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# Wave Set-Up on a Natural Beach

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Wave set-up, the superelevation of mean water level owing to the presence of breaking incident waves, was measured at the shoreline of a natural beach. Offshore pressure sensors monitored incident wave conditions. The set-up of the shoreline was found to be about $0.17H_{s,\infty}$, where $H_{s,\infty}$ is the significant wave height in deep water.

## INTRODUCTION

During the 1938 hurricane which hit the east coast of the United States, observations showed the maximum mean shoreline water elevation was 1 m greater at the relatively exposed area of Narragansett Pier, where wave energy was dissipated as surf, than at the calmer waters, off Newport. This difference could not be explained by variations in storm surge height, and the idea was put forth that the breaking waves at Narragansett caused the mean level change. This speculation was verified by a series of model tests showing superelevations of mean level near the shoreline that were a significant fraction of the incident wave height [*Savage*, 1957; *Fairchild*, 1958; *Saville*, 1961]. Soon after these model tests, it was shown theoretically that a small depression of mean surface occurs seaward of the break point, and a larger superelevation shoreward, because of the changes in momentum flux associated with shoaling and subsequent wave breaking [*Dorrenstein*, 1961; *Longuett-Higgins and Stewart*, 1962, 1963, 1964]. A careful and detailed laboratory study [*Bowen et al.*, 1968] with monochromatic incident waves verified the theoretical suggestion that the set-up slope inside most of the surf zone is proportional to the beach slope, and obtained an explicit form for the constant of proportionality, $K$,

$$\frac{\partial \bar{\eta}}{\partial x} = -K\frac{\partial h}{\partial x} \qquad K = (1 + 2.67\,\gamma^{-2})^{-1} \tag{1}$$

where $h$ is the still water depth, $\bar{\eta}$ the change in mean depth owing to waves, $x$ is the offshore coordinate, and $\gamma = H/(\bar{\eta} + h)$ is of the order of 1 constant relating the broken wave height, $H$, to the total mean water depth. Both theory and laboratory measurements show that the maximum set-up, $\bar{\eta}_M$, occurs at the shoreline. These simple monochromatic theories predict [*Battjes*, 1974]

$$\frac{\bar{\eta}_M}{H_b} = 0.3\gamma \tag{2}$$

where $H_b$ is the wave height at breaking. *Battjes* [1974] displays laboratory data from several different sources showing $\gamma$ increasing from approximately 0.8 for (monochromatic) spilling breakers to about 1.3 for plunging breakers, so that

$$0.24 < \bar{\eta}_M/H_b < 0.39 \tag{3}$$



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The extension of the monochromatic set-up theory to cover incident wave spectra [*Battjes*, 1974; *Battjes and Janssen*, 1978] necessarily involves assumptions about the distribution of wave heights in the surf zone and the amount of energy lost by individual waves during the breaking process. These complications preclude explicit analytic statements, similar to (3), about the size of $\bar{\eta}_M$ relative to $H_b$, which is now a statistical quantity. These models, however, are not very sensitive to variation in beach slope, and when properly normalized, are only weakly dependent on details of the offshore incident wave spectra. Theory and laboratory experiments with random waves show that

$$0.14 < \frac{\bar{\eta}_M}{H_{s,\infty}} < 0.21 \tag{4}$$

where $H_{s,\infty}$ is the significant wave height in deep water [*Battjes*, 1974]. The range of values given in (4) corresponds to an order of magnitude variation in incident wave steepness.

If the beach face slope, $\tan \beta$, is approximately constant, then the mean swash location is moved landward a distance $\bar{\eta}_M/\tan \beta$ because of set-up. If $H_{s,\infty} \cong 2$ m, then (4) predicts $\bar{\eta}_M \cong 35$ cm. If $\tan \beta = 0.03$, then the mean swash location should be moved shoreward about 12 m.

