

# Radar Altimeter Mean Return Waveforms from Near-Normal-Incidence Ocean Surface Scattering

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**Abstract**—Under assumptions common in radar altimetry, the mean backscattered return power for a short-pulse radar and near-normal-incidence scattering from a rough ocean surface is given by the convolution of several terms. For a nearly Gaussian transmitted pulse shape scattered from a nearly Gaussian distributed sea surface, a small-argument series expansion of one of the terms within the convolution leads to a several-term power series expansion for the mean return waveform. Specific expressions are given for the first four terms. These results, which require much less computer time than would the otherwise necessary numerical convolution, are useful for data analysis from current or past radar altimeters and for design studies of future systems. Several representative results are presented for an idealized SEASAT radar altimeter.

## I. INTRODUCTION

TWO RECENT NASA earth orbiting satellites, GEOS-3 and SEASAT-1, have carried pulsewidth-limited radar altimeters with provisions for sampling a number of points in the individual radar return waveforms.<sup>1</sup> For these and similar systems, the mean return waveform is [1] the convolution of a) the average impulse response of the quasi-calm sea surface, b) the sea surface elevation distribution, and c) the radar system point-target response (transmitted pulse as affected by the receiver bandwidth). The first term a) includes the effects of the antenna beamwidth and the off-nadir pointing angle; "quasi-calm" emphasizes that an incoherent surface scattering process is assumed but that the sea surface elevation distribution is separately written in b).

A number of papers have described the extraction of ocean significant waveheight (SWH) from altimeter waveform samples [2]–[7]. SWH is proportional to the surface elevation distribution's second moment; there is also a preliminary altimeter measurement [5] of the surface skewness which is related to the elevation distribution's third moment. These papers all use the same basic procedure: a specific probability distribution form (usually a simple Gaussian) is assumed for the ocean surface; amplitude and timing biases are removed from waveform sampler data if necessary; a suitable waveform sample averaging time is chosen; and the average sampled return waveform is best-fitted by a theoretical template through some process of varying template parameters until some typical least-squares error criterion is satisfied.<sup>2</sup> In several cases [3],

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<sup>1</sup> GEOS-3 had 16 waveform sample-and-hold (S&H) gates at 6.25-ns spacing, while SEASAT-1 had 60 S&H gates at 3.125-ns spacing.

<sup>2</sup> One exception to the template fitting procedure is described in [8] in which Priester and Miller describe work intended to estimate the surface height probability density function without the necessity of assuming a specific functional form; this method is difficult to implement and interpret and will not be considered for the purposes of this paper.

[7] the differences between adjacent sampled waveform values are fitted to the derivative of the theoretical return waveform, but this is a minor variation of the basic fitting procedure. In the general template fitting, either 1) a simplified form is assumed for the different convolution terms so that the final expression will be simple and suited to short computer running times, or 2) a more general, more complete template is used at the expense of large computation times from having to do several numerical convolutions for each sample point within the line fitting process.

This paper presents general expressions for the radar mean return waveform for cases when the ocean surface elevation distribution and the radar system point-target response can each be represented by a modified Gaussian form including skewness and kurtosis terms, and the antenna pattern can be assumed to be Gaussian in angle; these expressions give the mean return waveform at the times of interest in the region of the risetime portion ("ramp portion") of the waveform for small off-nadir angles. The expressions should be useful either for waveform parameter estimation from experimental data, or for waveform generation as part of design studies for future radar altimeters. Comparable results could be obtained from numerical convolution, but this paper's expressions have the advantages of reducing computational time and of avoiding the detailed questions of sample density and roundoff problems which would be part of any numerical convolution modeling study.

