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Ocean Current Wave Interaction Study

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A numerical model has been developed to incorporate refraction of ocean surface gravity waves by major ocean currents. The model is initialized with directional wave spectra and verified with aircraft synthetic aperture radar *X* band spectra, laser profilometer spectra, and pitch and roll buoy data. Data collected during the Marineland test experiment are used as surface truth observations for the wave-current study. Evidence of Gulf Stream refraction and trapping of surface waves as well as caustics in the current is shown and modeled assuming a nonuniform Gulf Stream distribution. Frequency and directional resolution of the wave spectral distribution and the current refraction patterns illustrates the need for further study of ocean current-wave interaction in wave refraction studies.

INTRODUCTION

The refraction of ocean surface waves on the continental shelf is an important factor in hindcast and in forecast wave studies. Practical techniques for developing wave refraction diagrams were initiated by *Johnson et al.* [1948], *Arthur et al.* [1952], *Munk and Arthur* [1952], *Pierson* [1951], *Pierson et al.* [1953, 1955], *Longuet-Higgens* [1956, 1957], and others. The graphical techniques for ray construction used in these papers are called the 'orthogonal methods.'

More recent numerical techniques for wave refraction have been used by *Griswold* [1963], *Harrison and Wilson* [1964], *Wilson* [1966], *Dobson* [1967], *Chao* [1970], and others. Numerical calculations of wave orthogonals can be quickly plotted by computer more accurately and faster than they can be by hand. Wave studies that include refraction owing to complex topography and refraction that covers large geographical areas is more easily accomplished through numerical techniques.

Wiegel [1964], *Kinsman* [1965], and *Neumann and Pierson* [1966] have reviewed earlier work on the important shoaling characteristics of surface waves. The basic refraction theory used for shallow water wave refraction studies is a combination of Snell's law for phase speed variations and the theory on conservation of wave energy. Experimental verification of this approach has been proven in several studies. These refraction techniques use the basic concept that wave energy is conserved between wave orthogonals, where the orthogonals of the wave rays are perpendicular to the bending wave crests. The rays indicate the direction that the shoaling wave will progress, and the speed of the wave depends on the depth of the water at a particular location. It is assumed that the wave period will remain constant as waves refract, but the wave length, the velocity, and the height will change. Complicated wave patterns such as crossed orthogonals and caustics were studied by *Chao* [1970], using an asymptotical solution of the Airy function derived by *Ludwig* [1966].

The effects of an opposing ocean current on the surface propagation of gravity waves were first researched by *Unna* [1942]. This early work concentrated on tidal effects, while further investigation by *Johnson* [1947] found that major ocean currents, such as the Gulf Stream, may have effects on the height, the length, and the direction of propagation of intersecting waves, while the possibility for reflection of ocean waves also is assumed to exist.

Kenyon [1971] reported that 'applications to the ocean indicate that both trapping of surface waves in currents and the total reflection of waves by ocean currents should be possible under the appropriate conditions.' The verification of wave-current interaction theories remained to be proven. Until recently, high-resolution observational data have not existed in sufficient quantity to verify the numerical refraction techniques. An ocean experiment, named the Marineland test experiment, held off of Marineland, Florida, in December 1975, offered surface measurements in sufficient quantity and accuracy to verify numerical techniques of wave refraction and ocean current interaction. The basic concept was (1) to define the deep-water directional wave spectrum, (2) to define the ocean current and the near-shore bathymetry, and (3) to attempt to verify the numerical model, the refraction, and the wave-current interaction.

THE MARINELAND EXPERIMENT

To obtain surface measurement verification for shallow water refraction and ocean current refraction, a major oceanographic experiment was conducted offshore of Marineland, Florida, in December 1975. The objectives of this experiment were described by *Shemdin et al.* [1978]. Three surface measurement stations instrumented at 10, 20, and 60 m depths plus one deep-water station were used to collect wave and wind data during this experiment. Complementing the surface measurements were aircraft data, including *X* and *L* band synthetic aperture radar (SAR) flown in patterns over the test area and across the Gulf Stream. Figure 1 illustrates the location of the surface stations and the aircraft flight paths.

