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VERIFICATION OF A BAYESIAN METHOD FOR ESTIMATING DIRECTIONAL SPECTRA FROM HF RADAR SURFACE BACKSCATTER

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A Bayesian method for estimating directional wave spectra from the Doppler spectra obtained by HF radar is examined using data acquired during the SCAWVEX project. Applicability, validity and accuracy of the Bayesian method are demonstrated compared with the directional spectrum observed by a directional buoy. In addition the estimated spectra are compared with Wyatt (1990) and the Bayesian method is found to be more robust against noise. Necessary conditions of the Doppler spectral components to be used to estimate a reliable directional spectrum for the present method are also discussed.

Keywords: HF radar: VHF radar; remote sensing; directional spectrum: wave observation; wave data analysis; current measurement.

1. Introduction

The observation of ocean surface currents by means of HF radar is already in practical use, and a number of actual applications have been reported so far. On the other hand, the method for estimating directional wave spectra using HF radar still requires further studies, and is not yet in practical use. Several estimation methods of the directional spectrum from HF radar surface backscatter have been proposed in the US, Australia and Europe, but so far the only successful cases that have been published in any detail are results from the EC MAST SCAWVEX (Surface Current And Wave Variability EXperiments) project (Wyatt *et al.*, 1999) and the EuroROSE (European Radar Ocean Sensing) project (Wyatt *et al.*, 2002).

In Japan, Hisaki (1996) and Hashimoto and Tokuda (1999) proposed methods for estimating the directional spectra from HF radar surface backscatter. However, they only studied the theoretical possibility of the methods without examining the applicability of them to actual observation data.

In this study, we apply the Bayesian method proposed by Hashimoto and Tokuda (1999) to some of the above-mentioned SCAWVEX data, from which some reliable estimated directional wave spectra are reported. We verify the applicability and accuracy of the Bayesian method and discuss some results on further improvements of the method for future practical use in operational wave measurements. Some comparisons with the Wyatt (1990) method for extracting directional spectra from HF radar surface backscatter are also presented. This was the method used during the SCAWVEX and EuroROSE projects where good agreement with buoys has been demonstrated.

2. Estimation of Directional Wave Spectra Using HF Radar Surface Backscatter

The Doppler spectrum $\sigma(\omega)$ obtained by HF ocean radar represents the energy distribution of the radio wave signal back-scattered at the angular frequency ω by the ocean surface waves, and is expressed by the summation of the first-order scattering component $\sigma^{(1)}(\omega)$ and the second-order scattering component $\sigma^{(2)}(\omega)$, i.e. $\sigma(\omega) \approx \sigma^{(1)}(\omega) + \sigma^{(2)}(\omega)$. Each component can be expressed by the following equations for deep-water conditions, respectively (Barrick, 1972):

$$\sigma^{(1)}(\omega) = 2^6\pi k_0^4 \sum_{m=\pm1} S(-2mk_0, 0)\delta(\omega - m\omega_B) \tag{1}$$

$$\sigma^{(2)}(\omega) = 2^6\pi k_0^4 \sum_{m_1,m_2=\pm1} \iint_{-\infty}^{\infty} |\Gamma|^2 S(m_1\mathbf{k}_1)S(m_2\mathbf{k}_2) \times \delta(\omega - m_1\sqrt{gk_1} - m_2\sqrt{gk_2})dpdq \tag{2}$$

where k_0 is the absolute value of the wavenumber vector $\mathbf{k}_0$ of the radar waves, $S(\mathbf{k}) = S(k_x, k_y)$ is the wavenumber spectrum, $\omega_B(= \sqrt{2gk_0})$ is the Bragg angular frequency, and $\delta()$ is the delta function. The independent variables, p and q, of the integration of Eq. (2) represent coordinates, parallel and orthogonal to the radar beam, respectively. The wavenumber vectors, $\mathbf{k}_1$ and $\mathbf{k}_2$, are related to these variables by the following equations:

$$\mathbf{k}_1 = (p - k_0, q), \qquad \mathbf{k}_2 = (-p - k_0, -q) \tag{3}$$

These relations indicate the Bragg's resonance condition expressed by

$$\mathbf{k}_1 + \mathbf{k}_2 = -2\mathbf{k}_0 \tag{4}$$

The coupling coefficient, Γ, shows the degree of the contribution from the wave components having the wavenumber $\mathbf{k}_1$ and $\mathbf{k}_2$ to the second-order energy distribution of the back-scattered radar signal, and is commonly expressed by the summation of the electromagnetic scattering effect, Γ_E, and the hydrodynamic scattering effect, Γ_H, i.e. $\Gamma = \Gamma_E + \Gamma_H$ (Barrick, 1972).

Since the first-order scattering component $\sigma^{(1)}(\omega)$ and second-order scattering components $\sigma^{(2)}(\omega)$ appear at different frequencies in the Doppler spectrum $\sigma(\omega)$, they can be separated easily even when they are small in magnitude. Consequently, valuable oceanographic information such as surface currents, waves and wind direction can be obtained from the respective components of the Doppler spectrum. Current measurement techniques are discussed by Paduan and Graber (1997). Wave techniques are discussed by Wyatt (1997) and Graber and Heron (1997), whereas the method for extracting wind direction is discussed by Fernandez *et al.* (1997).

As is evident from Eq. (2), two-component waves having the wavenumber vector $\mathbf{k}_1$ and $\mathbf{k}_2$ are related to the second-order scattering component $\sigma^{(2)}(\omega)$. There are an infinite number of combinations of $\mathbf{k}_1$ and $\mathbf{k}_2$ relevant to the corresponding Doppler frequency ω under the constraint of δ function in Eq. (2) and the resonance condition of Eq. (4). This means that Eq. (2) includes the contributions of an infinite numbers of component waves having different frequency ω and propagation direction θ, and hence in principle, we can estimate the directional spectrum based on this information. When we estimate the directional spectrum based on Eq. (2), however, the following problems arise:

(1) Because of the constraint of the δ function, the integration of Eq. (2) must be executed along a curve on the "frequency-direction" plane into which the wavenumber plane is transformed by the dispersion relationship. The digitization of the integral of Eq. (2) is therefore complicated.
(2) This is a so-called incomplete inverse problem in which the number of unknown parameters is much larger than that of equations obtained from the measurements. This sometimes causes the problem that even a small measurement error would seriously deteriorate the reliability of the estimate.

