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An optimal interpolation scheme for the assimilation of spectral wave data

S. Hasselmann
Max-Planck-Institut für Meteorologie, Hamburg, Germany

P. Lionello
Department of Physics, University of Padua, Padua, Italy

K. Hasselmann
Max-Planck-Institut für Meteorologie, Hamburg, Germany

Abstract. An optimal interpolation scheme for assimilating two-dimensional wave spectra is presented which is based on a decomposition of the spectrum into principal wave systems. Each wave system is represented by three characteristic parameters: significant wave height, mean propagation direction, and mean frequency. The spectrum is thereby reduced to a manageable number of parameters. From the correction of the wind-sea system a correction of the local wind is derived. A 2-month test of the system using wave spectra retrieved from ERS 1 synthetic aperture radar wave mode data in the Atlantic yielded consistent corrections of winds and waves. However, the corrected wind data alone, although valuable in identifying wind errors in critical high wind speed regions, are too sparsely distributed in space and time to be used in isolation and need to be combined with other data in an atmospheric data assimilation scheme. This emphasizes the need for the development of combined wind and wave data assimilation schemes for the optimal use of satellite wind and wave data.

1. Introduction

The availability of near-real-time global wind and wave data from the all-weather microwave sensors on board the ERS satellites, together with the planned continuation of such measurements on future Earth-observing missions, has provided a strong impetus for the development of wind and wave data assimilation schemes. The operational provision of these data through ERS was motivated by the demonstration of the feasibility of obtaining global wave data from satellites pioneered by Seasat and further space missions such as Geosat and the SIR C shuttle experiment.

An essential prerequisite for the development of an effective wave data assimilation scheme is the existence of a reliable wave model, such as the third-generation WAve Model 3G-WAM (or simply WAM). Motivated by a critical assessment of the shortcomings of previous first- and second-generation models [*Sea Wave Modeling Project (SWAMP)*, 1985], the WAModel represents a first attempt to incorporate our present understanding of the physics of ocean waves [cf. *Komen et al.*, 1994] into the basic transport equation governing the evolution of the wave spectrum. The model is currently run operationally at a number of forecasting centers. It has been verified extensively under fetch-limited wave growth conditions, a variety of extreme conditions (storms and hurricanes) and on a statistical operational basis against buoy and satellite data [e.g., *Wave Model Development and Implementation (WAMDI) Group*, 1988; *Hasselmann et al.*, 1988; *Zambreski*, 1989, 1991; *Bauer et al.*, 1992; *Romeiser*, 1993; *Komen et al.*, 1994].

Despite the good overall agreement between model and observations, systematic errors were nevertheless found in certain situations. For example, the significant wave heights tend to be underestimated in the high wind regions of the southern Pacific during the winter months and for the monsoon region of the Indian Ocean for the entire year, while the wave heights are sometimes overestimated in the strong wind regions of the northern hemisphere in winter. In view of the successful model verifications under well-documented wind conditions, pronounced discrepancies between modeled and observed wave fields in individual cases can most likely be attributed to errors in the wind fields. Thus we may anticipate that a simultaneous wind and wave data assimilation scheme should lead to an improvement in both wind and wave forecasts. This would be of great

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value for many applications, such as ship routing, fisheries, coastal warning, and offshore activities.

Prior to the availability of satellite data, operational wave information has been available essentially only from buoys and a few platforms. These were sparsely distributed over the oceans and mostly limited to coastal (on shelf) areas. Visual ship observations are more widely available but are restricted to estimated significant wave heights of limited accuracy. With the launch of Earth-observing satellites, global wave data are now continuously provided operationally in the form of significant wave heights from radar altimeters and two-dimensional wave spectra retrieved from the ERS SAR synthetic aperture radar (SAR) wave mode image spectra. In addition, global surface winds over the ocean are available from the ERS scatterometer.

While the significant wave height information can be extracted directly from the altimeter measurements without prior knowledge of the sea state, the retrieval of a wave spectrum from a SAR image spectrum requires as input a first-guess wave spectrum. This is needed to resolve the 180° angular ambiguity unavoidably associated with a snapshot frozen image spectrum as well as to augment the spectrum beyond the azimuthal wave number cutoff arising from the nonlinear velocity bunching mechanism, the displacement of the backscattering elements in the image plane due to the Doppler shifts induced by the orbital velocities of waves.

Because of the strong imaging distortions induced by velocity bunching, it was long questioned whether quantitative measurements of ocean waves by a SAR were feasible. However, the imaging mechanism is now well understood (compare review by *K. Hasselmann et al.* [1985]), and following the derivation of a closed nonlinear transform relation for the mapping of an ocean wave spectrum into a SAR image spectrum [*Hasselmann and Hasselmann*, 1991], operational algorithms have now been developed for inverting the mapping relation to retrieve ocean wave spectra from SAR image spectra [*Hasselmann and Hasselmann*, 1991; *Krogstad*, 1992; *S. Hasselmann et al.*, 1994, 1996; *K. Hasselmann et al.*, 1996; *Brüning et al.*, 1990, 1993, 1994; *Bao et al.*, 1994; *Engen et al.*, 1994] (see also *Komen et al.* [1994, chap. 5] for a more detailed review).

The assimilation of wave data into wave models differs in several respects from the more familiar problem of the assimilation of meteorological data into atmospheric models:

1. A correction of the wave field needs to be accompanied by a simultaneous correction of the local wind field. Otherwise, the wind sea relaxes back rapidly into the incorrect state generated by the incorrect wind field.

2. The region of influence of a wave observation depends strongly on the sea state. Thus wind-sea errors are limited to the generation region, while the correlation scale of errors in long swell components is significantly larger and can extend up to ocean basin scales.

3. The dynamical shock problem of meteorological data assimilation does not arise in wave data assimilation. Wave corrections at individual locations propagate into the forecast without dynamical coupling to neighboring grid points.

4. The data are normally incomplete satellites yielding, for example, spectral integrals (significant wave heights) or spectra with directional ambiguities, and the assimilation scheme has to augment this information to obtain full spectral updates.

Two approaches to wave data assimilation have been pursued so far:

1. The first approach is optimal interpolation schemes. These ignore the dynamical model constraints and simply generate an updated wave field by distributing the information from the observed data within a given time window over the entire model grid. This is computed from a linear superposition of the model first guess and a weighted sum of the errors between the model and the observations.

2. The second approach is dynamical schemes. These insert observations under the constraints of the model dynamics, either sequentially (Kalman filtering [*Kalman*, 1960]) or through a variational method. The latter have been implemented using adjoint techniques [*De Valk and Calkoen*, 1989; *de las Heras and Jansen*, 1992; *de las Heras et al.*, 1994] or a Green function method [*Bauer et al.*, 1996]. The schemes require large computing resources, and more experience will be needed before they can be used generally in operational data assimilation.

Simple optimal interpolation assimilation schemes for radar altimeter wave height data, tested against Seasat and Geosat data, have been developed by *Hasselmann et al.* [1988], *Bauer et al.* [1992], *Thomas* [1988], *Janssen et al.* [1989], *Lionello and Jansen* [1990], *Lionello et al.* [1992, 1995], *Burgers et al.* [1990], and *Francis and Stratton* [1990]; see also *Komen et al.* [1994, chap. 6]. These schemes are now running operationally using ERS radar altimeter data at the United Kingdom Meteorological Office and at the European Centre for Medium Range Weather Forecasting (ECMWF).

A basic difficulty in the assimilation of significant wave height data is the extension of the single piece of information on the total wave energy into a necessarily ad hoc algorithm for the modification of the full two-dimensional wave spectrum. The present paper presents a generalization of the simple optimal interpolation schemes developed for wave height data to the assimilation of the full two-dimensional spectral information retrieved from the SAR. Here we are faced with the opposite problem. Because of the large amount of information retrieved from the ERS 1 wave mode data (one two-dimensional wave spectrum every 200 km along the satellite swath or every 30 s), a reduction of the data to a manageable subset is mandatory for a feasible operational assimilation scheme. This is achieved by applying the spectral partitioning of *Gerling* [1992] in the modified form of *Brüning et al.* [1994] and *S. Hasselmann et al.* [1996]. The spectrum is represented as a superposition of a relatively small number of wave systems, each of which is characterized by three integral spectral parameters: significant wave height H_s, mean direction

Θ, and a mean frequency $\bar{f}$. These parameters are then used as the basic variables of the optimal interpolation scheme.

