# **RESONANT WAVE REFLECTION BY TRANSVERSE BEDFORMS AND ITS RELATION TO BEACHES AND OFFSHORE BARS**

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## ABSTRACT

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On an erodible sand bed, there may be a coupling between the reflection of incident wave energy and the development of transverse bedforms. The nature of this interaction is examined, with particular reference to a laboratory experiment in which a veneer of sand was introduced into a pre-existing, fixed, pattern of bars. The observed motion of the mobile sand suggested potential development of this existing bar system in the up-wave direction, but not in the down-wave direction. It is argued here that this phenomenon may be significant in terms of the growth of shore-parallel bars off beaches and for coastal protection in general.

## INTRODUCTION

When surface waves are incident upon a region of undulating sea bed, it is well known that the wave energy may be reflected and scattered by the topography. The general problem of wave-energy reflection by sea-bed topography has been examined by Long (1973). However, at the edge of the sea, waves may also be reflected by beaches (Carter et al., 1973; Suhayda, 1974). In some cases this may lead to the formation of shore-parallel bars which in turn may reflect more of the incoming wave energy.

While these features have been widely reported in the literature (e.g., Greenwood and Davidson-Arnott, 1975, 1979; Goldsmith et al., 1982; Bowman and Goldsmith, 1983) the processes which control their formation are not clear (e.g. see Davidson-Arnott and Randall, 1984). Thus experimental (Short, 1975) and theoretical evidence (Lau and Travis, 1973) has suggested a standing wave mechanism while Bowen and Inman (1971) and Holman and Bowen (1982) have suggested that edge waves may be important. Other mechanisms include the interaction between breaking waves and rip currents (Greenwood and Davidson-Arnott, 1979) and harmonics generated during wave shoaling (Boczar-Karakiewicz et al., 1981, in Davidson-Arnott and Randall, 1984).

In this paper we examine the case in which waves interact with a preexisting pattern of bedforms, such as shore-parallel bars on beaches, and the way in which this interaction may lead to further growth of the bedforms.

## THEORETICAL CONSIDERATIONS

In some recent papers Davies (1980, 1982) has examined wave reflection from a patch of transverse bedforms on an otherwise flat bed. These results were obtained for sinusoidal bedform profiles, but can be extended quite simply to other bedforms. It was shown that the amplitude of the reflected surface wave,  $a_{\rm R}$ , was related to that of the incident wave,  $a_{\rm I}$ , by the reflection coefficient:

$$\frac{a_{\rm R}}{a_{\rm I}} = \frac{2bk}{(2kh + \sinh 2kh)} (-1)^m \frac{2k/l}{(2k/l)^2 - 1} \sin 2kL \tag{1}$$

where:

b = bedform amplitude,  $l = 2\pi/\lambda_B = \text{bedform wavenumber } (\lambda_B = \text{bedform wavelength}),$  m = the number of sinusoidal bedforms in the patch, $2L = 2m\pi/l = \text{the overall length of bedform area},$ 

h = the mean water depth above the regions of flat bed,

and  $k = 2\pi / \lambda_w$  = surface wavenumber ( $\lambda_w$  = surface wavelength).

These definitions are illustrated in Fig.1. Equation 1 is valid for all combinations of values of l and k, except l = 2k. When the surface wavelength,  $\lambda_w$ , is exactly twice the bedform wavelength,  $\lambda_B$ , eq.1 is replaced by:



Fig.1. Surface elevations for the near-resonant case in which m = 10, L = 500 cm, b = 5 cm,  $\lambda_B = 100$  cm, h = 41.7 cm and  $\lambda_w = 203$  cm (2k/l = 0.985). The instantaneous profiles of wave elevation (A, B, C, D) are plotted, together with the envelope of wave elevation  $\pm E$ . The development of a partially standing wave structure between  $x = \pm L$  and x = -L is evident. Further details of these calculations are given in Davies and Heathershaw (1983).

$$\frac{a_{\rm R}}{a_{\rm I}} = \frac{2bk}{(2kh + \sinh 2kh)} \cdot \frac{m\pi}{2} \qquad \left(\frac{l=2k}{a_{\rm R}} < a_{\rm I}\right) \tag{2}$$

If the critical condition l = 2k is approximately satisfied, eqs.1 and 2 reveal that there is a resonant Bragg-type interaction between the surface waves and the bedforms which may give rise to a substantial back-reflected wave. In particular, eq.2 indicates that the reflected wave amplitude increases linearly with the number of bedforms m, and depends on the dimensionless expressions bk and kh, and upon the incident wave amplitude  $a_{I}$ .