## II. EVALUATION OF MEAN RETURN WAVEFORM

This work is based on Brown's paper [1] which reviewed the assumptions and limitations of the convolutional model for near-normal-incidence rough surface backscattering of short-pulse radar waveforms. The general square-law-detected waveform  $W(t)$  is given by the convolution

$$W(t) = P_{FS}(t) * q_s(t) * s_r(t) \quad (1)$$

where  $P_{FS}(t)$  is the average flat surface impulse response,  $q_s(t)$  is related to the surface elevation probability density of scattering elements (specular points), and  $s_r(t)$  is the radar system point-target response. Each of the three terms in (1) will be separately discussed; then through a small-argument series expansion for Brown's  $P_{FS}(t)$  the  $W(t)$  of (1) will be expressed as a power series for which the first four terms<sup>3</sup> are evaluated in this paper.

<sup>3</sup> The number of power series terms needed for adequate waveform approximation is a function of satellite altitude, off-nadir angle, and radar pulsewidth, and of how late on the return waveform the evaluation is desired. Figs. 1 and 2, and the discussion of these figures in Section III, indicate the convergence of the series expansion in a typical application.

### A. Average Flat Surface Impulse Response Function

For practical satellite-carried radar altimeters the flat surface impulse response is given by [1, eq. (9)] as

$$P_{FS}(t) = A \exp(-\delta t) I_0(t^{1/2} \beta) U(t) \quad (2)$$

in which

$$\delta = (4/\gamma)(c/h) \cos(2\xi) \quad (3)$$

and

$$\beta = (4/\gamma)(c/h)^{1/2} \sin(2\xi). \quad (4)$$

In (2),  $U(t)$  is a unit step function,  $I_0(t^{1/2}\beta)$  is a modified Bessel function, and  $\xi$  is the absolute off-nadir pointing angle. In (3) and (4),  $c$  is the speed of light,  $h$  is the satellite altitude, and  $\gamma$  is an antenna beamwidth parameter defined as in Brown's [1, eq. (4)] by a Gaussian approximation to the antenna gain of the form

$$G(\theta) = G_0 \exp[-(2/\gamma) \sin^2 \theta]. \quad (5)$$

If  $\theta_w$  is the usual antenna beamwidth, i.e., the angular full width at the half-power points, (3) and (4) can be written as

$$\delta = \frac{\ln 4}{\sin^2(\theta_w/2)} (c/h) \cos(2\xi) \quad (6)$$

and

$$\beta = \frac{\ln 4}{\sin^2(\theta_w/2)} (c/h)^{1/2} \sin(2\xi). \quad (7)$$

The amplitude term  $A$  in (2) contains several constants:

$$A = \frac{G_0^2 \lambda^2 c \sigma^\circ(0)}{4(4\pi)^2 L_p h^3} \exp\left(-\frac{4}{\gamma} \sin^2 \xi\right) \quad (8)$$

where  $\lambda$  is the radar wavelength,  $\sigma^\circ(0)$  is the ocean surface backscattering cross section at normal incidence,  $G_0$  is the radar antenna boresight gain, and  $L_p$  is the two-way propagation loss over and above the free-space loss.

In (8), the possible variation of  $\sigma^\circ$  with the angle of incidence (or, equivalently, with increasing time in the return waveform) has been ignored; this is a reasonable assumption for sea surface scattering in the short-pulse (<20 ns pulsewidth) radar altimeters considered here. Because radar return signals are usually normalized by an automatic gain control system, we ignore all individual terms within the  $A$  in (8) and use  $A$  as a simple amplitude scaling term.

### B. Radar-Observed Surface Elevation Density Function

Equation (1) includes a term  $q_s(t)$  which is the surface specular point density function written in the altimeter's time domain; the conversion factor from surface elevation measurements in meters to two-way ranging time in nanoseconds is  $c/2 = 0.15$  m/ns. For a satellite altimeter above the ocean, a lowering of the ocean surface would lead to an increase in the altimeter's ranging time. Hence this paper's time-domain-described surface elevation distribution has opposite signs for its odd moments compared to the oceanographer's elevation

distribution defined for elevation positive upward out of the surface. In particular, this paper's surface skewness will have a sign opposite to the oceanographer's skewness. This paper makes the usual assumption that the surface specular point density function is identical to the true (geometric) surface elevation density function, but the possibility of a difference between the radar-observed and the true geometric elevation densities remains an open issue for future work.<sup>4</sup>