The estimation of wave conditions from the Doppler spectra observed by HF radar surface backscatter was first carried out by Barrick (1972 and 1977). He proposed the equations for calculating the significant wave height and period by: (1) removing the wavenumber spectrum with the larger wavenumber included in Eq. (2)

from the integral by assuming that it was known from the wave component that caused the first-order scattering, and (2) obtaining an approximate integral equation by linearizing Eq. (2) with respect to the wavenumber spectrum having the smaller wavenumber.

Lipa (1977), Wyatt (1990) and Howell and Walsh (1993) proposed a method for estimating the directional wave spectrum based on the Barrick's linearized integral equation. Wyatt (2000) also extended her method to apply it to the nonlinear integral Eq. (2) without linearization although demonstrating that any improvement in accuracy does not warrant the increase in computational complexity. The Wyatt method is an iterative method that makes a first guess for the directional spectrum and adjusts this at each iteration using the difference between the measured Doppler spectrum and the spectrum obtained by integrating Eq. (2) with the latest adjusted directional spectrum. Howell and Walsh (1993) developed a singular values decomposition method to invert the matrix equation that results from a discretization of Eq. (2). Lipa (1977) solved the matrix equation using a regularization method.

Recently in Japan, Hisaki (1996) proposed an alternative method for estimating the directional spectrum from the nonlinear integral Eq. (2). In addition to the Eq. (2), he added *a priori* conditions that are desirable for the directional spectrum to solve the above problems (1) and (2), and solved the nonlinear inverse problem by an optimization method with a perturbation technique. His approach was similar to a Bayesian approach developed by Hashimoto *et al.* (1987) for estimating the directional spectrum from the data of *in situ* measurements (e.g directional wave buoys). In Hisaki's method, however, the number of the *a priori* conditions exceeded that of the unknown parameters. This was because the *a priori* conditions included not only the expectation that the directional spectra were smooth continuous functions, but also the conditions that the spectrum values changed at a known ratio with respect to the frequency and the direction, and that they avoided taking negative energy values. The problem of the proper setting of the weighting factors introduced in each *a priori* condition was also left unsolved.

To solve such problems implied in the Hisaki method, Hashimoto and Tokuda (1999) applied the Bayesian approach to develop a method for estimating the directional spectrum applicable to HF radar. To satisfy the condition that the directional spectra avoid taking negative values, they assumed that the directional spectrum had exponential forms having piecewise-constant functions with respect to the frequency and the directional angle. They also added an *a priori* condition that the piecewise-constant values of the exponential parts were on smooth continuous functions. The number of the *a priori* conditions thus became equal to that of the unknown parameters. In addition to the above, they introduced the Akaike Bayesian information criterion ABIC (Akaike, 1980) to balance the degree of satisfying Eq. (2) with that of satisfying the *a priori* condition that the directional spectra were smooth continuous functions. They then minimized the ABIC to estimate the optimum directional spectrum so that the desirable weighting factors were automatically acquired from both the viewpoints of the certainty of the solution and the smoothness. The following is a brief explanation of the Bayesian method (Hashimoto and Tokuda, 1999).

3. A Bayesian Method for Estimating Directional Spectrum from HF Radar

As mentioned above, the directional wave spectrum $S(f,\theta)$ as a function of frequency f and direction θ is assumed to be an exponential piecewise-constant function over the directional range from 0 to 2π and the frequency range from $f_{\min}$ to $f_{\max}$.

$$S(f,\theta) = \alpha \sum_{i=1}^{M} \sum_{j=1}^{N} \exp(x_{i,j}) \delta_{i,j}(f,\theta) \tag{5}$$

where $x_{i,j} = \ln\{S(f_i,\theta_j)/\alpha\}$, M is the number of segments Δf of frequency f, N is the number of segments $\Delta\theta$ of direction θ, and

$$\delta_{i,j}(f,\theta) = \begin{cases} 1: & f_{i-1} \le f < f_i \text{ and } \theta_{j-1} \le \theta < \theta_j \\ 0: & \text{otherwise} \end{cases} \tag{6}$$

α is a parameter introduced for normalizing the magnitude of $x_{i,j}$, and is given by

$$\alpha = \frac{\int_{f_{\min}}^{f_{\max}} \int_0^{2\pi} S(f,\theta) df d\theta}{\int_{f_{\min}}^{f_{\max}} \int_0^{2\pi} df d\theta} \tag{7}$$

The numerator on the right hand side of Eq. (7) is approximately given by the following equation (Barick, 1977):

$$\int_{f_{\min}}^{f_{\max}} \int_0^{2\pi} S(f,\theta) df d\theta \approx \frac{2\int_{-\infty}^{\infty} \{\sigma^{(2)}(\omega)/W(\omega/\omega_B)\} d\omega}{k_0^2 \int_{-\infty}^{\infty} \sigma^{(1)}(\omega) d\omega} \tag{8}$$

where $W(\omega/\omega_B) = 8|\bar{\Gamma}^2|/k_0^2$ is a weighting function and $\bar{\Gamma}$ is an approximate coupling coefficient of Γ (Barick, 1977).

The frequency f and the direction θ are discretized by the following equations, respectively.

$$\mu_i = \ln f_i = \ln f_{i-1} + \Delta f, \quad \theta_j = \theta_{j-1} + \Delta\theta \tag{9}$$

Substituting Eq. (5) into Eq. (2) with the transformation of the variables from wavenumber **k**-plane to (f,θ)-plane using the dispersion relationship yields an integral equation including unknown variables, $\mathbf{X} = (x_{1,1}, \ldots, x_{M,N})^t$. Finally, after some manipulations, by taking into account the errors ε_k of the Doppler spectrum $\sigma^{(2)}(\omega_k)$, the integral Eq. (2) can be approximated by the nonlinear algebraic equation including the unknown $\mathbf{X} = (x_{1,1}, \ldots, x_{M,N})^t$, and is expressed by

$$\sigma_k^{(2)} = F_k(\mathbf{X}) + \varepsilon_k \tag{10}$$

where the suffix k indicates a value at the Doppler frequency ω_k $(k = 1, \ldots, K)$.