While the resonance at l = 2k is the most important aspect of the interaction between the surface waves and the bedforms, it may be seen from eq.1 that  $a_{\rm R}$  is oscillatory in the quotient of the length of bedform area, 2L, and the surface wavelength  $\lambda_{\rm w} = 2\pi/k$  and that the reflected wave amplitude has secondary maxima at harmonics of the critical wave.

These results, for the overall reflection coefficient of an area of bedforms, have been extended recently (Davies and Heathershaw, 1983, 1984) to some considerations of the detailed nature of the wave field over the bedforms themselves. An extensive set of laboratory observations was undertaken (Heathershaw, 1982, 1983; Davies and Heathershaw, 1983, 1984) to test all aspects of the theory over a wide range of parameter settings. The observations were made for laminar flow conditions over a fixed bed containing a number of transverse sinusoidal bars. It was found that there was good agreement between eqs.1 and 2 and measurements of the reflected wave amplitude. It was also demonstrated, theoretically and experimentally, that the partially standing wave pattern which exists, at resonance, on the up-wave side of the bedforms gives way, in an almost linear manner over the patch itself, to a purely progressive (transmitted) wave on the down-wave side.

This is illustrated in Fig.1 for a typical near-resonant case, with parameter settings relating to one of the laboratory experiments already referred to above: bar amplitude b = 5 cm, bar wavelength  $\lambda_{\rm B} = 100$  cm, the number of bars m = 10, the water depth h = 41.7 cm, surface wavelength  $\lambda_{\rm w} = 203$  cm, from which it follows that 2k/l = 0.985. The predicted reflected wave amplitude is given from eq.1 by  $a_{\rm R} = 0.509 a_{\rm I}$ .

Results are presented for the predicted surface elevation  $(\eta)$  normalised in each case by the incident wave amplitude  $a_{I}$ . The two outer curves,  $\pm E$ , indicate the envelope of wave elevation, and the inner curves, A to D, indicate successive instantaneous profiles of surface elevation at wave phase angles separated by one quarter of a wave period. In Fig.1 it should be noted that, as a first approximation, the incident waves have been assumed to be unattenuated across the bedforms. Further details of this calculation are given in Davies and Heathershaw (1983, 1984).

The horizontal velocity field associated with the waves in Fig.1 is shown in Fig.2. Here the amplitude of the horizontal velocity is shown as a function of horizontal distance x, for discrete values of the normalised depth Y, namely Y = 0 (free surface), Y = -0.5, -0.75 and -1.0 (bed). The values of the horizontal velocity  $\hat{u}$  have been normalised by  $U_0 = ga_I k/\sigma$ , the horizontal velocity amplitude of the incident waves at the free surface (g = acceleration



Fig.2. Amplitude of horizontal velocity  $\hat{u}$ , normalized by  $U_0 = ga_I k/\sigma$ , for the nearresonant case in Fig.1 and for the levels Y = 0, -0.5, -0.75 and -1.0. Further details of these calculations are given in Davies and Heathershaw (1983).

of gravity, and  $\sigma = 2\pi/T$  = wave frequency where T = wave period). On both the up-wave and down-wave sides of the bedforms there is a decrease in velocity amplitude from peak values at the free surface to minimum values at the bed. However, it should be noted that the velocity maxima do not necessarily coincide with the crests of the bedforms. The implications of this result for the movement of sediment beneath wave patterns of the type shown in Fig.1 are discussed in the following section, in relation to bedform stability on an erodible bed.

## LABORATORY EXPERIMENT WITH AN ERODIBLE BED

We have illustrated in Figs.1 and 2 how, for the simple case of sinusoidal bedforms, the reflection of incident surface waves at resonance  $(k \approx l/2)$  gives rise to a partially standing wave pattern on the up-wave side of a system of transverse bars. If the bed is erodible, it is possible that new bars will develop on the up-wave side of the bedforms as a result of this wave pattern. Intuitively, accumulation of material might be expected at positions on the bed with the smallest bottom velocity amplitudes, and erosion at positions with the greatest velocity amplitudes (see Fig.2). It is argued below, however, that the situation may be more complicated than this. Ultimately, for there to be a coupling between wave reflection and bedform growth, accumulation and erosion must occur on the *existing* transverse bars in a way which suggests bar growth, rather than bar destruction, by the wave action.

