GEOPHYSICS, VOL. 41, NO. 5 (OCTOBER 1976), P. 985–996; 4 FIGS., 2 TABLES

SHEAR-WAVE VELOCITY VERSUS DEPTH IN MARINE SEDIMENTS: A REVIEW

EDWIN L. HAMILTON*

The objectives of this paper are to review and study selected measurements of the velocity of shear waves at various depths in some principal types of unlithified, water-saturated sediments, and to discuss probable variations of shear velocity as a function of pressure and depth in the sea floor. Because of the lack of data for the full range of marine sediments, data from measurements on land were used, and the study was confined to the two "end-member" sediment types (sand and silt-clays) and turbidites.

The shear velocity data in sands included 29 selected in-situ measurements at depths to 12 m. The regression equation for these data is: $V_s = 128D^{0.28}$, where V_s is shear-wave velocity in m/sec, and D is depth in meters. The data from field and laboratory studies indicate that shear-wave velocity is proportional to the 1/3 to 1/6 power of pressure or depth in sands; that the 1/6 power is not reached until very high pressures are applied; and that in most sand bodies the velocity of shear waves is proportional to the 3/10 to 1/4 power of depth or pressure. The use of a depth exponent of 0.25 is recommended for prediction of shear velocity versus depth in sands.

The shear velocity data in silt-clays and turbidites include 47 selected in-situ measurements at depths to 650 m. Three linear equations are used to characterize the data. The equation for the 0 to 40 m interval ($V_s = 116 + 4.65D$) indicates the gradient (4.65 sec^{-1}) to be 4 to 5 times greater than is the compressional velocity gradient in this interval in comparable sediments. At deeper depths, shear velocity gradients are 1.28 sec^{-1} from 40 to 120 m, and 0.58 sec^{-1} from 120 to 650 m. These deeper gradients are comparable to those of compressional wave velocities. These shear velocity gradients can be used as a basis for predicting shear velocity versus depth.

INTRODUCTION

In a previous report (Hamilton, 1971a), the presence and causes of dynamic rigidity (shear modulus) and shear waves in water-saturated land and marine sediments were reviewed. It was concluded that almost all marine sediments possess enough rigidity to allow transmission of shear waves. In that report, the variation of shear velocity with depth in the sediments was not considered.

This report is a short review and study of selected, published measurements of the velocity of low-strain (less than about 10^{-5}) shear waves at various depths and pressures in some principal types of unlithified, water-saturated sediments. Some associated studies of compressional wave velocity versus pressure in quartz sands are briefly noted. The purposes of this report are to detail and discuss probable shear velocity gradients in the sea floor. Because of the lack of data for the full range of marine sediments, data from the literature of land geophysics were used, and the study confined to the two "end-member" sediment types (sand and silt-clays) and turbidites. Turbidites are usually composed of thicker layers of silt-clays with thinner intercalated layers of silt and sand.

The values and gradients of shear-wave velocity and dynamic rigidity (shear modulus), with depth in the sediments of the sea floor, are important physical properties of earth materials and are used in both basic and applied geophysical and engi-

Manuscript received by the Editor May 28, 1975; revised manuscript received April 23, 1976.
* Naval Undersea Center, San Diego, Calif. 92132.

neering studies. The shear modulus is usually determined from measurements of shear velocity and density. Given shear velocity, density, and compressional wave velocity, all of the other elastic properties can be computed. The shear modulus is critically important in studies of foundation vibrations, the effects of earthquakes, slope stabilities, and other practical applications. The velocity of shear waves is also important in several geophysical studies such as the propagation of Rayleigh waves, and the interaction of sound (compressional waves) with the sea floor.

METHODS AND RESULTS

Introduction

There are several principal methods for measuring shear-wave velocities. One of the best methods involves wave-generating sources on the sediment surface and instruments in boreholes. Warrick (1974) summarized these techniques, including the excellent work at the Earthquake Research Institute of the University of Tokyo. Several measurements in Tables 1 and 2 were made between boreholes (White and Sengbush, 1953), in

Table 1. Shear wave velocities in sands.

Material	Shear-wave velocity, m/sec	Depth, m	Reference and method (see footnotes)
Multiple measurements at each site			
Marine medium sand	53	0.2	Cunny and Fry (1973)—1
	287	7.2	
Marine medium sand	119	0.4	Cunny and Fry (1973)—1
	165	3.3	
	210	5.2	
Marine silty sand	82	0.1	Cunny and Fry (1973)—1
	148	3.7	
Marine fine sand	88	0.2	Cunny and Fry (1973)—1
	116	2.0	
	168	4.2	
Marine coral sand	140	0.4	Cunny and Fry (1973)—1
	253	7.0	
Single measurements at each site			
Fine sand	131*	1.3**	Volin et al (*in* Molotova and Vassil'ev, 1960)—1
Fine sand	98*	1.0**	
Fine sand	85*	0.9**	
Sand	152	0.9	Stokoe and Woods (1972)—2
Gravelly coarse sand	125	0.4	Barnes et al (1973)—2
Medium sand	197	3.4†	Hamilton et al (1970)—1
Fine sand	154	2.7†	
Medium sand	101	2.2†	Bucker et al (1964)—1
Fine sand	110	1.1**	Barkan (1962)—1(?)
Medium sand	160	1.6**	Barkan (1962)—1(?)
Fine sand	190	2.8†	Heukelom (1961)—1
Sand	255	3.8†	
Sand	230	3.4†	
Sand	227	3.3†	
Loose sand	264	12	Jolly (1956)—3
Fine sand	270	12	Shima et al (1968)—3
Alluvial sand	173	5	Kawasumi et al (1966) and Shima et al (1968)—3
Some maximum values (not in Figure 1)			
Loose sand	518	18	White and Sengbush (1953)—4
	579	27	
Sand and gravel	521	0.4	Barnes et al (1973)—2
Fine sand	524	0.4	
Medium sand	475	0.4	
Fine sand	567	0.4	
Fine sand	390	12	Molotova (1964)—5

* V_s computed from V_p/V_s (V_p given).

** Depth computed as one-half length of surface wave (assumed frequency of 50 Hz).

† Depth computed as one-half length of Stoneley or Rayleigh wave.