The errors ε_k $(k = 1, \ldots, K)$ at every Doppler frequency ω_k are assumed to be independent of each other and their occurrence probabilities can be expressed by a normal distribution having a zero mean and variance λ^2. Then, for a given $\sigma_k^{(2)}$ $(k = 1, \ldots, K)$, the likelihood function of $\mathbf{X}$ and λ^2 is given by

$$L(\mathbf{X}; \lambda^2) = \frac{1}{(\sqrt{2\pi}\lambda)^K} \exp\left[-\frac{1}{2\lambda^2}\sum_{k=1}^{K}\{\sigma_k^{(2)} - F_k(\mathbf{X})\}^2\right] \tag{11}$$

Note that the directional wave spectrum, $S(f, \theta)$, has thus far been expressed by a piecewise-constant function, with the correlation between the wave energy of each segment of $\Delta f \times \Delta\theta$ not yet having been taken into account. However, $S(f, \theta)$ is generally considered to be a continuous and smooth function. This allows an introduction of an additional condition that the local variation of $x_{i,j}$ $(i = 1, \ldots, M;$ $j = 1, \ldots, N)$ can be well approximated by a smooth surface so that the value given in Eq. (12) is expected to be small.

$$x_{i,j+1} + x_{i+1,j} + x_{i,j-1} + x_{i-1,j} - 4x_{i,j} \tag{12}$$

In the upper boundary $(i = M)$ and the lower boundary $(i = 1)$ of the frequency f, the value given in Eq. (13) is expected to be small as an *a priori* condition.

$$x_{i,j+1} - 2x_{i,j} + x_{i,j-1} \tag{13}$$

These additional conditions lead to

$$\sum_{i=2}^{M-1}\sum_{j}(x_{i,j+1} + x_{i+1,j} + x_{i,j-1} + x_{i-1,j} - 4x_{i,j})^2 + \sum_{j}(x_{1,j+1} - 2x_{1,j} + x_{1,j-1})^2$$
$$+ \sum_{j}(x_{M,j+1} - 2x_{M,j+1} + x_{M,j-1})^2 \rightarrow \text{small} \tag{14}$$

where $x_{i,0} = x_{i,N}$ and $x_{i,-1} = x_{i,N-1}$.

In a matrix form, Eq. (14) can be written as

$$\|\mathbf{DX}\|^2 \rightarrow \text{small} \tag{15}$$

where $\|\cdots\|$ is the Euclid norm, and $\mathbf{D}$ is the coefficient matrix of Eq. (14).

It is, therefore, surmised that the optimal estimate of $S(f, \theta)$ is the one maximizing the likelihood function of Eq. (11) under the condition of Eq. (15). More precisely, the most suitable estimate is given as a set of $\mathbf{X} = (x_{1,1}, \ldots, x_{M,N})^t$ which maximizes the following equation for a given hyperparameter u.

$$\ln L(\mathbf{X}; \lambda^2) - \frac{u^2}{2\lambda^2}\|\mathbf{DX}\|^2 \tag{16}$$

The hyperparameter u is a type of weighting coefficient which represents the smoothness of $\mathbf{X}$, where large or small values of u, respectively, give an estimate of the directional wave spectrum having either smooth or rough shapes. It should be noted that Eq. (16) corresponds to the Bayesian relationship expressed by the following equation when we consider the exponential function having the power of Eq. (16).

$$p_{\mathrm{POST}}(\mathbf{X}|u^2,\lambda^2) = L(\mathbf{X};\lambda^2)p(\mathbf{X}|u^2,\lambda^2) \tag{17}$$

where $p_{\mathrm{POST}}(\mathbf{X}|u^2,\lambda^2)$ is the posterior distribution, and $p(\mathbf{X}|u^2,\lambda^2)$ is the prior distribution of $\mathbf{X} = (x_{1,1},\ldots,x_{I,J})^t$ expressed by

$$p(\mathbf{X}|u^2,\lambda^2) = \left(\frac{u}{\sqrt{2\pi}\lambda}\right)^M \exp\left\{-\frac{u^2}{2\lambda^2}\|\mathbf{DX}\|^2\right\} \tag{18}$$

The estimate $\mathbf{X}$ obtained by maximizing Eq. (16) can be considered as the mode of the posterior distribution $p_{\mathrm{POST}}(\mathbf{X}|u^2,\lambda^2)$.

Now, if the value of u is given, then regardless of the value of λ^2, the values of $\mathbf{X}$ that maximize Eq. (16) can be determined by minimizing

$$\sum_{k=1}^{K}\{\tilde{\sigma}_k^{(2)} - F_k(\mathbf{X})\}^2 + u^2\|\mathbf{DX}\|^2 \tag{19}$$

The determination of u and the estimation of λ^2 can be automatically performed by minimizing the following ABIC (Akaike's Bayesian Information Criterion, Akaike, 1980) from the viewpoint of the suitability and smoothness of the estimate of $\mathbf{X}$.

$$\mathrm{ABIC} = -2\ln\int L(\mathbf{X}|\lambda^2)p(\mathbf{X}|u^2,\lambda^2)d\mathbf{X} \tag{20}$$

The details of computing this method are described in Hashimoto and Tokuda (1999).

4. Surface Current And Wave Variability EXperiments Project (SCAWVEX)

SCAWVEX was an EC joint research project conducted by the researchers from four European countries including the UK. In SCAWVEX, various measuring instruments, including an HF radar system, Synthetic Aperture Radar (SAR), microwave altimeter, X-band radar system, wave gauges, current meters, water pressure gauges, and Acoustic Doppler Current Profiler (ADCP), were used to measure the time and space variations of the waves and the currents. The characteristics of each measuring instrument were also studied. There were four SCAWVEX experiments, two at Holderness in the UK and two in the Netherlands at Maasmond and Petten. For the work reported here, we used only the data obtained with the Ocean Surface Current Radar (OSCR) HF radar at 14:00 hours on December 21, 1995 during the second Holderness experiment which ran from December 1995 to January 1996, since the quality of the data was considered to be reliable enough to analyze and was surely confirmed by Wyatt *et al.* (1999).