Considering the obvious importance of wave set-up to coastal design criterion and beach morphologies, surprisingly few field measurements have been made. *Dorrestein* [1961] made field observations of set-up at various locations across the surf zone, with significant offshore wave heights between 0.8 and 1.6 m. *Battjes* [1974] shows these data to agree with predictions similar to (4). This agreement, however, may be fortuitous because the mean sea level results were based on a 72-s averaging time, which may not be long enough to filter out surf beat. The present measurements show that successive estimates of $\bar{\eta}_M$, based on a 64-s averaging time, frequently vary by more than the mean value of $\bar{\eta}_M$ based on a 4096-s average. The possible serious contamination of *Dorrestein*'s [1961] mean values by low frequency motions is obvious in the sample time histories of water elevations shown in that work. *Sonu* [1972] presented mean sea level measurements from the field which showed maximum set-up seaward of the break point and the lowest mean sea level at the shoreline. This contradicts all laboratory measurements and theoretical results.

In summary, there are presently several theoretical studies dealing with set-up using both monochromatic and random

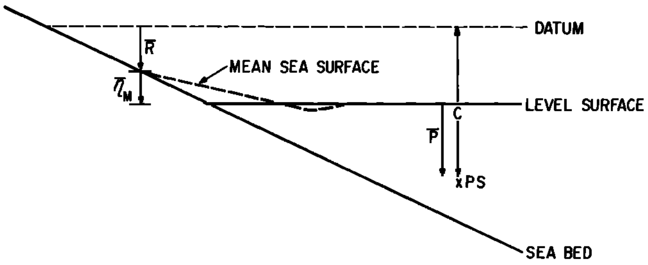


Fig. 1. Definition sketch. P.S. is a pressure sensor, located distances $C$ and $\bar{P}$ below datum and mean sea surface, respectively. The mean swash is $\bar{R}$ below datum, and the shoreline set-up is $\bar{\eta}_M$.

waves, fewer useful laboratory studies, and at most one useful set of field data. The present field experiments have deficiencies (discussed below) but nevertheless yield some useful qualitative results and suggest areas for future work.

## MEASUREMENTS

Experiments were conducted at Torrey Pines Beach, San Diego, California, during November 1978. This is a gently sloping ($\tan \beta \approx 0.02$) moderately sorted, fine grain (mean diameter 0.1 mm) sandy beach. The beach profile [*Guza and Thornton,* 1980] does not show any well-developed bar structure and is remarkably free from longshore topographic inhomogeneities. Winds during the experiments were slight and variable in direction. Shadowing by offshore islands and offshore refraction limits the angles of wave incidence in 10 m depth to less than 15°. Typical incident wave energy spectra typically exhibit maxima between 0.1 and 0.065 Hz (Figure 4) with higher frequency energy (~0.14 Hz) energy occasionally generated by local events [e.g., *Guza and Thornton,* 1980, Figure 9]. During the experiments, significant offshore wave heights varied between 60 and 160 cm. The condition of nearly normally incident, spilling (or mixed plunging-spilling) waves, breaking in a continuous way across the surf zone, prevailed during most of the experiments discussed below. An extensive array of instruments was deployed to study nearshore wave dynamics. The calibration and other details of the electromagnetic Marsh-McBirney current meters and the temperature compensated Stathem pressure transducers are discussed in *Guza and Thornton* [1980].

Mean water level at the shoreline was measured with an 80-m long, dual resistence (nichrome) wire run-up meter employing the very stable electronics described in *Flick et al.* [1979]. The run-up meter wires were supported about 3 cm above the sand level by nonconducting supports spaced every 10 m along the wires and thus conformed to the beach topography. The horizontal run-up location is thus defined by the sensor as being the most shoreward point where the depth is at least 3 cm. The run-up meter was calibrated before, and sometimes after, each data run (at high tide) by shorting the wires at different locations and recording the voltages. When transformed to vertical elevations, the error introduced by drift of the run-up meter electronics into the measured shoreline set-up ($\bar{\eta}_M$) was typically less than 3 cm. Beach profiles were measured relative to a fixed bench mark by using a rod and transit. On several days, beach surveys were taken by two different survey crews at different times during a data run, using different surveying equipment. In four of six cases, the resultant difference in estimated mean run-up vertical elevation was less than 3 cm. In the remaining two cases, the discrepancy was about 6 cm, and an average profile was used. In one case, November 19, the only profile taken was obviously bad, and the profile of the next day was used. The total run-up meter error in estimating shoreline mean water level is estimated as being generally less than 5 cm.