In order to examine these possibilities, a single experimental trial was carried out as part of the overall sequence of laboratory observations described earlier. These observations were carried out in a glass-walled wave tank,  $45.72 \times 0.91 \times 0.91$  m (nominally  $150' \times 3' \times 3'$ ) at the Coastal Engineering Research Center, Fort Belvoir, Virginia. Some preliminary results of this work were described by Heathershaw (1982) and full details of the experimental set-up are given in Davies and Heathershaw (1983, 1984). In the single trial, sand of mean diameter  $235 \,\mu$ m was distributed in a thin (<0.05 cm) uniform layer throughout the (fixed) bar area, and for a distance of 2 m in the down-wave direction and 3.5 m in the up-wave direction. The trial was carried out with m = 2 bars, water depth h = 15.6 cm, ripple amplitude b = 5 cm, bar wavelength  $\lambda_{\rm B} = 100$  cm, and a (resonant) surface wavelength of 206.7 cm. The measured value of the reflected wave amplitude with these parameter settings was  $a_{\rm R} \approx 0.34 \, a_{\rm I}$ .

The trial commenced with the water in the tank at rest. The stroke of the wave generator was then increased until sand motion was initiated. The subsequent development of ripples of small wavelength [O(5 cm)] was then recorded photographically at intervals over approximately 130 min, after which no further changes could be detected. Small sand ripples were first observed to occur both on the crests of the 1 m wavelength bars, and also in the up-wave direction in patches approximately 1 m apart (i.e., with the same spacing as the original bars). At the outset, these patches of small ripples formed under the nodes of the partially standing waves on the upwave side, where the wave-induced bed velocities were greatest. With increasing time, ripples started to form up-wave of these nodes, with the largest ripples forming about half way between the nodes and antinodes. The ripple heights increased with increasing distance from the nodes such that, at the nodes, heights were typically 0.1-0.2 cm while, midway between the nodes and antinodes, heights were typically about 1.5 cm and wavelengths were 5.5 cm (see Fig.3). These ripples were asymmetrical, having their steepest faces in the down-wave direction. Despite this, there was a net movement of sediment in the up-wave direction, associated with the action of vortex shedding from the ripple crests. In general, it might be expected that, for net sediment accumulation to occur close to an antinode of surface elevation, ripple heights would increase in this direction, even though the water particle excursions due to the waves decrease. The situation on the up-wave side of the bars may be contrasted with that on the down-wave side, where small ripples, asymmetric in the down-wave direction, were observed to grow and spread over the entire bed, with uniform height and spacing. Typical ripple heights were about 0.5-0.8 cm, and ripple wavelengths were about 4 cm.

The development of the patch of small sand ripples on the up-wave side of the original bars is shown schematically in Fig.3, and a sequence of photographs illustrating the evolution of the ripple patches during the 130 min period is shown in Fig.4a—f. A photograph is also included of the ripple sheet on the down-wave side (Fig.4g). To enable clear photographs to be taken in still water, the wave generator was stopped for each photograph and then restarted. The position of the ripple patches in relation to the standing wave nodes and antinodes is illustrated schematically in Fig.3. The positions of the nodes and antinodes themselves were obtained from earlier independent



Fig.3. Schematic diagram showing the formation of short wavelength ripple patches beneath the partially standing wave pattern resulting from reflection by long wavelength (1 m) transverse bars. Ripples appear first in small patches beneath the nodes (N), and these patches then grow in extent towards the antinodes (A). Within a ripple patch the wavelength decreases from about 5.5 cm near the antinode (a) to about 2 cm near the node (b). There is a corresponding decrease in ripple height from about 1.5 to 0.2 cm within a patch. Ripple asymmetry throughout the test section, and on either side of it, was found to be in the down-wave direction.

measurements on two ripples with the same ripple amplitude to water depth ratio (b/h = 0.32). Details of the wave field giving rise to the observed sediment motion (Fig.4a—f) were also determined from these earlier measurements made without any sand in the tank. The absence of sand, and hence small ripples, in the short section of the tank near the transverse bars might have led to slightly different wave conditions compared with those when sand was present, due to increased wave-energy dissipation. However, this effect was probably very small.

In Fig.5, theoretical results for the horizontal velocity field are shown for the parameter settings of the above experiment. The run was close to resonance with 2k/l = 0.968 and with the reflected wave amplitude predicted from eq.1 as  $a_{\rm R} = 0.455a_{\rm I}$ . This latter figure may be compared with the measured value of  $a_{\rm R} = 0.34a_{\rm I}$ . The discrepancy between these values is consistent with uncertainties in the experimental results arising from back reflection of small amounts of wave energy by the wave absorbing beach at the down-wave end of the wave tank (see Davies and Heathershaw, 1983). As in Figs.1 and 2, the incident waves have been assumed to be unattenuated in amplitude across the barred test section, the results for attenuated incident waves, in this case, being very similar. Velocity amplitudes are plotted as functions of horizontal distance (x) for the heights Y = 0 (free surface), -0.5and -1.0 (bed). The velocities at these heights are nearly in phase in respect of variations in x but, as in Fig.2, are attenuated in the vertical direction rather differently over the flat and rippled parts of the bed. [Note that the divergences in the predicted bed velocity field at the ends of the patch (x =