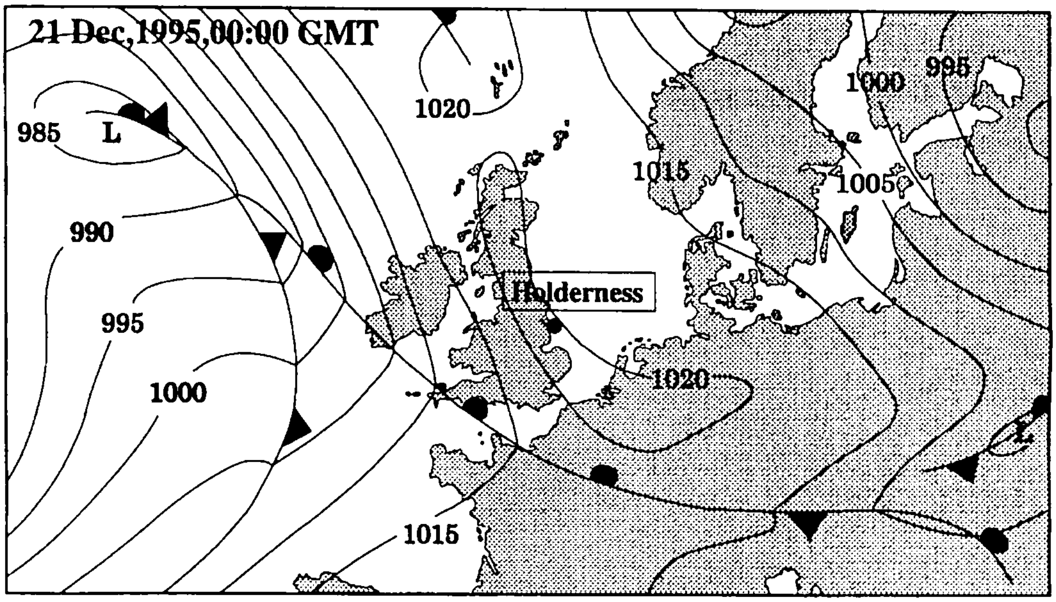

Fig. 1. European weather map (21 December, 1995, 00:00GMT) (Unit: hPa).

Figure 1 shows the European weather map at 00:00 hours on December 21, 1995. Before and after this date, the low-pressure system was stationary in the sea area west of the UK. At Holderness, located on the east coast of the UK and facing the North Sea, the bi-directional wave fields, consisting of swell from the north and wind waves from the southeast, were formed. Figure 2 shows the observation area of the HF ocean radar system. An OSCR HF radar system was installed at the Master and Slave points in Holderness as shown in the figure, and observations were made almost continuously for about one month. The OSCR HF radar system is of the pulse type and different from the one adopted in Japan which uses the Frequency-Modulated Continuous-Wave (FMCW) system (ex. Nadai *et al.*, 1997) for the determination of the observation distance. The observation was carried out for 5 minutes at each station and repeated every twenty minutes providing 896 coherent samples at each measurement point. To estimate the Doppler spectra, 512 sample FFTs were used with a 75% overlap to provide 4 spectra for each five minute period. Three successive five minute collections were then averaged to provide an hourly averaged (from 12 individual) Doppler spectrum. The mark ⊚ in Fig. 2 indicates a wave observation point by a buoy. In SCAWVEX, the method developed by Longuet-Higgins *et al.* (1961) was used to analyze the directional wave data measured by the buoy. The resultant Fourier coefficients for the directional spectra have been preserved as the parameters of the directional spectra. Based on these Fourier coefficients, we applied the method developed by Kim *et al.* (1994) to obtain the directional spectra using the maximum entropy principle method (MEP) proposed by Kobune and Hashimoto (1986), and compared them with the results estimated from HF radar.

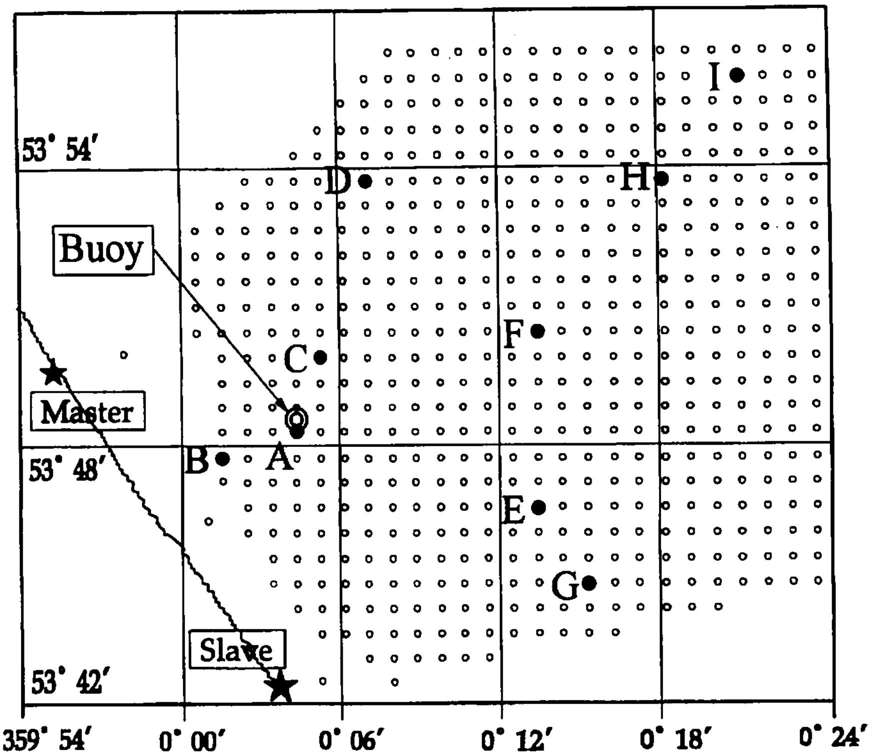

Fig. 2. Observation area of the OSCR HF ocean radar system. Backscatter is measured for all locations indicated with a small open circle. The letters denote points where wave spectra are compared.

5. Directional Spectrum Estimation

Figure 3 shows an example of the Doppler spectra of the site A in Fig. 2 measured at the Master and Slave points with the OSCR HF radar (at a frequency of 25.4 MHz). The angular frequency on the horizontal axis is normalized by the Bragg angular frequency, i.e. $\tilde{\omega} = \omega/\omega_B$. The Doppler spectrum on the vertical axis is also normalized by the area of the larger first order component, i.e. $\tilde{\sigma}(\tilde{\omega}) = \sigma(\tilde{\omega})/\int \sigma^{(1)}(\tilde{\omega})d\tilde{\omega}$. As discussed before, the second-order scattering components of the Doppler spectra represent the contribution of an infinite number of combinations of the wave components having different frequencies and propagation directions, and are related to the directional spectrum by the nonlinear integral equations. It is therefore possible that the inverse computation of the directional spectrum from the Doppler spectra becomes unstable and that the estimated values of the directional spectrum may vary depending on which frequency components of the Doppler spectra we use. We therefore examined the Doppler spectra by dividing the four frequency ranges into the domains I, II, III, and IV shown in Fig. 3, excluding the neighborhood of the first-order scattering frequencies ($\tilde{\omega} = \pm 1$) and the zero frequency of $\tilde{\omega} = 0$. Then, we estimated the directional spectra by changing the frequency range in each