Five offshore pressure sensors at mean depths between 7 and 10.5 m were used to obtain a still water reference level in the absence of wave set-up, well outside the surf zone. Atmospheric pressure fluctuations were compensated for using data from the Scripps pier recording barometer, located about 2 miles down coast from the study site. Drift in the individual pressure sensors was estimated by examining changes in the mean depth difference between sensors. Since the sensor elevations relative to a vertical datum did not change, the depth differences should remain constant. For the first 7 data days, the change in depth difference (based on 4096 s averages) between any two sensors was always less than 2 cm and usually less than 1 cm. At this point, during instrument maintenance, two of the pressure sensors appeared to become about 5 cm deeper relative to the other three sensors. Since the pressure sensor mean depths are averaged together, this yields an estimated uncertainty in offshore mean sea level of about 2 cm. The total errors in shoreline and offshore mean sea levels is therefore about 7 cm for November 6–17 and 9 cm for November 18–22.

## EXPERIMENTAL RESULTS

Ideally, measurements of offshore and run-up mean water level would be referenced to a common datum, directly yielding $\bar{\eta}_M$. As shown in Figure 1, with $C$ the surveyed pressure sensor location below any datum, $\bar{P}$ the measured mean fluid depth at the pressure sensor, and $\bar{R}$ the measured mean swash location below datum, the shoreline set-up is given by $\bar{\eta}_M = C - \bar{P} - \bar{R}$. Unfortunately, it was not possible to reference the offshore pressure sensors vertical locations to a shore datum because the large depth of water precluded accurate surveys. Thus, $\bar{P}$ and $\bar{R}$ were measured as discussed above, but $C$ was unknown. To eliminate $C$, it is necessary to assume a functional form for the dependence of set-up on offshore wave parameters. The existing theory ((4)) and laboratory data for set-up suggests that $\bar{\eta}_M$ depends approximately linearly on $H_{s,\infty}$, with proportionality constant $p$

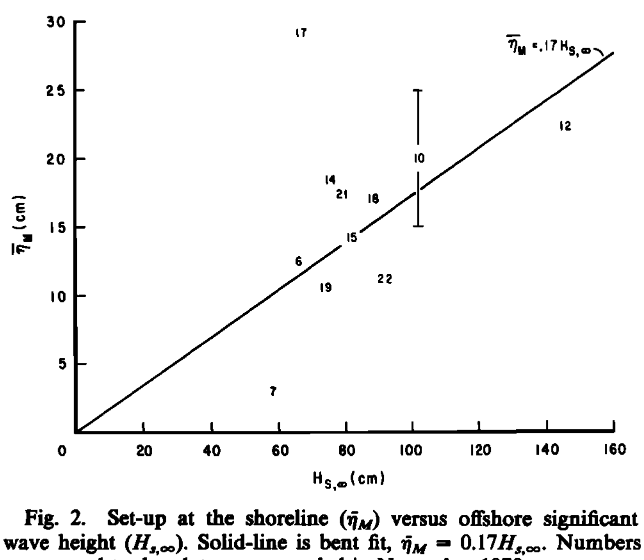


Fig. 2. Set-up at the shoreline ($\bar{\eta}_M$) versus offshore significant wave height ($H_{s,\infty}$). Solid-line is bent fit, $\bar{\eta}_M = 0.17H_{s,\infty}$. Numbers correspond to day data was recorded in November 1979.

$$\bar{\eta}_M = pH_{s,\infty} \tag{5}$$

with measured values differing by an unknown constant $C$

$$\bar{\eta}_M^* = pH_{s,\infty} - C = -\bar{P} - \bar{R} \tag{6}$$

Data runs were obtained on 11 different days, the shortest run being 128 min. The constants $C$ and $p$ ((6)) were solved for, using least squares. The resulting $C = 840$ cm represents the average depth of the offshore pressure sensors, relative to the shore datum (a long-term mean sea level), and $p = 0.17$ gives the desired dependency of shoreline set-up on offshore wave height.