The paper is organized as follows: The WAModel is described briefly in section 2. Section 3 presents the general theory of optimal interpolation and its application to the assimilation of two-dimensional spectral information derived from ERS 1 SAR wave mode data. The wind retrieval algorithm, following *Lionello et al.* [1992], is described in section 4, and the results are presented in section 5.

2. The WAModel

The WAModel [*WAMDI Group*, 1988; *Komen et al.*, 1994] is a full spectral wave model which incorporates our present understanding of the physics of wave evolution and propagation in deep or shallow water in the basic wave transport equation, without additional ad hoc assumptions regarding the spectral shape. It runs on any spherical latitude-longitude grid on the globe or an arbitrary region of the oceans, with the option of multinested grids. It is currently used worldwide by many research and operational institutions.

The model integrates the wave transport equation on a sphere [*WAMDI Group*, 1988]:

$$\frac{\partial F}{\partial t} + (\cos\phi)^{-1}\frac{\partial(\dot{\phi}\cos\phi F)}{\partial\phi} + \frac{\partial(\dot{\lambda}F)}{\partial\lambda} + \frac{\partial(\dot{\theta}F)}{\partial\theta} = S \quad (1)$$

where

$$S = S_{in} + S_{ds} + S_{nl} + S_{bot} \quad (2)$$

and $F = F(\lambda, \phi, f, \theta, t)$ represents the two-dimensional wave spectrum as a function of frequency (f), direction (θ) (measured clockwise relative to true north) at the location λ (longitude), ϕ (latitude), and time t; the terms on the right-hand side represent the various source function contributions determining the rate of change of the spectrum in a coordinate system moving with the wave group velocity, and the propagation of the waves with group velocity $v = g/4\pi f$ along a great circle path is governed by the equations (in deep water and without currents):

$$\begin{aligned} \dot{\phi} &= \frac{d\phi}{dt} = vR^{-1}\cos\theta \\ \dot{\lambda} &= \frac{d\lambda}{dt} = v\sin\theta(R\cos\phi)^{-1} \\ \dot{\theta} &= \frac{d\theta}{dt} = v\sin\theta\tan\phi R^{-1} \end{aligned}$$

where R is the radius of the Earth and g is the acceleration of gravity.

The wind input source term S_{in} is discussed by *Janssen* [1991]. It is based on an extension of Miles's theory [*Miles*, 1957] of the generation of ocean waves by wind, including the modification of the wind profile by the growing waves.

The dissipation of waves S_{ds} due to white capping is based on the quasi-linear theory of *Hasselmann* [1974], as realized in the model proposed by *Komen et al.* [1984], with some modifications to balance the modifications of the original *Komen et al.* [1984] input term introduced by *Janssen* [1991].

The nonlinear energy transfer S_{nl} is represented by the discrete interaction parameterization of *S. Hasselmann et al.* [1985]. This approximates the full Bolzmann-integral [*Hasselmann and Hasselmann*, 1985] by reducing the five-dimensional interaction phase space to a two-dimensional continuum obtained by applying a scale change and rotation to two basic resonant quadruplets.

Current and depth refraction terms together with an additional bottom dissipation term, S_{bot}, taken from the Joint North Sea Wave Project (JONSWAP) study [*Hasselmann et al.*, 1973], can be included as option in the model but will not be considered here.

Details are given by *WAMDI Group* [1988], *Komen et al.* [1994], and the WAM manual [*Günther et al.*, 1991].

3. The Optimal Interpolation Scheme

3.1. Theory

The optimal interpolation method is a statistical interpolation technique without consideration of model constraints. The analyzed or best estimate $\mathbf{x} = (x_i)$ of the true state vector $\mathbf{x}^t = (x_i^t)$ of the system is represented as a linear superposition of the model first-guess vector $\mathbf{x}^f$ and the weighted errors between the observed data $\mathbf{d}^o = d_\alpha^o$ and the corresponding data values $\mathbf{d}^f = \mathbf{d}^f(\mathbf{x}^f)$ computed from the first-guess model field:

$$x_i = x_i^f + \sum_\alpha W_{i\alpha}\left(d_\alpha^o - d_\alpha^f\right) \quad (3)$$

The analyzed field is updated at each analysis time level, typically every 6 hours, using all observed data within a 6-hour time window centered on the analysis time.

The weight matrix $W_{i\alpha}$ is determined by the condition that the statistical mean square error $<(x_i - x_i^t)^2>$ for each component of the analyzed field is minimized (angle brackets denote ensemble means). This yields

$$W_{i\alpha} = D_{\alpha\beta}^{-1} < x_i^f d_\beta^f > \quad (4)$$

where

$$D_{\alpha\beta} =< (d_\alpha^f - d_\alpha^o)(d_\beta^f - d_\beta^o) >=< d_\alpha^f d_\beta^f > + < d_\alpha^o d_\beta^o > . \quad (5)$$

We note that the computation of the weight matrix $W_{i\alpha}$ is separable with respect to the index i of the state vector components.

It has been assumed in the second part of the equation that the errors in the first-guess field and the observations are uncorrelated [cf. *Komen et al.*, 1994, chap. 6]. A correlation between errors of the first-guess field and observations can in principle arise through the transmission of past errors in the observed field through data assimilation into the first-guess field. However, we shall assume later that the errors in the observations are uncorrelated, so that this effect does not arise. In practice, observation errors, particularly of satellite data, can be correlated, for example, when they are due to environmental influences which are not included in the retrieval algorithms. However, the degree and scale of such error correlations are difficult to assess, and they are therefore normally ignored.

Equation (3) is applicable for any form of data, such as radar altimeter wave height observations, buoy data, etc. The computation of the error covariance matrices $< x_i^f d_\beta^f >$ and $< d_\alpha^f d_\beta^f >, < d_\alpha^o d_\beta^o >$ generally requires extensive statistics, which are not always available. In most applications the expressions are therefore replaced by simple functional forms, using estimated correlation scales, which can be easily handled analytically.

3.2. Assimilation of Radar Altimeter Wave Heights

The optimal interpolation scheme for SAR wave mode spectral data described later represents a generalization of the scheme currently applied operationally at ECMWF for the assimilation of radar altimeter wave heights in their global wave forecasts using the WAModel [*Lionello and Jansen*, 1990; *Lionello et al.*, 1992, 1995].

It is assumed that the data errors are uncorrelated, while for the model error correlation functions, simple exponential expressions are taken. Explicitly, the error covariance functions are given by

$$D_{\alpha\beta} =< (\zeta_\alpha^f - \zeta_\alpha^o)(\zeta_\beta^f - \zeta_\beta^o) >= v^f \exp\left(-\frac{r_{\alpha\beta}}{L}\right) + v^o \delta_{\alpha\beta}, \quad (6)$$

$$< x_i^f d_\alpha^f >=< \zeta_i^f \zeta_\alpha^f >= v^f \exp(-\frac{r_{i\alpha}}{L}) \quad (7)$$

where $\zeta_i^f, \zeta_i^o, \zeta_\alpha^o$ denote the first-guess (index f) or observed (index o) wave heights at the model grid points r_i or observation locations r_α, $r_{\alpha\beta}$ and $r_{i\alpha}$ represent distances $|r_\alpha - r_\beta|$ and $|r_i - r_\alpha|$, respectively, between these locations, v^f and v^o denote the error variances of the modeled and observed wave heights, respectively, and L represents an empirical correlation length scale, which is set at $L = 1000$ km.

The optimal interpolation algorithm yields an updated field of significant wave heights at all model grid points at each analysis time. The total energies of the model first-guess spectra are then adjusted to agree with the analyzed wave heights. For wind-sea spectra (defined by the condition that the direction of the highest spectral peak lies within 15° of the wind direction and the peak frequency is larger than the Pierson-Moskowitz frequency), this is achieved by simultaneously adjusting the local wind speed and both the frequency and energy scales of the spectra using general nondimensional relations for growing wind seas [*Hasselmann et al.*, 1976]. For swell spectra the scale adjustments are carried out under the side condition that the wave steepness is conserved. Details are given by *Lionello et al.* [1995].