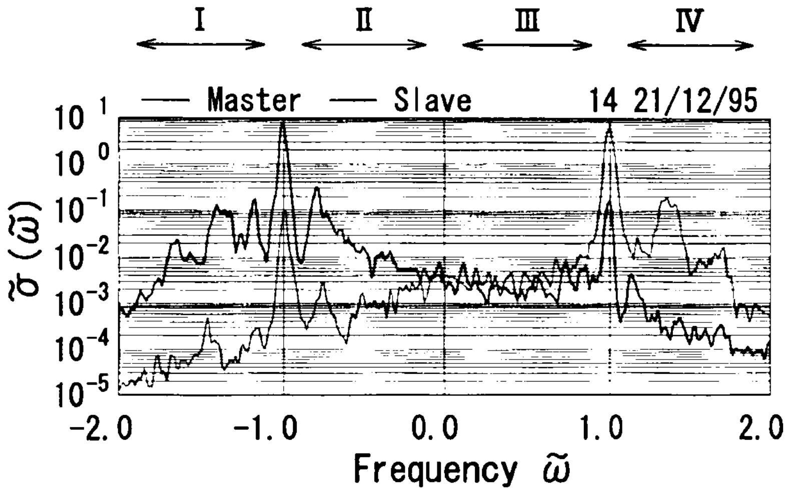

Fig. 3. An example of the normalized Doppler spectra of the backscatter at the site A from the two radars.

domain, and investigated the conditions under which an accurate directional spectrum could be estimated. We also estimated the directional spectrum by changing the combination of the frequency domains I, II, III, and IV. In the estimation of the directional spectrum through iterative computations, we used the same initial value of zero for all the computations for practical convenience.

5.1. *Necessary frequency ranges of Doppler spectra for estimation of directional spectrum*

The necessary conditions of the components of the Doppler spectra were examined by changing the frequency range criterion used in each domain for the reliable estimation of the directional spectra. The following are the results:

(1) We used a fixed frequency range of $1.85 \geq |\tilde{\omega}| \geq 1.15$ for the domains I and IV. For the domains II and III, we used a frequency range of $0.85 \geq |\tilde{\omega}| \geq \tilde{\omega}_{\min}$, and changed the value $\tilde{\omega}_{\min}$ from 0.15 to 0.75 to find its permissible range. As a result, we found that the upper limit of the value $\tilde{\omega}_{\min}$ was about 0.45, and that the directional spectra would diverge in the high frequency side if we set the value of $\tilde{\omega}_{\min}$ larger than this value. We could thus estimate a stable directional spectrum by setting the value of $\tilde{\omega}_{\min}$ smaller than 0.45.

(2) We used a fixed frequency range of $0.85 \geq |\tilde{\omega}| \geq 0.15$ for the domains II and III. For the domains I and IV, we used a frequency range of $\tilde{\omega}_{\max} \geq |\tilde{\omega}| \geq 1.15$, and changed the value $\tilde{\omega}_{\max}$ from 1.2 to 3.0 to find its permissible range. As a result, we found that the lower limit of the value $\tilde{\omega}_{\max}$ was about 1.4, and that the directional spectra would diverge in the low frequency side if we set the value of $\tilde{\omega}_{\min}$ smaller than this value. We could estimate a stable directional spectrum by setting the value of $\tilde{\omega}_{\max}$ larger than 1.4.

(3) We used a frequency range of $1.45 \geq |\tilde{\omega}| \geq \tilde{\omega}_{\min}$ for the domains I and IV, and that of $\tilde{\omega}_{\max} \geq |\tilde{\omega}| \geq 0.4$ for the domains II and III, and changed the values of $\tilde{\omega}_{\min}$ and $\tilde{\omega}_{\max}$ from 1.05 to 1.35, and from 0.95 to 0.65, respectively, to find the ranges of the second scattering components to be considered in the neighborhood of the first-order scattering frequency ($|\tilde{\omega}| \approx 1$). As a result, we found that the upper and lower limits of the value $\tilde{\omega}_{\min}$ in the domains I and IV were about 1.17 and about 1.07, respectively, and that the upper and lower limits of the value $\tilde{\omega}_{\max}$ in the domains II and III were about 0.93 and about 0.83, respectively. The directional spectra would diverge in the low frequency side if we set the values of $\tilde{\omega}_{\min}$ and $\tilde{\omega}_{\max}$ beyond these ranges. We could estimate stable directional spectra if we set the values of $\tilde{\omega}_{\min}$ and $\tilde{\omega}_{\max}$ within these ranges.

5.2. *Necessary combinations of Doppler spectra for estimation of the directional spectrum*

Using the examined results of the above (1)–(3) of Sec. 5.1 as a reference, we used a fixed frequency range of $1.15 \leq |\tilde{\omega}| \leq 1.85$ for the domains I and IV, and that of $0.15 \leq |\tilde{\omega}| \leq 0.85$ for the domains II and III, and checked the accuracy and the stability of the directional spectra with respect to combinations of the Doppler spectra in the domains from I to IV. Findings were as follows:

(1) When using Doppler spectral components in all the domains from I to IV, we could estimate stable directional spectra.
(2) We could estimate stable directional spectra as in the case of (1) only when we used Doppler spectra in the domains containing second-order scattering components that sandwiched the larger first-order scattering component of the two.
(3) When we used only second-order scattering components that sandwiched the smaller first-order scattering component, the computations became unstable.
(4) When using the Doppler spectra containing second-order scattering components that sandwiched the larger first-order scattering component for one of the two stations, and using second-order scattering component that sandwiched the smaller first-order scattering component for the other station, we could estimate the directional spectra, but their peaks were estimated in the wrong directions.

The conclusion of the above studies in Secs. 5.1 and 5.2 is that estimation of stable directional spectra is possible if we use second-order scattering components in a proper range of the Doppler spectra containing the necessary information for the computation of the directional spectra, i.e. in the neighborhood of second-order scattering components that sandwich the larger first-order scattering component. It is also understood that the second-order scattering components in the range far away from the first-order scattering components hardly improve the accuracy of