Figure 2 shows the inferred $\bar{\eta}_M (\bar{\eta}_M = \bar{\eta}_M^* - C)$ versus $H_{s,\infty}$, and the best fit line $\bar{\eta}_M = 0.17\ H_{s,\infty}$. $H_{s,\infty}$ was calculated by using standard linear shoaling theory to back the waves out to deep water from the deepest pressure spectrum, measured in about 10 m depth as described in *Guza and Thornton* [1980]. The outlying data point for November 17 was not used in calculating $p$, and no reason for the anamalously large set-up could be found. It is noted that wind was measured during the experiment, and its contribution to set-up was negligible. The observed best fit value of $p = 0.17$ is within the range of values suggested by laboratory and theoretical studies ((4)) with random waves.

Some insight into the mechanism resulting in the measured set-up can be gained by using current meters and pressure sensors distributed between the shoreline and 10 m depth. The fundamental theoretical equation for wave-induced changes in mean sea level ($\bar{\eta}$) is based on the condition that in the steady state, the shoreward ($x$ coordinate) flux of momentum must be constant [*Dorrestein*, 1961]. If the waves can be approximated as normally incident and as having linear dynamics, the wave induced shoreward momentum flux density is given by

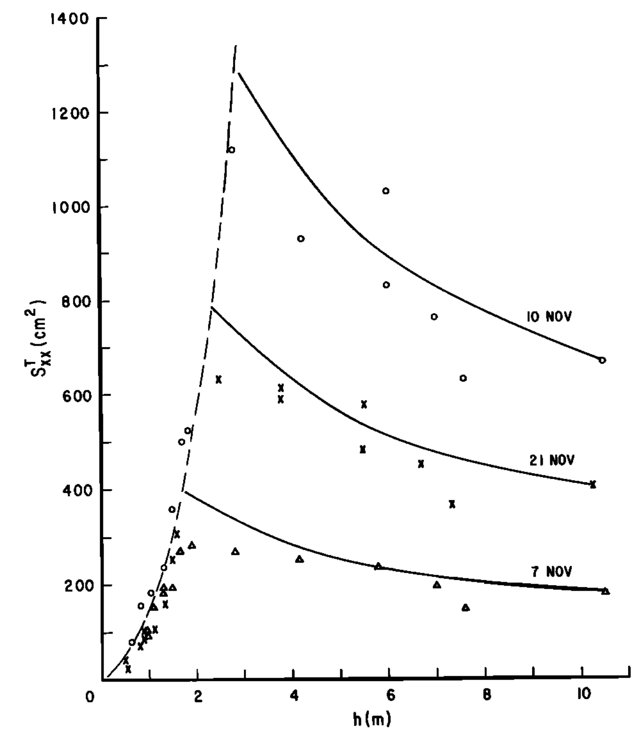


Fig. 3. Symbols are total measured radiation stress between 0.05 and 0.3 Hz versus depth, based on 4096 s of data. Solid lines are predictions based on linear shoaling theory; dashed line is saturation ((10)), $\gamma' = 0.4$).

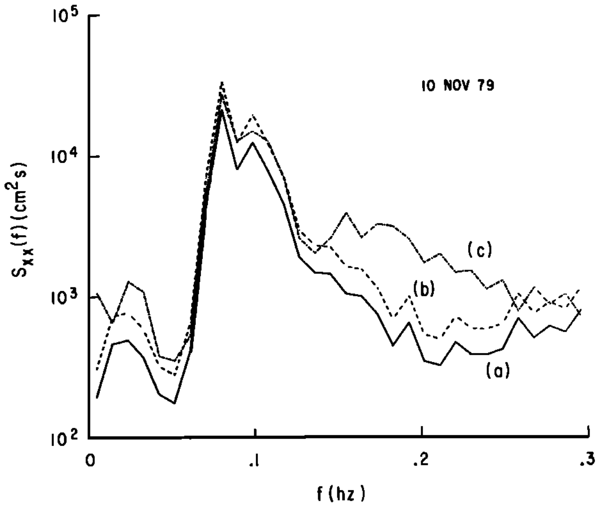


Fig. 4. Radiation stress density spectra: (*a*) measured in 1040 cm total depth; (*b*) predicted at $h = 425$ cm, (*c*) observed at $h = 425$ cm. Data record is 128 min, 64 degrees of freedom.

