Formulas for ambient noise level and coherence

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This paper investigates the various approximations commonly made in noise and noise coherence models and shows that in many cases a very simple ray approach can produce the same answers as full wave treatments such as RANDI-2. The solution presented here takes the form of a single angle integral which is valid for range-independent environments. Some closed-form solutions are presented, and the approach makes it very easy to understand such phenomena as the ''noise notch.'' The method can be extended to range dependence, and demonstrations are given of performance near a boundary (inhomogeneous field) and in the presence of nonuniform horizontal distributions of noise sources. © *1996 Acoustical Society of America.*

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INTRODUCTION

A fundamental aim of this paper is to throw some light on what is, at first sight, a complicated subject, namely, modeling ambient noise in the sea, its vertical directionality, and spatial coherence. A review of existing models and techniques is given by Hamson,[1,2] and Kennedy *et al.*[3] include references to a number of noise models. Much of the literature[4–12] either assumes isovelocity or resorts to numerical methods which are often computer intensive. Alternatively some authors[13] modestly believe their approach to be oversimplistic when, in fact, they may be perfectly adequate for more general regimes. Here we try to clarify some of the assumptions that have been made and reconcile a simple ray approach with normal mode approaches, more suitable for low frequencies. In passing we find some closed-form solutions for range-independent environments, and we present some numerical comparisons with results from the literature. The method can deal with uniform noise source distributions as one might find with wind and rain, and nonuniform distributions more typical of shipping. It is important to realize that these solutions do not require any normal mode or ray tracing calculations. The main derivation is equally valid for range-dependent environments, and these are pursued elsewhere (Harrison[14]). It is perfectly possible to obtain absolute noise level from these formulas by employing experimental noise source levels (see, for instance, Kuperman and Ferla[15]) and this is discussed by Harrison.[16]

I. REVIEW OF POSSIBLE ASSUMPTIONS

We will consider the following assumptions. In order to find a neat solution we will initially adopt assumptions A, B, C, F, but we will show that it is possible to drop them all without making the solution too cumbersome.

Assumption A: ray treatment
Assumption B: closely spaced hydrophone pair (1—in depth, 2—in range)
Assumption C: neglect multipath modal interference
Assumption D: range-independent environment
Assumption E: isovelocity environment
Assumption F: azimuth-independent environment
Assumption G: neglect boundary reflections

Currently no formulas exist for arbitrary horizontal range separation (B) nor for range-dependent environments (D) though, of course, numerical solutions are possible. With various combinations of these approximations we end up with well known results, as follows: Cron and Sherman[4] assumed (A+B+C+D+E+F+G), Buckingham[5] assumed (D+E+F), and, Kuperman and Ingenito[6] assumed (D+F). It is stressed that *all* the above formulas can be derived from the simple ray approach of this paper. This gives one more insight into the environmental dependence of ambient noise and the behavior of phenomena like the ''noise notch.'' Also, the model has scope for covering more complicated effects. In particular we will concentrate on the azimuth-independent (F), range-independent case (D) with closely spaced hydrophones (B), and with the aid of Appendices A and B we will show precisely how to extend the formula to include modal effects, thus avoiding assumption C. In Sec. V we briefly drop assumption F and investigate azimuth dependence.

II. DERIVATION

Now we will derive a formula for the un-normalized spatial coherence function ρ initially making assumptions A, B, C, F (ray treatment in an arbitrary environment). Later in this section we will show that the effect of assumption C (multipaths) is trivial and can be dropped. Similarly it will be shown that it is possible to drop assumption B and assume wide vertical separations. Thus we will arrive at essentially the result of Kuperman and Ingenito,[6] and we will have scope for range-dependent extensions.

The amplitude from a unit source at horizontal range r and depth zero measured by a receiver at depth z_r can be written as a summation over all paths,

$$\psi(z_r,r)=\sum_p A_p(z_r,r,\theta_r)e^{iB_p(z_r,r,\theta_r)}, \tag{1}$$

where p is the path index, θ_r is arrival angle at the receiver, A is the (real) amplitude for each path, and B is the phase for each path. Although this is a ray treatment (approximation A) it allows for refraction and reflection at both boundaries.

The relevant geometry is shown in Fig. 1 by the slice at azimuth ϕ containing a noise source (at r,ϕ) and the center

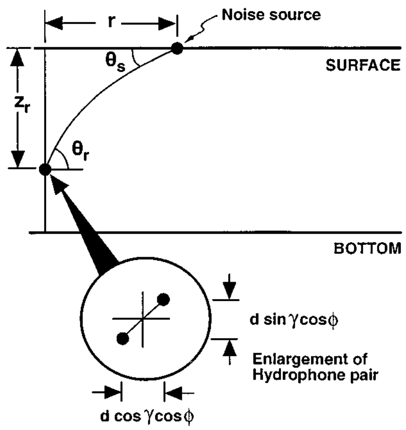

FIG. 1. Geometry of the hydrophone pair.

point of the hydrophone pair. One hydrophone is slightly above the plane of the paper, the other just below. We adopt Cron and Sherman's notation for hydrophone spacing d and elevation-angle γ of the line joining the pair. Thus the hydrophone coordinates relative to the source are

$$z_1 = z_r + (d/2)\sin\gamma\cos\phi,$$

$$r_1 = r + (d/2)\cos\gamma\cos\phi,$$

$$z_2 = z_r - (d/2)\sin\gamma\cos\phi,$$

$$r_2 = r - (d/2)\cos\gamma\cos\phi,$$

and the coherence function is

$$\rho(d,\gamma) = q\int_0^\infty \int_0^{2\pi} \psi(z_1,r_1)\psi^*(z_2,r_2)g^2(\theta_s)r\,dr\,d\phi. \tag{2}$$

We have allowed for a noise source (amplitude) directionality $g=\sin^m\theta_s$ as in Cron and Sherman and we integrate over the entire sea surface. The number of these sources per unit area is represented by q.

Note that in normal mode treatments (see Appendix A) one typically assumes a unit (nondirectional) point source at a depth z_0 below the surface which results in the dipole $4k^2z_0^2\sin^2\theta_s$. Thus $q/4k^2z_0^2$ would represent the number of this type of source per unit area. Furthermore, one could equally well talk of a power per unit area resulting from integration over all θ_s. Thus unit power per unit area would be represented by the beam pattern $((2m+1)/2\pi)\times\sin^{2m}\theta_s$. In subsequent equations in this paper we will always take q to be 1.

Now we neglect interference between multipaths (approximation C) which means we power-add ray contributions and neglect cross terms in the double sum. We assume that the two hydrophones are close together (approximation B) so that rays arriving from the same noise source via the same family of rays are parallel at the hydrophones. The complex amplitudes received at the hydrophones have the same modulus but differ in phase by $kd\cos\xi$ where k is the wave number at the hydrophones and $\cos\xi$ is the direction cosine between the incoming ray and the line joining the hydrophone pair. Note that k is a function of hydrophone depth, but in the following analysis there is never any ambiguity so we avoid using a subscript:

$$\rho(d,\gamma) = \int_0^\infty \int_0^{2\pi} \sum_p |A_p(z_r,r,\theta_r)|^2 \times e^{ik\,d\cos\xi} g^2(\theta_s)r\,dr\,d\phi. \tag{3}$$

The phase term can be written in terms of the angles from Fig. 1 as

$$kd\cos\xi = kd(\sin\theta_r\sin\gamma + \cos\theta_r\cos\gamma\cos\phi). \tag{4}$$

The term $|A|^2$ is just the ray intensity which can be written for each path as

$$|A|^2 = \frac{\cos\theta_r}{r|(dr/d\theta_r)\sin\theta_s|}QP_n, \tag{5}$$

where P_n represents the cumulative (power) boundary and absorption losses after the nth complete ray cycle. Note that $P_0=1$:

$$P_n = \prod_{j=1}^{n} R_s(\theta_{sj})R_b(\theta_{bj})e^{-as_{cj}}. \tag{6}$$

Here, R_s and R_b are surface and bottom power reflection coefficients at the surface and bottom grazing angles, a is the volume absorption, and s_c is the path length of a single ray cycle. The subscript j acknowledges that in a range-dependent environment the angles and cycle distance could change from cycle to cycle. Additionally, Q takes care of the losses along the first part cycle and includes a bottom turning point loss for initially downward rays.