Method of measurement: 1 = Rayleigh or Stoneley wave observation; 2 = between probes; 3 = surface source, borehole receiver; 4 = between boreholes; 5 = in boreholes or wells; and 6 = conversion compressional to shear waves.

Table 2. Shear wave velocities in silts, clays, and turbidites.

Material	Shear wave velocity, m/sec	Depth, m	Reference and method (see footnotes)
Multiple measurements at each site			
Saturated clay	180	10	Zhadin (in Vassil'ev and Gurevich, 1962)—5
Saturated clay	300	60	
Saturated clay and sand	450	250	
Sat. clay, siltstone, sandstone	700	650	
Bay mud	90	6	Warrick (1974)—3
Clay with sand and gravel layers	270	26	
	380	112	
Wet clay	120*	10	Zhadin (in Molotova and Vassil'ev, 1960)—5
Wet clay	289*	35	
Wet clay	359*	63	
Wet clay	194*	12	Berdennikova (in Molotova and Vassil'ev, 1960)—4
Wet clay	337*	26	
Wet clay	170*	5	Zhadin (in Molotova and Vassil'ev, 1960)—5
Wet clay	250*	18	
Wet clay	320*	35	
Deep-sea sediment	50	0	Davies (1965)—1
Deep-sea sediment	110	4	
Deep-sea sediment	190	16	
Silty clay	137	1	Cunny and Fry (1973)—1
Silty clay	165	4	
Clayey silt	174	9	
Silt	192	11	
Clay, firm	203	17	
Sandy silt	235	21	
Shaley clay	239	27	
Alluvial silt	102	16	Kudo and Shima (1970)—3
	120	22	
	156	28	
	200	32	
	270	42	
Sandy clay	225	30	Ohta (1967)—3
	250	38	
	275	51	
Silty clay with sand, shell	84‡	5	Aisiks and Tarshansky (1969)—4
	257‡	31	
Single measurement at each site			
Clayey silt	201	5	Stokoe and Woods (1972)—2
Clayey silt	101	2†	Hamilton et al (1970)—1
Clayey silt	108	2†	
Clayey silt	88	2†	
Clay	230	10	Molotova (1963)—5
Clay	354	20	Molotova (1966)—5
Turbidites	400	150	Sutton et al (1971)—6
Saturated clay	430	140	Berzon et al (1967)—5
Plastic clay	310	37	Molotova (1964)—5
Clay	175	10	Shima et al (1968)—3
Silt	300	17	
Clay	122	10	Ohta and Shima (1967)—3

* V_s computed from V_p/V_s (V_p given).
** Depth computed as one-half length of surface wave (assumed frequency of 50 Hz).
† Depth computed as one-half length of Stoneley or Rayleigh wave.
‡ V_s computed from shear modulus and density (Trask and Rolston, 1951).

Method of measurement: 1 = Rayleigh or Stoneley wave observation; 2 = between probes; 3 = surface source, borehole receiver; 4 = between boreholes; 5 = in boreholes or wells; and 6 = conversion compressional to shear waves.

boreholes (Berzon et al, 1967), or between probes (Stokoe and Woods, 1972; Barnes et al, 1973). Cunny and Fry (1973) reviewed the extensive work by the Waterways Experiment Station using surface vibrators and measurements of the frequency and wavelengths of Rayleigh waves (from which shear-wave velocities can be derived). Bucker et al (1964) and Hamilton et al (1970) described measurements off San Diego of the velocity of Stoneley waves, from which shear-wave velocities can be derived. Davies (1965) reported the shear velocity profile (from Stoneley-wave measurements) to 16 m in the Somali Basin in the western Indian Ocean. In these Stoneley-wave studies, the explosive source and a line of geophones were both on the sediment surface. Shear-wave velocities have also been measured between reflecting boundaries after conversion of compressional to shear waves (Sutton et al, 1971).

Most of the measurements in the tables had depths or depth intervals listed in the referenced work. Where an interval was recorded, the depth to the midpoint of the interval was used with the interval shear velocity. Where the original report concerned Rayleigh waves, and a depth was not given, the derived shear-wave velocity is listed and plotted at a depth equal to one-half the wavelength of the Rayleigh wave. This procedure followed common practice in soil mechanics experimental work (e.g., Cunny and Fry, 1973), and is partially (at least) supported theoretically (Richart et al, 1970, p. 113). Where the frequency was not given, it was assumed to be 50 Hz. In the measurements by Bucker and his colleagues there was insufficient information to determine the depth of penetration of the Stoneley waves (at 20 or 25 Hz: Bucker et al, 1964) into the sediments. In these cases, the derived value of the shear wave was listed and plotted at one-half the length of the Stoneley wave.

In-situ shear-wave data on sands

One purpose of this study was to determine reasonable in-situ values of shear velocity as a function of depth in the upper 10 to 20 m of various natural marine sands. The best way to determine these gradients would be to measure shear velocities at several depths at the same site in various homogeneous sand layers. Cunny and Fry (1973) have published the only such measurements, in situ, in natural marine sands. The velocity range in their study, in five different areas, was 53 to 287 m/sec in the depth range of 0.1 to 7.2 m. To supplement their relatively few measurements (Table 1), it was decided to include all available measurements within this velocity range, but to increase the depth to 12 m to include two measurements by Jolly (1956) and Shima et al (1968). A selection of higher shear velocities, not included in Figure 1 or in the regression equation, is listed in Table 1 to indicate maximum values. To have included all of these in the statistical analysis would have resulted in such wide scatter as to render the resulting gradient a statistical artifact. The in-situ gradient determined after this selection procedure is verified by laboratory measurements in sands as discussed in a later section.

In selecting shear-wave velocity data to represent marine sands, those measurements made in some mixed-grain sediment types, such as sandy silt and sand-silt-clay, were not used. Also excluded were measurements in mixed lithologies such as "alluvial sands, gravels, and clays."

All measurements (Table 1, Figure 1) are in situ except one, which was in a large bin in the laboratory (Stokoe and Woods, 1972). Measurements both above and below the water table were used in these sands, as discussed in a later section.