3.3. Generalization to Two-Dimensional Wave Spectra

The main difficulty in the assimilation of wave height data into a spectral wave model is the adjustment of the spectra to the updated wave heights. This is necessarily a rather arbitrary process. In particular, the assumption that the wave spectrum can be regarded as pure wind sea or swell is generally not valid for open ocean spectra, which typically contain both wind-sea components and a variety of swell systems (between one and six), some of which may be merged with the wind sea (compare global distributions of wave systems shown by *Brüning et al.* [1994] or *Komen et al.* [1994, Figure 5.11]). (For details, see P. Heimbach et al., A three year statistical comparison of ERS-1 SAR wave mode data with the WAModel, manuscript in preparation, 1997) (hereinafter referred to as Heimbach et al., manuscript in preparation, 1997).

This indeterminacy problem does not arise in the generalization presented in this paper of the optimal interpolation scheme of *Lionello et al.* [1992] to spectral data, since the observed data are matched to the model data. However, we are faced now with the computational problem that the assimilation of one or two hundred spectral data for every satellite SAR wave mode image spectrum obtained every 30 s (200 km) along the satellite track cannot be readily processed on an operational basis. We have accordingly reduced the spectral data set by partitioning each wave spectrum into a relatively small number of wave systems (typically three or four). Each wave system can then be described by a few characteristic parameters. We have selected three parameters: the significant wave height, mean wave frequency, and mean propagation direction.

The partitioning technique used here is a modification [*Brüning et al.*, 1994; *S. Hasselmann et al.*, 1996] of a method originally proposed by *Gerling* [1992]. Each wave system is defined as an "inverse catchment area" of the wave spectrum. This can be expressed by the simple rule that each component of the wave spectrum is assigned to the same wave system as the highest of the four immediately neighboring spectral components in the spectral wavenumber grid. If all neighboring spectral components are smaller than the component considered, the component represents a spectral peak, which defines a given wave system. Spectral peaks which lie close to one another are coalesced to a single wave system. Details are given by *Komen et al.* [1994] and *S. Hasselmann et al.* [1994, 1996].

In the optimal interpolation scheme, each wave system is then characterized by a few integral parameters. We have chosen three parameters: the significant wave height, the mean propagation direction, and the mean frequency. The later two may also be expressed in terms of mean wavenumber components $\overline{k}_x, \overline{k}_y$ (details of how the spectral averages are defined are given by *S. Hasselmann et al.* [1996]).

On the basis of these parameters and some additional characteristics, the wave systems may be classified into wind sea and swell or mixed wind-sea-swell systems. We test for a wind-sea system by considering the phase velocity of the peak of the wave system. If the component of the wind velocity in the direction of the phase velocity is greater than 1.3 × the phase speed (corresponding to a typical value for a fully developed wind sea), the wave system is classified as wind sea. Otherwise, the wave system is regarded as swell or as a mixed wind-sea-swell system. We refer for details again to *S. Hasselmann et al.* [1996]. The classifications are useful for the interpretation of the figures shown later but are in fact irrelevant for the assimilation algorithm.

The general optimal interpolation relation (3) can now be applied to the assimilation of wave spectral data, expressed in terms of the parameters of the partitioned wave systems, by simply identifying the state vector $\mathbf{x}$ with the parameters of the partitioned wave systems. However, to relate the observed and modeled partitioned wave systems, we must first define a method for cross assigning the separate wave systems obtained for the modeled and observed wave spectra at the same or different locations.

For this purpose we introduce a dimensionless square "distance" between two wave systems $(i), (j)$,

$$\Delta^2 = \frac{(\overline{k}_x^i - \overline{k}_x^j)^2 + (\overline{k}_y^i - \overline{k}_y^j)^2}{(\overline{k}_x^{i\,2} + \overline{k}_x^{j\,2}) + (\overline{k}_y^{i\,2} + \overline{k}_y^{j\,2})} \tag{8}$$

and "cross assign" two wave systems, i.e., regard them as representing the same physical wave system, if $\Delta^2 < 0.75$. The cross-assignment threshold value 0.75 was determined empirically as an acceptable compromise between the conflicting requirements of being able to differentiate between similar but separate wave systems on the one hand and slightly modified but nonetheless matching wave systems on the other.

In general, it is possible that a given system of one of the spectra of a spectral pair can satisfy the cross-assignment condition (8) for more than one system of the other spectrum. However, this occurs rather seldom, since multiple wave systems within a spectrum which lie close to one another are already coalesced, as mentioned above, during the partitioning operation [*S. Hasselmann et al.*, 1996], prior to cross assignment. Nevertheless, to define a unique cross-assignment algorithm, the distances of all combinations of wave system pairs satisfying the condition (8) are ordered in a descending sequence, and the cross assignment is carried out following down this sequence. This can leave finally individual systems in either spectrum for which no match is found.

Wave systems of a first-guess spectrum without a match in the observed spectrum are transferred unchanged into the analyzed spectrum. This occurs in the cross assignment of first-guess wave spectra and SAR retrieved spectra in the iterative wave spectral retrieval algorithm for SAR data [cf. *S. Hasselmann et al.*, 1996] mainly for wave systems of the first-guess spectrum which lie beyond the azimuthal cutoff of the SAR retrieved spectrum. However, in the present application to data assimilation, the retrieved spectra have already been augmented with these wave systems, and relatively few cases of this mismatch are found.

Conversely, if the observed wave spectrum contains a system not contained in the first-guess spectrum, the system is superimposed on the first-guess spectrum. Thus, finally, a one-to-one correspondence exists between all wave systems of the analyzed and observed spectra.

The success level of this "cross-assignment" algorithm is strongly dependent on the correlation length used in the optimal interpolation scheme (see below) and the distance from the observation point. In general, first-guess wave systems, in particular wind wave systems, at grid points further away from the observation points differ more strongly from the observed systems and are more difficult to cross assign. In our experiments, 87% of the wave systems at all grid points and 95% of the systems at grid points collocated with the observation points could be cross assigned.

Application of the optimal interpolation algorithm yields a set of updated (analyzed) fields of wave system parameters. We correct not only the wave parameters but also the wind field, applying the same algorithm as *Lionello et al.* [1992] to the parameters of the wind-sea wave systems (see next section). However, to allow for the small correlation scale of wind field corrections in intense low-pressure systems, with which we shall be primarily concerned, we chose a smaller correlation length scale of $L = 200$ km rather than $L = 1000$ km as given by *Lionello et al.* [1992]. The optimal interpolation scheme was furthermore applied only to grid points within one correlation scale of the measurement location (normally termed "data selection" in meteorological applications). A more sophisticated scheme could be readily implemented using different correlation scales for wind-sea and swell systems, but this was not tested in the present first investigation.

Each model first-guess partitioned wave system $F_i(f, \theta)$ is rotated and rescaled such that the mean propagation direction, energy, and mean frequency of the new analyzed wave system $F_i'(f, \theta)$ agree with the corresponding parameters derived from the optimally interpolated parameter fields:

$$F_i'(f, \theta) = AF_i(Bf, \theta'). \tag{9}$$

where $\theta = \theta' + \text{const}$ and A and B are suitably defined scaling constants. At each model grid point the corrected wave systems are then superimposed to yield an analyzed spectrum. This requires some adjustment along the wave system boundaries in the spectral frequency-direction grid. Since the rotation and frequency rescaling differ for different wave systems, the modified wave systems will generally no longer fit together to exactly cover the frequency-direction grid: along some wave system boundaries, there will be some overlap between adjacent wave systems, while along other boundaries, gaps will arise.

No special measures are undertaken in areas of overlap. The spectral systems are simply superimposed, as in other regions of the spectral grid. Since overlapping occurs in regions of low energy and the spectral densities are smoothed through interpolation onto the prescribed grid after rotation and frequency rescaling, no serious local energy distortions were found to be incurred through this simple approach.