In previous work the ocean surface elevation density function  $q_s(t)$  has been assumed to have the skewed Gaussian form given in the time domain by

$$q_s(t) = \frac{1}{\sqrt{2\pi} \sigma_s} \left[ 1 + \frac{\lambda_s}{6} H_3(t/\sigma_s) \right] \exp\left[-\frac{1}{2} (t/\sigma_s)^2\right] \quad (9)$$

where  $\sigma_s$  is the surface rms waveheight,  $\lambda_s$  is the skewness, and  $H_3$  is a Hermite polynomial. Pierson and Mehr [9] discussed this form for radar altimeter analyses; this is a low-order case of a general probability function by Longuet-Higgins [10] for a random variable that is weakly nonlinear. The Hermite polynomials for argument  $z$  needed in this paper are

$$H_3(z) = z^3 - 3z \quad (10)$$

$$H_4(z) = z^4 - 6z^2 + 3$$

and

$$H_6(z) = z^6 - 15z^4 + 45z^2 - 15.$$

Under the assumption of a Gaussian sea surface height distribution the SWH is approximately four times the rms waveheight. It is related to  $\sigma_s$  by

$$\text{SWH} = 4(c/2)\sigma_s \quad (11)$$

in which the  $c/2$  converts from the ranging time to the surface elevation. Most of the radar altimeter SWH measurements to date have been based on the pure Gaussian resulting from setting  $\lambda_s = 0$ , but (9) is the surface elevation density function used in attempts to recover a surface skewness [5].

Equation (9) results from taking the first two terms only in a general Gram-Charlier series [11]. Recent work by Huang and Long [12], based on laboratory measurements of the surface elevation density function for a wind-generated wave field, suggests that (9) is an inadequate form for the surface elevation density, and that it is necessary to use the four-term series given by

$$q_s(t) = \left[ 1 + \frac{\lambda_s}{6} H_3(t/\sigma_s) + \frac{\kappa_s}{24} H_4(t/\sigma_s) + \frac{\lambda_s^2}{72} H_6(t/\sigma_s) \right] \times \frac{1}{\sqrt{2\pi} \sigma_s} \exp\left[-\frac{1}{2} (t/\sigma_s)^2\right], \quad (12)$$

<sup>4</sup> Radar measurements by Yaplee *et al.* [15] can be interpreted [16] as showing that the mean of the scattering distribution is displaced downward from the true water level by 20 percent of the rms waveheight, and recent theoretical work by Jackson [17], assuming infinitely long-crested waves, is in agreement with these measurements. However, preliminary results from the aircraft-borne Surface Contour Radar experiment [18] indicate that this bias may be in the range of 4-8 percent of the rms waveheight with more measurements currently underway [19].

where  $\kappa_s$  is the kurtosis with other quantities as already defined. The general waveform result to be derived in the following portions of this paper will be based on (12), but  $\lambda_s^2$  terms will be kept separate from the  $\lambda_s$  terms so that the final result can easily be converted to a surface elevation density of form (9) instead of (12) if desired.

### C. Effect of Radar System Point-Target Response

The radar system point-target response  $s_r(t)$  is primarily the transmitted radar pulse shape but also includes effects of the bandwidth of the receiver. We take  $s_r(t)$  as nearly Gaussian and use the same general form already discussed for the surface elevation density:

$$s_r(t) = \left[ 1 + \frac{\lambda_r}{6} H_3(t/\sigma_r) + \frac{\kappa_r}{24} H_4(t/\sigma_r) + \frac{\lambda_r^2}{72} H_6(t/\sigma_r) \right] \times \frac{1}{\sqrt{2\pi}\sigma_r} \exp \left[ -\frac{1}{2} (t/\sigma_r)^2 \right]. \quad (13)$$