The 29 selected measurements in sands at depths from 0.1 to 12 m are listed in Table 1 and plotted against depth in Figure 1. The regression equation for these data is

$$V_s = 128D^{0.28}, \tag{1}$$

where

V_s is shear-wave velocity, m/sec,
D is depth in the sand body, m.

In situ shear-wave data on silt-clays and turbidites

There are more data on shear velocity versus depth in unlithified silt-clays and turbidites than in sands. Almost all, however, are from terrestrial measurements.

Data were selected which were known to be in fully saturated, unlithified material of a type, and with physical properties, which might be present in thick, marine silt-clay, or turbidite sections.

Data were not used if shear velocity was measured above the water table, as discussed in a later section. Measurements in definitely lithified materials such as shale were not considered.

Data were eliminated if the compressional velocity, accompanying the shear velocity, was too high for the indicated depth in the sediments. For

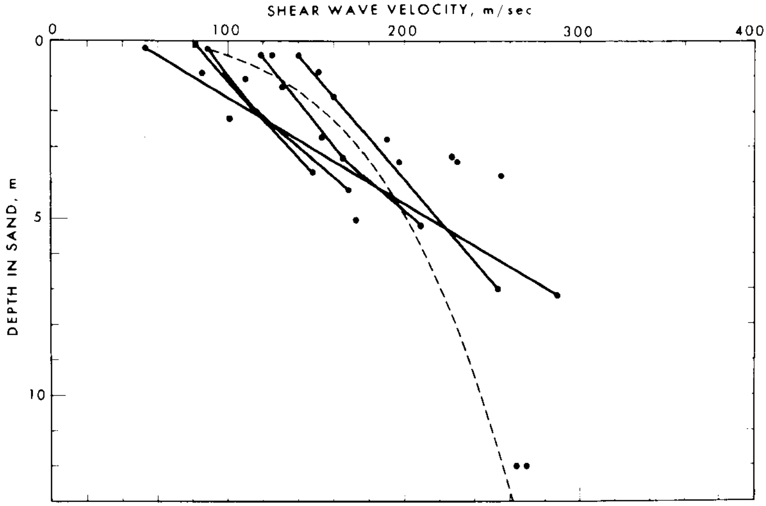

FIG. 1. Shear-wave velocity versus depth in selected water-saturated sands. Multiple measurements at the same site are connected by solid lines. The dashed line is the regression equation: $V_s = 128\,(D)^{0.28}$.

example, a compressional wave velocity of 2400 m/sec is unlikely at a depth of 50 m in a marine silt-clay or turbidite section. This criterion was based on recent studies (e.g., Houtz et al, 1968; Hamilton et al, 1974) which related compressional velocity with depth in sediments in various areas of the world's ocean. If compressional velocity is too high for the indicated depth, then shear-wave velocity is also apt to be high. Some causes involve removal of overburden through slumping or erosion, and/or there has been some cementation (lithification) so that the material is no longer a soft sediment.

Data were eliminated if density was too high or porosity too low in relation to an average marine silt-clay or turbidite section (Hamilton, 1976). For example, a porosity of 30 percent is unlikely at a depth of seven meters in such materials. Data were also eliminated for mixed lithologies such as alluvial sands, gravels, and clays.

All selected measurements were in situ in natural, saturated sediments at depths from 1 to 650 m. Of the 47 measurements, 39 were at depths less than 42 m. Below 40 m, data are scarce (as selected). It was decided to use the excellent measurements by Warrick (1974) between 36 and 120 m. Warrick's measurements were taken in a borehole in the shore of San Francisco Bay. The two values used were in the older sediments under the Bay Mud. These older sediments are clay with sand and gravel layers. Except for the gravel, the sediments are similar in type to deeper-water turbidites.

In the depth range of 120 to 650 m, the profile and gradient were based on five measurements, including one by Warrick at 112 m. Two measurements are from Zhadin (in Vassil'ev and Gurevich, 1962): one in water-saturated clays and sands at 250 m, and the other in water-saturated clays, sandstones, and aleurolites (siltstone) at 650 m. The compressional velocity of 2100 m/sec in the deeper measurements indicates that the section is essentially unlithified. A third Soviet measurement (Berzon et al, 1967), in water-saturated clays (V_p = 1640 m/sec, V_s = 430 m/sec, at 140 m), was based on acoustic logs and downhole surveys in a 280-m thick layer. The fifth measurement, in the Delgado Fan in the deep-sea off California, was made with ocean-bottom seismometers (Sutton et al, 1971). The shear velocities were approximated by using delay times from compressional to shear-wave conversions at the base of the sediments. Although the compressional velocity (2100 m/sec) in the 0 to 300 m interval is too high for most marine turbidite sections, the shear velocity measurement (400 m/sec) was used because it is in a deep-sea fan, and is only one of two in deep-sea sediments. A shear velocity measurement in the interval of 300 to 1300 was not used because the compressional velocity (2700 m/sec) indicated probably lithified material. This was probably confirmed by the Deep Sea Drilling Project which encountered altered basalts, chert, and andesites at four sites in the Delgado Fan at depths from 212 to 384 m (McManus et al, 1970; Kulm et al, 1973).