Gaps are filled by parabolic interpolation in accordance with the general relation

$$y = a_0 + a_{10}x_1 + a_{20}x_2 + a_{11}{x_1}^2 + a_{22}{x_2}^2 + a_{12}x_1x_2 \quad (10)$$

where y represents the spectral energy at the frequency-direction grid points x_1, x_2 within a connected gap region. The coefficients a_{ij} are computed from a least squares fit to the energies of a set of grid points i surrounding the gap region consisting of the boundary grid points and their next nongap neighbors,

$$J = \sum_i (y_i - y)^2 = \min \quad (11)$$

The conditions $\delta J/\delta a_{ij} = 0$ yield six equations for the six unknown coefficients.

4. Wind Retrieval

The correction applied to the wind-sea wave system can be used to derive a correction for the local wind velocity. Following *Lionello et al.* [1992], it is assumed that the total wave energy E, the mean frequency $\bar{f}$ of the wind sea, and the local wind speed U (at 10-m height) are interrelated in accordance with the general nondimensional power laws for a growing wind-sea spectrum under quasi-equilibrium growth conditions [*Hasselmann et al.*, 1976], i.e., $E^* = af^{*b}$, where $E^* = (g^2/U_{10}^4)E$, $f^* = (U_{10}/g)f$. This yields the empirical power law

$$U = \left[Ag^{B2}E^{-1}\bar{f}^{B}\right]^{B4} \quad (12)$$

where $B2 = B + 2$ and $B4 = -1/(B + 4)$. The constants A and B were adjusted separately by *Lionello et al.* [1992] for winds of order 10 m/s and 20 m/s using the WAModel, yielding for 10 m/s winds $A = 2.2510^{-5}$ and $B = -2.6$ and for 20 m/s winds $A = 1.76\ 10^{-5}$ and $B = -2.6$. A linear superposition of the winds computed with both sets of constants was then formed, with weights depending on the first-guess wind speeed.

Subsequently, the wind field corrections derived from the algorithm of *Lionello et al.* [1992] were combined with the first-guess wind field in an optimal interpolation scheme to yield an update of the wind field. An exponential correlation function was used as in the wave height optimal interpolation scheme. However, in order to limit the impact of this second smoothing operation, following the smoothing already incurred through the optimal interpolation of the wave field, the horizontal correlation scale was reduced now to 50 km (the grid size of the ECMWF wind field) and the interpolation was applied only to neighboring grid points. A more realistic analyzed wind field was obtained by requiring that the divergence of the wind field remained small, which was achieved by a cost function minimization.

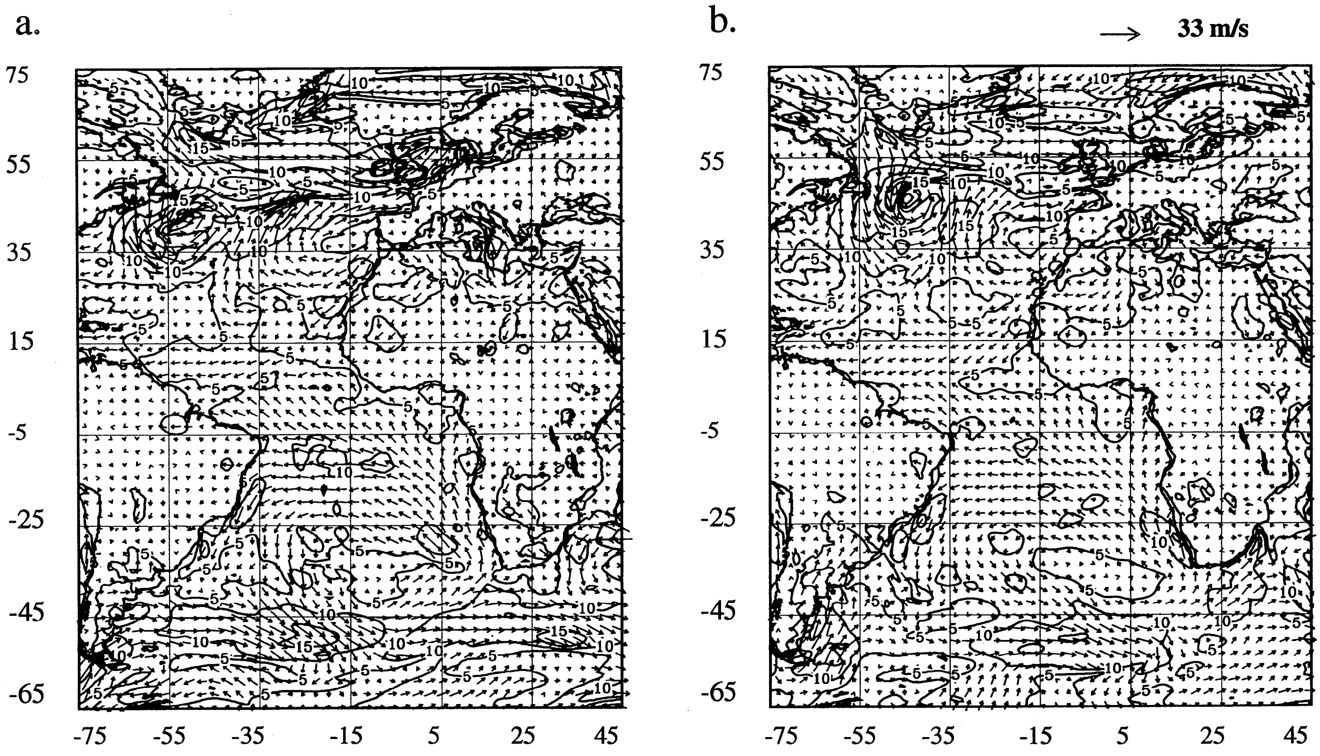

Figure 1. ECMWF analyzed U_{10} wind fields for the time windows centered at 1200 UT on (a) November 2 and (b) November 3, 1992. Contour intervals are 5 m/s starting at 5 m/s.

5. Results

The data assimilation scheme was tested in the Atlantic for the 2 months November 1992 and March 1993. Wave spectra retrieved from ERS 1 SAR image spectra using the improved retrieval algorithm of *S. Hasselmann et al.* [1996] were assimilated for each 6-hour assimilation time window at synoptic times centered at 0000, 0600, 1200 and 1800 UT. The WAModel was run on a 1° × 1° grid driven by ECMWF analyzed U_{10} winds given on a 0.5°×0.5° grid. Several extreme storms crossed the Atlantic during both of these months. We consider both a particular extreme storm event in early November 1992 and the statistics of the assimilation scheme over the 2-month analysis period.

5.1. The November 1992 Storm

The analyzed wind fields for 1200 UT, November 2 and November 3, 1992 (Figures 1a and 1b, respectively) show a strong storm moving across the North Atlantic with wind speeds up to 30 m/s. The rate of movement of the storm happens to match the 1-day displacement of the satellite passes, and on each day, one of the descending passes fortunately lies close to the center of the storm. The analyses for each day were mutually supporting but were carried out separately. An interaction between the two analyses would become effective only at a second stage of the assimilation exercise (not pursued here), in which the corrected wind fields are assimilated in an atmospheric model and used to predict an improved wave field.