Figures 2, 3, 4, and 5 illustrate the meteorological conditions analyzed by the U.S. Navy Fleet Numerical Weather Center (FNWC) during the December 11–14 time period. These polar stereographic projections, particularly Figures 2 and 3, depict the pressure pattern near Newfoundland. This system generated a wave system which would propagate toward the east coast of the United States on December 11 and 12. The FNWC numerical products and surface measurements appear to verify these wind and wave systems. Figures 4 and 5 show that on December 14, the local wind at Marineland, Florida (29°40′N, 81°13′W) was onshore from the east. This generated a local sea with an 8 s period peak in the wave spectrum.

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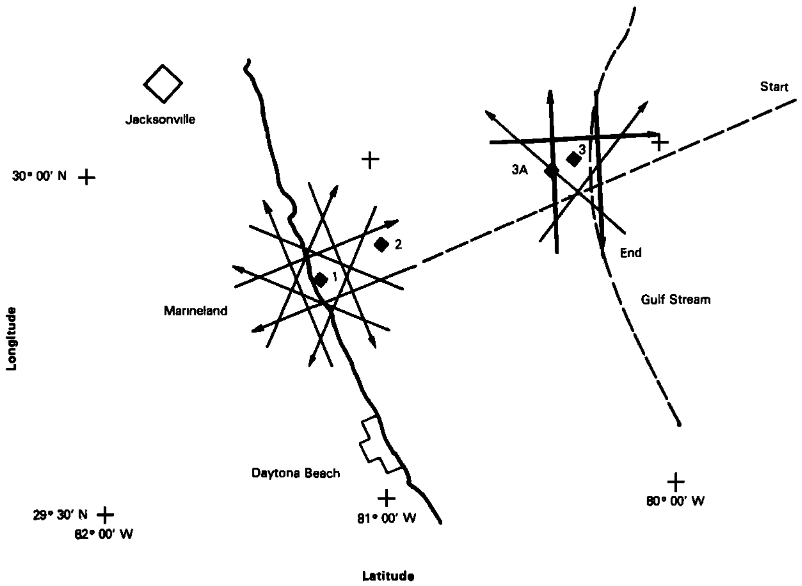

Fig. 1. Flight paths and station locations for Marineland experiment.

Therefore the deep-water wave field consisted primarily of a two-peak wave system. SAR and laser profilometer spectra calculated from aircraft overflight data at approximately 29°56′47″N and 80°16′55″W are shown in Figure 6. This figure shows one peak in the wave spectrum at 0.078 Hz. This swell is propagating in a direction toward 240°, originating from the Newfoundland weather patterns described previously. The second peak at 0.12 Hz propagates in a westerly direction as a result of the high pressure pattern centered off the east coast near Cape Hatteras. Therefore the resulting wave field consisted of two independent wave systems, including swell generated in the north and the northeast, and an additional sea-generated wave system generated on December 14 propagating onshore from the east.

The following section will discuss the effect of ocean currents on the refraction of waves and how, in this particular case, it caused significant alteration of the wave field.

Ocean Current Refraction

Early work by *Johnson* [1947] pointed out that major ocean currents such as the Gulf Stream may have an appreciable effect on the height, length, and direction of waves approaching the shore and, under some circumstances, may cause almost complete reflections. The need for a description of the wave-current interaction recently has been a topic of considerable interest. *Schuman* [1975, 1976] has illustrated that wave records taken in the Agulhas current have shown extreme wave steepness and amplification caused by opposing ocean currents. Other observed incidents of waves breaking at the surface near ocean current boundary have been qualitatively described but have not been quantitatively measured.

Kenyon [1971] points out that this phenomenum of wave-current interaction may cause trapping of waves at certain incident angles. He also shows that the radius of curvature is a characteristic length scale for the refraction and that it decreases with increasing current vorticity and with decreasing group speed relative to the current. He continues, 'the sign of R (radius of curvature) is the same as the sign of the (vorticity), $R = C_g'/\xi$ shows that for waves that propagate with (against) a variable current the rays will bend in the direction of decreasing (increasing) current speed. Therefore, under the right conditions it is possible for waves that propagate with (against) a current to be trapped about a local minimum (maximum) in current speed. It is also possible that waves that propagate with (against) a current could be reflected from a local maximum (minimum) in current speed.'