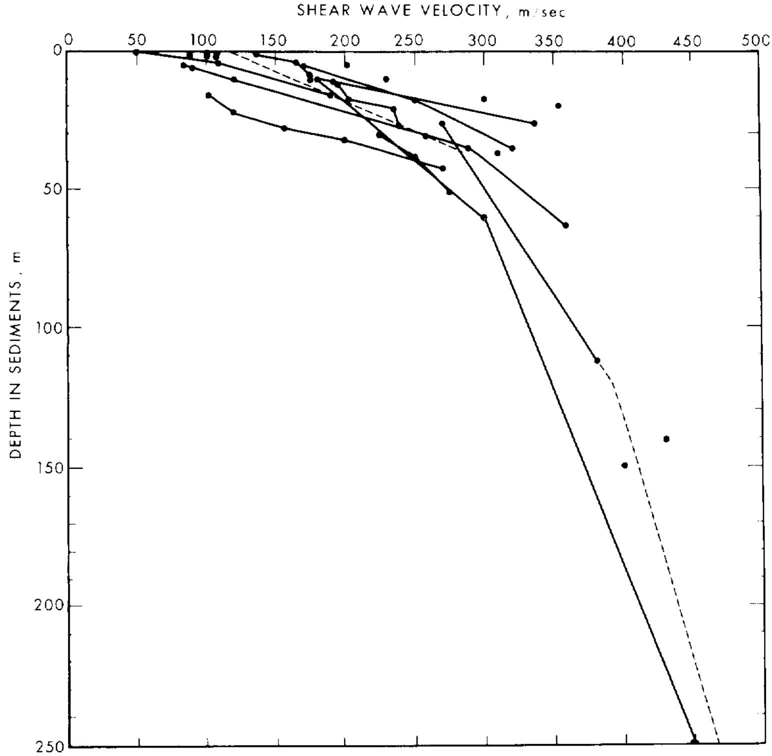

FIG. 2. Shear-wave velocity versus depth in selected water-saturated silt-clays and turbidites. Multiple measurements at the same site are connected by solid lines. The dashed lines are three regression equations. One measurement (V_s = 700m/sec at 650 m) is not shown.

The selected data for silt-clays and turbidites are listed in Table 2 and plotted in Figure 2. After study and trial of various nonlinear regression equations, it was apparent that none properly characterized the data. It was thus decided to express the velocity-depth relations (Figure 2) with three linear equations. All data at less than 42 m yield a linear equation for use between the depths indicated (V_s is in m/sec, and depth D in m): 0 to 36 m:

$$V_s = 116 + 4.65D. \quad (2)$$

Based on only two measurements, but unusually good ones (Warrick, 1974), from 36 to 120 m:

$$V_s = 237 + 1.28D. \quad (3)$$

Based on five measurements, including one by Warrick (1974) at 112 m, from 120 to 650 m:

$$V_s = 322 + 0.58D. \quad (4)$$

DISCUSSION AND CONCLUSIONS

Effects of moisture content

Many of the measurements of compressional and shear-wave velocities in the literature have been made on land through the weathered zone above the water table. When the position of the water table is not stated, it can be inferred that the measurement was above the water table when the compressional velocity is significantly lower than in water of the appropriate physical properties—e.g., in all cases less than about 1300 to 1400 m/sec.

The water table has little influence on shear waves in sands, with velocities only slightly less in completely saturated sands than in completely dry materials (e.g., Figures 3 and 4). Hardin and Richart (1963) noted that the addition of as little as 1.4 percent moisture can reduce the shear modulus by as much as 15 percent in round-grained sands (apparently because of grain lubrication),

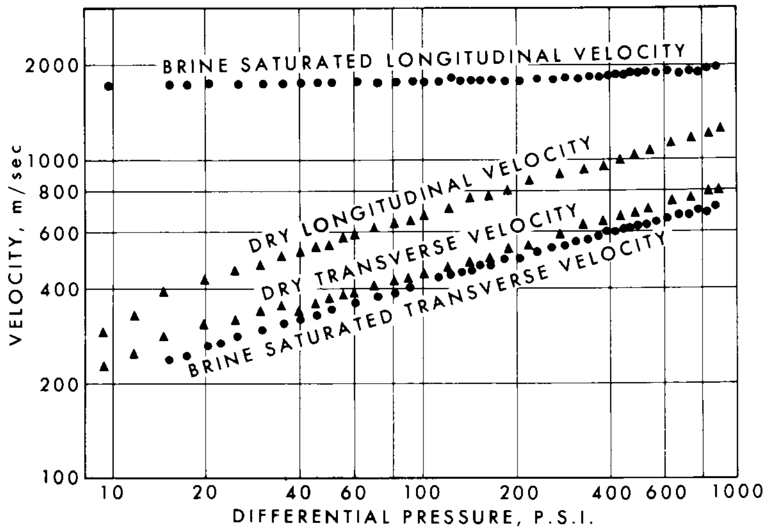

FIG. 3. Velocity of compressional (longitudinal) and shear (transverse) waves versus pressure in dry and brine-saturated (25 gm/liter NaCl) St. Peter fine sand (100-120 mesh, or 0.125-0.149 mm). By permission of the Shell Development Co. (personal communication, 1965); from Hamilton (1971a).

while angular sands are not affected. Most natural sands are angular, and even above the water table will usually have small amounts of moisture. Below the water table, the sediment pore water does not transmit shearing motions, but merely lubricates the grains. Consequently, it has been determined that shear and Rayleigh wave velocities in sands are little affected by the water table (e.g., Richart et al, 1970).

The position of the water table in cohesive sediments (such as silt-clays) is important in shear-wave measurements. Above the water table, silt-clays are apt to contract and harden when drying, and shear moduli are higher than in the section below the water table. Consequently, measurements in silt-clays above the water table must be eliminated for studies of saturated sediments.

Relationships between velocities of Rayleigh and shear waves

In some reports (e.g., Cunny and Fry, 1973), the Rayleigh wave velocity is taken as the shear velocity for practical engineering purposes. The Rayleigh wave velocity can be related to the shear-wave velocity through Poisson's ratio or the V_p/V_s ratio (e.g., Knopoff, 1952; Kolsky, 1963; White, 1965; Richart et al, 1970; Jones, 1958). A survey of Poisson's ratio in surficial, saturated marine sediments (this study; Hamilton, 1971a, 1974) and to depths to which the Rayleigh wave velocity is measured in soil mechanics studies, indicates that almost all fall between 0.45 and 0.50. This means that, usually, the velocity of Rayleigh waves in the upper levels of saturated marine sediments will be about 5 percent less than the velocity of shear waves. Thus the approximation noted above is reasonable. A closer approximation (if insufficient data were available for more precise computations) for fully saturated, unlithified sediments would be to add 5 percent to measured Rayleigh waves velocities to estimate shear-wave velocities, or deduct 5 percent from measured shear velocities to approximate Rayleigh wave velocities. In the case of the data of Cunny and Fry (1973), the shear-wave velocities were used as reported. Use of "corrected" velocities made no significant difference in the regression equations.