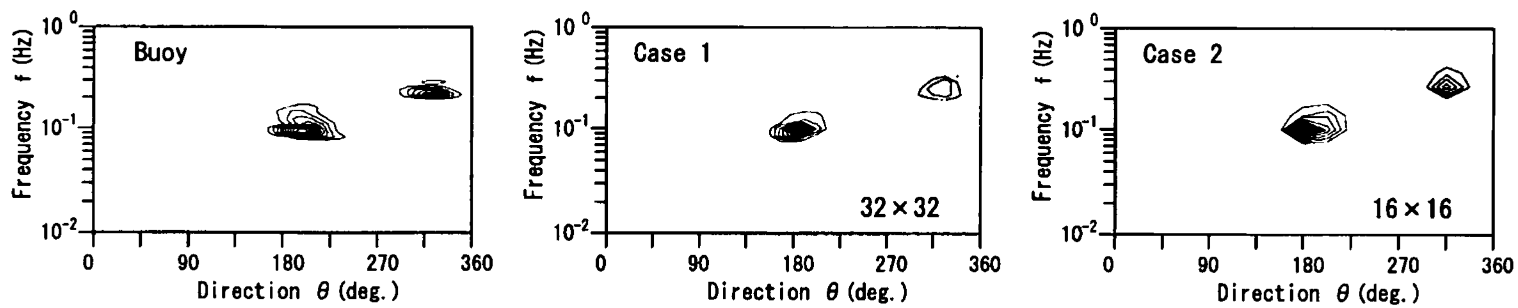

Fig. 4. Comparison of the directional spectra observed by the buoy with those estimated by the Bayesian method. (The contour lines are drawn for every 1/10 of the range from 0 to the maximum value of the directional spectrum.)

the directional spectra. Therefore, we set a frequency range that gives convenient computation time without losing the stability of the estimated values.

5.3. *Necessary number of segments for frequency/directional angle of directional spectrum*

Since the Bayesian approach assumes that the directional spectra are expressed by piecewise-constant function with respect to the frequency and the directional angle, we need to solve the equation containing $M \times N$ unknown parameters where M and N are the numbers of the frequency segments and the directional segments respectively. For practical convenience to limit the computation time, we cannot set the number of the segments too large. We hence calculated the two cases of $M = N = 32$ and $M = N = 16$, and compared their accuracy. With respect to the directional angle, we separated the range equally between 0 and 2π, and with respect to the frequency, we separated the range between 0.01 and 1.0 (Hz) on the logarithmic scale as shown in Eq. (9).

Figure 4 compares the directional spectra observed by the buoy with those estimated by the Bayesian method, in the cases of $M = N = 32$ (Case 1) and $M = N = 16$ (Case 2). The result of the buoy in Fig. 4 shows the bi-directional wave field where the swell and the wind waves propagating in the directions of about 200° and about 320° respectively. The results of both methods for the HF radar in Cases 1 and 2 show that the two energy peaks were estimated at the proper directions.

Figure 5 compares the frequency spectra and the directional functions in Case 2 with the estimated values observed by the buoy. The solid and dotted lines in Fig. 5 show the estimated values observed by buoy and those observed by HF radar respectively. As shown in Fig. 5, the two distinctive peaks in the frequency spectrum by HF radar appeared at the same frequencies in that by buoy. On the other hand, the estimated peak directions in the directional functions by HF radar appeared at the same directions in those by buoy except the case of $f = 0.398$ (Hz) where wave energy is insignificant in the frequency spectrum. In spite of setting a small number of segments in frequency and direction, the results above seem to be acceptable.

In the estimation of the directional spectrum using a Bayesian approach, an ordinary personal computer takes tens of seconds to compute the directional spectrum in the case of $M = N = 16$, which is permissible for practical use. On the other hand, it takes several minutes to compute in the case of $M = N = 32$, which is presently impractical for real-time processing. We need to further improve the method to reduce the computation time.

5.4. *Other examples of the directional spectra estimated from different Doppler spectra*

To study the applicability and accuracy of the Bayesian method, we estimated the directional spectra from the Doppler spectra of all 565 measurement points in the

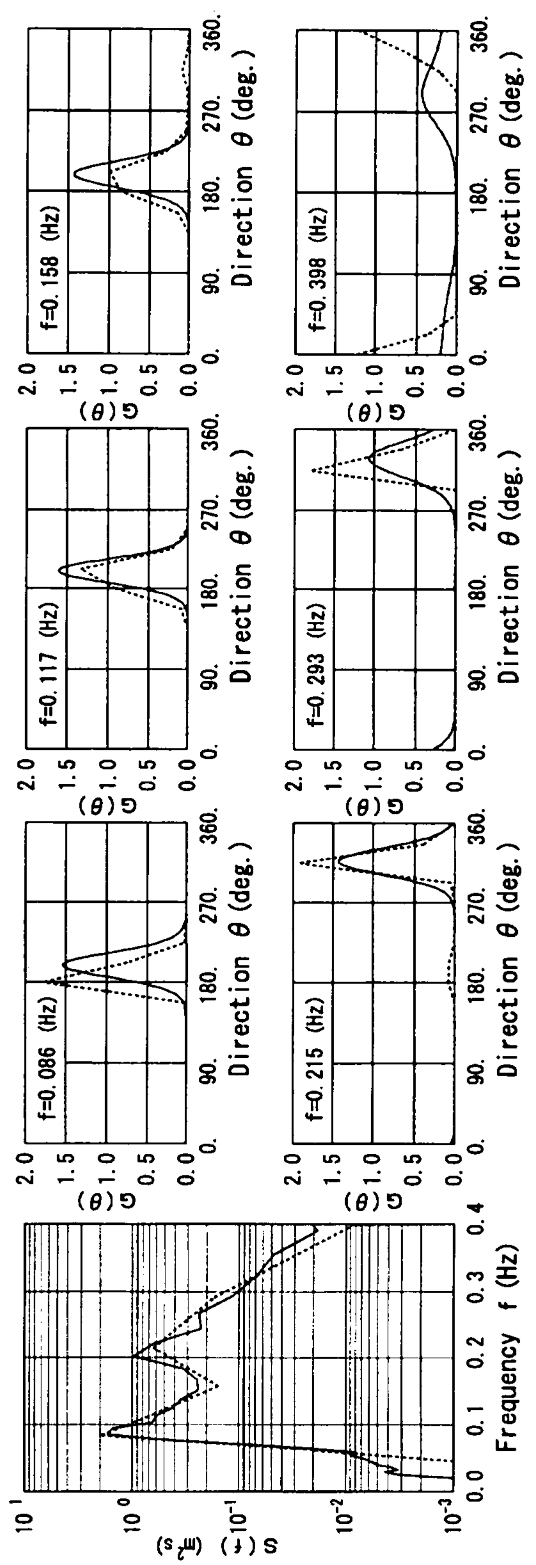

Fig. 5. Comparison of the frequency spectra and the directional functions observed by the buoy (solid lines) with those estimated by the Bayesian method (dotted lines).