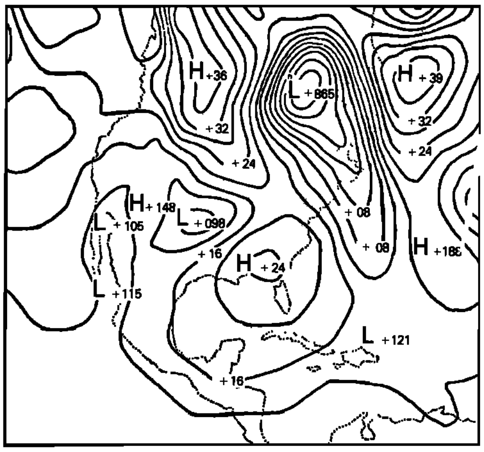

Fig. 2. FNWC 1200Z surface pressure analysis for December 11, 1975.

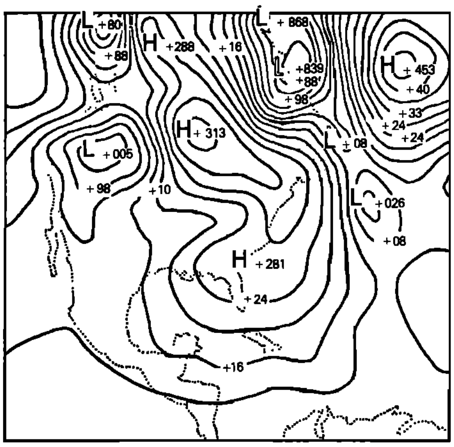

Fig. 3. FNWC 1200Z surface pressure analysis for December 12, 1975.

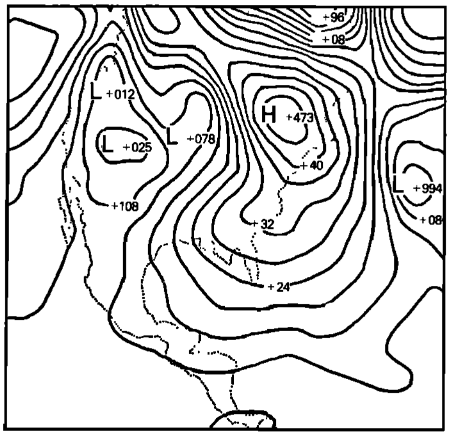

Fig. 4. FNWC 1200Z surface pressure analysis for December 13, 1975.

Later studies by *Longuet-Higgins and Stewart* [1960, 1961] describe the energy transfer between the surface waves and the ocean current as

$$\nabla \cdot [E(C_g + U)] + \tfrac{1}{2} S_{ij}\left[\frac{\partial Ui}{\partial x_j} + \frac{\partial Uj}{\partial x_i}\right] = 0 \qquad (1)$$

where E is the energy density and S_{ij} is the radiation stress tensor as defined by

$$S_{ij} \begin{bmatrix} E\left(\frac{2C_g}{C} - \frac{1}{2}\right) & 0 \\ 0 & E\left(\frac{C_g}{C} - \frac{1}{2}\right) \end{bmatrix} \qquad (2)$$

where C_g is the group velocity and U is the current velocity.

Surface waves possess momentum, which is proportional to the square of the wave amplitude, and is in the direction of wave propagation. The stress or momentum flow is defined as the excess flow of momentum owing to the presence of waves. *Longuet-Higgins and Stewart* [1960] define the change in angle of a refracted wave to be the same as that reported by *Johnson* [1947]; that is,

$$\text{Sin}\,\theta = \frac{\text{Sin}\,\theta_0}{[l - (V/C_0)\,\text{Sin}\,\theta_0]^2} \qquad (3)$$

where Θ is the refracted angle, θ_0 is the incident angle, V is the current speed, and C_0 is the incident phase velocity.

Kenyon points out that a significant fraction of the wave energy that enters the Gulf Stream from the south could be reflected by the current, and also a significant fraction of the wave energy that propagates against the Gulf Stream could be trapped in the current.

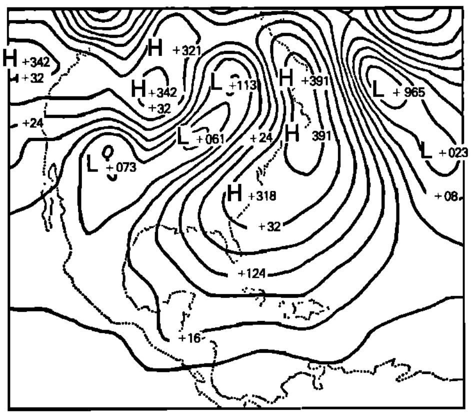

Fig. 5. FNWC 1200Z surface pressure analysis for December 14, 1975.