Pressure effects

In discussing pressure effects in water saturated sediments it is important to define terms. The total pressure P (also called the overburden pressure or the geostatic pressure) is the sum of the fluid pressure P_f in the sediment pore spaces and the effective pressure P_e. In the upper levels of the sea floor, pore-water pressure is usually hydrostatic. Effective pressure (also called intergranular or differential pressure) is formed by the buoyed weight of the mineral grains and is transmitted through the sediment mineral frame. Effective pressure in natural, water-saturated sediments is

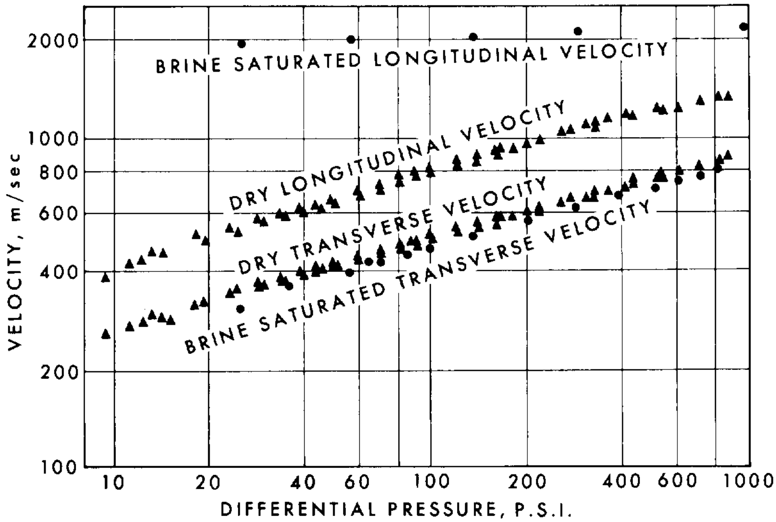

FIG. 4. Velocity of compressional (longitudinal) and shear (transverse) waves versus pressure in dry and brine-saturated (25 gm/liter NaCl) St. Peter coarse sand (20-30 mesh, or 0.59 to 0.84 mm). By permission of the Shell Development Co. (personal communication, 1965); from Hamilton (1971a).

easily computed: (saturated bulk sediment density minus density of pore water) times the depth in the sediments. In symbols: $P = P_f + P_e$. Any text book on soil mechanics has additional detailed discussions (e.g., Lambe and Whitman, 1969, p. 241–250).

In discussing the effects of intergranular (effective) pressure on shear and compressional wave velocities, a sharp distinction has to be made between sand bodies and higher porosity silt-clay layers. In sands, there is very little reduction of porosity because of pressures in the upper tens of meters (e.g., Roberts in Lambe and Whitman, 1969, p. 157, 298). Consequently, porosity changes due to pressures can be eliminated as a factor in velocity changes at shallow depths in sands. Effective pressure in silt-clays reduces porosity, which causes marked changes in the elastic properties of the material, and the increase of both compressional and shear-wave velocities. Consequently, in the following discussions, pressure relationships in sands will be stressed, but not those in silt-clays.

Before discussions of laboratory studies, a note should be made of the differing pressures used. In the Shell Development Co. study and the one by Gardner et al (1964), differential or effective pressure was used. In the studies by Hardin and Richart (1963) and Hardin and Black (1966, 1968), the pressures are "effective confining pressure," discussed in these references and by Richart et al (1970, p. 137, 169). The effective confining pressure depends on the submerged unit weight and depth in saturated sediments, and on Poisson's ratio; it is difficult to evaluate in the field (see Richart et al, 1970, p. 169–170 for calculation and discussion). Average values of Poisson's ratio in various types of saturated sands probably varies between 0.46 and 0.49 [Hamilton, 1974; Shell Development Co. data, (personal communication, 1965), Figures 3, 4]. Effective confining pressure for these values of Poisson's ratio would be 90 to 97 percent of normal effective pressure due to overburden.

Shear-wave velocity versus pressure and depth in sands

Hardin and Richart (1963) used a resonant-column technique to derive empirical equations relating shear velocity and the shear modulus to pressure and void ratio e [porosity $= e/(1 + e)$] for round-grained and angular quartz sands. These data were supplemented and manipulated by Hardin and Black (1966, 1968) and the resulting empirical equations were recommended by Richart et al (1970, p. 154) for use in approximate engineering computations. Pressures to about 7 kg/cm^2 were used in these studies. The equations for angular-grained materials (sands and some clays, wet or dry) are

$$G_{\mathrm{psi}} = \frac{1230(2.973 - e)^2}{1 + e} P_{c,\mathrm{psi}}{}^{0.5}, \qquad (5)$$

$$V_{s,\mathrm{fps}} = [159 - (53.5)e]P_{c,\mathrm{psf}}{}^{0.25}, \qquad (6)$$

where

G is the rigidity or shear modulus, psi;
V_s is shear wave velocity, feet per second (fps);
e is void ratio; and
P_c is effective confining pressure (in the units shown: pounds per square inch (psi), or pounds per square foot (psf).

Equations (5) and (6) have been shown to approximate some field measurements (e.g., Whitman et al, 1969; Richart et al, 1970, p. 169).

The results of pulse technique measurements of compressional and shear-wave velocities in coarse and fine quartz sands by the Shell Development Co. were published with their permission by Hamilton (1971a); these results are reproduced herein as Figures 3 and 4. Equations were computed for the Shell velocity data in the form: $V = KP^n$ (where P is effective pressure and K is a constant) for compressional (V_p) and shear wave (V_s) velocities in the brine-saturated condition (25 gm/liter NaCl).

For fine sand (100 to 120 mesh, or 0.149 to 0.125 mm) from 1.4 to 28.1 kg/cm² (Figure 3),

$$V_p = 1736P^{0.013}, \qquad (7)$$

$$V_s = 237P^{0.28}. \qquad (8)$$

For coarse sand (20 to 30 mesh, or 0.84 to 0.59 mm) from 1.4 to 7.0 kg/cm² (Figure 4),

$$V_p = 1939P^{0.016}, \qquad (9)$$

$$V_s = 256P^{0.31}, \qquad (10)$$

and for coarse sand from 7.0 to 28.1 kg/cm²,

$$V_p = 1991P^{0.003}, \qquad (11)$$

$$V_s = 285P^{0.26}, \qquad (12)$$

where V_p and V_s are in m/sec, and effective pressure P is in kg/cm².