Substituting Eqs. (4)–(6) into Eq. (3) we obtain

$$\rho(d,\gamma) = \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} e^{ik\,d\sin\theta_r\sin\gamma} e^{ik\,d\cos\theta_r\cos\gamma\cos\phi} \times\left(\sum_{n=0} P_n g^2(\theta_{sn})/\sin\theta_{sn}\right) Q\cos\theta_r\,d\theta_r\,d\phi. \tag{7}$$

Remarkably the $r\,dr$ cancels out. What is happening is that the weakening of intensity due to ray spreading is exactly compensated by the simultaneous increase in number of noise sources with surface area. This was noted by Chapman[7] for an isovelocity environment but is actually true even for range-dependent environments. Note that we have cancelled the modulus of $r\,dr$ with $r\,dr$. This simply means that in the integral over θ_r all elements of the integrand have to contribute positive amounts to the total. This actually happens quite naturally, and no special precautions are required. The most important point is that Eq. (7) is valid for a range-dependent environment with or without reflecting boundaries. We shall return to the problem of evaluating ΣP_n in a companion paper.[14] For the time being we will stick to range-independent environments. In this case ΣP_n is simply a geometric series (as Chapman observed for an isovelocity environment), and we can drop the n subscript for θ_s and then perform the integration in ϕ. Thus

$$\Sigma P_n = \frac{1}{1 - R_s(\theta_s) R_b(\theta_b) e^{-a s_c}}.$$

To accommodate Q we separate the angle integral into two halves and recombine them to an integral over half the angle range, $0 \rightarrow \pi/2$, with the result

$$\rho(d,\gamma) = 2\pi \int_0^{\pi/2} [1 - R_s R_b e^{-a s_c}]^{-1} (e^{ikd \sin\theta_r \sin\gamma} e^{-a s_p} + R_b e^{-ik\, d \sin\theta_r \sin\gamma} e^{-a(s_c - s_p)}) \times J_0(kd \cos\theta_r \cos\gamma) \sin^{2m-1}\theta_s \cos\theta_r \, d\theta_r, \quad (8)$$

where s_p is the part cycle path length (from receiver depth to surface for one upward going ray.

This integral is easy to evaluate numerically, and some examples are shown later. To investigate its behavior we will look at several cases.

Notice that there is no explicit water depth dependence and that receiver depth dependence is also weak, its main effect being through limiting ray angles, as will be seen in the examples in Sec. III.

In theory, as all losses tend to zero, there is a possibility of a singularity in Eq. (8) if the term in square brackets goes to zero. This will be familiar to astronomers as a 2-D version of Olbers' paradox where one might expect an infinitely bright night sky to result from a uniform distribution of stars! Even with finite losses that tend to zero only at grazing incidence the $m=0$ case may result in a singularity, but the $m \neq 0$ cases will usually (but not always) avoid it. These cases will be noted later.

For simplicity we assume that a is small and the reflection coefficients R_s and R_b do not deviate far from unity. Furthermore we will assume that either R_s or R_b behaves like $e^{-\alpha \sin\theta}$ where θ is the appropriate boundary angle. [Analytical solutions are still possible for large losses (by assuming $R = 1 - \alpha \sin\theta$) but the solutions are not so neat.] This is the same assumption (i.e., reflection loss proportional to $\alpha \sin\theta$) that results in mode stripping and a signal fall-off with range of $r^{-3/2}$ (see Appendix A). In the case where boundary loss dominates, Eq. (8) reduces to

$$\rho(d,\gamma) = \int_0^{\pi/2} 4\pi\alpha^{-1} \cos(kd \sin\theta_r \sin\gamma) \times J_0(kd \cos\theta_r \cos\gamma) \frac{\sin^{2m-1}\theta_s}{\sin\theta} \cos\theta_r \, d\theta_r. \quad (9)$$

Already there are some obvious simple cases for noise level alone ($d=0$). For instance, dominance by surface loss with a dipole noise source results in an integral only dependent on θ_r and therefore not dependent on refraction details. Similar simplification results in isovelocity water. Before investigating these cases in detail we briefly review the effects of relaxing the approximations B and C. From here on we will only deal with dipole sources, $m=1$.

A. Allowances for large hydrophone separations

The derivation in Appendix A assumes arbitrarily spaced hydrophones at depths z_1 and z_2, and shows that for sufficiently long ranges the vertical phase terms simply become $\int_{z_1}^{z_2} k \sin\theta \, dz$ instead of $k(z_2 - z_1)\sin\theta_r = kd \sin\gamma \sin\theta_r$. This range restriction is comparable to the restriction on validity of the discrete normal mode sum. No such restriction is required for closely spaced hydrophones.

B. Inclusion of multipath interference

Equation B(2) in Appedix B includes the multipath terms for a single ray phase speed. If we were to integrate this in range allowing for different up- and down-going ray angles to each hydrophone we would obtain a condition equivalent to Eq. (A4) in Appendix A. This states from normal mode arguments that, provided the decay (including boundary losses) within one cycle is small one can neglect cross terms or multipath interference. The resulting ray or mode formula is therefore demonstrably incoherent. Despite this there is still a systematic depth effect which manifests itself as the mode shape [e.g., Eq. (A5) or (A8) in Appendix A] or the first cosine term in Eq. (9). The only effects that are ignored as a result of approximation C are the slight differences within a distance $\lambda/\sin\theta_c$ of the boundary (Buckingham's "quasihomogeneity") and the discretization of the angle integral due to modal propagation (see Appendix A). Furthermore, these effects are very easy to put back into Eq. (8) without spoiling its simplicity.

III. SPECIAL CASES

A. Surface dominated losses

We assume $R_b = 1$, $a = 0$, and $R_s = e^{-\alpha_s \sin\theta_s}$, where α_s (assumed small) is related to the dB loss per radian per bounce α_{dB_s} through $\alpha_{dB_s} = 4.343\ \alpha_s$. We allow for these conditions to apply over a limited angle range by generalizing the integration limits to θ_1 and θ_0. As will be seen, these angle limits are simply determined by Snell's law for the given sound-speed profile (SSP). Equation (9) becomes

$$\rho(d,\gamma) = \int_{\theta_0}^{\theta_1} \frac{4\pi}{\alpha_s} \cos(kd \sin\theta_r \sin\gamma) \times J_0(kd \cos\theta_r \cos\gamma) \cos\theta_r \, d\theta_r. \quad (10)$$

The absolute noise level I_N for an environment with an arbitrary SSP consistent with the assumptions is simply the unnormalized correlation function evaluated at $d=0$. This can be integrated to give

$$I_N = \frac{4\pi}{\alpha_s} (\sin\theta_1 - \sin\theta_0). \quad (11)$$

In a surface duct, for instance, the only dependence on the SSP is through θ_1 which is the steepest ray angle at the receiver sustainable by the duct (of maximum velocity $c_{\max}$):

$$\theta_1 = \cos^{-1}(c(z_r)/c_{\max}).$$

If, despite being in a surface duct, there happens to be a velocity maximum c_u above the receiver, such that

$c_{\max} > c_u > c(z_r)$, then θ_0 is nonzero and given by

$$\theta_0 = \cos^{-1}(c(z_r)/c_u).$$

Otherwise θ_0 is zero.