Then the convolution in (1) of  $q_s(t) * s_r(t)$  can immediately be written as

$$B(t) = q_s(t) * s_r(t) = \left[ 1 + \frac{\lambda}{6} H_3(t/\sigma) + \frac{\kappa}{24} H_4(t/\sigma) + \frac{\lambda^2}{72} H_6(t/\sigma) \right] \times \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{1}{2} (t/\sigma)^2 \right] \quad (14)$$

in which  $\sigma$ ,  $\lambda$ , and  $\kappa$  (with no subscripts) are the composite risetime, skewness, and kurtosis; these composite quantities can easily be shown to be related to the contributing surface elevation and point target quantities by

$$\sigma^2 = \sigma_s^2 + \sigma_r^2, \quad (15)$$

$$\lambda = \lambda_s(\sigma_s/\sigma)^3 + \lambda_r(\sigma_r/\sigma)^3, \quad (16)$$

and

$$\kappa = \kappa_s(\sigma_s/\sigma)^4 + \kappa_r(\sigma_r/\sigma)^4. \quad (17)$$

In a practical radar altimeter the waveform sampling gates will be positioned by a range tracker having its own range noise (jitter) characteristics, and the track jitter probability density function must be included as an additional term in the convolutional description of the mean return waveform. For this paper the jitter will be assumed negligible in comparison to the other terms.<sup>5</sup>

### D. Expansion and Solution for $W(t)$

The waveform  $W(t)$  is given by the convolution of the  $P_{FS}(t)$  of (2) with the  $B(t)$  of (14):

$$W(t) = P_{FS}(t) * B(t) = \int_{-\infty}^{\infty} P_{FS}(z) B(t-z) dz \quad (18)$$

<sup>5</sup> Usually, when the jitter cannot be treated as negligible, it can be described by a near-Gaussian form having its own  $\sigma_j$ ,  $\lambda_j$ , and  $\kappa_j$ ; then (15), (16), and (17) will each have one additional term of the same form as the two terms already written. For instance, (15) will become  $\sigma^2 = \sigma_s^2 + \sigma_r^2 + \sigma_j^2$ .

but the presence of the Bessel function  $I_0$  in  $P_{FS}(t)$  prevents the easy integration of (18). For the waveform regions of interest in the case of short-pulse radar altimeters in this paper, it is possible to expand the  $I_0$  in the small argument series expansion [13]:

$$I_0(z) = \sum_{n=0}^{\infty} (z^2/4)^n \left( \frac{1}{n!} \right)^2 \quad (19)$$

which leads to the term-by-term integration in (18) with the result

$$W(t) = \frac{A}{6} \exp[-d(\tau + d/2)] \sum_{n=0}^{\infty} \left( \frac{1}{n!} \right)^2 \left( \frac{\beta^2 \sigma}{4} \right)^n C_n(t) \quad (20)$$

where

$$C_n(t) = C_{n0} + \kappa C_{n1} + \lambda^2 C_{n2}, \quad (21)$$

$$C_{n0} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau} (\tau - z)^n [6 + \lambda H_3(z + d)] e^{-z^2/2} dz, \quad (22)$$

$$C_{n1} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau} (\tau - z)^n [H_4(z + d)] e^{-z^2/2} dz, \quad (23)$$

and

$$C_{n2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau} (\tau - z)^n [H_6(z + d)] e^{-z^2/2} dz \quad (24)$$

with

$$\tau = \frac{t - t_0}{\sigma} - d \quad (25)$$

and

$$d = \delta \sigma. \quad (26)$$

Notice that an arbitrary time origin shift  $t_0$  has been written in (25); in template fitting for parameter recovery from altimeter data  $t_0$  is one of the parameters to be varied since in practical altimeters a range tracker moves the waveform sampling gates back and forth in a (time) position relative to the true mean waveform.