 $\pm L$ , Y = -1.0) have no general significance, merely being associated with discontinuities in the bed slope at both ends of the patch.] For the present purpose, we need only be concerned with the velocity amplitude at the bed, and what is of interest is the relationship between this quantity and the threshold velocity amplitude of the sand size in question (235  $\mu$ m). For monochromatic waves with the experimental wave period, and for a flat bed, Komar and Miller's (1975) formula gives the threshold velocity amplitude as 11.4 cm s<sup>-1</sup>, or  $\hat{u}/U_0 = 0.89$  in normalised form as plotted in Fig.5. It may be seen that, on the down-wave side, the threshold velocity amplitude is less than the predicted bed velocity amplitude. This is consistent with the observed formation of a sheet of ripples of short wave length on the region of flat bed. On the up-wave side of the bars, where the reflected wave gives rise to a partially standing wave pattern, the predicted velocities are greater than the threshold velocity in those parts of the bed marked "E" and less in those marked "D". In the former, sediment movement should occur and erosion may be expected, particularly near maxima of the bed velocity amplitude. Over the barred test section itself predictions of this kind, based upon Komar and Miller's threshold velocity results, are complicated by the fact that the bed is not flat; in particular, on sloping parts of the bed, significantly different values of the threshold velocity may be expected. If we ignore this complication and treat the matter in the same way as for a flat bed, we again arrive at the prediction of regions of deposition and erosion on the bed. For both the flat and rippled regions of the bed, the predictions of deposition "D" and erosion "E" were reasonably well borne out by the experimental observations (Fig.5). On account of the grain size in use, sediment motion occurred as bed load only; if there had been a suspended load, deposition would possibly have occurred throughout the regions marked "D".

Previous observations of patches of rippled, and of unrippled, sand on a flat erodible bed beneath partially standing waves have been made in the laboratory by Kennedy and Falcon (1965). However, the situation described by these workers was rather different from that in the present experiment. Firstly, the partially standing wave pattern in their experiment was caused by the superimposition of incident waves and waves reflected by a beach. Secondly, their observations were made in conditions exceeding the threshold of sediment motion over the entire bed. Furthermore, the positioning of the patches of ripples and of the flat bed in their experiment was complicated both by the existence of significant drift velocities in the tank, and by the asymmetrical nature of the bottom velocity field associated with the (relatively steep) incident waves which were generated. Despite these differences with the present experiment, the observations of Kennedy and Falcon provide an interesting, and contrasting, example of the effects of a partially standing wave structure on an erodible bed.

# INTERPRETATION OF EXPERIMENTAL RESULTS

The predicted and observed partially standing wave pattern on the up-wave side of the bedforms in the present experiment suggests that new ripples may



Fig.4a and b. Caption on p.331.





Fig.4c and d. Caption on p.331.







Fig.4. Sequence of photographs showing the formation of patches of ripples of short wavelength in 235  $\mu$ m sand, beneath the partially standing wave pattern up-wave of two transverse sinusoidal bars of 5 cm amplitude and 1 m wavelength. The water depth h was 15.6 cm and the surface wavelength  $\lambda_w$  was 206.5 cm. The measured reflection coefficient of the barred test section was  $|a_{\mathbf{R}}/a_{\mathbf{I}}| \approx 0.34$ . The following sequence shows how the ripple patches evolved with increasing time t:(a) t = 0. Sand lying in a thin layer (<0.05 cm) throughout the barred test section and on either side of it. (b) t = 10 min. Small ripple patches on the crest of the first bar, and at a distance of 1 m up-wave of this crest. (c) t = 30 min. Formation of three ripple patches increasing in extent. (e) t = 90 min. Ripple patches increasing in extent. (f) t = 130 min. Ripple patches increasing in extent.

Finally, in (g), a photograph shows the continuous sheet of ripples of short wavelength, at least 1 m in extent, formed on the down-wave side of the barred test section.