The normalized coherence function C is

$$C = \int_{\theta_0}^{\theta_1} \cos(kd \sin\theta_r \sin\gamma) J_0(kd \cos\theta_r \cos\gamma) \times \cos\theta_r \, d\theta_r / (\sin\theta_1 - \sin\theta_0). \tag{12}$$

In the special case where $\theta_1 = \pi/2$ and $\theta_0 = 0$ we can rewrite the integral in its original form,

$$\int_{-\pi/2}^{\pi/2} \int_0^{2\pi} e^{ik\,d\,\cos\xi} \cos\theta \, d\theta \, d\phi.$$

Noting that this is an integral over all solid angles and changing coordinates so that the polar axis is aligned with ξ, we obtain

$$\int_0^{\pi} e^{ikd\cos\xi} 2\pi \sin\xi \, d\xi = 2\pi 2 \frac{\sin kd}{kd}$$

This result is, in fact, a version of the Sommerfeld integral.[17]

Thus the coherence function with surface dominated losses and no angle restriction is

$$C = \frac{\sin kd}{kd}. \tag{13}$$

At first sight it is surprising that this is the same as the result for a uniform volume distribution of sources (see Cron and Sherman[4]) and there is no dependence on hydrophone pair orientation γ. The reason for this is that the bottom loss law, the dipole law, and the ray spreading jointly result in an effectively angle-independent emission by the noise sources, and regarding the multipaths as issuing from many image planes we have surrounded the hydrophones by a uniform volume of sources. Note that this result is valid for any SSP with dominant surface losses provided the sound speed at the receiver is greater than at any point in the SSP above it.

For other values of θ_1, θ_0 we can solve the vertical array case ($\gamma = \pi/2$), and

$$C = \frac{\sin(kd \sin\theta_1) - \sin(kd \sin\theta_0)}{kd(\sin\theta_1 - \sin\theta_0)}. \tag{14}$$

For large kd Eq. (12) can be solved in many cases by the method of stationary phase.

B. Bottom dominated losses

We assume $R_s = 1$, $a = 0$, and $R_b = e^{-\alpha_b \sin\theta_b}$ where α_b is small. Ignoring the effect of R_b in the numerator of Eq. (8) so that Eq. (9) still stands, and, again, allowing for a general upper and lower angle limit θ_1, θ_0 we obtain

$$\rho(d,\gamma) = \int_{\theta_0}^{\theta_1} \frac{4\pi}{\alpha_b} \cos(kd \sin\theta_r \sin\gamma) \times J_0(kd \cos\theta_r \cos\gamma) \frac{\sin\theta_s}{\sin\theta_b} \cos\theta_r \, d\theta_r. \tag{15}$$

Clearly all relevant rays must hit the surface and bottom, so the lower limit is determined by the maximum velocity in the entire SSP.

It is obvious from Eq. (15) that any environment with $\theta_s = \theta_b$ behaves in exactly the same way as the surface loss dominated environment. Also, the role of a critical angle θ_c in the bottom loss is the same as θ_1 in Eq. (11). In fact for isolvelocity we have the simple formula

$$I_N = \frac{4\pi}{\alpha_b} (\sin\theta_c - \sin\theta_0).$$

By invoking Snell's law the absolute noise level I_N can be written as

$$I_N = \frac{4\pi}{\alpha_b} \frac{c_s}{c_b} \int_{\sin\theta_0}^{\sin\theta_1} \left(\frac{A + X^2}{B + X^2} \right)^{1/2} dX, \tag{16}$$

where

$$A = \frac{c_r^2}{c_s^2} - 1, \quad B = \frac{c_r^2}{c_b^2} - 1.$$

According to Gradshteyn and Rhyzik[18] (p. 276) this can be evaluated in terms of elliptic integrals of the first and second kind F and E under six conditions depending on the signs of A and B and the relative magnitude of A and B. These correspond to the six ways of ordering c_r, c_s, and c_b. Here we show two of these solutions, one for upward refraction, and one for downward refraction. To keep presentation neat we show the integrals with the lower limit θ_0 set either to zero or to sine of the appropriate limiting angle. In other words we assume the minimum phase velocity to be $\max[c_r, c_s, c_b]$. From these, the integrals with any limit can be found. In each case θ_{s1}, θ_{b1} are the surface and bottom angles related by Snell's law to θ_1.

1. $c_b > c_r > c_s$: Upward refraction

$$I_N = \frac{4\pi}{\alpha_b} \frac{c_s}{c_b} \left[\frac{c_r^2}{c_s c_b} \frac{\sin\theta_{s1} \sin\theta_{b1}}{\sin\theta_1} + (A - B)^{1/2} \times (F(\mu,\nu) - E(\mu,\nu)) \right], \tag{17}$$

where

$$\cos\theta_0 = c_r / c_b,$$

$$\mu = \arccos((-B)^{1/2} / \sin\theta_1),$$

$$\nu = (1 - B/A)^{-1/2}.$$

2. $c_s > c_r > c_b$: Downward refraction

$$I_N = \frac{4\pi}{\alpha_b} \frac{c_s}{c_b} \left[\frac{c_r^2}{c_s c_b} \frac{\sin\theta_{s1} \sin\theta_{b1}}{\sin\theta_1} - (B - A)^{1/2} E(\mu,\nu) \right], \tag{18}$$

where

$$\cos\theta_0 = c_r / c_s,$$

$$\mu = \arccos((-A)^{1/2} / \sin\theta_1),$$

$$\nu = (1 - A/B)^{-1/2}.$$

C. Absorption dominated losses

A simple possibility is to assume isovelocity water with $R_s = R_b = 0$ and $a \neq 0$. Equation (9) then becomes

$$\rho(d,\gamma) = 2\pi \int_0^{\pi/2} e^{ikd \sin\theta_r \sin\gamma} J_0(kd \cos\theta_r \cos\gamma) \times e^{-az_r \csc\theta_r} \sin\theta \cos\theta_r \, d\theta_r. \tag{19}$$

The absolute noise level I_N reduces to the exponential integral E_3 (Abromowitz and Stegun[19]):

$$I_N = 2\pi E_3(az_r). \tag{20}$$

If we take instead $R_s = R_b = 1$ and make the approximation that as_c is small, so that $1 - \exp(-as_c) \sim as_c$, we obtain

$$\rho(d,\gamma) = 4\pi \int_0^{\pi/2} \frac{1}{as_c} \cos(kd \sin\theta_r \sin\gamma) \times J_0(kd \cos\theta_r \cos\gamma) \sin\theta_s \cos\theta_r \, d\theta_r, \tag{21}$$

where s_c is a function of angle. We might consider two cases, isovelocity and surface duct.

For isovelocity we have

$$s_c = 2H/\sin\theta$$

and the absolute noise level is

$$I_N = \frac{4\pi}{a} \int_0^{\pi/2} \frac{\sin^2\theta}{2H} \cos\theta \, d\theta = \frac{2\pi}{aH} \left[\frac{\sin^3\theta}{3} \right]_0^{\pi/2} = \frac{2\pi}{3aH}. \tag{22}$$

For a surface duct we have

$$s_c = 2 \frac{c}{c'} \tan\theta_s.$$

For small angles this gives the same result as for surface losses except for a multiplication factor

$$I_N = \frac{4\pi}{a} \frac{c'}{2c} \int_0^{\theta_1} \cos\theta_r \, d\theta_r = \frac{2\pi c'}{ac} \sin\theta_1, \tag{23}$$

where we assume arrival angles in the range $|\theta_r| < \theta_1$.

IV. GENERAL CASE

A. Rules of thumb

We start by deriving some rules of thumb that enable complex environments to be tackled by the simple formulas already derived; then we move on to some numerical evaluations of Eq. (8) for realistic environments. If we consider an arbitrary range-independent environment with any combination of a, R_s, and R_b then the only possible regimes are shown diagrammatically in Fig. 2 in terms of phase speed (a) and noise intensity versus angle (b).

Four velocities are shown: the value at the receiver c_r, the maximum value in the SSP above the receiver c_u, the maximum value in the entire water column c_{max}, and the bottom critical angle c_c. These translate into the angles θ_0, θ_1, and θ_2 where $\cos\theta_0 = c_r/c_u$, $\cos\theta_1 = c_r/c_{max}$, and $\cos\theta_2 = c_r/c_c$, as shown in Fig. 2(b). Clearly we must always have $c_r \leqslant c_u \leqslant c_{max}$, so potentially there are no more than four phase speed bands or angle ranges. From c_r to c_u (if $c_u \geqslant c_r$) there may be a noise notch (NN); from c_u to c_{max} (if $c_{max} > c_u$) there may be a surface duct with upward refraction (SD); from c_{max} to c_c (if $c_c > c_{max}$) there may be low loss surface and bottom reflected paths (SB); and above c_c there will always be direct paths (D) and high bottom loss paths (HBL).