$$S_{xx}(f) = E(f)\ (2kh/\sinh 2kh + \tfrac{1}{2}) \tag{7}$$

with $h$ the local mean depth, $E(f)$ the wave energy spectral density, and $k$ the wave number given by the usual linear gravity wave dispersion relation. Assuming there are no mean onshore-offshore currents, gradients in the total $S_{xx}$ must be balanced by changes in the mean sea level

$$\frac{\partial S_{xx}{}^T}{\partial x} + \rho g(\bar{\eta} + h)\ \frac{\partial \bar{\eta}}{\partial x} = 0 \tag{8a}$$

where

$$S_{xx}{}^T = \int S(f) df \tag{8b}$$

All set-up theories basically hypothesize $x$ dependencies for $E(f)$ and thereby for $S_{xx}(f)$ ((7)), and integrate (8) for $\bar{\eta}$. Outside the surf zone, wave energy flux is usually assumed to be conserved, which leads to a predicted shoreward increase in $S_{xx}(f)$ ((7)) and a negative gradient, or depression, in mean level ((8)). Inside the surf zone, monochromatic set-up theory relies on the similarity parameter ($\gamma$) to related $S_{xx}$ to the local depth

$$S_{xx} = \frac{3}{2} E = \frac{3}{16}\ \rho g H^2 = \frac{3}{16}\ \rho g[\gamma(\eta + h)]^2 \tag{9}$$

which, using (8), yields (1). The qualitative correctness of these standard assumptions, applied to the present field data, is shown in Figure 3, where $S_{xx}{}^T$ is plotted as a function of depth for 3 representative data days.

Well seaward of the surf zone, the observed values of $S_{xx}{}^T$ agree well with predictions based on linear shoaling theory with the deepest observed $S_{xx}(f)$ spectrum as an input condition (solid lines). In the theoretical predictions of $S_{xx}{}^T$, the real topography can be acceptably modeled as plane parallel contours, so that shoaling effects can be treated analytically. Depth profiles, and a more complete justification of the neglect of long-shore topographic inhomogeneities, are given in *Guza and Thornton* [1980]. $S_{xx}{}^T$ was calculated from onshore-offshore velocities and/or pressure sensors by using (7) and (8*b*) coupled with the linear dispersion equation and theoreti-

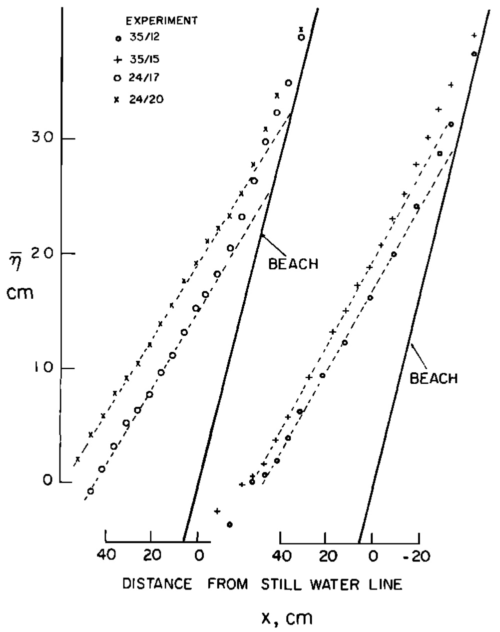


Fig. 5. Set-up on a laboratory beach, showing a constant set-up slope over most of surf zone, but increasing in very shallow water [*Bowen et al.*, 1968].