Fig.5. Predicted horizontal velocity amplitudes are compared with observations of sediment movement in the wave tank, for the case in which m = 2, b = 5 cm,  $\lambda_B = 100$  cm, h = 15.6 cm and  $\lambda_w = 206.7$  cm (2k/l = 0.968). The predicted reflection coefficient of the barred test section was  $a_R/a_I = 0.455$ , which may be compared with the measured value of  $|a_R/a_I| \approx 0.34$ . Horizontal velocity amplitudes (see Fig.2) are plotted for the levels Y = 0, -0.5 and -1.0 and the predicted bed velocity amplitude (Y = -1.0) is compared with the threshold velocity for a 235- $\mu$ m grain size. Thus regions of deposition (D) and erosion (E) are predicted, and these are compared with the laboratory observations.

develop in patches on the region of flat bed as a result of wave reflection by the existing, longer wavelength, bedforms. In the present experiment, it was found that deposition "D" and erosion "E" of sand occurred in regions of the bed where the predicted horizontal bed velocity amplitudes were minimum and maximum, that is beneath the antinodes and nodes of elevation, respectively. However, as argued by Davies (1980), this may not always be the case as the residual circulation cells resulting from bottom friction under a standing wave have a rather complicated structure (Longuet-Higgins, 1953; Noda, 1969; Johns, 1970; Liu and Davis, 1977). In particular, for a smooth flat bed and a purely standing wave, the direction of the residual velocity changes at a certain height above the bed. If the boundary layer is laminar, this height is equal to 0.93  $\delta_*$  (Longuet-Higgins, 1953) where  $\delta_*$  is the Stokes' layer thickness (=  $\sqrt{2\nu_w/\sigma}$  in which  $\nu_w$  is the kinematic viscosity). If it is turbulent, the height is considerably larger than this (Johns, 1970). In both cases, water particle residual motions immediately above the bed, in the "inner" layer, are towards the positions of greatest horizontal motion, that is towards the nodes of elevation. This has been demonstrated by Noda (1969), who found that vinyl pellets moving on a *smooth* bed accumulated beneath the nodes of a standing wave. In the upper, or "outer", layer immediately above, dye moved in the opposite direction. The implications of this rather complicated velocity structure for sediment movement have been discussed by Johns (1970). For rough beds, and for laminar flow in the boundary layer, Johns has suggested that any material in motion near the bed will probably be present in the "outer" layer (by virtue of the very small "inner" layer thickness which, for the conditions shown in Fig.4, was of the order of  $\delta_{\mu}$  = 0.07 cm), and that the residual velocity in this layer will probably give an indication of the direction and magnitude of sediment transport. In the laminar case, therefore, the influence of the residual velocity is consistent with our results. However, in the physically more interesting case of a turbulent boundary layer, the greater thickness of the inner layer suggests that sediment motion may be confined to this layer within which the residual motion of water particles, and hence sediment, is towards positions of greatest horizontal motion, that is towards the nodes of surface elevation. In any practical situation the true result will depend to a large extent on the sediment grain size involved and whether the sediment is transported as bed load only, or as both bed load and suspended load.

The laboratory experiments of Nielsen (1979) demonstrated bar growth beneath standing waves, with sediment accumulation occurring at the antinodes of surface elevation. For very fine sand (~80  $\mu$ m) moving in suspension, Nielsen observed upwardly convected clouds of grains above the evolving bar crests, and interpreted his observations on the basis of the residual transport pattern for a laminar boundary layer described above. In the present experiment, there was a partially standing wave structure on the up-wave side of the bars, rather than a purely standing wave as in Nielsen's experiments. Also, the boundary layer was laminar, by virtue of its low wave Reynolds number (see Davies and Heathershaw, 1983), and sediment motion occurred as bedload only. Laminar flow was also confirmed with dye tests. Our observations of incipient ripple formation are, therefore, consistent with the expected directions of sediment movement for relatively large grains and laminar flow.

# COUPLING BETWEEN WAVE REFLECTION AND GROWTH OF BEDFORMS

It was argued earlier that, for there to be a coupling between wave reflection and bar growth, accumulation and erosion of sediment must occur on the existing bars in a way which suggests growth rather than destruction by the wave action. In the discussion of Fig.5, it was suggested that, if the effects of bed slope were ignored, deposition and erosion should occur at the positions marked "D" and "E", respectively. This result was quite well supported by the experimental observations. However, these observations also suggested either the destruction of the two existing bars or, possibly, their overall movement in the up-wave direction, by erosion of sand from the regions both of their crests and their down-wave slopes, and deposition on their upwave slopes. Unfortunately, with the fixed bed in the present experiments, it was not possible to establish which of these alternatives would have been the true outcome on a fully erodible bed of single grain size.

There are further considerations which may be relevant to the question of the stability of the existing bedforms. For example, Sleath (1974, 1976) has shown that both a uniform oscillation, and a progressive wave motion, over a rippled bed give rise to residual circulation cells in which the fluid near the bed is transported towards the ripple crests. In the present context, the correct interpretation of this result for bedform stability and growth again depends rather critically upon the manner in which sediment is transported. As a result of all these uncertainties, and also of the possible effects of differing grain sizes in natural sediments it would be premature to conclude that the existing bars in the present experiment were either stable or unstable.