Carrying out the integrations indicated in (22), (23), and (24) for any given  $n$  will produce results in terms of  $G(\tau)$  and  $P(\tau)$ , where  $G(\tau)$  is the Gaussian,  $G(\tau) = (2\pi)^{-1/2} \exp(-\tau^2/2)$ , and  $P(\tau)$  is the probability integral;<sup>6</sup> two new functions of time,  $D_{nm}$  and  $E_{nm}$ , are then defined by

$$C_{nm} = D_{nm} P(\tau) + E_{nm} G(\tau), \quad \text{for } m = 0, 1, \text{ and } 2. \quad (27)$$

These  $D_{nm}$  and  $E_{nm}$  have been worked out for  $n = 0, 1, 2$ ,

<sup>6</sup> Often the error function  $\text{erf}(\tau)$  is written instead of  $P(\tau)$ . These are related by  $\text{erf}(\tau) = 2P(2^{1/2}\tau) - 1$ .  $P(\tau)$  can easily be implemented on a computer by a series expansion given in [13].

and 3, with the following results:

$$\begin{aligned}
 D_{00} &= 6 + \lambda d^3 \\
 E_{00} &= \lambda(1 - 3d^2 - 3d\tau - \tau^2) \\
 D_{01} &= d^4/4, \\
 E_{01} &= (d - d^3) + \left(\frac{3}{4} - \frac{3}{2}d^2\right)\tau - d\tau^2 - \frac{\tau^3}{4} \\
 D_{02} &= d^6/12 \\
 E_{02} &= \left(-\frac{3}{2}d + \frac{5d^3}{3} - \frac{d^5}{2}\right) + \left(-\frac{5}{4} + \frac{15d^2}{4} - \frac{5d^4}{4}\right)\tau \\
 &\quad + (3d - \frac{5}{3}d^3)\tau^2 + \left(\frac{5}{6} - \frac{5}{4}d^2\right)\tau^3 \\
 &\quad - \frac{d}{2}\tau^4 - \frac{\tau^5}{12}
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 D_{10} &= -3\lambda d^2 + (6 + \lambda d^3)\tau \\
 E_{10} &= (6 + 3\lambda d + \lambda d^3) + \lambda\tau \\
 D_{11} &= -d^3 + \frac{d^4}{4}\tau \\
 E_{11} &= \left(-\frac{1}{4} - 3d^2 + \frac{d^4}{4}\right) + d\tau + \frac{\tau^2}{4} \\
 D_{12} &= -\frac{d^5}{2} + \frac{d^6}{12}\tau \\
 E_{12} &= \left(\frac{1}{4} - \frac{5}{4}d^2 + \frac{5}{4}d^4 + \frac{d^6}{12}\right) + \left(-\frac{3}{2}d + \frac{5}{3}d^3\right)\tau \\
 &\quad + \left(-\frac{1}{2} + \frac{5}{4}d^2\right)\tau^2 + \frac{d}{2}\tau^3 + \frac{\tau^4}{12}
 \end{aligned} \tag{29}$$

$$\begin{aligned}
 D_{20} &= (6 + 6\lambda d + \lambda d^3) - 6\lambda d^2\tau + (6 + \lambda d^3)\tau^2 \\
 E_{20} &= (-2\lambda - 6\lambda d^2) + (6 + \lambda d^3)\tau \\
 D_{21} &= \left(3d^2 + \frac{d^4}{4}\right) - 2d^3\tau + \frac{d^4}{4}\tau^2 \\
 E_{21} &= (-2d - 2d^3) + \left(-1 + \frac{d^4}{2}\right)\tau \\
 D_{22} &= \left(\frac{5}{2}d^4 + \frac{d^6}{12}\right) - d^5\tau + \frac{d^6}{12}\tau^2 \\
 E_{22} &= (d - \frac{10}{3}d^3 - d^5) + \left(\frac{1}{2} - \frac{5}{2}d^2 + \frac{d^6}{12}\right)\tau \\
 &\quad - d\tau^2 - \frac{\tau^3}{6}
 \end{aligned} \tag{30}$$