cal transfer functions between velocity, pressure, and wave amplitude. The use of pressure sensors to calculate $S_{xx}$ requires that the local waves be close to normally incident, as is in the present experiments [*Guza and Thornton*, 1980]. This is further demonstrated by the similar values of $S_{xx}{}^T$ obtained from a current meter and pressure sensor at the same horizontal location, shown in Figure 3 as two points at the same depth (i.e., November 21, $h$ = 380 cm, 550 cm). A typical measured $S_{xx}(f)$ in 10 m depth, and predicted and observed $S_{xx}(f)$ in 4.25 m depth, are shown in Figure 4. Nonlinear shoaling effects not accounted for in the theory lead to a gross underprediction of $S_{xx}(f)$ at frequencies near the second harmonic of the spectral peak, but this is partially compensated for by an overprediction at the spectral peak. The calculated and observed $S_{xx}{}^T$ differ by 15%. Because $S_{xx}(f)$ is proportional to $E(f)$, the extensive discussion in *Guza and Thornton* [1980] about local and shoaled energy densities is trivially extended to local and shoaled radiation stress densities. The relevant conclusions are (1) linear theory adequately relates local $S_{xx}(f)$ measurements from pressure, sea surface elevation and velocity sensors to each other, and (2) linear shoaling theory does a qualitatively good job of predicting $S_{xx}{}^T$ outside the breaker zone, given an input spectrum at 10 m. Thus, Figure 3 leads to the expectation that seaward of the breaker zone, the model predictions for set-down are based on valid assumptions about the spatial dependence of $S_{xx}{}^T$. None of the present measurements is accurate enough to measure set-down.

In the shallow water of the breaker zone, Figure 3 shows that data from the 3 days approximately collapse to a single curve, suggesting saturation. An extensive discussion of saturation and related phenomenon is given in *Thornton and Guza* [1981]; only the relevant essentials are given here. When (9) is generalized to a random wave field in shallow water,

$$S_{xx}{}^T = \frac{3}{2}E = \frac{3}{16}\rho g H_{rms}{}^2 = \frac{3}{32}\rho g H_s{}^2 = \frac{3}{32}\rho g \gamma'^2(\bar{\eta}+h)^2 \qquad (10)$$

where $H_{rms}$, $H_s$ are root mean square and significant wave heights, respectively, and $\gamma' = H_s/(\bar{\eta}+h)$ relates the significant wave height to mean depth. It is assumed $H_s = \sqrt{2}\,H_{rms}$, which is valid for surface elevations described by a Gaussian probability distribution function (pdf) and wave heights with a Rayleigh pdf. It is shown in *Thornton and Guza* [1981] that this is a reasonable approximation even for the nonlinear surf zone region. The dashed curve in Figure 3 is equation (10), $\gamma'$ = 0.4. Note that this does not imply that maximum bore heights are limited to 0.4 $(\bar{\eta}+h)$ but rather that the statistical distribution of wave heights is such that $H_s = 0.4\ (\bar{\eta}+h)$. Given that the data approximately follow (equation (10), $\gamma'$ = 0.4)

$$S_{xx}{}^T \cong 0.015\rho g(\bar{\eta}+h)^2 \qquad h < h_b' \qquad (11)$$

where $h_b'$ is the depth of maximum $S_{xx}{}^T$, (8*a*) yields

$$\frac{\partial\bar{\eta}}{\partial x} = -0.03\,\frac{\partial h}{\partial x} \qquad h < h_b' \qquad (12)$$

This is the same functional form as the classical monochromatic result ((1)), but the constant of proportionality $K \cong$ 0.3 (equation (1), $\gamma \cong 1$) is an order of magnitude greater in the monochromatic case. Integrating (12) from $h_b'$ to the shoreline yields