However, we have carried out a brief theoretical investigation of this problem and preliminary results have been given in Davies and Heathershaw (1983). The stability of the bars depends on the coincidence or otherwise of the crests with the positions of velocity maxima. Davies and Heathershaw have shown that this may depend critically on the ratio 2k/l at or close to resonance. In particular for a fixed pattern of 10 by 1 m wavelength bars it was found that velocity maxima and minima were displaced in the down-wave direction from the crest and trough positions by up to 25 cm and 45 cm, respectively, as 2k/l increased from 0.95 to 1.05, over which range of values wave reflection was still strongly resonant. These displacements were greatest on the up-wave section of the bar system and in general decreased in the down-wave direction. These results indicated that for values of 2k/l just below resonance  $(2k/l \approx 0.95)$  the velocity maxima were situated over the bar crests, suggesting that they might not be stable. For 2k/l values above resonance  $(2k/l \approx 1.05)$  the velocity maxima were displaced towards the troughs in which position they might be expected, intuitively, to maintain a stable bar system. These results also suggested that the bar system might, in some circumstances, migrate in an up-wave direction. However, further experimental work is required to determine the stability conditions for bedforms on an erodible bed.

For a bed comprising a mixture of grain sizes, the question of the stability of a bar system is more involved, as demonstrated by Scott (1954) in a laboratory study concerned mainly with onshore/offshore sediment transport on a beach. Scott commenced his experiments with an initial beach profile, and then allowed waves to "mold the beach until an approximate equilibrium profile was obtained". The beach profile changes involved the formation of a series of offshore bars, and in his description of this process Scott makes the following observations: "Reflections from the beach were visible from the start of each run, but a (pronounced) standing wave did not become visible before the offshore bars had formed. Evidently the bars caused additional reflections which built up the amplitude of the standing wave .... The nodes of the standing waves were in all cases over the offshore bars ...". In other words, the bars (i.e. bar crests in our earlier terminology) were subjected to greater horizontal velocities than were the regions between the bars (troughs), and yet the bars were stable. This was explained by Scott in terms of the observed tendency for coarser grains to be found on top of the bars, and for finer grains to be found in the troughs between the bars. Clearly, this phenomenon is highly relevant to the earlier discussion of the stability of bottom ripples, though it is not a matter which we are able to pursue further here.

In the sea further complications may arise if mixtures of sediments containing high proportions of carbonate material are present. These sediments, which are less dense than quartz, would be expected to move at lower threshold velocities. However, the shape of carbonate grains is likely to influence their overall mobility. This aspect is considered to be beyond the scope of the present paper, although we do hope to examine this problem in future experimental work.

## APPLICATIONS IN NATURE

The most obvious application of the present results in the natural environment concerns the formation of shore-parallel bars off beaches. These features have been described in tidal as well as in tideless seas (e.g., Zenkovich, 1967; King, 1972) although only in the latter case do the bars achieve a fully developed state. Sand bar formation is generally considered to be due to wave breaking (see Komar, 1976), the position of a bar representing the average break point for certain wave conditions. Keulegan (1948, in Schwartz, 1972), in a series of laboratory experiments, showed that the bar crests occurred just seaward of the break point as a result of the combination of forward mass transport in the shoaling wave and a seaward-directed scour in the breaking wave. Bar position was shown to be controlled by wave height and wave steepness.

However, of more relevance to this study are those cases of bar formation in water too deep for wave breaking to be a controlling factor. One such example is given by Zenkovich (1967) and quoted in Carter et al. (1973), in which shore-parallel bars have been eroded into a limestone platform at the foot of a beach in the Black Sea comprising non-cohesive sediments. These features extended a distance of 1500 m from the shore, with regular wave length, and were thus well beyond the effects of breaking waves. Similarly Saylor and Hands (1971) have reported bars at depths where breaking waves did not occur. Under these circumstances it is necessary to look at alternative bar building mechanisms.

Waves of relatively low amplitude and steepness may be reflected by an initially plane beach (Carter et al., 1973) leading to standing wave patterns which may extend some distance from the shore (Suhayda, 1974). Alternatively, standing waves may be generated within the surf zone as a result of the time varying breakpoint forcing mechanism which has been described by Symonds et al. (1982). In either of these cases, sediment accumulation might be expected to occur at or near the antinodes of elevation of the standing wave as a result of residual circulations induced by bottom friction (see earlier). As described previously, experimental evidence for this effect,

involving wave reflections from a plane wall, was provided by Nielsen (1979). However, experiments on model beaches (Carter et al., 1973) have shown that sediment accumulation and ridge or bar building might occur beneath the nodes of elevation, a result similar to Noda's (1969) and due to sediment motion being influenced by the residual circulation in an "inner" layer. This result may be compared with our own finding, for reflection from a pattern of pre-existing bars, that sediment accumulation occurred at a point midway between a node and an antinode and that, due to grain size, this was influenced by circulation in the "outer" layer. Theoretical studies by Lau and Travis (1973) also showed that a standing wave mechanism might explain the offshore dependence of bar spacing, and they found good agreement between theory and observations from sites in Lake Michigan, the Black Sea and Escambia Bay, Florida.