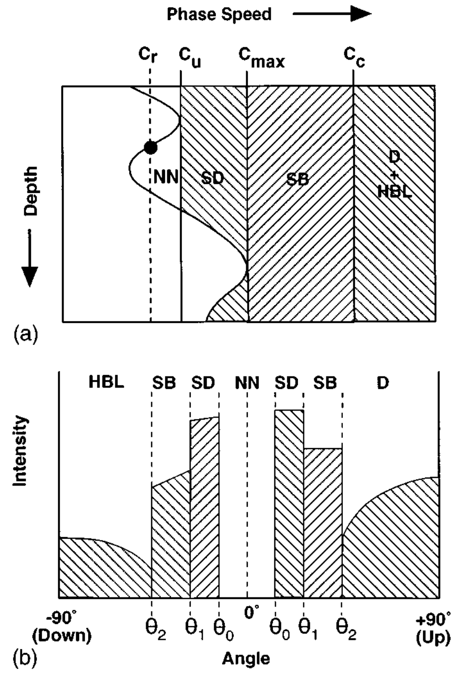

FIG. 2. (a) Diagram showing the only possible phase speed bands for surface noise sources in a range-independent environment. NN=noise notch, SD=surface duct, SB=surface/bottom reflected paths, D=direct path, HBL =high bottom loss paths. (b) Intensity contributions in the angle bands corresponding to (a).

In Fig. 2(b) the SD and SB contributions are shown as flat topped for upward rays and slightly drooping for downward rays because of the extra R_b term [see Eq. (8)]. In addition, the intensity of the steep upward and downward rays is proportional to sin θ, since the quantity plotted is the integrand of Eq. (8) (omitting the cos θ because this is part of the solid angle element).

Obviously the surface duct will never be influenced by bottom reflections, but the SB band can be dominated by surface, bottom or absorption. Since these losses are additive in the denominator of Eq. (8), the surface/bottom change over is given by equating surface and bottom losses:

$$\alpha_s \sin\theta_s = \alpha_b \sin\theta_b.$$

From Snell's law this translates to an angle at the receiver of $\theta_r = \theta_{sb}$:

$$\cos\theta_{sb} = c_r \left(\frac{\alpha_s^2 - \alpha_b^2}{c_s^2 \alpha_s^2 - c_b^2 \alpha_b^2} \right)^{1/2} \tag{24}$$

and the phase speed c_{sb} is

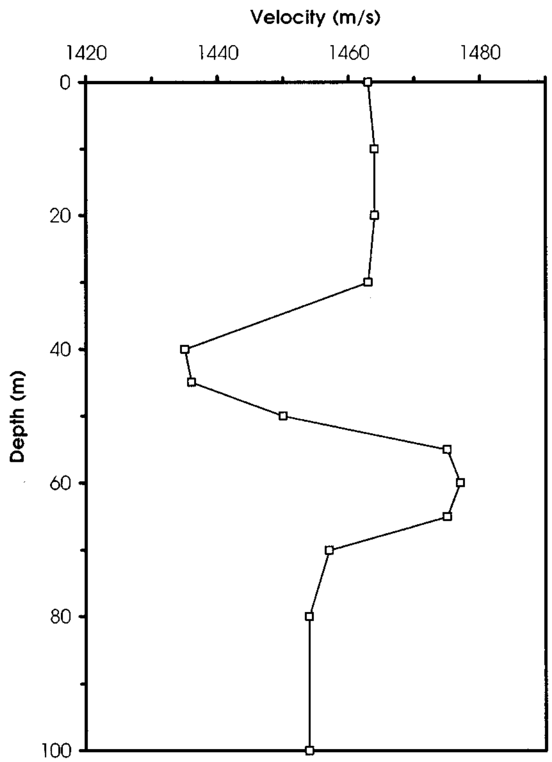

FIG. 3. Sound-speed profile for a Baltic environment.

$$c_{sb} = c_r \sec \theta_{sb} = \left(\frac{c_s^2 \alpha_s^2 - c_b^2 \alpha_b^2}{\alpha_s^2 - \alpha_b^2} \right)^{1/2}. \tag{25}$$

Use of this approach means that we can deal with arbitrary environments (noise and coherence) without resorting to normal mode calculations or ray traces since all that is required is Eq. (8) with appropriate given R_s, R_b, a, and an approximate formula for the ray cycle complete path length s_c and partial path length s_p. For instance we could use

$$s_p = z_r / \sin((\theta_r + \theta_s)/2)$$

and

$$s_c = 2H / \sin((\theta_b + \theta_s)/2), \quad \text{for bottom rays}$$
$$s_c = (2c/c') \sin \theta_s, \quad \text{for surface only rays.}$$

B. Numerical examples

In the following examples we have evaluated Eq. (8) numerically with given values of $d, \gamma, \alpha, R_s(\theta_s), R_b(\theta_b)$, and velocity profile. The cycle path lengths s_p and s_c were calculated by a ray trace for one half-cycle from the sea surface, respectively, down to the receiver depth and down to the lower turning point (whether refracted or reflected).

The environment is a Baltic case investigated at a frequency of 800 Hz by Hamson[7] using RANDI-2. RANDI[11,12] is a noise model, based on a full wave solution, that can include near-field effects as well as discrete point sources. Figure 3 shows the sound-speed profile, and the bottom loss, shown in Fig. 4, has been recalculated using SAFARI.[20] Following Hamson we take surface losses to be zero, and we

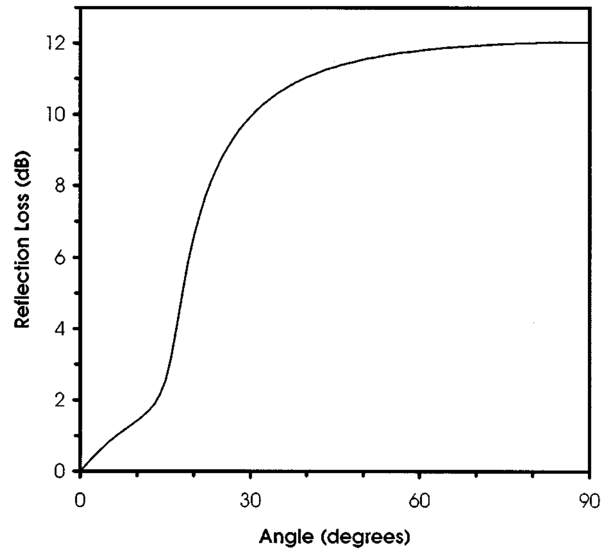

FIG. 4. Bottom loss for the Baltic environment calculated with SAFARI.

have a volume absorption of 0.0505 dB/km. The absolute noise source level per unit area for wind sources follows Kuperman and Ferla[15] as does RANDI-2.

1. Noise intensity versus angle

Figure 5 shows noise intensity versus angle [i.e., the integrand of Eq. (8) with $d = 0$ excluding the final cos θ_r which is part of an element of solid angle] plotted for three depths 20, 40, and 80 m. Referring to the SSP, these are, respectively, above the trough, in the trough, and below the peak in the SSP. There are several interesting features here. One is that the picture is asymmetrical because bottom losses dictate that there is more energy coming from upward than downward. Another is that at 40 and 80 m there is a clear noise notch, and one can easily see the correspondence between the parts of this graph and Fig. 2(b). The narrow spikes for receivers above the peak in the SSP are low loss contributions from the surface duct. The small hump at around 15° corresponds to relatively low loss bottom re-

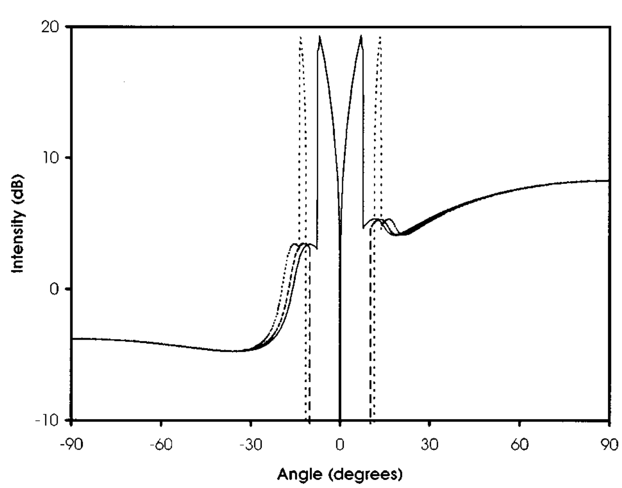

FIG. 5. Noise intensity versus angle for three receiver depths: 20 m (solid), 40 m (dotted), and 80 m (dashed).