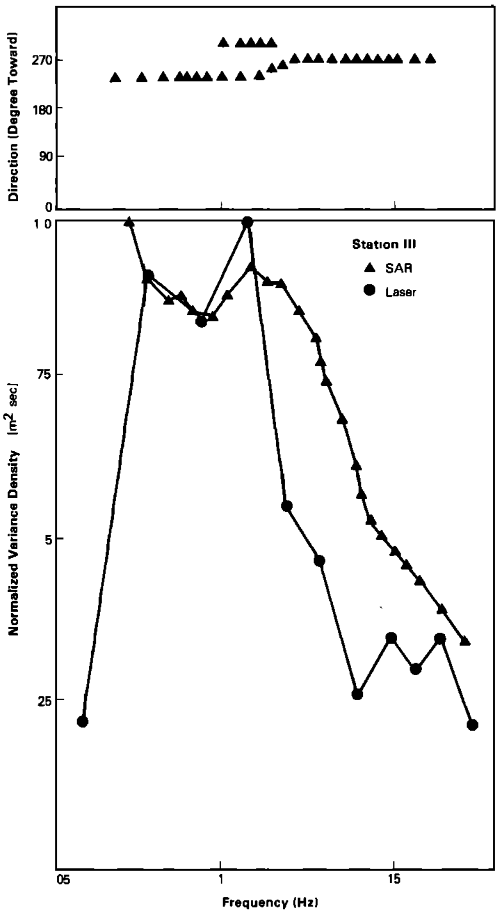

Fig. 6. Station 3 spectral comparisons. Laser is an average of four spectra at 29°51′N and 80°25′W with 42 degrees of freedom. 95% confidence factors are 0.679, 1.6.

Interactions between steady nonuniform ocean currents and gravity waves have been tested in laboratory wind and wave tanks by *Huang et al.* [1972] and *Long and Huang* [1976]. These studies concentrated on kinematic aspects of the influence that currents have on waves and show that the current-wave angle alters the shape of the wave spectra. Changes observed in these studies can be used to explain the drastic changes in sea state resulting when waves and currents interact along shear boundaries such as the Gulf Stream. The additional dissipation affects of wave breaking will occur under conditions where incident angles are greater than the critical angle of reflection.

The phenomenum of wave-current refraction has been discussed in many theoretical papers and qualitatively explained, but its effect has not been measured prior to the Marineland experiment [*Hayes*, 1977]. The laser profilometer and SAR spectra, shown in Figure 6, indicated that near the western boundary of the Gulf Stream there were two peaks in the directional spectra. While in the shelf region west of the western boundary of the Gulf Stream, only one directional peak could be detected in the directional spectra, as is shown in Figure 7.

The wave-current model, described by *Hayes* [1977], uses (3) to define the refraction angle of the wave field after entering the ocean current. The model was run by using directional

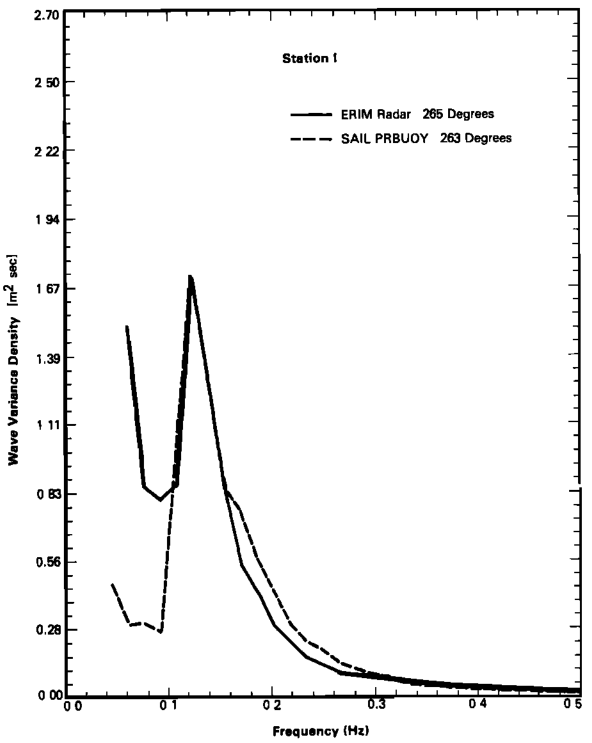

Fig. 7. Station 1: Radar spectra were derived from the geometry corrected Fourier transforms of the Erim SAR imagery. The pitch-and-roll buoy (PRBUOY) spectra is the average of nine 34-min spectra where the variance density of the radar spectrum was matched to the spectrum peak of the PRBUOY.