The SAR observations within the vicinity of the storm exhibit large deviations from the first-guess wave spectra computed by the WAModel. This is evident in Figure 2 from a comparison of the significant wave heights and propagation directions of the wave systems of the modeled first-guess (Figure 2a) and SAR retrieved (Figure 2b) wave spectra (indicated as directed bars) along the satellite swaths for the descending passes of November 3, 1992 (time window 1200 UT). Similar results are found for November 2, 1992. Figure 3 shows the corresponding wave height/direction difference vectors for the time window 1200 UT of November 3. The wave heights near the center of the storm in the northwest Atlantic are overestimated by as much as 5 m. This is an unusually high error. Statistically, good agreement has been found between SAR retrieved spectra and both the WAModel and observed altimeter wave heights [cf. *Brüning et al.*, 1994; E. Bauer and P. Heimbach, An intercomparison of ERS-1 and TOPEX-POSEIDON altimeter wave heights, manuscript in preparation,1997 (hereinafter referred to as Bauer and Heimbach, manuscript in preparation, 1997); Heimbach et al., manuscript in prepa-

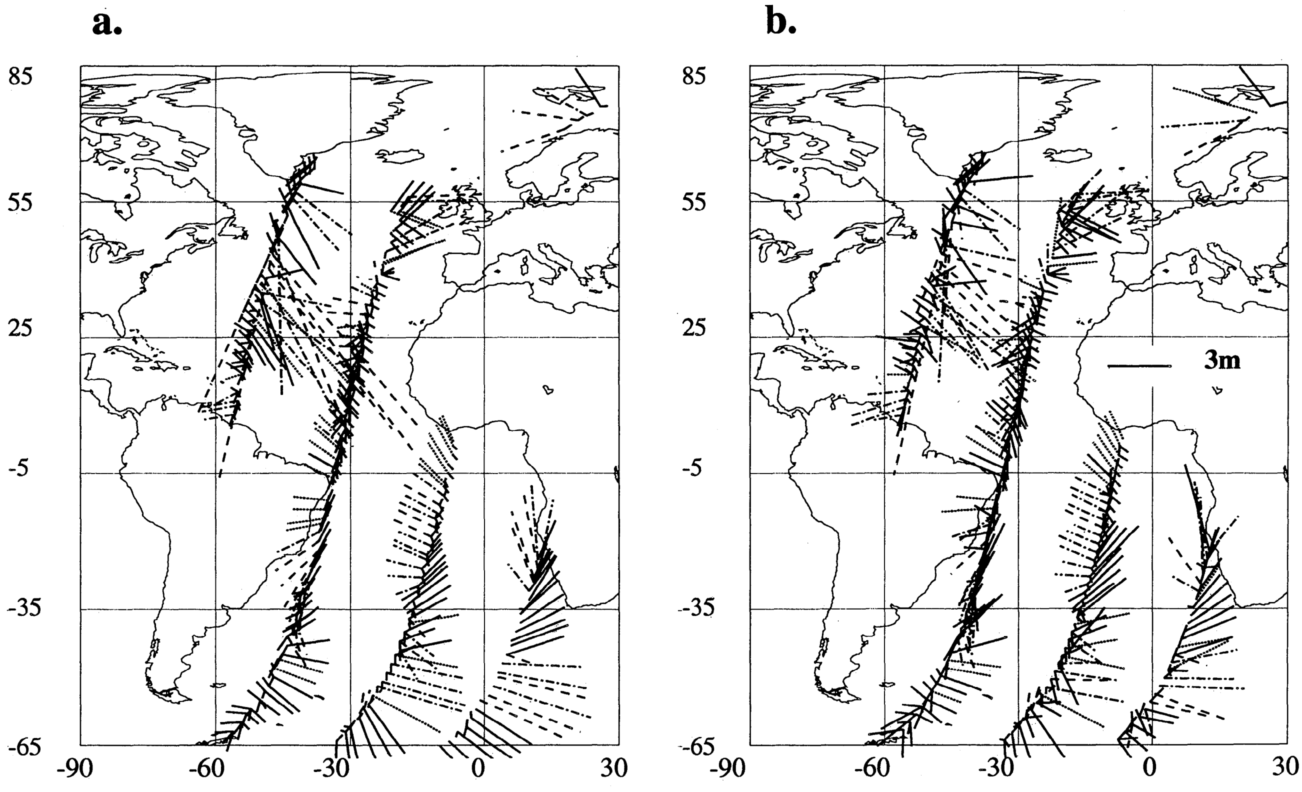

Figure 2. Wave systems of partitioned wave spectra for the Atlantic for the time window centered at 1200 UT, November 3, 1992: (a) WAM first guess: (b) spectral retrievals from ERS 1 SAR wave mode spectra. WAM spectra are collocated at the nearest grid point and time to the retrieved spectra. The bars are proportional in length to the significant wave heights and indicate the mean propagation direction of the wave systems. Swell is denoted by solid lines, wind sea is denoted by dashed lines, mixed wind sea swell is denoted by dots, and old wind sea is denoted by dot-dashed lines.

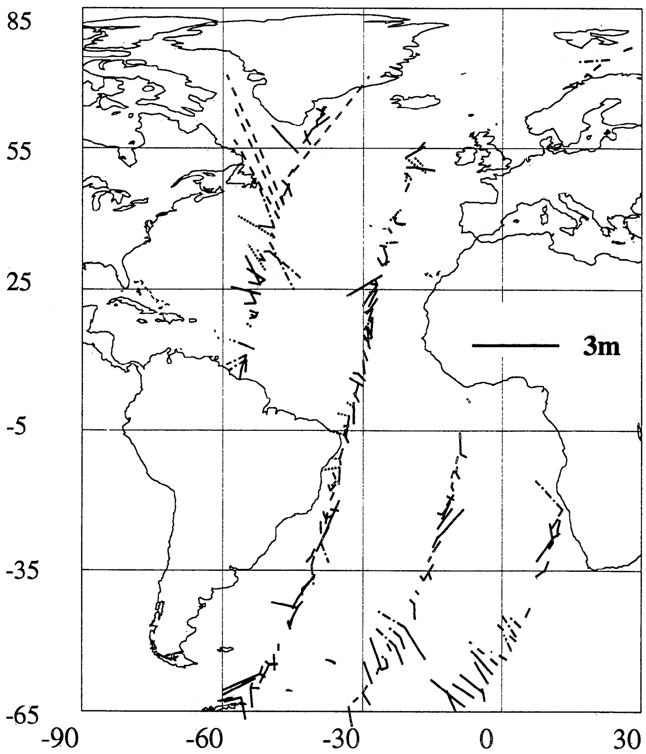

Figure 3. Wave height/direction error vectors (indicated by bars as in Figure 2) representing the SAR retrieved minus WAM first-guess wave height/direction vectors for 1200 UT, November 3, 1992.

ration, 1997], as well as with buoy data (M. Toporek et al., A statistical comparison of ERS-1 SAR retrieved wave spectra and buoy data, manuscript in preparation, 1997; S. Hasselmann et al., A spectral wave data assimilation scheme applied to SWADE data, manuscript in preparation, 1997 (hereinafter referred to as Hasselmann et al., manuscript in preparation, 1997)). However, it has sometimes been found (P.A.E.M. Janssen, personal communication, 1996) that the ECMWF atmospheric model does not provide accurate wind field analyses in areas of high gradient off the east coast of the United States. Unfortunately, our attempts to obtain independent data to corroborate the inferred errors in the wind field in the present case were unsuccessful. However, in a recent analysis of the Surface Wave Dynamics Experiment (SWADE) data (S. Hasselmann et al., manuscript in preparation, 1997), good agreement was found between buoy winds and the wind field corrections of analyzed wind fields inferred using the present assimilation scheme.

The wind velocity correction vectors derived from the corrected wind-sea systems are shown in Figure 4. The wind corrections are distributed over neighboring points around the observation points through optimal interpolation; for clarity, only every second vector (2°) is plotted. Wind corrections up to 10 m/s are found in the storm region. The pattern of wind corrections indicates that the errors cannot be attributed to a misplacement of the location of the storm but represent a genuine overestimation of the strength of the storm.

The assimilation exercise was carried out for all time windows for both months using prescribed first-guess wind and wave fields from the ECMWF analysis and a WAModel integration. The assimilation results were not used to update the first-guess wind or wave fields in the course of the integration. To investigate the impact of the improved wind fields, the WAModel integration was subsequently repeated using the corrected wind fields. The wave field was again not corrected in the course of the integration in order to separate the impact of correcting the wind field from the influence of a running update of the wave field. The errors in wave height, in particular in the wind-sea region for the time window of 1200 UT, November 3, 1992 (Figure 5), are reduced for the run carried out with the corrected wind field, but not as strongly as one may have expected intuitively.

The limited impact of the wind field correction can be explained by the fact that wind field errors in the storm area are corrected only within the 6-hour time window in which the satellite passes close to the storm. The next descending overpass near the storm occurs 24 hours later (the ascending overpass 12 hours later misses the storm). Thus the SAR wave mode data provide a correction of the wind field in the storm region only for about 1/4 of the time. This can be seen on Figure 6, which shows first-guess and corrected wind speeds for seven points along the storm track for the period 1200 UT, November 1, to 0000 UT, November 4. Wind corrections were obtained only for the two time windows centered at 1200 UT, November 2, and 1200 UT, November 3.