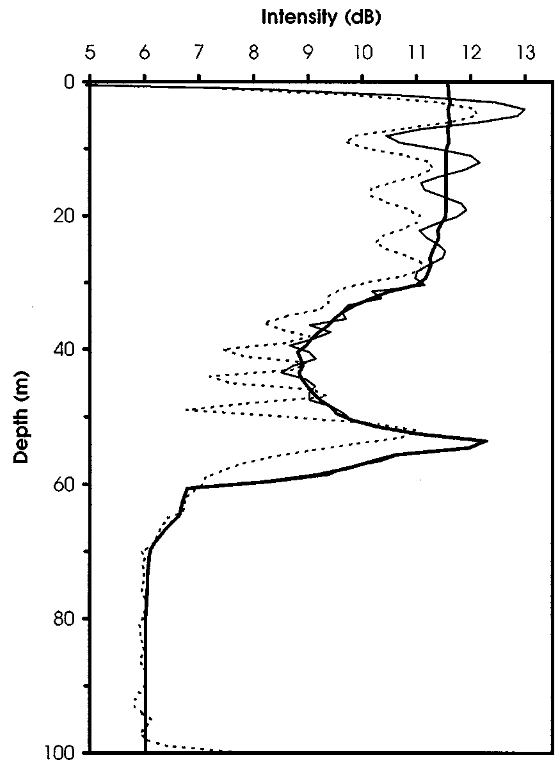

FIG. 6. Noise intensity versus depth calculated from: Eq. (8) (thick solid line); Eq. (8) modified with additional cos term (thin solid line); and RANDI-2 after Hamson (dashed line).

flected paths, and the kink at about $+20°$ is the transition between linear bottom loss and high fixed loss (the effective critical angle).

2. Noise intensity versus depth

The thick solid line in Fig. 6 is a plot of noise intensity versus depth for the same environment. Notice that there is a resemblance between the shape of the intensity and the SSP. The main reason for this is that the width of the noise notch increases, and therefore the intensity decreases, when the receiver enters a local minimum in the SSP. Hamson's RANDI-2 results are superimposed as a dotted line in Fig. 6. The overall shape is the same, but there are more oscillations because of wave interference effects, and ambient noise falls to zero at the sea surface. As discussed in Appendices A and B, it is possible to put back the second cos term [Eq. (A8), Appendix A] in our numerical integral, and the result is the thin solid line in Fig. 6. Considering the simplicity of the approach (still a single integral) agreement is extremely good. The remaining discrepancies are due to the discreteness of the mode angles, which is clearly not represented in the integral. The only penalty is that the extra term can be highly oscillatory with angle, since the argument is 2 $kz_r \sin\theta_r$ rather than $kd \sin\theta_r$, and this requires finer sampling.

3. Coherence versus hydrophone separation

Equation (8) can be evaluated for fixed receiver depth, fixed hydrophone elevation angle γ, but varying hydrophone spacing d. For horizontal pairs ($\gamma=\pi/2$) the imaginary part

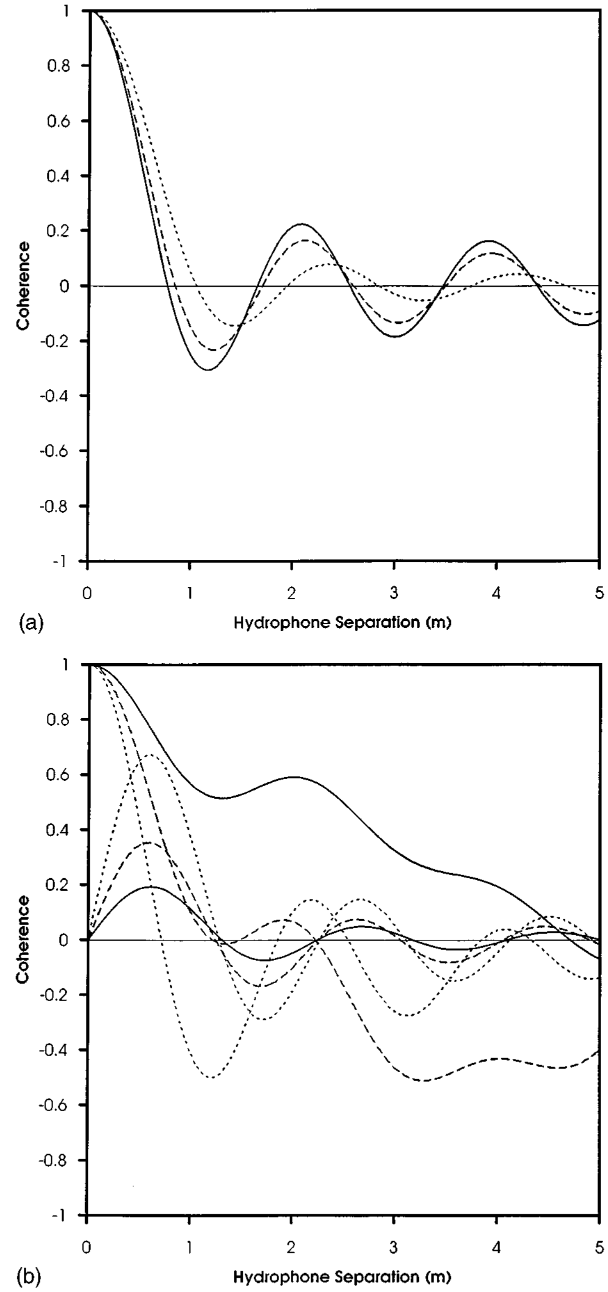

FIG. 7. Coherence versus hydrophone spacing for (a) a horizontal pair and (b) a vertical pair at three depths: 20 m (solid), 40 m (dashed), and 80 m (dotted).

of ρ is identically zero, and even in this Baltic environment the variations are small as seen in Fig. 7(a) for depths 20, 40, and 80 m. The corresponding plots for vertical pairs ($\gamma=0$) are more interesting as seen in Fig. 7(b).

V. NONUNIFORM SPATIAL DISTRIBUTIONS

Up until now we have assumed a uniform distribution of noise sources in range and azimuth. So the earlier results are suitable for wind and rain, but not directly for shipping. We deal below separately with nonuniformity in range and azimuth.

A. Range nonuniformity

We define our "nonuniform" distribution as uniform within a thick annulus bounded by a minimum range r_1 and a maximum range r_2. For the purpose of noise calculation (or vertical array coherence) the fact that this is symmetrical in azimuth is irrelevant, and the ships could just as easily be concentrated in a limited spread of azimuth.

By arguments similar to that leading to (A4) in Appendix A it is easy to show that multipath interference can still be dropped under certain conditions. For instance, if we were to localize the ships in range with a displaced Gaussian probability distribution of width w, the condition for validity would be that for all elevation angles the cycle distance r_c should be less than the modal decay distance δ_n^{-1} (i.e., $r_c\delta_n \ll 1$) *and* that the cycle distance should be less than the Gaussian's width (i.e., $r_c \ll w$). Thus this condition of validity translates to a minimum thickness for the annulus: $r_2 - r_1 \gg r_c$ for all elevation angles. This is likely to be satisfied in shallow water but possibly a poor approximation in deep water when there may be convergence zones of 60 km or more spacing.