$$\begin{aligned}
 D_{30} &= (-6\lambda - 9\lambda d^2) + (18 + 18\lambda d + 3\lambda d^3)\tau \\
 &\quad - 9\lambda d^2\tau^2 + (6 + \lambda d^3)\tau^3 \\
 E_{30} &= (12 + 18\lambda d + 2\lambda d^3) - 9\lambda d^2\tau + (6 + \lambda d^3)\tau^2 \\
 D_{31} &= (-6d - 3d^3) + (9d^2 + \frac{3}{4}d^4)\tau - 3d^3\tau^2 + \frac{d^4}{4}\tau^3 \\
 E_{31} &= \left(\frac{3}{2} + 9d^2 + \frac{d^4}{2}\right) - 3d^3\tau + \frac{d^4}{4}\tau^2 \\
 D_{32} &= (-10d^3 - \frac{3}{2}d^5) + \left(\frac{15}{2}d^4 + \frac{d^6}{4}\right)\tau - \frac{3d^5}{2}\tau^2 + \frac{d^6}{12}\tau^3 \\
 E_{32} &= \left(-\frac{1}{2} + \frac{15}{2}d^2 + 10d^4 + \frac{d^6}{6}\right) \\
 &\quad + \left(3d - \frac{3d^5}{2}\right)\tau + \left(\frac{1}{2} + \frac{d^6}{12}\right)\tau^2.
 \end{aligned} \tag{31}$$

### III. SUMMARY AND EXAMPLES

In brief review, the waveform is given by (20), with the number of terms required being a function of time, pointing angle, and antenna beamwidth. For instance, a single term is adequate for the GEOS-3 altimeter waveforms at any time within the span covered by the individual sampling gates for off-nadir angles within  $1^\circ$ , while SEASAT-1 with its smaller beamwidth requires the first three terms in (20) for any position within its sampling gate span for nadir-angles within  $1^\circ$ . The decision of how many terms are required is made in each application by comparing the relative contribution of each successive higher term to the total; Figs. 1 and 2 as discussed in the next paragraph provide examples for an idealized SEASAT altimeter. For each term  $C_n(t)$ , (21) gives  $C_n(t)$  in terms of  $C_{n0}$ ,  $C_{n1}$ , and  $C_{n2}$  with these three quantities in turn given by (27) which uses (28), (29), (30), or (31) for  $n = 0, 1, 2$ , or  $3$ , respectively. This procedure may appear cumbersome and does assume the use of a computer, but these results are much easier to use and require far less computer time than the otherwise necessary numerical convolution procedures.

Figs. 1-6 provide examples of mean return waveforms calculated from (20) for a radar altimeter having an antenna beamwidth  $\theta_w = 1.6^\circ$  at a height  $8 \times 10^5$  m above the ocean and having a pure Gaussian radar system point-target response of 3.125 ns full width at 1/2 height so that  $\sigma_r = 1.327$  ns, and  $\lambda_r = \kappa_r = 0$  in (14). These numbers are nominal SEASAT-1 values [14], and these figures predict SEASAT-1 measured mean return waveforms for the limit of infinite averaging time.<sup>7</sup> To investigate how many terms to include in the waveform expansion for this case, define  $\Delta_k$  as

$$\Delta_k = W_k - W_{k-1} \tag{32}$$

<sup>7</sup> The actual SEASAT-1 mean return waveforms will differ from this paper's figures because of a non-Gaussian sidelobe structure in the transmitted pulse shape. MacArthur [14, p. 4-50] describes an attempt to treat this sidelobe structure by increasing the effective width of the Gaussian used to represent the transmitted pulse; however, a discussion of the full consequences of the actual SEASAT-1 pulse shape is beyond the scope of this paper.

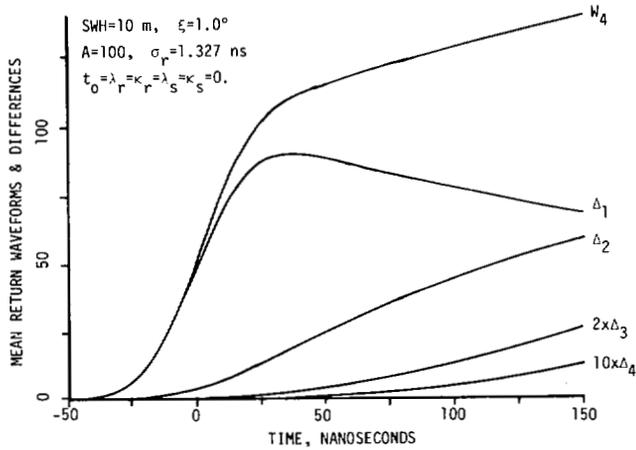


Fig. 1. Idealized SEASAT radar altimeter mean return waveforms, at  $1.0^\circ$  off-nadir angle, showing effects of adding terms to waveform expansion.