The Shell Co. investigators considered that, overall, shear velocity varied with the 1/4 power of pressure in both dry and saturated sand. To explain the departure from the theoretical 1/6 power relationship, the Shell investigators, in a Progress Report of the Shell Development Company dated 1958 (personal communication, 1965) favored a theory (attributed to A. S. Ginzbarg of Shell) in which the number of stress-bearing grain-to-grain contacts is not a constant, as assumed by contact theory, but increases linearly with strain. Shear velocity dependence in both dry and saturated sand, and compressional velocity in dry sand, then varies from 3/10 to 1/6 power of effective pressure as pressure increases to the point where all contacts are made.

The Shell data did not include porosity or density values. Assuming that the Shell coarse sand had a porosity of 36 percent and the fine sand a porosity of 46 percent, and that there was no reduction of porosity due to effective pressure to depths of several tens of meters, pressure was computed for various depths to 20 m and equations were computed relating depth in the sands to compressional and shear-wave velocities.

For the fine sand (0 to 20 m),

$$V_p = 1681D^{0.013}, \qquad (13)$$

$$V_s = 120D^{0.28}. \qquad (14)$$

For the coarse sand (0 to 20 m),

$$V_p = 1869D^{0.016}, \qquad (15)$$

$$V_s = 127D^{0.31}. \qquad (16)$$

Many of the measurements listed in Table 1 had insufficient data on grain sizes, density, or porosity, but the "average" grain size of all these sands was probably in the fine sand category. The regression equation (1) on these data ($V_s = 128D^{0.28}$) is very close to Shell equation (14) for fine sand ($V_s = 120D^{0.28}$), and is close to Shell equation (16) for coarse sand ($V_s = 127D^{0.31}$). Computations and comparisons between the Hardin and Richart equation (6), and the Shell equation for fine sand (8) and coarse sand (10) at the same assumed pressures and porosities, indicates less than 10 percent differences in shear velocity. Thus, these three separate investigations mutually support each other. A conclusion is that the Hardin and Richart (1963) equation (6) is a good, general predictor of shear velocity as a function of depth in sand, and has the advantage of including void ratio (or porosity) as a variable factor. However, many measured values of near-surface shear velocities in sands are greater than might be predicted by equation (6), as in Table 1. In these cases, the exponent of depth can be used to derive shear velocities at greater depths in the sands (as discussed in a later section).

Pilbeam and Vaisnys (1973) have recently per-formed some experiments and have reviewed the relationships between pressure and velocity in granular aggregates. Their experiments are in agreement with previous work (reviewed in 1963 by Hardin and Richart) in that they showed compressional and shear wave velocities to be proportional to the 1/3 to 1/6 power of pressure in dry and partially saturated (lubricated) material. They point out that contact theory calls for the 1/6 power (reviewed by White, 1965), but note that tests with steel balls by Duffy and Mindlin (1957) and by Duffy (1959) showed pressure exponents varying from 1/3 to 1/4 to pressures of about 0.7 kg/cm^2, and thence to the 1/6 power at greater pressures. Pilbeam and Vaisnys (1973) derived a model calling for velocity proportional to the 1/3.3 (0.303) power of pressure at lower pressures. Their model was similar to that of Ginzbarg and required an increasing number of interparticle contacts as pressure increases until a pressure is reached at which no new contacts were being made. Thus, one should expect changes in the exponent with increasing pressures.

Gardner et al (1964) used a resonant-column vibration technique to measure relations between velocities of compressional and shear waves and pressure in dry and partially saturated quartz sand and glass beads. They concluded that both compressional and shear-wave velocities varied with the 1/4 power of effective pressure to pressures of 1000 psi (70 kg/cm^2) and gradually approached the 1/6 power above 5000 psi (352 kg/cm^2).

Summary.-The relationship between shear velocity and pressure in sands is important in predicting shear velocity, the shear modulus, and other elastic moduli with depth in a sand body.

A great deal of theoretical and experimental work reviewed in the cited studies indicates that shear velocities in sands cannot be fit by a simple power law, as the exponent in such a power law decreases from 1/3 to 1/6 as the pressure increases. It is apparent from the referenced studies that the 1/6 power will probably not be attained in sands until pressures of the order of 350 kg/cm^2 are achieved (Gardner et al, 1964).

Gardner et al (1964) and the Shell investigators went to pressures of 60 to 70 kg/cm^2 and found velocity related to the 1/4 power of pressure. Effective pressure in sands increases by approximately 1 kg/cm^2 per 10 m increase in depth. Pressures to 70 kg/cm^2 would be found in sand bodies about 700 m thick. Few loose, uncemented sands (especially in the sea floor) are this thick. For practical purposes, therefore, the cited studies show that variation of shear velocity with depth or pressure can be fit by a power law with an exponent of 1/3 to 1/4 in most natural sand bodies. The present study found an exponent of 0.28 to a depth of 12 m [equation (1)].

In view of the several careful studies in which numerous variable factors were considered, and the fact that these variables could not be measured or known in the field, the writer recommends use of a depth (or pressure) exponent of 1/4 (0.25) in computations and predictions of shear velocity as a function of depth in sands. There is a possibility that 0.3 may be closer in the upper 10 m of sands, but this requires further experimental verification.

Shear-wave velocity gradients in silt-clays and turbidites

The relations between shear-wave velocity and depth in silt-clays and turbidites (Figure 2) are expressed by three linear equations. The first, equation (2), for depths between 0 and 36 m, has a slope, or gradient, of 4.65 (m/sec)/m (or sec^{-1}). This gradient is about 4 to 5 times greater than the gradient for compressional waves in comparable marine sediments (Houtz et al, 1968; Hamilton et al, 1974). The shear-wave velocity gradient from 36 to 120 m is 1.28 sec^{-1}, equation (3), which is comparable to the gradient for compressional velocity (about 1.1 sec^{-1}) in the same interval in the average marine section (Hamilton et al, 1974). The shear velocity gradient in the 120-m to 650-m interval is 0.58 sec^{-1} [equation (4)]. The average compressional wave velocity gradient in this interval is about 0.8 sec^{-1} (Hamilton et al, 1974).