Similar evidence has been provided by Short (1975), from multiple bars in the Chukchi Sea, North Alaska, who found that the positions of bar crests agreed well with standing wave predictions. More importantly, Short was able to show that measured infragravity wave spectra contained significant amounts of wave energy at the correct frequencies for the observed bar spacings (wave periods of 75–100 s).

The significance of our results, therefore, is that once bars have been formed, as a result of the standing wave mechanism outlined previously, resonant interaction of incoming waves with existing bars can, potentially at least, provide a mechanism for further bar growth in the seaward direction. Since the timescales involved in bar formation are large compared with the tidal period, this effect is likely to be most significant in areas where the tidal range is small and where wave conditions are relatively constant. Changes in bedform wavelength will not take place as rapidly as changes in the surface wavelength due to different wave conditions or changes in water depth. Thus wave energy which was previously reflected by the bars, might propagate across them and result in serious coastal erosion. This effect was observed in Lake Michigan by Saylor and Hands (1971), following a change in mean water level. Similarly, as pointed out by Lau and Travis (1973), removal of offshore bars by dredging may produce equally disastrous results.

# CONCLUSIONS

On the basis of both theoretical considerations, and a single laboratory experiment, it has been demonstrated that, potentially, wave reflection from bottom undulations provides a mechanism for the growth of existing bedforms in the up-wave direction. This conclusion is based upon observations of areas of erosion and deposition on the up-wave side of a barred test section, which exhibited the same spacing as the original transverse bars. Since the laboratory experiment was conducted with a thin veneer of mobile sand placed on a fixed immobile bar system, it was not possible to determine whether the (fixed) region of bars was a stable, or an unstable, feature on the bed. Further theoretical and experimental studies are needed in this connection. For there to be a coupling between wave reflection by an existing bar structure and the growth of new bars on its up-wave side, it is clearly necessary for the existing bars to be stable features on the bed.

Both the observations (Fig.4a-f), and the theory (Fig.5) suggest that the crests of the bars in the present experimental trial may have been unstable, that is that the crests may have been subject to erosion. This would almost certainly have been the case for a bed comprising a single grain size. However, it would not necessarily have been so for a bed comprising a mixture of grain sizes. Previous work has indicated that, in the latter case, relatively coarse grains may migrate to the bar crests, thus making the crests relatively resistant to erosion. In this connection, some detailed calculations of the effects of bottom friction are needed to determine the roles of the bottom stress, and of residual velocities, in situations in which there may be both bed-load and suspended-load motion. Ideally, such work should be supported by a comprehensive experimental study with a fully erodible bed, in which both the wave parameters, and the sediment size, are varied over wide ranges. If a coupling between wave reflection and bedform growth was to be clearly demonstrated by such an exercise, this would have most significant implications for coastal protection, not least in connection with the formation of shore-parallel bars off beaches.

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#### REFERENCES

- Boczar-Karakiewicz, B., Paplinska, B. and Winiecki, J., 1981. Formation of sandbars by surface waves in shallow water. Laboratory Experiments. Rozpr. Hydrotech., 41: 111-125.
- Bowen, A.J. and Inman, D.L., 1971. Edge waves and crescentic bars. J. Geophys. Res., 76: 8662-8671.
- Bowman, D. and Goldsmith, V., 1983. Bar morphology of dissipative beaches: An empirical model. Mar. Geol., 51: 15-33.
- Carter, T.G., Lui, P.L. and Mei, C.C., 1973. Mass transport by waves and offshore sand bedforms. Proc. Am. Soc. Civ. Eng., J. Waterways, Harbors Coastal Eng. Div., WW2: 165-184.
- Davidson-Arnott, R.G.D. and Randall, D.C., 1984. Spatial and temporal variations in spectra of storm waves across a barred nearshore. In: B. Greenwood and R.A. Davis, Jr. (Editors), Hydrodynamics and Sedimentation in Wave-Dominated Coastal Environments. Mar. Geol., 60: 15-30.
- Davies, A.G., 1980. Some interactions between surface water waves and ripples and dunes on the seabed. Inst. Oceanogr. Sci., Rep., 108: 134 pp.

