^2/h^3 > 40$ (Hedges, 1995). Nevertheless, Booij (1981) suggested that the substitution $Z = H/2$ in Eq. (1) gave better agreement than $Z = H$ between predicted celerities and measured values in the breaker zone:

$$C = \sqrt{\frac{gL}{2\pi}\tanh\frac{2\pi}{L}\left(h+\frac{H}{2}\right)}. \tag{6}$$

Note, however, that the issue of substituting $Z = H/2$ had arisen earlier in discussion of Hedges' original paper (Hedges, 1977; Lewis, 1977). Adopting the subscript ‘$b$’ for values at the breaking point, this expression is reproduced in the paper under discussion. It is the expression which the authors use for predicting $C_b$.

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## 2. Further modifications to the Airy expression for wave celerity

Re-writing Eq. (6) using the wave number, $k = 2\pi/L$, gives:

$$C = \sqrt{\frac{g}{k}\tanh(kh+\varepsilon)} = \frac{gT}{2\pi}\tanh(kh+\varepsilon) \quad \text{(7a and 7b)}$$

in which $\varepsilon = kH/2$ and $T$ is the wave period. Notwithstanding its usefulness in this form, Hedges (1987) further modified the expression to agree with Stokes' second-order solution for deep water, writing:

$$C = \sqrt{\frac{g}{k}\left(1+\varepsilon^2\right)\tanh\left(\frac{kh+\varepsilon}{1+\varepsilon^2}\right)} = \frac{gT}{2\pi}\left(1+\varepsilon^2\right)\tanh\left(\frac{kh+\varepsilon}{1+\varepsilon^2}\right). \quad \text{(8a and 8b)}$$

When $\varepsilon$ is at its maximum value of about 0.45, Eq. (8a) gives a celerity in deep water which is about 10% greater than that predicted by Airy theory for a small-amplitude wave of the same length, while Eq. (8b) gives a celerity in deep water which is about 20% greater than that predicted by Airy theory for a small amplitude wave of the same period. Li and Lee (2002) provided an explicit approximation so that Eqs. (8a and 8b) could be evaluated without iteration. The authors may wish to consider employing this approximation in estimating $C_b$.

Fig. 1 compares the values given by Eqs. (7b) and (8b) for $\varepsilon = 0.2$. According to Airy theory, $C/(gT/2\pi) - \tanh(kh)$ is zero. Thus, the figure illustrates the different allowances for wave nonlinearity provided by the two expressions. Note that, when $\varepsilon = 0.2$, waves may be expected to break once $kh < 0.5$ (approximately) as H/h then exceeds 0.8.

Eqs. (1) to (8a and 8b) relate to waves of permanent form. However, Hedges and Kirkgoz (1981) measured the speeds of wave crests at the breaking point, $C_b$, and used Eq. (1) with $Z = \eta_b$, the breaking wave crest elevation, to compare with the measured speeds. They wrote:

$$C_b = \sqrt{\frac{gL_b}{2\pi}\tanh\frac{2\pi}{L_b}(h_b+\eta_b)} \quad (9)$$

in which $h_b$ is the water depth at the breaking point (so that $h_b + \eta_b$ is the depth of water beneath the wave crest) and $L_b = C_bT$. This expression reduces to the nonlinear shallow water solution, $C_b = [g(h_b + \eta_b)]^{0.5}$, when $h_b + \eta_b << L_b$. It provided reasonable agreement with Hedges' and Kirkgoz's laboratory measurements of the crest speeds of breaking waves. However, use of this equation clearly requires knowledge of the breaking wave crest elevation, $\eta_b$.

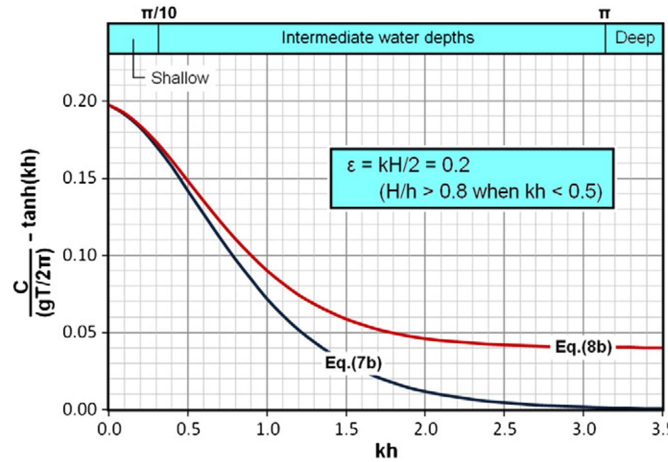


**Fig. 1.** Allowances for wave nonlinearity provided by Eqs. (7b) and (8b) for $\varepsilon = 0.2$.

## 3. Concluding comment

In summary, the authors have employed Hedges' (1976) modification of Airy wave theory, with $Z = H/2$, in order to estimate $C_b$, the breaking wave celerity. Ascertaining $C_b$ is fundamental to evaluating their breaking index. This discussion highlights additional modifications to the Airy theory expression for wave celerity, which are designed to improve agreement with measured values.

## References

Booij, N., 1981. Gravity waves on water with non-uniform depth and current. Research Report, Department of Civil Engineering, Delft University of Technology, The Netherlands.
Hedges, T.S., 1976. An empirical modification to linear wave theory. Proceedings of the Institution of Civil Engineers 61, 575–579.
Hedges, T.S., 1977. Discussion of 'An empirical modification to linear wave theory'. Proceedings of the Institution of Civil Engineers 63, 252–253.
Hedges, T.S., 1987. An approximate model for nonlinear dispersion in monochromatic wave propagation models, by J.T. Kirby and R.A. Dalrymple: discussion. Coastal Engineering 11, 87–89.
Hedges, T.S., 1995. Regions of validity of analytical wave theories. Water, Maritime and Energy Journal, Proceedings of the Institution of Civil Engineers 112, 111–114.
Hedges, T.S., Kirkgoz, M.S., 1981. An experimental study of the transformation zone of plunging breakers. Coastal Engineering 4, 319–333.
Lewis, A.W., 1977. Discussion of 'An empirical modification to linear wave theory'. Proceedings of the Institution of Civil Engineers 63, 251–252.
Li, R.-J., Lee, D.-Y., 2002. A weakly nonlinear water wave model taking into account dispersion of wave phase velocity. Chinese Journal of Oceanology and Limnology 20, 107–112.