In the earlier derivation leading to Eq. (8) we summed a geometric series for the noise arrivals from zero to infinity. The only difference now is that the series goes from n_1 to n_2 where

$$n_1 = \mathrm{INT}(r_1/r_c), \tag{26}$$

$$n_2 = \mathrm{INT}(r_2/r_c), \tag{27}$$

and clearly n_1 and n_2 are range dependent. We now simply substitute a new geometric series in Eq. (8):

$$\sum_{n=n_1}^{n_2} R^n = \frac{R^{n_1} - R^{n_2+1}}{1-R}, \tag{28}$$

giving

$$\rho(d,\gamma) = 2\pi \int_0^{\pi/2} \left[\frac{R^{n_1} - R^{n_2+1}}{(1-R)}\right] (e^{ikd \sin\theta_r \sin\gamma} e^{-as_p} + R_b e^{-ikd \sin\theta_r \sin\gamma} e^{-a(s_c - s_p)}) \times J_0(kd \cos\theta_r \cos\gamma) \sin^{2m-1}\theta_s \cos\theta_r \, d\theta_r, \tag{29}$$

where $R = R_s(\theta_s) R_b(\theta_b) e^{-as_c}$.

The effect of the extra R^n terms is to emphasize low angles; it is precisely the "mode stripping" effect,[21] as is easily seen. Assuming reflection loss to be $\alpha \sin\theta$ we have terms like $R^n = e^{-n\alpha \sin\theta}$, but n is range dependent according to Eqs. (26) and (27). For isovelocity we have $r_c = 2H \cot\theta$ so the term R^n becomes $\exp(-r\alpha \sin\theta \tan\theta/2H)$. It is the roughly Gaussian behavior (in θ) that leads after integration to mode stripping. Clearly numerical integration is again straightforward, but for analytical purposes the Gaussian can be thought of as a cutoff in angle at $\theta \simeq (2H/r\alpha)^{1/2}$, providing a (range dependent) value for θ_1 in the earlier integrals.

1. Numerical example

Taking the earlier environment we can demonstrate the effect of range nonuniformity on intensity versus angle. We assume the noise source distribution to be uniform except for a circular area above the receiver where there are no sources. Figure 8 shows the effect of enlarging this area for a receiver at 20-m depth. Narrow angle surface duct returns are largely unaffected whereas the wide angle bottom reflections are cut out by mode stripping. The effect is shown as a function of range at three depths in Fig. 9. The deepest depth, 80 m, is attenuated most severely. The other two, 20 and 40 m, being dominated by surface duct, fall off exponentially because of absorption since $R = \exp(-as_c)$ and $n = r/r_c$ giving $R^n \simeq \exp(-ar)$.

By convolving these curves, for uniformly distributed wind sources but nonuniformly distributed shipping, with a vertical line array beam pattern one can calculate an array response, and favorable comparisons have been made[16] with RANDI-2 calculations by Hamson.[9] The array response in

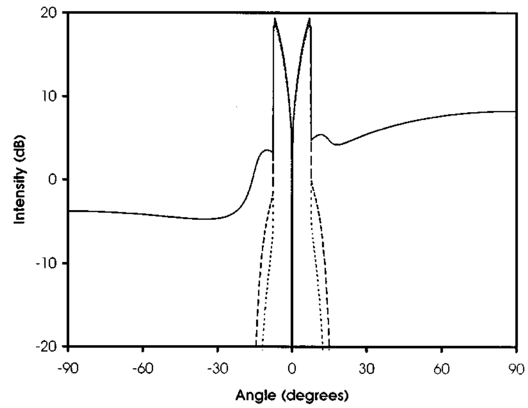

FIG. 8. The effect on noise directionality of removing noise sources from ranges less than: 0 km (solid), 5 km (dotted), 10 km (dashed) for a receiver at 20-m depth.

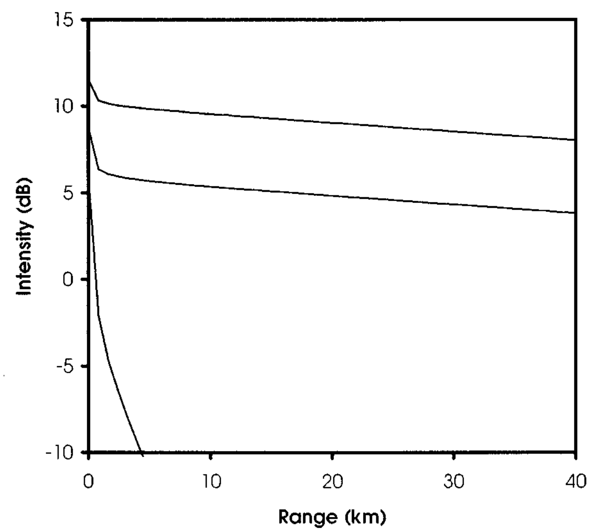

FIG. 9. The dependence of noise level on the range of the closest noise source in a distribution that otherwise extends uniformly to infinity. Curves for three receiver depths are shown: 20 m (upper), 40 m (intermediate), and 80 m (lower).

any environment (assuming the array has enough angular resolution) will always reflect the features of Fig. 8 (with or without a surface duct) since distant ships will result in tall peaks near to the horizontal, whereas uniformly distributed wind sources will give a broader response.

B. Azimuth nonuniformity

Returning to Eqs. (3) and (4) we see that it is the cos ϕ term in the exponent that leads to the Bessel function $2\pi J_0$ in Eq. (7) after azimuth ϕ integration. If we had superimposed an azimuthal dependence $p(\phi-\beta)$, where β is a given constant, we would have obtained instead

$$I(A,\beta)=\int_0^{2\pi} e^{iA\cos\phi} p(\phi-\beta)\,d\phi. \tag{30}$$

Numerically it is easy to tabulate I against A for a given constant β and to store the function $I(A,\beta)$ *independently* of any other integrals in elevation angle.

Analytically we can solve Eq. (30) when p is a Gaussian beam of width w, i.e., $p(\phi)=\exp(-\phi^2/w^2)$, since the complex exponential can be expanded in a series of Bessel functions[19] (9.1.44, 9.1.45):

$$\begin{aligned}I(A,\beta)=w\pi^{1/2}\Bigg[&\Bigg(J_0(A)+2\sum_{j=1}^{\infty}(-1)^j J_{2j}(A)\\&\times\cos(2j\beta)\exp(-j^2w^2)\Bigg)\\&+i\Bigg(2\sum_{j=1}^{\infty}(-1)^{j+1}J_{2j-1}(A)\cos((2j-1)\beta)\\&\times\exp(-(j-1/2)^2w^2)\Bigg)\Bigg].\end{aligned} \tag{31}$$

These series converge rapidly especially for large w (e.g., $w\sim1$ rad). For very small w it is more convenient to use the equivalent formula

$$\begin{aligned}I(A,\beta)=&\,w\pi^{1/2}\exp(iA\cos\beta)\\&\times\exp(-(A^2w^2\sin^2\beta)/4).\end{aligned} \tag{32}$$

C. A shipping lane

A simple scenario is a uniform distribution of noise sources (e.g., ships) on one side only of a horizontal straight line displaced by a distance y from the receiver. From this we can evaluate the effect of a finite width lane by subtraction. Taking the origin of ϕ as orthogonal to the shipping lane we find R^n is given by

$$R^n=\exp(-\alpha y\sin^2\theta\sec\phi/2H). \tag{33}$$

This is dependent on θ and ϕ so we need to reintroduce the original ϕ integral. However, it can be seen that the main contributions come from small θ but a relatively large range of ϕ, say, ϕ_0. Thus roughly we can separate the function into exp $(-\alpha y\sin^2\theta/2H)$ and an angle limit of $\pm\phi_0$ in the ϕ integral. Using the earlier expansion of the complex exponential[19] generalized to exp $(iA\cos\phi)$ we can solve the ϕ integral.