Prediction of shear velocity profiles

The velocity of compressional waves in surficial marine sands and silt-clays can be predicted within narrow limits because of numerous laboratory and in-situ measurements (Hamilton, 1970, 1971b, 1974, and references therein). The relatively few shear-wave measurements, and their wide range in values (as in Tables 1 and 2), make the prediction of shear velocity in surficial sediments difficult and tenuous.

To predict shear velocity as a function of depth requires a value at or near the sediment surface and the gradient of velocity. In the absence of a measured value in surficial sediments, one can be approximately computed, or taken from tables and corrected to in situ (Hamilton, 1971a,b), assumed from measurements in similar materials (as in equations (1) and (2), or Tables 1 and 2), or computed from appropriate equations [such as equations (5) and (6) for sands]. Given a value near the sediment surface, shear velocity as a function of depth can be approximated by using the gradients of velocity as indicated above. For sands, an exponent of depth of 1/4 is recommended; for silt-clays, the linear gradients of equations (2), (3), and (4) are recommended.

Compressional velocity versus depth in sands

A review of compressional velocity gradients in sands is not an objective of this report, but it is appropriate to note compressional velocity variations with depth in the Shell sand data [Figures 3 and 4, and equations (13) and (15)].

Some misleading data have entered the literature in regard to compressional wave velocity in "saturated" sands; saturation in this context implies 100 percent saturation. At 100-percent saturation, compressional velocities in sands are well above those in water and in the usual range of 1500 to 1900 m/sec in both laboratory and field (e.g., Figures 3, 4; Hamilton, 1970, 1974; Hamilton et al, 1970; McCann and McCann, 1969). When compressional velocity in "saturated sand" is reported in the range of several hundred m/sec, the reader should immediately conclude that the sands were incompletely saturated.

In dry or partially saturated sand, compressional wave velocity varies with effective pressure in about the same way as shear velocity: the exponent of pressure usually varies between 1/3 and 1/6 with an approximate 1/4 at lower pressures (e.g., Figures 3 and 4; Gardner et al, 1964). When the sands are fully saturated, the compressional velocity increases dramatically, and there is a slight increase of compressional velocity with pressure (e.g., Figures 3 and 4). Equations (7), (9), (13), and (15) indicate that for the Shell data, in the upper 10s of meters, the compressional wave velocity in the saturated sands varies with the 0.013 or 0.016 power of pressure and depth. In computations or predictions of compressional wave velocity versus depth in saturated sands, an exponent of pressure, or depth, of 0.015 is indicated by the Shell data.

ACKNOWLEDGMENTS

This work was supported by the Naval Sea Systems Command (code 06H14), by the Naval Electronic Systems Command (code 320), and the Office of Naval Research (code 486).

REFERENCES

Aisiks, E. G., and Tarshansky, I. W., 1969, Soil studies for seismic design of San Francisco Transbay Tube, *in* Vibration effects of earthquakes on soils and foundations: Am. Soc. Test. Materials, Spec. Publ. 450, p. 138–166.

Barkan, D. D., 1962, Dynamics of bases and foundations: New York, McGraw-Hill Book Co., Inc.

Barnes, B. B., Corwin, R. F., Hildebrand, T. G., Jackson, L., Kessler, R., Takeyama, W., Hornick, M., and Jenkins, R., 1973, Geophysics applied to geotechnical problems in a marine environment, a case study: Monterey Bay, California: An. Rep. 1972–1973, Washington, D. C. Environmental Res. Labs., NOAA.

Berzon, I. S., Ratnikova, L. I., and Mitronova, V. A., 1967, Shear waves reflected from a thin high-speed layer, *in* Seismic wave propagation in real media: I. S. Berzon, Ed.; English translation, Consult. Bur., New York, 1969, p. 154–162; orig. public., Nauka Press, Moscow, 1967.

Bucker, H. P., Whitney, J. A., and Keir, D. L., 1964, Use of Stoneley waves to determine the shear velocity in ocean sediments: J. Acoust. Soc. Am., v. 36, p. 1595–1596.

Cunny, R. W., and Fry, Z. B., 1973, Vibratory in situ and laboratory soil moduli compared: J. Soil. Mech. and Fdn. Div., Am. Soc. Civil Engin., v. 99, SM12, p. 1055–1076.

Davies, D., 1965, Dispersed Stoneley waves on the ocean bottom: Bull SSA, v. 55, p. 903–918.

Duffy, J., 1959, A differential stress-strain relation for the hexagonal close-packed array of elastic spheres: J. Appl. Mech., v. 26, p. 88–94.

Duffy, J., and Mindlin, R. D., 1957, Stress-strain relations and vibrations of a granular medium: J. Appl. Mech., v. 24, p. 585–593.

Gardner, G. H. F., Wyllie, M. R. J., and Droschak, D. M., 1964, Effects of pressure and fluid saturation on the attenuation of elastic waves in sands: J. Petrol. Tec., v. 16, p. 189–198.

Hamilton, E. L., 1970, Sound velocity and related properties of marine sediments, North Pacific: J. Geophys. Res., v. 75, p. 4423–4446.

——— 1971a, Elastic properties of marine sediments: J. Geophys. Res., v. 76, p. 579–604.

——— 1971b, Prediction of in-situ acoustic and elastic properties of marine sediments: Geophysics, v. 36, p. 266–284.

——— 1974, Prediction of deep-sea sediment properties: state of the art, *in* Deep-sea sediments, physical and mechanical properties: A. L. Inderbitzen, Ed., New York, Plenum Press, p. 1–43.

——— 1976, Variations of density and porosity with depth in deep-sea sediments: J. Sediment. Petrol., v. 46, p. 280–300.

Hamilton, E. L., Bucker, H. P., Keir, D. L., and Whitney, J. A., 1970, Velocities of compressional and shear waves in marine sediments determined in situ from a research submersible: J. Geophys. Res., v. 75, p. 4039–4049.

Hamilton, E. L., Moore, D. G., Buffington, E. C., Sherrer, P. L., and Curray, J. R., 1974, Sediment velocities from sonobuoys: Bay of Bengal, Bering Sea, Japan Sea, and North Pacific: J. Geophys. Res., v. 79, p. 2653–2668.