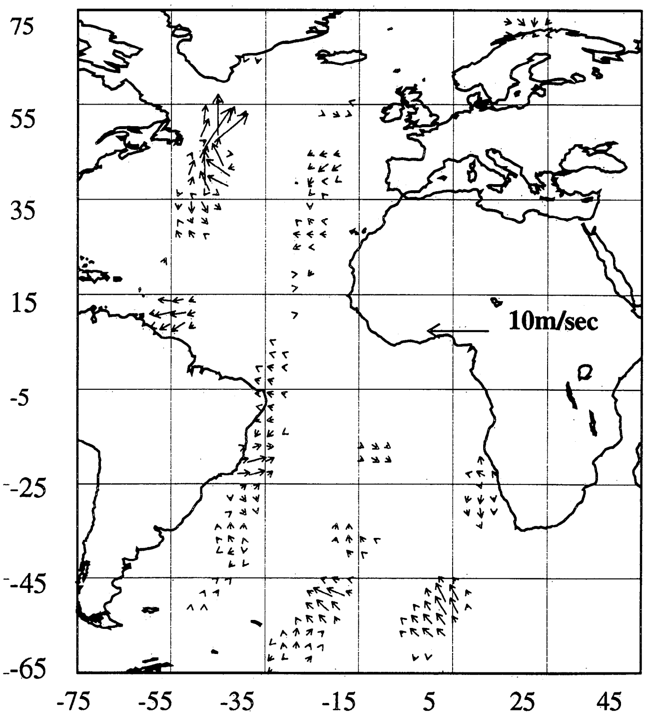

Figure 4. Wind corrections computed from wind-sea corrections for the 6-hour time window centered at 1200 UT, November 3, 1992. Only every second correction vector is shown. The maximal wind correction is 10 m/s.

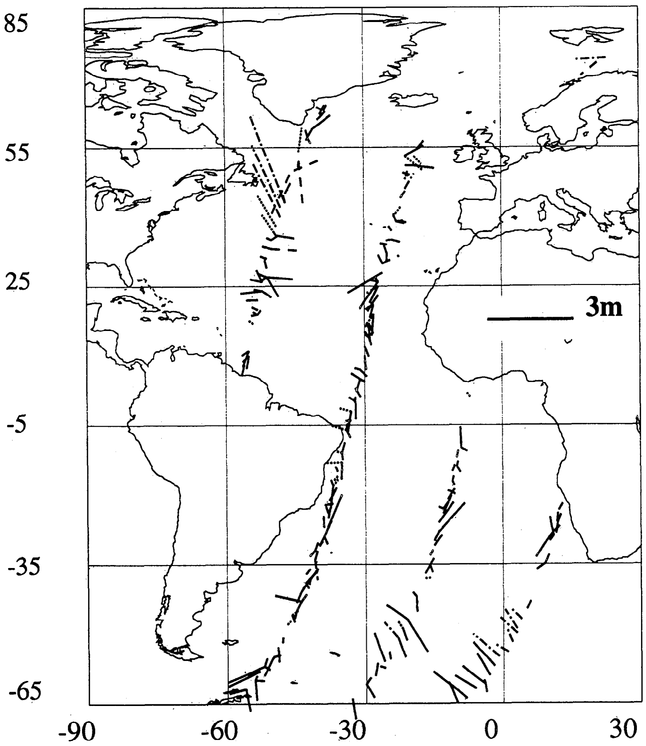

Figure 5. Wave height/direction correction vectors representing the SAR retrieved minus WAM modeled wave height/direction vectors, computed after correction of the wind field, for the 6-hour time window centered at 1200 UT, November 3, 1992.

5.2. Statistical Analysis

The limitations of the space-time coverage of the wind corrections is demonstrated also by Figure 7, which shows the mean wind corrections, averaged over 2° bins for the month of November 1992 (Figure 7a), and the number of wind corrections for each grid point (Figure 7b). Most grid points contain only about five wind corrections, with a maximal number of 30 corrections. Areas of strong mean wind corrections do not necessarily coincide with areas of frequent wind corrections. This emphasizes the need to imbed satellite wind and wave data assimilation schemes in more comprehensive data assimilation schemes based on both global wave models and atmospheric circulation models. The wind corrections derived from satellite wave data can then be used to initialize the atmospheric model, thereby modifying not only the instantaneous wind field but also the future wind field predicted by the atmospheric model. A significantly stronger impact of SAR wave mode data assimilation on marine weather and global wave forecasts may be anticipated if the derived wind corrections are assimilated in such a more comprehensive scheme, particularly since the SAR data can pinpoint errors in critical areas of strong winds.

Detailed statistical intercomparisons between modeled, SAR retrieved, and buoy wave height data were carried out generally for both assimilation months November 1992 and March 1993. The results were similar for both months, and we show in the following only the results for November 1992.

Figure 8 shows scatter diagrams of significant wave heights computed from the WAModel versus wave heights measured by 18 operational wave buoys located in the northeastern Atlantic and the Norwegian Sea (Figure 8a) and versus wave heights computed from ERS 1 SAR retrieved wave spectra (Figure 8b). The box around the high wave heights above 7 m in Figure 8b contains only 0.1% of the data, typically with only one data point per bin. Moreover, the locations of these data do not appear in the set of collocated SAR and buoy data points. The points correspond mostly to the strong storm case discussed above, for which we concluded that the ECMWF wind fields were too high, and they may therefore be regarded as statistically irrel-

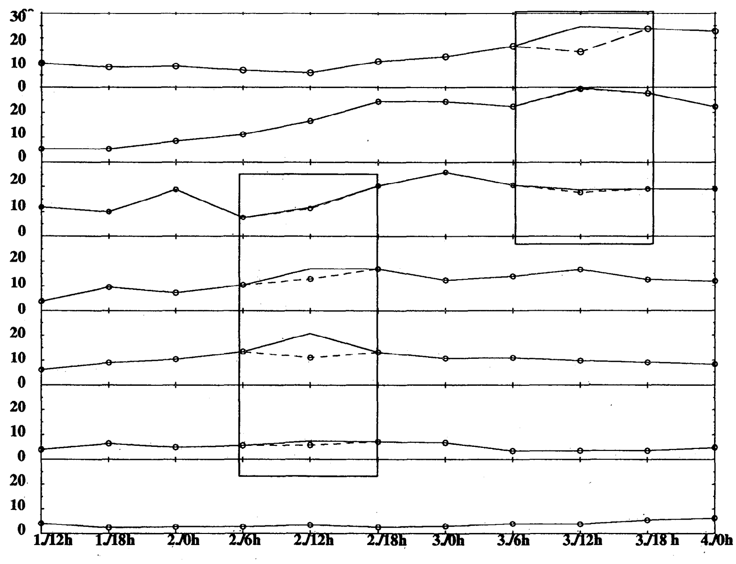

Figure 6. First-guess wind and corrected wind at seven points along the storm track starting at 1200 UT, November 1, 1992. Corrections are applied only within the framed periods.