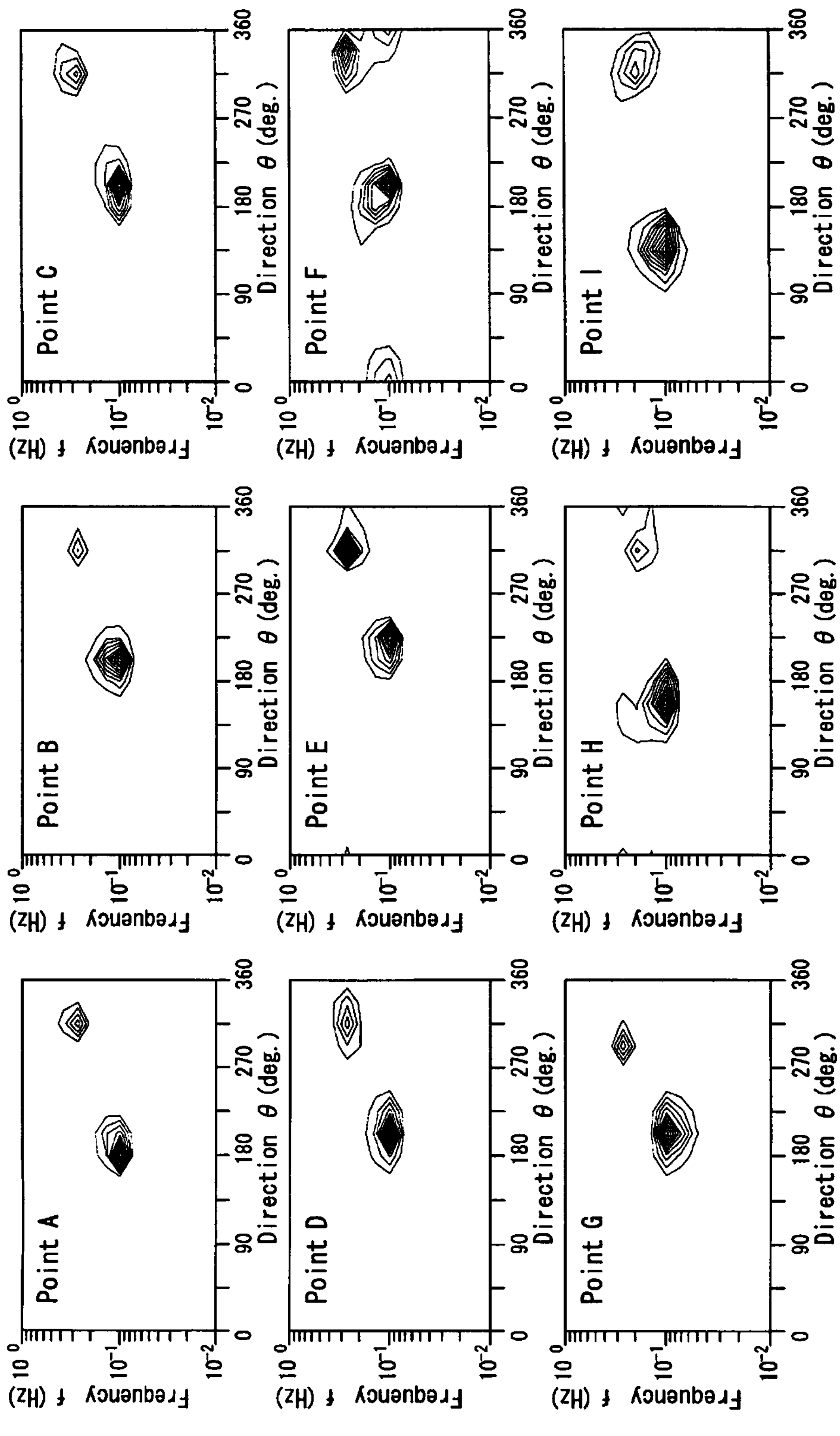

Fig. 6. Examples of the directional spectra estimated by the Bayesian method. (The contour lines are drawn for every 1/10 of the range from 0 to the maximum value of the directional spectrum.)

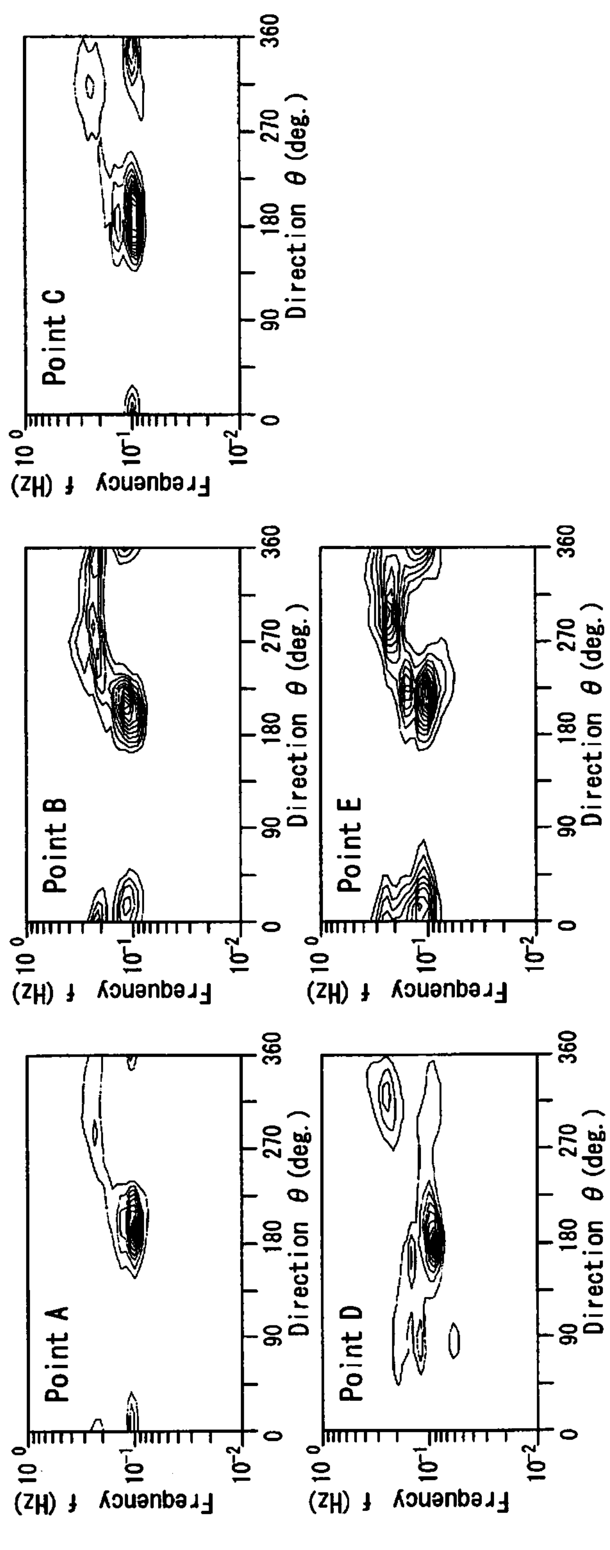

Fig. 7. Examples of the directional spectra estimated by the Wyatt method. (The contour lines are drawn for every 1/10 of the range from 0 to the maximum value of the directional spectrum.)