$$\bar{\eta}_M = 0.03\ h_b' \qquad (13)$$

where the small set-down at $h_b'$ is neglected. From Figure 3, $h_b'$ varies from roughly 1.6 m. on November 7 to 2.8 m on November 10 which suggests ((13)) $\bar{\eta}_M$ values of 4.8 and 8.4 cm, respectively. This is substantially less than the values inferred from the measurements (Figure 2). A possible explanation for the discrepancy between measured set-up, and the smaller value calculated on the basis of measured values of $S_{xx}{}^T$ and integration of the momentum equation, is the invalidity of the assumption of constant set-up slope across the surf zone. Figure 5, reproduced from *Bowen et al.* [1968], shows laboratory measurements in which the set-up slope markedly increases very close to the shoreline. An estimation of $\bar{\eta}_M$ based on a constant set-up slope, shown by the dashed lines, would result in a substantial underprediction. Similar results are apparent in the laboratory measurements of *Van Dorn* [1976]. Some of those results show that the point of maximum rundown is higher on the beach face than the mean swash location inferred from extrapolation of a constant slope set-up. In both these data sets the true mean swash location is obviously higher than the extrapolated estimate, but the extent of the superelevation appears to vary with beach slope and incident wave parameters. The data in Figure 5 suggest that extrapolation underestimates $\bar{\eta}_M$ by 30–50%, while Van Dorn's work indicates possible errors ranging from negligible to 100% or more. Other laboratory measurements of set-up do not extend far enough shoreward to detect this effect or else do not profile $\bar{\eta}$ across the surf zone.

## DISCUSSION

Figure 6 summarizes the important results, using typical data runs. The individual closed points seaward of $h = 0$ represent $\bar{\eta}$ values inferred from the measured $S_{xx}{}^T$ and a crude numerical integration of (8*a*). The $S_{xx}{}^T$ values are from 4096 s of data. The smooth lines are drawn to guide the eye. Offshore there is a small set-down. The approximately constant set-up slope, except very near the shoreline, reflects the fact that (10), which predicts a constant slope ((12)), provides a reasonably good fit to the observed $S_{xx}{}^T$ (Figure 3). The in-

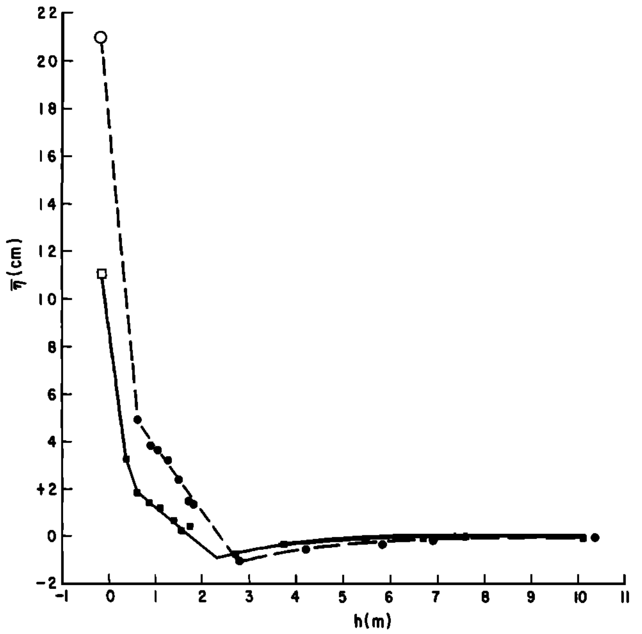


Fig. 6. Closed symbols show $\bar{\eta}$ obtained by numerical integration of (8*a*), using measured $S_{xx}{}^T$. Open symbols are $\bar{\eta}_M$ measured by the run-up meter. Box, November 19; circle, November 10.

ferred set-up slope is substantially smaller than that observed in laboratory studies, primarily because the present values of $\gamma$ are less than half of typical laboratory values. Set-up was measured only at the shoreline, and these data points are shown as open. The conclusion that the set-up slope markedly increases very close to the shoreline is supported by laboratory data with monochromatic incident waves (Figure 5).

Data from all runs suggests $\bar{\eta}_M = 0.17\ H_{s,\infty}$ (Figure 2). This result could be different on beaches with a different porosity, or with a topographic (bar) structure which alters the spatial variation of $S_{xx}{}^T$. The effects on the results of measuring the maximum run-up 3 cm above the bed is unclear. Experiments are planned on a steeper beach with a common datum for beach and offshore sensors.

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