spectra verified in deep water for the December 14 data set. The refracted rays were recombined in shallow water, and the spectra were compared to those spectra obtained from the SAR, the laser profilometer, and the pitch and roll buoy (PRBUOY). To represent the shear effect of the interaction of the waves with the currents, a simple geometric approach, originally suggested by *Johnson* [1947], was used to model the wave-current interaction. The Gulf Stream current profile (Figure 8) was described from data files of ship observations (G. Neumann, personal conversation, 1977). The wave-current model was initialized with the frequency and the direction components as specified by the deep-water observations and U.S. Navy wave products for December 14. The wave-current model was run for frequency and directional bands observed by the SAR.

Ray diagrams were calculated at 5° increments to examine the refraction patterns at the current interface. The theoretical incident angles for total reflection also were observed for each frequency where the incident angle exceeded the refraction angle resulting in the caustic patterns illustrated in Figure 9. This caustic pattern, first quantitatively described by *Chao* [1970], suggests the elimination of the swell waves from the northeast.

Figure 9 illustrates how the peak frequency in the spectrum at 0.078 Hz was apparently eliminated by the wave interaction with the Gulf Stream. The apparent elimination of the 0.078 Hz peak only can be attributed to the selective elimination of this wave system owing to the angle of incidence exceeding the angle of total reflection as indicated by (3). The orientation of the Gulf Stream at station 3 as shown in SAR images align the current in a near north-south position. Therefore the local wave system at 0.124 Hz penetrated the Gulf Stream at normal incidence, allowing transmission through the current with no alteration, while the 0.078 Hz system was filtered out by the current system. However, Figure 10 illustrates the refraction effect the Gulf Stream has on wave rays where wave-current interaction occurs, but it does not eliminate the rays. Ray diagrams similar to Figure 10 were generated for each 5° directional increment and all wave frequencies measured on December 14. The characteristic curvature of the rays illustrates the current-wave interaction. Further investigation of other wave-current interactions is underway for the Cape Hatteras area. This appears to be the first quantitative measurement of ocean current-wave interaction.

SUMMARY

The preceding results support the existing theories of wave-current interaction. Quantitative comparisons between the results of our wave-current refraction model and the measurements at four stations between the Gulf Stream and the shore support these theories.

It appears that the Gulf Stream did interact with the 0.078 Hz wave train directed toward 240° while the waves were in deep water. This wave energy apparently was generated from the high-pressue system in the North Atlantic between December 11 and 14. The Gulf Stream interaction with this wave train resulted in total reflection or breaking of the wave energy at the western boundary of the Gulf Stream.

The radar images of ocean waves provided consistent information with the theory of wave-current interaction. The SAR evidently has the potential of offering high quality images showing wave-current refraction patterns. These images also

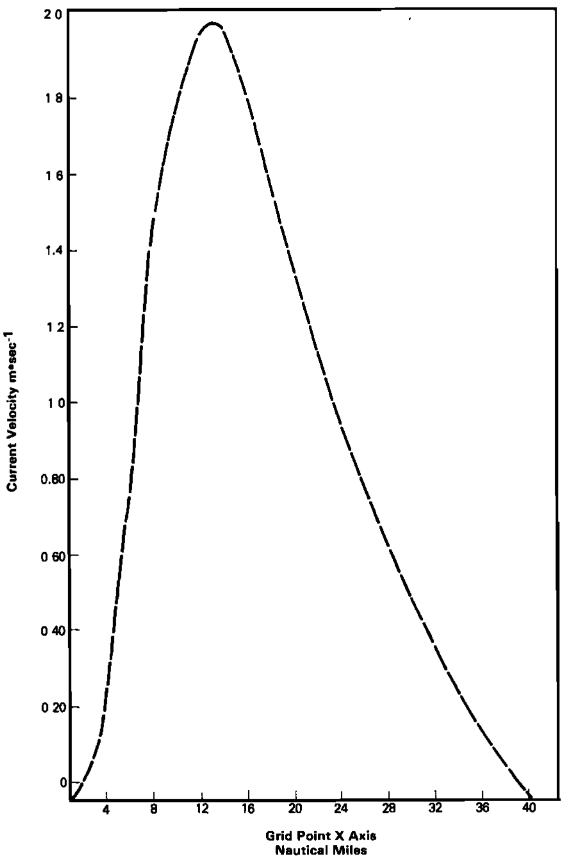

Fig. 8. Gulf Stream current distribution.