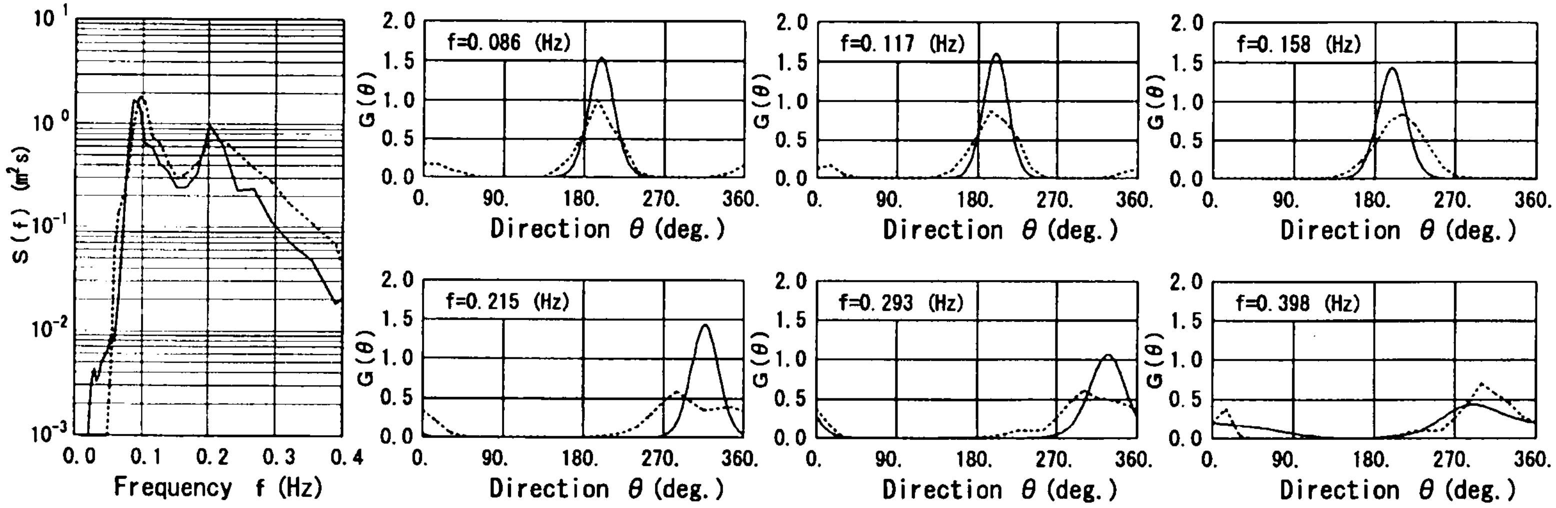

Fig. 8. Comparison of the frequency spectra and the directional functions observed by the buoy (solid lines) with those estimated by the Wyatt method (dotted lines).

observation area shown in Fig. 2. There was only one case for which the iterative computations diverged. In the cases of the other 564 measurement points, we could get convergent values by the Bayesian method. But not all of the estimated spectra were similar to those shown in Fig. 4. There were a variety of distributions showing a single peak directional spectrum or a directional spectrum having more than two peaks, etc.

As shown in Fig. 3, the energy levels of the second-order scattering components of the Doppler spectra are much lower than those of the first-order scattering components, and the second-order scattering components seem to be vulnerable to noise. There is, therefore, no guarantee that we can estimate the proper directional spectra from the second-order scattering components measured at all the observation points. The results of our study, however, showed that the measurement points at which proper directional spectra were estimated were widely distributed in the measurement area shown in Fig. 2. Figure 6 shows examples of the estimated directional spectra similar to that shown in Fig. 4. Each measurement point corresponds to the point from A to I shown in Fig. 2. As seen in Fig. 6, reliable directional spectra are successfully estimated at points widely distributed in the area.

Figure 7 shows examples of the directional spectra estimated by using the Wyatt (1990) method. Note that in the implementation of the Wyatt method, the noise level is checked and inversion proceeds only if there is sufficient signal (Wyatt, 2000). As a result for these data, analysis only proceeded at points A to E. The examples shown in Figs. 6 and 7 suggest that the Bayesian method is more robust in the presence of noise both in terms of extracting useful information at lower signal to noise ratios (at points F to I) and in terms of reducing noise in the measured spectra (e.g. at points D, E and B).

Incidentally, Fig. 8 shows corresponding results of Fig. 5. The frequency spectra and the directional functions estimated by using the Wyatt (1990) method are compared with the estimated values observed by buoy. The solid and dotted lines in Fig. 8 show the values observed by the buoy and those estimated by the Wyatt method respectively. Although the frequency spectrum estimated by the Wyatt method shows two distinctive energy peaks at the same frequencies in that of the buoy, the energy distribution is overestimated especially at the higher frequency side. In comparison with Figs. 5 and 8, the directional resolution of the Bayesian method seems to be higher than that of the Wyatt method, which underestimates the energy peaks of the directional distribution functions and shows some energy leakage around the peaks as seen in Fig. 8.

6. Conclusion

We applied a Bayesian approach for estimating the directional spectrum from the Doppler spectra acquired at the European project SCAWVEX. We have calculated a number of directional spectra using the Bayesian method and thus verified its validity and applicability. The results showed that the Bayesian method is more robust than the Wyatt (1990) method in the presence of noise both in terms of extracting useful information at lower signal to noise ratios and in terms of reducing noise in the measured spectra. We have also investigated and clarified the necessary conditions of the Doppler spectral components to be used to estimate a reliable directional spectrum with respect to (1) necessary frequency ranges of Doppler spectra, (2) necessary combinations of Doppler spectra, and (3) necessary number of segments for frequency/directional angle of directional spectrum. Although the drawback of the Bayesian method is that it requires a time-consuming iterative computation, the computation time can be reduced without losing the stability of the estimated values by taking into account those necessary conditions.

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