The complete coherence function becomes

$$\begin{aligned}\rho(d,\gamma)=\int &F(kd\cos\gamma\cos\theta_r,\beta)e^{-\alpha y\sin^2\theta_r/2H}\\&\times[1-R]^{-1}(e^{ikd\sin\theta_r\sin\gamma}e^{-as_p}\\&+R_b e^{-ikd\sin\theta_r\sin\gamma}e^{-a(s_c-s_p)})\sin^{2m-1}\theta_s\\&\times\cos\theta_r\,d\theta_r,\end{aligned} \tag{34}$$

where the function F is the result of the ϕ integral with $\phi_0=\pi/2$:

$$\begin{aligned}F(A,\beta)=\pi J_0(A)+2i\sum_{j=1}^{\infty}&\frac{2}{2j-1}\\&\times\cos(2j-1)\beta J_{2j-1}(A).\end{aligned} \tag{35}$$

If $\beta=\pi/2$ so that the hydrophone pair is aligned with the shipping lane the function F reduces to $\pi J_0(A)$ and Eq. (34) differs from Eq. (8) only in the multiplier of π (rather than 2π) and the mode stripping term exp $(-\alpha y\sin^2\theta/2H)$ which is determined by the closest point of approach, y. Thus, but for the factor of 2, the solution is the same as that in Sec. V A with $n_2=\infty$ and $n_1=y/r_c$.

If $\beta=0$ so that the hydrophone pair points across the shipping lane, the real part of π and the absolute noise level are unchanged, but there is an additional imaginary part consisting of ever-decreasing odd-order Bessel functions.

VI. CONCLUSIONS

This paper demonstrates that a simple ray approach is capable of modeling sophisticated noise level and coherence effects, such as the noise notch, mode stripping, near boundary inhomogeneity, and nonuniform source distributions. The solution is essentially the same as the wave solution in RANDI-2. Some closed-form solutions are given for special cases including noise level and coherence in an arbitrary surface duct [Eqs. (11) and (13)]. The general case can be solved by a single numerical integral without the need for detailed ray tracing or calculation of normal modes. It is shown that the noise notch is quite a simple phenomenon whose existence is determined by Snell's law and the excess in sound speed above the receiver. In an arbitrary range-independent environment there can never be more than three angle bands contributing to the noise, as shown in Fig. 2. Figure 5 and 6 demonstrate that absolute noise source levels, whose implementation is described in Harrison,[16] can easily be included in this approach. Harrison[14] extends this simple treatment to range-dependent environments.

ACKNOWLEDGMENTS

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APPENDIX A: RELATION BETWEEN NORMAL MODE AND RAY CALCULATION OF NOISE

We give a brief derivation of the discrete mode formula for the coherence between two hydrophones in a stratified medium in order to compare it with the ray treatment. It is suggested that this derivation is much simpler than that of Buckingham[5] or Kuperman and Ingenito.[6] We start with the usual discrete mode sum with a noise source at depth z_0 near the surface such that the mode function

$$u_n \simeq (2/H)^{1/2} \sin(\pi z_0 n/H) \simeq \pi z_0 n (2/H^3)^{1/2}.$$

The decay constant is δ_n:

$$\psi(z,r) = (2\pi)^{1/2} e^{i\pi/4} \sum_n^N \frac{u_n(z)}{(K_n r)^{1/2}} \left(\frac{\pi z_0 n}{H}\right) \times \left(\frac{2}{H}\right)^{1/2} e^{iK_n r} e^{-\delta_n r}. \tag{A1}$$

The coherence function is

$$\rho = \int_0^{2\pi} \int_0^\infty \psi_1(z_1,r_1)\psi_2^*(z_2,r_2) r\, dr\, d\phi$$
$$= 2\pi \int_0^{2\pi} \int_0^\infty \sum_n^N \sum_m^N \frac{u_n(z_1)u_m(z_1)}{(K_n K_m r_1 r_2)^{1/2}} \left(\frac{\pi z_0}{H}\right)^2 \frac{2nm}{H} \times \exp[i(K_n r_1 - K_m r_2)] \exp[-\delta_n r_1 - \delta_m r_2] r\, dr\, d\phi. \tag{A2}$$

The relationship between r (on which the polar coordinate system is based) and r_1 and r_2 is shown in the top view in Fig. A1.

Provided r is large compared with the horizontal projection of the array ($d \cos\gamma$) we can put $r = (r_1 r_2)^{1/2} = (r_1 + r_2)/2$ and $r_1 - r_2 = d \cos\gamma \cos\phi$. The imaginary exponent in Eq. (A2) can be rewritten as

$$K_n r_1 - K_m r_2 = (K_n - K_m)(r_1 + r_2)/2 + (K_n + K_m)(r_1 - r_2)/2$$
$$= (K_n - K_m) r + ((K_n + K_m)/2) d \times \cos\gamma \cos\phi.$$

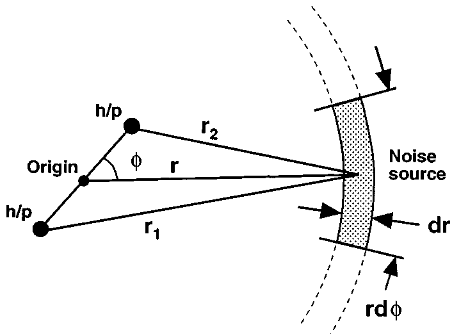

FIG. A1. Top view of hydrophone pair and noise source.

Similarly the decay exponent is

$$\delta_n r_1 + \delta_m r_2 = (\delta_n + \delta_m)(r_1 + r_2)/2 + (\delta_n - \delta_m)(r_1 - r_2)/2$$
$$= (\delta_n + \delta_m) r + (\delta_n - \delta_m) d \cos\gamma \cos\phi,$$

the last term of which we can safely neglect. Thus integrating in ϕ we obtain

$$\rho = (2\pi)^2 \int_0^\infty \sum_n^N \sum_m^N \frac{u_n(z_2)u_m(z_1)}{(K_n K_m)^{1/2}} \left(\frac{\pi z_0}{H}\right)^2 \frac{2nm}{H} \times e^{i(K_n - K_m)r} e^{-(\delta_n + \delta_m)r} J_0((K_n + K_m) d \cos\gamma/2) dr. \tag{A3}$$

We now rewrite the double sum as a single sum of the terms that have $n=m$ plus the double sum $\Sigma_{n=1}^N \Sigma_{m=n+1}^N$. When we integrate the single sum in r the only range dependence is the decay term which results in a factor $(2\delta_n)^{-1}$ to be multiplied by the other terms. The equivalent range dependence in the double sum is the term $2\cos((K_n - K_m)r)\exp(-(\delta_n + \delta_m)r)$. It is easy to show that the integral of this quantity is

$$\int_0^\infty 2\cos((K_n - K_m)r)\exp(-(\delta_n + \delta_m)r) dr = \frac{2(\delta_n + \delta_m)}{(\delta_n + \delta_m)^2 + (K_n - K_m)^2}. \tag{A4}$$

Now $K_n - K_m \simeq 2\pi(n-m)/r_c$ where r_c is the ray cycle distance. So the integral is

$$2(\delta_n + \delta_m)^{-1}[1 + (2\pi(n-m)/r_c(\delta_n + \delta_m))^2]^{-1},$$

and since we can assume that the decay over a ray cycle $r_c \delta_{n,m}$ is small it reduces to

$$2(\delta_n + \delta_m)^{-1} \times (r_c(\delta_n + \delta_m)/2\pi(n-m)).$$

This is necessarily much smaller than $(2\delta_n)^{-1}$ for the single sum so we are left with just the single sum, and the cross terms can be neglected:

$$\rho = (2\pi)^2 \sum_n^N \frac{n^2 u_n(z_2) u_n(z_1)}{\delta_n K_n H} \left(\frac{\pi z_0}{H}\right)^2 J_0(K_n d \cos\gamma). \tag{A5}$$

This is essentially the discrete mode part of Kuperman and Ingenito's solution. Buckingham assumes isovelocity water with a vertical array ($\gamma = \pi/2$) and $u_n(z) = (2/H)^{1/2} \times \sin(\pi z n/H)$. He also shows from the boundary conditions that δ_n is proportional to n^2. The n^2 in the denominator and numerator cancel, K_n is more or less constant ($\simeq k$), and we are then left with the simple sum of the mode product.