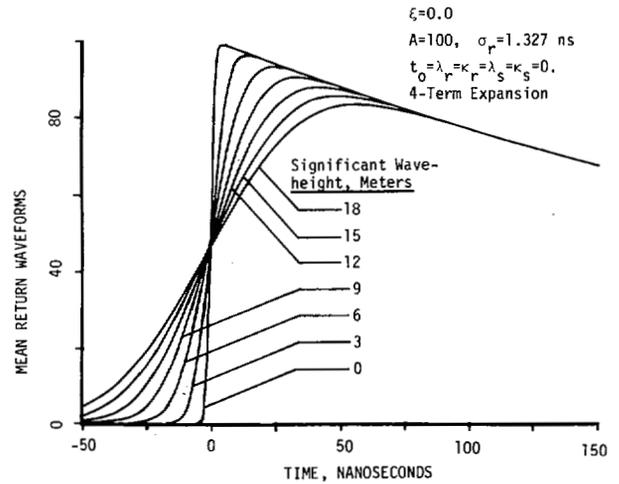


Fig. 4. Idealized SEASAT radar altimeter mean return waveforms, showing effects of different ocean significant waveheights.

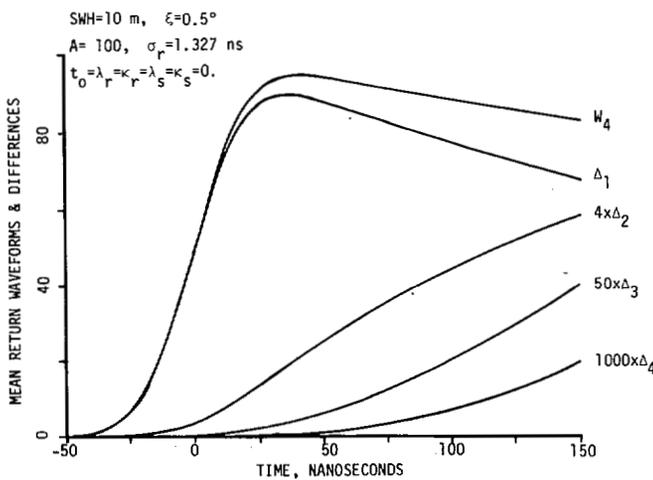


Fig. 2. Idealized SEASAT radar altimeter mean return waveforms, at  $0.5^\circ$  off-nadir angle, showing effects of adding terms to waveform expansion.

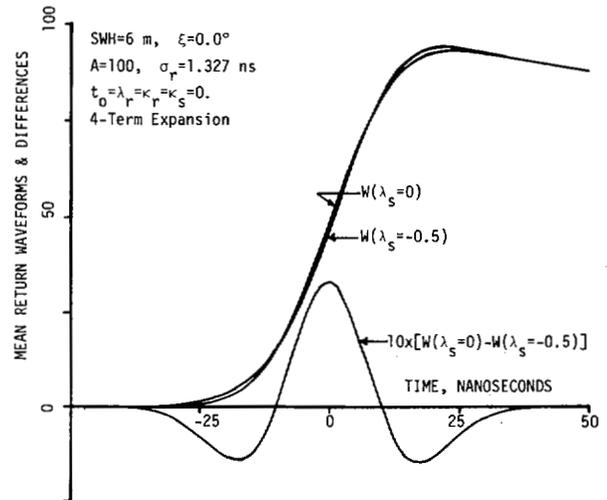


Fig. 5. Idealized SEASAT radar altimeter mean return waveforms, showing effects of skewness in surface elevation probability density function.