Hardin, B. O., and Black, W. L., 1966, Sand stiffness under various triaxial stresses: J. Soil Mech. and Fdn.

Div., Am. Soc. Civil Engin., v. 92, No. SM2, p. 27–42.
——— 1968, Vibration modulus of normally consolidated clay: J. Soil Mech. and Fdn. Div., Am. Soc. Civil Engin., v. 94, SM2, 353–369.
Hardin, B. O., and Richart, F. E., Jr., 1963, Elastic waves in granular soils: J. Soil Mech. and Fdn. Div., Am. Soc. Civil Engin., v. 89, SM1, p. 33–65; and discussions, SM5, p. 103–118.
Heukelom, W., 1961, Analysis of dynamic deflections on soils and pavements: Geotechnique, v. 11, p. 224–243.
Houtz, R. E., Ewing, J. I., and LePichon, X., 1968, Velocity of deep-sea sediments from sonobuoy data: J. Geophys. Res., v. 73, p. 2615–2641.
Jolly, R. N., 1956, Investigation of shear waves: Geophysics, v. 21, p. 905–938.
Jones, R., 1958, In situ measurement of the dynamic properties of soil by vibration methods: Geotechnique, v. 8, p. 1–21.
Kawasumi, H., Shima, E., Ohta, Y., Yanagisawa, M., Allam, A., and Miyakawa, K., 1966, S wave velocities of subsoil layers in Tokyo, I: Bull. Earthquake Res. Inst., Univ. of Tokyo, v. 44, p. 731–747.
Knopoff, L., 1952, On Rayleigh wave velocities: Bull. SSA, v. 42, p. 307–308.
Kolsky, H., 1963, Stress waves in solids: New York, Dover Publishing Co., Inc.
Kudo, K., and Shima, E., 1970, Attenuation of shear waves in soil: Bull. Earthquake Res. Inst., Univ. of Tokyo, v. 48, p. 145–158.
Kulm, L. D., von Huene, R., et al, 1973, Initial reports of the deep sea drilling project: Washington, U. S. Govt. Printing Office, v. 18.
Lambe, T. W., and Whitman, R. V., 1969, Soil mechanics: New York, John Wiley and Sons, Inc.
McCann, C., and McCann, D. M., 1969, The attenuation of compressional waves in marine sediments: Geophysics, v. 34, p. 882–892.
McManus, D. A., et al, 1970, Initial reports of the deep sea drilling project: Washington, U. S. Govt. Printing Office, v. 5.
Molotova, L. V., 1963, Velocity ratio of longitudinal and transverse waves in terrigenous rocks: English translation, Bull. Acad. Sci., USSR, Geophys. Ser., no. 12, p. 1769–1779.
——— 1964, The relationship between the frequency spectra of seismic vibrations and explosion conditions: English translation, Bull. Acad. Sci., USSR, Geophys. Ser., no. 12, p. 1059–1067.
——— 1966, Velocity dispersion of body waves in terrigenous rocks: English translation, Bull. Acad. Sci., USSR, Physics of the Solid Earth, no. 7, p. 500–506.
Molotova, L. V., and Vassil'ev, Yu. I., 1960, Velocity ratio of longitudinal and transverse waves in rock, II: English translation, Bull. Acad. Sci., USSR, Geophys. Ser., no. 8, p. 731–743.
Ohta, Y., 1967, Experimental study on generation and propagation of S-waves: III. Generation of SV-waves by means of a modified explosion source: Bull. Earthquake Res. Inst., Univ. of Tokyo, v. 45, p. 727–738.
Ohta, Y., and Shima, E., 1967, Experimental study on generation and propagation of S-waves: II. Preliminary experiments on generation of SV-waves: Bull. Earthquake Res. Inst., Univ of Tokyo, v. 45, p. 33–42.
Pilbeam, C. C., and Vaisnys, J. R., 1973, Acoustic velocities and energy losses in granular aggregates: J. Geophys. Res., v. 78, p. 810–824.
Richart, F. E., Jr., Woods, R. D., and Hall, J. R., Jr., 1970, Vibrations of soils and foundations: New Jersey, Prentice-Hall, Inc.
Shima, E., Ohta, Y., Yanagisawa, M., and Kudo, K., 1968, S wave velocities in subsoil layers in Tokyo, 3: Bull. Earthquake Res. Inst., Univ. of Tokyo, v. 46, p. 1301–1312.
Stokoe, K. H., II, and Woods R. D., 1972, In situ shear wave velocity by cross-hole method: J. Soil Mech. and Fdn. Div., Am. Soc. Civil Engin., v. 98, SM5, p. 443–460.
Sutton, G. H., Maynard, G. L., and Hussong, D. M., 1971, Widespread occurrence of a high-velocity basal layer in the Pacific crust found with repetitive sources and sonobuoys, *in* The structure and physical properties of the earth's crust: J. G. Heacock, Ed., Geophys. Monogr. no. 14, Washington, D. C., AGU, p. 193–209.
Trask, P. D., and Rolston, J. W., 1951, Engineering geology of San Francisco Bay, California: Bull. GSA, v. 62, p. 1079–1110.
Vassil'ev, Y. I., and Gurevich, G. I., 1962, On the ratio between attenuation decrements and propagation velocities of longitudinal and transverse waves: English translation, Bull. Acad. Sci., USSR, Geophys. Ser., no. 12, p. 1061–1074.
Warrick, R. E., 1974, Seismic investigation of a San Francisco Bay Mud site: Bull. SSA, v. 64, p. 375–385.
White, J. E., 1965, Seismic waves: radiation, transmission, and attenuation: New York, McGraw-Hill Book Co., Inc.
White, J. E., and Sengbush, R. L., 1953, Velocity measurements in near-surface formations: Geophysics, v. 18, p. 54–69.
Whitman, R. V., Holt, R. J., and Murphy, V. J., 1969, Discussion of : Vibration modulus of normally consolidated clay, B. O. Hardin and W. L. Black: J. Soil Mech. and Fdn. Div., Am. Soc. Civil Engin., v. 95, SM2, p. 656–659.