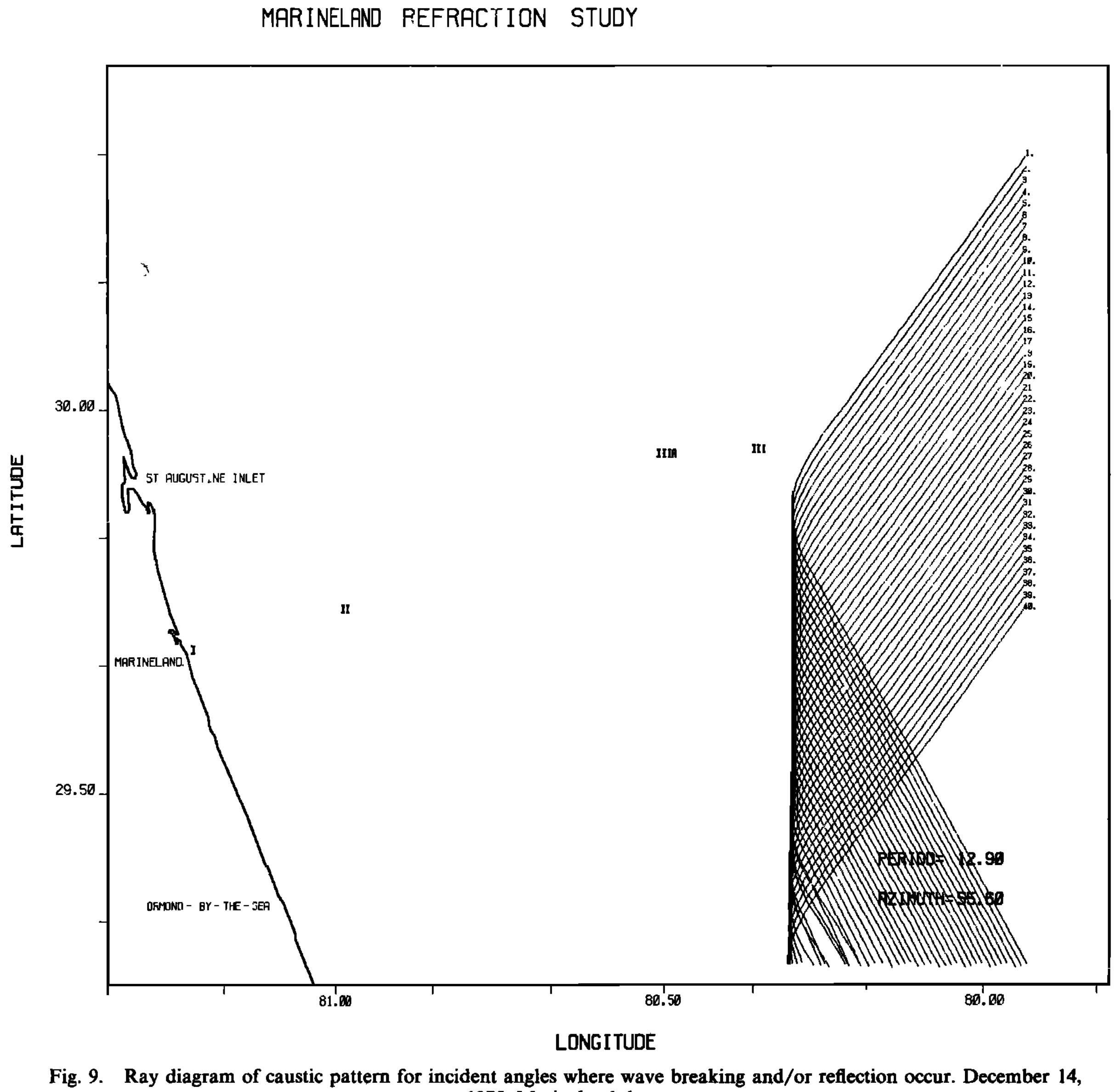

Fig. 9. Ray diagram of caustic pattern for incident angles where wave breaking and/or reflection occur. December 14, 1975, Marineland data.