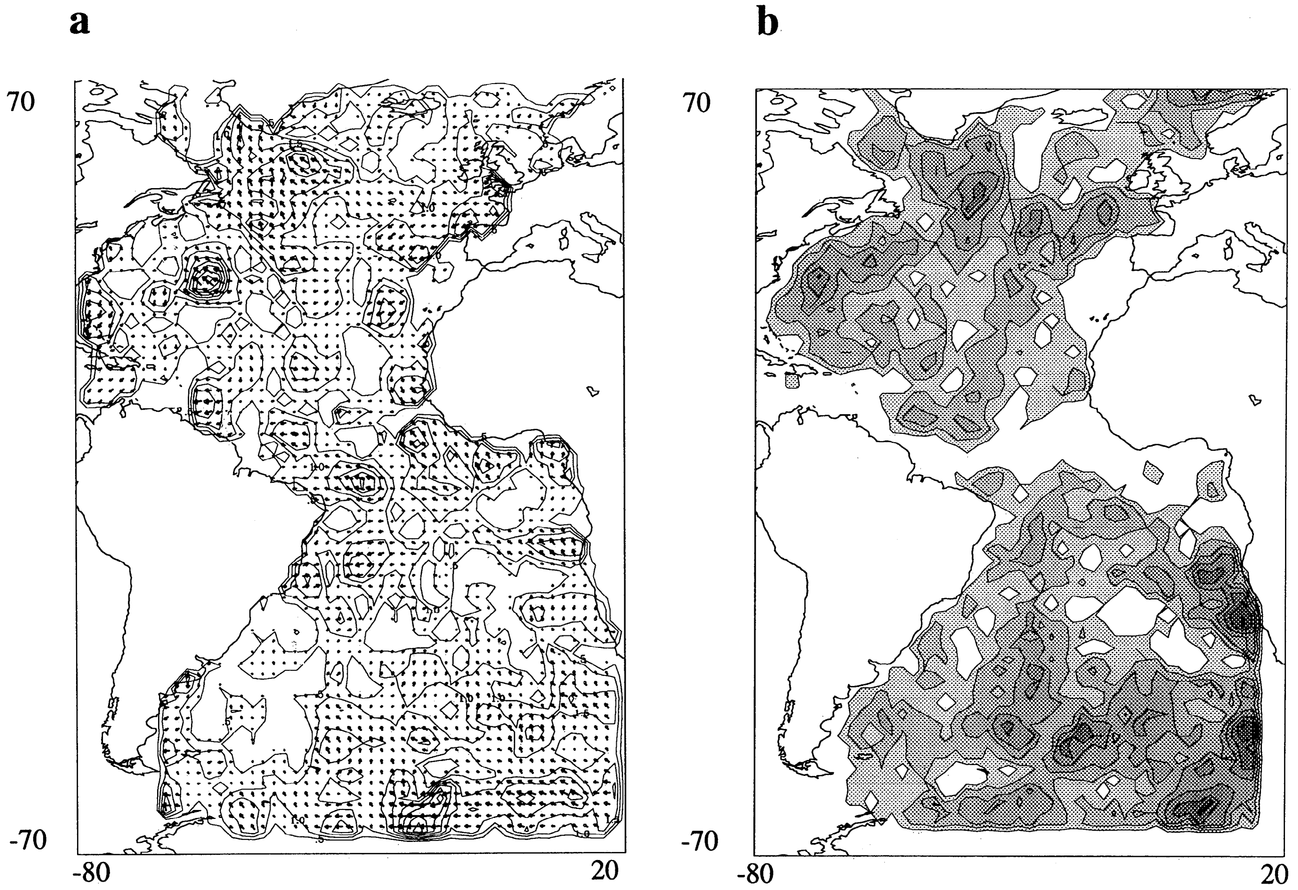

Figure 7. (a) Mean wind correction vectors averaged over 2° latidudinal-longitudinal bins for November 1992. Isolines indicate intervals of 0.5 m/s starting at 0.5 m/s. (b) Number of wind corrections for each model grid point for November 1992. Isolines indicate intervals of 5 m/s starting at 5 m/s.

evant in comparing Figures 8a and 8b. Excluding these data, both data sets show a good agreement with the WAModel wave heights.

The scatter diagram for the modeled and retrieved wind-sea wave heights (Figure 9a), however, shows that while low sea states show little scatter apart from a few outliers, the WAModel tends to overestimate wave heights for high wind seas. *S. Hasselmann et al.* [1996] and Heimbach et al. (manuscript in preparation, 1997) also found in an analysis of 3 years of global ERS 1 SAR wave mode data that the WAModel, when driven by ECMWF analyzed winds, tends to slightly overpre-

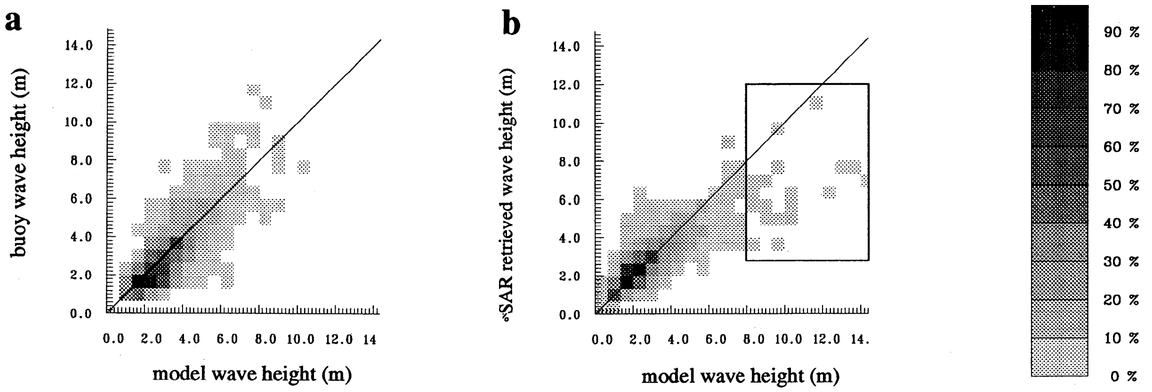

Figure 8. Scatter diagrams of WAM versus (a) buoy and (b) SAR-derived significant wave heights for November 1992. Eighteen operational wave buoys located in the northeastern Atlantic and the Norwegian Sea yield 3041; the SAR yields 2294 data points for the North Atlantic. The frame around wave heights higher than 7 m in Figure 8b contains only 0.1% of the data, normally one data point per bin. Shading denotes density classes in increments of 10%, beginning with the lowest density (light grey pixels, summing to 10% of all cases).

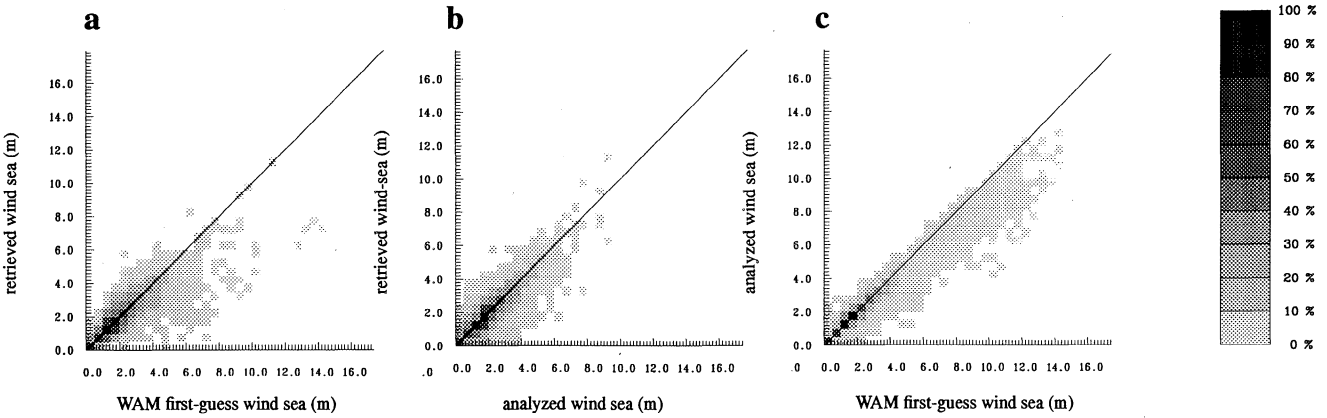

Figure 9. Scatter diagrams of significant wave heights of wind-sea systems for November 1992: (a) WAM first-guess versus SAR retrieved wind sea; (b) analyzed versus SAR retrieved wind sea; (c) WAM first-guess versus analyzed wind-sea systems. The data consist of 3415 points for Figures 9a and 9b and all 60,310 points within the correlation length scale for Figure 9c. Shading is as in Figure 8.

dict high wind-sea states (with respect to the SAR retrieved wave heights). The bias for the total wave heights, however, was found to be small, since the model tends to slightly underpredict swell, which compensates in the mean for the overpredicted wind seas (it has been pointed out by P.A.E.M. Janssen (personal communication, 1996) that this may be due to the assimilation of underestimated ERS 1 altimeter wave heights in the operational ECMWF WAModel). The bias in the high wave heights is considerably reduced after assimilation (Figure 9b). The scatter diagram of the corrected wind-sea wave heights versus the first-guess wave heights, (Figure 9c) is consistent with the scatter diagram of the corrected winds versus the ECMWF analyzed winds shown in Figure 10.