It is informative to translate Eq. (A5) into ray terms and then compare it with Eq. (9) in the main text. This can be done in a stratified medium (see, for instance, Brekhovskikh and Lysanov[22]), but we can make the same point more clearly by taking the isovelocity case where $n\lambda = 2H \sin\theta$. The Buckingham assumption is equivalent to a loss per bounce $\alpha \sin\theta$, as assumed in the text of this paper. The resulting intensity after many bounces is proportional to

$$\exp(-\alpha \sin\theta r/r_c) = \exp(-\alpha r \sin^2\theta/2H \cos\theta),$$

since $r_c=2H\cot\theta$. This gives the familiar mode-stripping, $r^{-3/2}$, propagation law.[21] From the above, the amplitude decay constant is

$$\delta_n=\alpha\sin^2\theta/4H\cos\theta=n^2\alpha\lambda^2/16H^3\cos\theta. \tag{A6}$$

In the normal mode solution a point noise source results in a dipole strength

$$(2\sin(kz_0\sin\theta))^2\approx 4k^2z_0^2\sin^2\theta. \tag{A7}$$

We therefore need to divide the normal mode solution by $4k^2z_0^2$ to obtain the ray solution, where we have assumed a source strength of simply $\sin^2\theta$. Making these substitutions into Eq. (A5) we obtain

$$\begin{aligned}\rho&=8\pi\alpha^{-1}\frac{\lambda}{2H}\sum_n^N\sin\left(\frac{n\pi z_1}{H}\right)\sin\left(\frac{n\pi z_2}{H}\right)\\&\quad\times J_0(kd\cos\theta\cos\gamma)\\&=4\pi\alpha^{-1}\frac{\lambda}{2H}\sum_n^N\left(\cos\frac{n\pi(z_1-z_2)}{H}\right.\\&\quad\left.-\cos\frac{n\pi(z_1+z_2)}{H}\right)J_0(kd\cos\theta\cos\gamma).\end{aligned} \tag{A8}$$

Remembering that $(n\pi/H)(z_1-z_2)=kd\sin\gamma\sin\theta$, we see that there are two differences between Eq. (A8) and Eq. (9). One is that we have an extra term that has insignificant effect unless we are near a boundary, as discussed in Appendix B. The other is that instead of $\int_0^{\pi/2}\cos\theta\,d\theta$ we have $(\lambda/2H)\Sigma_n^N$. Since $\sin\theta=n\lambda/2H$, if we were to treat n as a continuum we would obtain $\int_0^{\pi/2}\cos\theta\,d\theta=\lambda/2H\int_0^\infty$. Therefore this remaining difference is merely that the angles have been discretized by the modes in Eq. (A8) and Eq. (A5) whereas they are continuously distributed in Eq. (9). The ray approximations A and C are therefore very good. Although this demonstration assumes isovelocity, one can follow the argument through equally well in a refracting environment, and the arguments of the two cosine terms can be written in terms of differences and sums of WKB phases, for example, $\int_{z_1}^{z_2}k(z)\sin\theta(z)dz$.

APPENDIX B: NEGLECTION OF MULTIPATH INTERFERENCE

In this Appendix we identify and quantify terms that were missed in the derivation given in the main text by reinstating multipaths with phases. Apart from discretization of the ray angle the only effect is found close to a boundary.

To investigate all the terms in Eq. (1) we retain here phases and path losses (reflection and absorption) but otherwise relative amplitudes in the term ψ. The phase along a path s, making use of Snell's law, is

$$\int k\,ds=k\cos\theta r+\int k\sin\theta\,dz\equiv k\cos\theta r+\zeta, \tag{B1}$$

where ζ represents the vertical part of the phase. We assume that an upgoing ray from hydrophone 1 first hits the surface with $\zeta=\zeta_1$ and adds phase ζ_c after each subsequent surface reflection. We represent hydrophone 2's terms by subscript 2. We retain complex reflection coefficients R_s,R_b and amplitude absorption coefficient a. At sufficiently long range we can always assume there is also a down-going ray with the same angle connecting the same hydrophone and noise source. Combining up- and down-going rays for each θ we obtain

$$\begin{aligned}\psi_1\times\psi_2^*&=[e^{i\zeta_1}e^{-s_1a}+R_be^{i(\zeta_c-\zeta_1)}e^{-(s_c-s_1)a}]e^{ik\cos\theta r_1}\\&\quad\times\sum_{j=0}^\infty(e^{i\zeta_c}R_sR_be^{-as_c})^j[e^{-i\zeta_2}e^{-s_2a}\\&\quad+R_b^*e^{-i(\zeta_c-\zeta_2)}e^{-(s_c-s_2)a}]e^{-ik\cos\theta r_2}\\&\quad\times\sum_{j=0}^\infty(e^{-i\zeta_c}R_s^*R_b^*e^{-as_c})^j,\end{aligned} \tag{B2}$$

$$\begin{aligned}\psi_1\times\psi_2^*&=[(e^{i(\zeta_1-\zeta_2)}e^{-(s_1+s_2)a}\\&\quad+R_bR_b^*e^{-i(\zeta_1-\zeta_2)}e^{-(2s_c-s_1-s_2)a})\\&\quad+(R_be^{i(\zeta_c-\zeta_1-\zeta_2)}e^{-(s_c+s_2-s_1)a}\\&\quad+R_b^*e^{-i(\zeta_c-\zeta_1-\zeta_2)}e^{-(s_c+s_1-s_2)a})]\\&\quad\times e^{ikd\cos\gamma\cos\theta\cos\phi}\\&\quad\times\left[\left|\sum_{j=0}^\infty(e^{i\zeta_c}R_sR_be^{-as_c})^j\right|^2\right].\end{aligned} \tag{B3}$$

Remembering that ζ_c must be a multiple of 2π (and attenuations small) for modal propagation we see that the first square bracket in Eq. (B2) is simply the mode amplitude at z_1. For example with $R_b=-1$ we obtain $\sin\zeta_1$ which could be expanded using Eq. (B1) (essentially WKB). Therefore we obtain in Eq. (B2) precisely the product of mode values at z_1 and z_2 as in Kuperman and Ingenito.[6] When expanded as in Eq. (B3) we can see exactly what was missed in the main text by ignoring cross terms. The first two terms in the square brackets of Eq. (B3) correspond to the two terms in Eq. (8). In isovelocity water these would be $\cos((z_1-z_2)\pi n/H)$ for the nth mode. The third and fourth terms in the square brackets of Eq. (B3) are missing in Eq. (8), but are predicted by Buckingham[5] as $\cos((z_1+z_2)\pi n/H)$. The first term outside the square brackets is responsible for the J_0 and identical in Eq. (8). The last term in Eq. (B3) appears to contain cross terms, but in fact the mode derivation (Appendix A) shows that they are insignificant. Essentially the independence of the noise sources when integrated over range (even without the assistance of azimuth integration!) reduces the cross terms to zero.

So if we have a reasonably large number of modes (so that $\Sigma_n\simeq\int dn$) then the only effect of the neglection of cross terms in the ray treatment is the dropping of the $\cos(z_1+z_2)\pi n/H$ term from Eq. (B3). As Buckingham pointed out, this term makes the coherence function slightly depth dependent (''quasihomogeneous'') with particular effects near the boundaries. However, if there are many modes, as we have just assumed, this term will be highly oscillatory compared with the other terms in the angle intergrand, and its integral will be zero. Because this term does not depend on hydrophone spacing one might expect it to have a residual effect on noise level [i.e., $\rho(0)$]. Indeed it does, but a much more familiar way of stating this is that the noise is proportional to the square of the mode amplitude as demonstrated by Eq. (B2), i.e.,

$$1-\cos((z_1+z_2)n\pi/H)=2\sin^2((z_1+z_2)n\pi/2H).$$

This is a simple correction to bear in mind having made assumption C in the ray treatment of the main text. Buckingham's criterion for "quasihomogeneity" is that the distance to the nearest boundary z is given by $z/H \geqslant 3/(2m+1)$ assuming there are m modes in water depth H. An equivalent way of stating this is that

$$kz \sin \theta_c \geqslant 3\pi/2$$

or

$$z \geqslant \tfrac{3}{4}(\lambda/\sin \theta_c),$$

where θ_c is the steepest, or critical, angle in the duct. In this region, near the boundary, the noise intensity is reduced by the factor $f=(z(m+\frac{1}{2})\pi/H)^2=(kz \sin \theta_c)^2$.

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