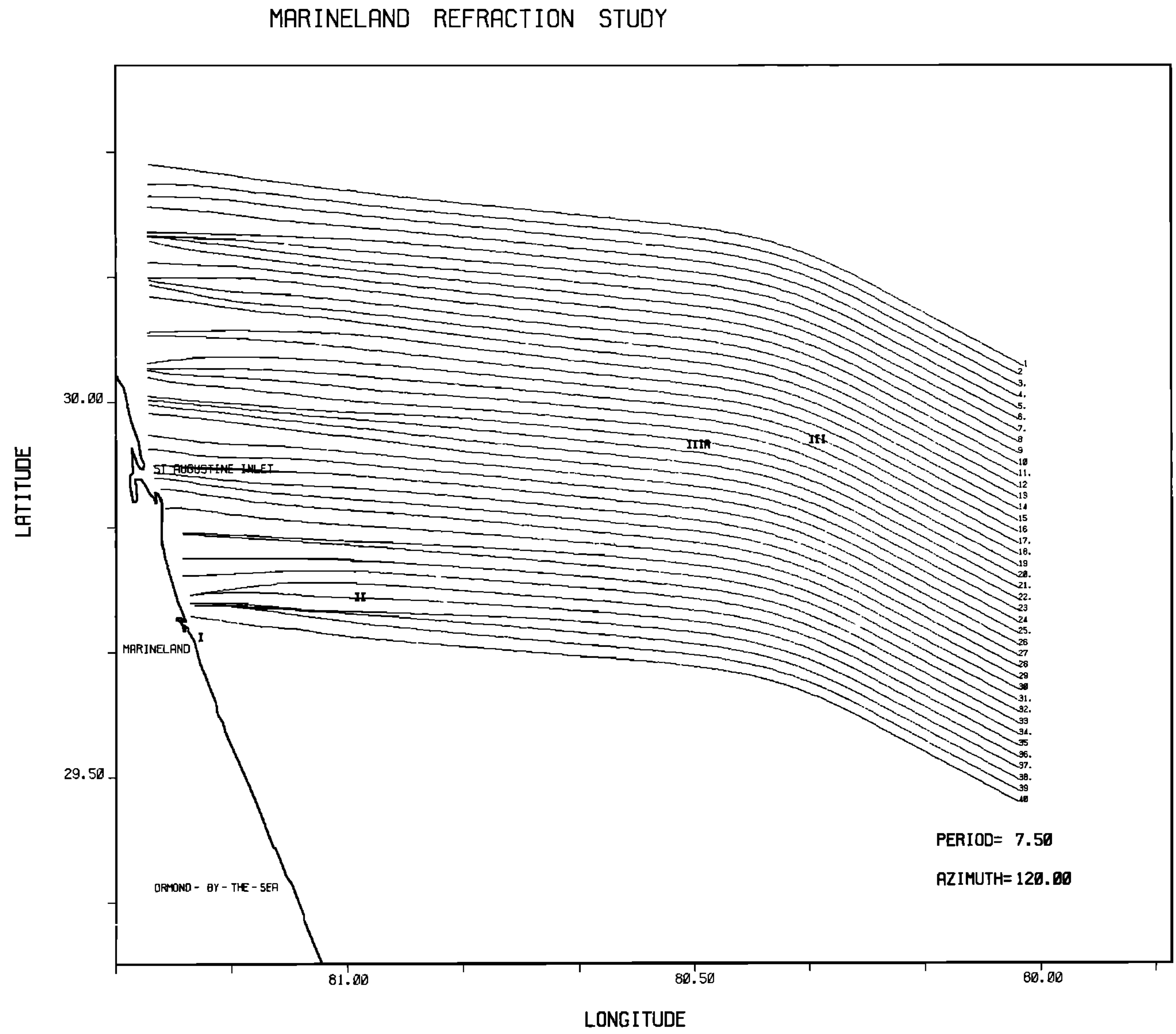

Fig. 10. Sample ray diagram illustrating wave-current interaction.

can be used in shallow waters for examination of wave bottom refraction effects.

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