The statistical impact of the corrected wind fields on the modeled wave heights for the month of November 1992 is depicted in the histogram of wave height errors before and after the wind field corrections in Figure 11.

The error distribution for the corrected wind field case is slightly narrower than the original distribution, the number of large errors being reduced (particularly noticeable for the largest errors for positive retrieved wave height above 4 m, Figure 11b). while the number of small errors is increased. In addition, however, the wind correction has tended to skew the errors toward negative values, suggesting that the wind field reductions in the November storm event may have been exaggerated, perhaps by optimally interpolating the corrections over a too large area.

The relatively small changes in the histograms confirm the conclusion of Figures 5, 6, and 7, namely that the impact of correcting the wind only within the 6-hour window corresponding to the satellite overpass is rather minor. Significant corrections are confined to large negative wave height errors, which can be attributed to overestimated winds in the strong storm of November 2 - 3, 1992. For the remaining period the wind field appeared to be more reliable. Most errors are found in the 2-m to 3-m wave height range and are not larger than 0.6 to 1 m, indicating that the analyzed wind fields and predicted wave heights were quite satisfactory for most of the month excluding the November 2 - 3 storm.

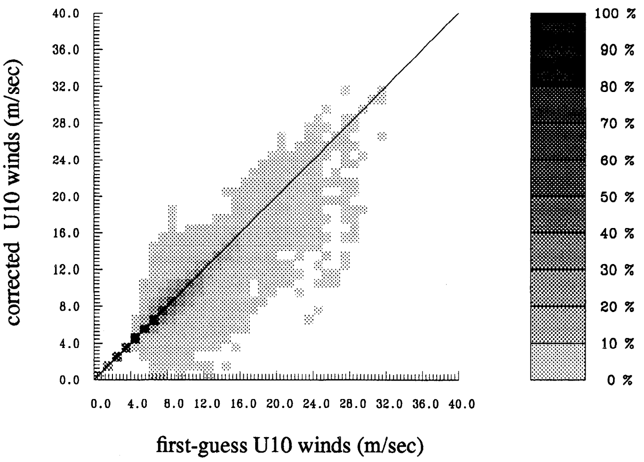

Figure 10. Scatter diagram of WAM first-guess winds versus corrected winds for all optimally interpolated points for November 1992. Shading is as in Figure 8.

6. Conclusions

An optimal interpolation method has been developed for the assimilation of two-dimensional wave spectra retrieved from ERS 1 SAR wave mode imagette spectra into the WAModel. The scheme is based on a decomposition of the wave spectrum into principal wave systems, which reduces the number of parameters which need to be updated during the assimilation cycle.

An application of the scheme to two winter months of ERS 1 data in the Atlantic was encouraging. Errors in the modeled wave spectrum were corrected, and errors in the analyzed ECMWF wind field inferred from errors in the wind-sea spectrum could be similarly identified and corrected. The updating of the two-dimensional wave spectrum based on detailed spectral information can be expected to yield a marked improvement over

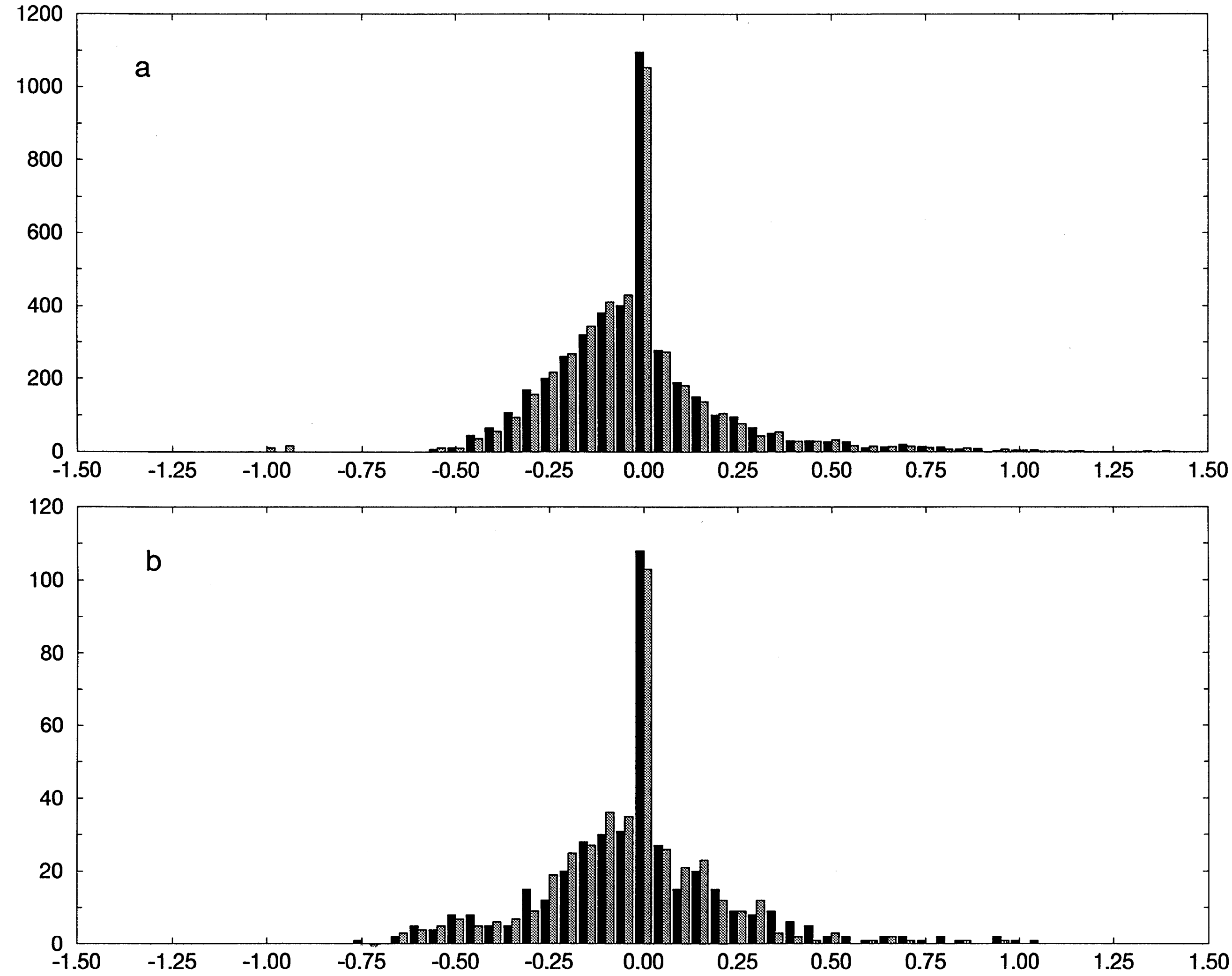

Figure 11. Histogram of the WAModel error H_s(WAM) - H_s(retrieved) for the first-guess WAModel run (solid columns) and for the WAModel run with corrected winds (shaded columns): retrieved wave heights (a) smaller and (b) greater than 4 m.

present wave data assimilation schemes designed for radar altimeter wave height data.

When the WAModel was forced by the corrected wind field, the errors between the model and SAR-retrieved wave spectra were reduced, but the residual errors were still appreciable. The assimilation test demonstrated that wind corrections inferred at SAR observation locations alone, without subsequent assimilation into an atmospheric model, are too sparsely distributed in space and time to provide reliable updated wind fields. However, when used in conjunction with other data, including satellite scatterometer winds, in an atmospheric assimilation scheme, they should provide valuable auxiliary information on wind field errors in critical regions of high winds. Our wave spectral assimilation scheme should therefore be viewed as a contribution toward the development of a more comprehensive system for the assimilation of satellite wind and wave data into atmospheric general circulation and global wave models.

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K. Hasselmann and S. Hasselmann, Max-Planck-Institut für Meteorologie, Bundesstraße 55, 20146 Hamburg, Germany. (e-mail: klaus.hasselmann@dkrz.de; susanne.hasselmann@dkrz.de)

P. Lionello, University of Padua, Department of Physics, Via Marcello 8, 35135 Padua, Italy. (e-mail: piero@borexo.pd.infn.it)

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