- Davies, A.G., 1982. The reflection of wave energy by undulations on the seabed. Dyn. Atmos. Oceans, 6: 207-232.
- Davies, A.G. and Heathershaw, A.D., 1983. Surface wave propagation over sinusoidally varying topography: theory and observation. Inst. Oceanogr. Sci., Rep., 159: 181 pp.
- Davies, A.G. and Heathershaw, A.D., 1984. Surface wave propagation over sinusoidally varying topography. J. Fluid. Mech., 144: 419-443.
- Goldsmith, V., Bowman, D. and Riley, K., 1982. Sequential stage development of crescentic bars: Hahoterim Beach, Southeastern Mediterranean. J. Sediment. Petrol., 52: 233-249.
- Greenwood, B. and Davidson-Arnott, R.G.D., 1975. Marine bars and near-shore sedimentary processes, Kouchibouguac Bay, New Brunswick. In: J. Hails and A.P. Carr (Editors), Nearshore Sediment Dynamics and Sedimentation. Wiley, New York, N.Y., pp. 123-150.
- Greenwood, B. and Davidson-Arnott, R.G.D., 1979. Sedimentation and equilibrium in wave-formed bars: a review and case study. Can. J. Earth Sci., 16: 312-332.
- Heathershaw, A.D., 1982. Seabed—wave resonance and sand bar growth. Nature, 296: 343-345.
- Heathershaw, A.D., 1983. Wave reflection from undulating seabed topography. Proc. 18th Coastal Eng. Conf., Cape Town, pp.543-554.
- Holman, R.A. and Bowen, A.J., 1982. Bars, bumps and holes: models for the generation of complex beach topography. J. Geophys. Res., 87: 457-468.
- Johns, B., 1970. On the mass transport induced by oscillatory flow in a turbulent boundary layer. J. Fluid. Mech., 43: 177–185.
- Kennedy, J.R. and Falcon, M., 1965. Wave-generated sediment ripples. Mass. Inst. Technol., Hydro. Lab. Rep., 86: 55 pp.
- Keulegan, G.H., 1948. An experimental study of submarine sand bars. US Army Corps of Engineers, Beach Erosion Board Tech. Rep., 3: 40 pp.
- King, C.A.M., 1972. Beaches and Coasts. Edward Arnold, London, 570 pp.
- Komar, P.D., 1976. Beach Processes and Sedimentation. Prentice-Hall, Englewood Cliffs, N.J., 429 pp.
- Komar, P.D. and Miller, M.C., 1975. Sediment threshold under oscillatory waves. Proc. 14th Coastal Eng. Conf., Copenhagen, pp.765-775.
- Lau, J. and Travis, B., 1973. Slowly varying Stokes waves and submarine longshore bars. J. Geophys. Res., 78: 4489-4497.
- Liu, A.-K. and Davis, S.H., 1977. Viscous attenuation of mean drift in water waves. J. Fluid Mech., 81: 63-84.
- Long, R.B., 1973. Scattering of surface waves by an irregular bottom. J. Geophys. Res., 78: 7861-7870.
- Longuet-Higgins, M.S., 1953. Mass transport in water waves. Philos. Trans. R. Soc. London, Ser. A, 245: 535–581.
- Nielsen, P., 1979. Some basic concepts of wave sediment transport. Tech. Univ. Denmark, Inst. Hydrol. Hydr. Eng., Ser. Pap., 20: 160 pp.
- Noda, H., 1969. A study of mass transport in boundary layers in standing waves. Proc. 11th Coastal Eng. Conf., London, pp.227-247.
- Saylor, J.H. and Hands, E.B., 1971. Properties of longshore bars in the Great Lakes. Proc. 12th Coastal Eng. Conf., Washington, D.C., pp.839-853.
- Schwartz, M.L., 1972. Spits and Bars. Dowden, Hutchinson and Ross, Stroudsburg, Pa., 452 pp.
- Scott, T., 1954. Sand movement by waves. US Army Corps of Engineers, Beach Erosion Board, Tech. Memo, 48: 37 pp.
- Short, A.D., 1975. Multiple offshore bars and standing waves. J. Geophys. Res., 80: 3838-3840.
- Sleath, J.F.A., 1974. Mass transport over a rough bed. J. Mar. Res., 32: 13-24.
- Sleath, J.F.A., 1976. On rolling-grain ripples. J. Hydraul. Res., 14: 69-81.
- Suhayda, J.N., 1974. Standing waves on beaches. J. Geophys. Res., 79: 3065-3071.
- Symonds, G., Huntley, D.A. and Bowen, A.J., 1982. Two-dimensional surf beat: Long wave generation by a time varying break point. J. Geophys. Res., 87: 492-498.
- Zenkovich, V.P., 1967. Processes of Coastal Development. Oliver and Boyd, London, 738 pp.