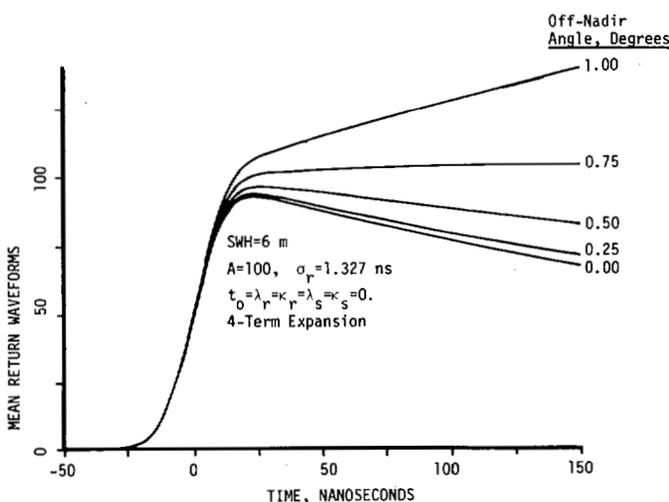


Fig. 3. Idealized SEASAT radar altimeter mean return waveforms, showing effects of different off-nadir angles.

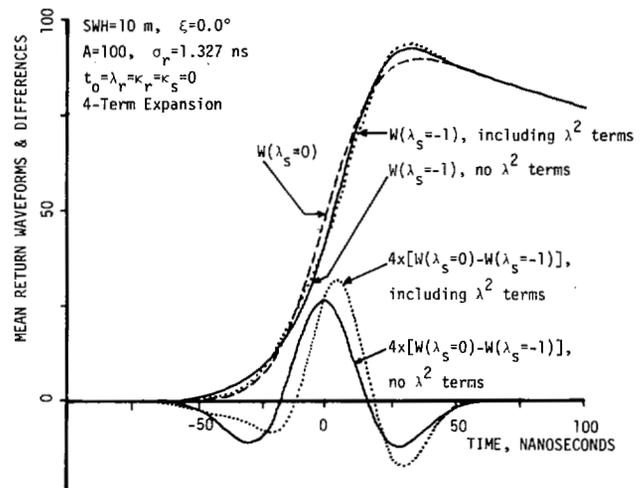


Fig. 6. Idealized SEASAT radar altimeter mean return waveforms, showing effects of including skewness squared terms in surface elevation probability density function.

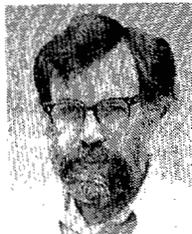
where  $W_k$  is this paper's waveform for the first  $k$  terms. For instance,  $W_1$  is the waveform including only the first term  $C_0(\xi)$  in (20). In this notation,  $W_0 = 0$  and  $\Delta_1 = W_1$ . Figs. 1 and 2 show  $W_4$  and  $\Delta_1 - \Delta_4$  for off-nadir angles of 1.0 and 0.5°, respectively. Fig. 1 shows that  $\Delta_4$  is less than one percent of  $W_4$  for times within 100 ns, the maximum SEASAT sampling gate span, and thus  $W_3$  is an acceptable description of a SEASAT waveform if one percent is acceptable as a maximum discrepancy. Fig. 2 shows that the agreement becomes much better when the off-nadir angle drops from 1.0 to 0.5°.

Figs. 3 and 4 show SEASAT waveform effects of varying the off-nadir angle,  $\xi$ , and the SWH, respectively, and Fig. 5 shows the effect of a nonzero surface skewness  $\lambda_s$ . Figs. 1-5 were plotted for no  $\lambda_s^2$  terms included in the surface distribution (12). Fig. 6 compares including  $\lambda_s^2$  terms to the result from no  $\lambda_s^2$  terms for the extreme value  $\lambda_s = -1$  (at least twice the  $|\lambda_s|$  value that might be expected for a real ocean surface). Even for this extreme case, neglecting  $\lambda_s^2$  produces fairly good agreement to including  $\lambda_s^2$ , and the agreement becomes rapidly better as  $|\lambda_s|$  approaches zero. Thus the  $\lambda_s^2$  terms in (12) can be neglected for an idealized SEASAT altimeter over the real ocean.

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