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Edge Waves and Beach Cusps

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Genetically, beach cusps are of at least two types: those linked with incident waves which are surging and mostly reflected (reflective systems) and those generated on beaches where wave breaking and nearshore circulation cells are important (dissipative systems). The spacings of some cusps formed under reflective wave conditions both in the laboratory and in certain selected natural situations are shown to be consistent with models hypothesizing formation by either (1) subharmonic edge waves (period twice that of the incident waves) of zero mode number or (2) synchronous (period equal to that of incident waves) edge waves of low mode. Experiments show that visible subharmonic edge wave generation occurs on nonerodable plane laboratory beaches only when the incident waves are strongly reflected at the beach, and this observation is quantified. Edge wave resonance theory and experiments suggest that synchronous potential edge wave generation can also occur on reflective beaches and is a higher-order, weaker resonance than the subharmonic type. In dissipative systems, modes of longshore periodic motion other than potential edge waves may be important in controlling the longshore scale of circulation cells and beach morphologies. On reflective plane laboratory beaches, initially large subharmonic edge waves rearrage sand tracers into shapes which resemble natural beach cusps, but the edge wave amplitudes decrease as the cusps grow. Cusp growth is thus limited by negative feedback from the cusps to the edge wave excitation process. Small edge waves can form longshore periodic morphologies by providing destabilizing perturbations on a berm properly located in the swash zone. In this case the retreating incident wave surge is channelized into breeches in the berm caused by the edge waves, and there is an initially positive feedback from the topography to longshore periodic perturbations.

INTRODUCTION

Rhythmical longshore patterns have often been observed in shoreline and nearshore morphologies, both in the field and in wave tanks [e.g., *Komar*, 1973; *Bowen and Inman*, 1971; *Dolan*, 1971; *Harris*, 1967; *Johnson*, 1919]. Cuspate patterns, which will be considered here, are concave seaward, are usually formed at the shoreline, and have longshore wavelengths varying from less than 1 m on lakeshores [*Komar*, 1973] to the scale of capes (10^5 m) or larger [*Dolan and Ferm*, 1968]. Dolan and Ferm enumerate some traditional names given cuspate features on ocean shorelines: (1) 'typical beach cusps,' 8–25 m; (2) 'storm cusps,' 70–120 m; and (3) 'giant cusps,' also known as 'shoreline rhythms,' 700–1500 m. A classification according to size is, however, genetically unconvincing. *Kuenen* [1948] stated that 'typical' and 'storm' cusps are 'practically the same' except for size. The present study centers on a single generating mechanism, edge waves, which have wavelengths ranging from centimeters on lakeshores to hundreds of meters (or more) near oceanic coastlines. Edge waves capable of producing cuspate morphologies are generated in the laboratory with wavelengths varying from tens of centimeters to 10 m, the wave basin size precluding longer waves.

Cusps are noted for their regularity and the distinctive circulations associated with them. *Johnson* [1919] summarizes early attempts to explain these rhythmic morphologies and cites *Lane* [1888] as hypothesizing that random irregularities on the beach become evenly spaced through some process of adjustment to equilibrium not clearly understood and that this equilibrium distance between cusps is related to the height of the waves. Johnson basically agreed with this hypothesis, as did later authors [e.g., *Kuenen*, 1948; *Otvos*, 1964], although some authors stressed the importance of depositional over erosional processes, and vice versa. This model stresses the importance of the interaction of wave motions and topographic changes in determining cusp spacing. While bottom interactive models are feasible on erodable beaches, they cannot explain later laboratory experiments [*Galvin*, 1965; *Harris*, 1967; *Bowen*, 1967; *Bowen and Inman*, 1969, 1971] on nonerodable plane beaches which showed the existence of longshore periodic wave motions (edge waves), some of which develop circulation cells. Either the edge waves or the associated circulation cells can impose their longshore periodicity on an erodable bed. The present experiments indicate that although the induced topographic changes eventually have a feedback to the waves and currents and thus alter the further rearrangement of sediment, the primary longshore periodic generative mechanism of beach cusps is sometimes present in the waves and currents occurring on nonerodable beds. Inclusion of the feedback of changing topography is found to be necessary to determine the equilibrium amplitude and the permanence of morphologic changes.

It has also been hypothesized that rhythmic topographies on an unbounded coast are 'sand wave' trains which result from an instability of the surf zone bed to perturbations by longshore currents [e.g., *Hom-ma and Sonu*, 1963; *Sonu*, 1968, 1973; *Schwartz*, 1972]. The analogy here is to the dunes and antidunes of fluvial systems; initially small bottom irregularities grow because of a reinforcing coupling to the longshore current. Their proponents hypothesize that sand waves influence the incident wave field in such a way as to produce the observed circulation cells. Like the earlier hypotheses, this mechanism cannot explain the longshore periodic motions observed on nonerodable beaches. Furthermore, many authors [e.g., *Johnson*, 1919; *Longuet-Higgins and Parkin*, 1962; *Russell and McIntire*, 1965] claim that natural cusps form only with normally incident waves, which generally do not produce unidirectional, coherent flows like those responsible for classical dune systems. Others have observed cusp formation with both normal and nonnormal incidence [*Worrall*, 1969; *Otvos*, 1964; *Mii*, 1959]. Clearly, any theory which requires a longshore current for cusp formation cannot explain a large portion of observed natural cusps. It should not be assumed, however, that cusps of large physical dimension formed at locations with large process scales (i.e., incident wave wavelengths) are necessarily genetically different from much smaller cusps formed elsewhere.

Edge waves are the normal trapped modes of longshore periodic wave motion that occur along the edge of water bodies, and they may be either standing or progressive. They exist equally well along straight or gently curving shorelines and for uniformly sloping, concavely or convexly curving offshore depth profiles. Numerical methods show the structure of edge waves on more realistic topographies [*Munk et al.*, 1964]. Edge waves are noteworthy because they are trapped; they cannot radiate energy out into deep water. Unlike ordinary swell waves, they cannot propagate offshore and away from a shallow water energy source. Their energy can only be dissipated through friction and through interaction with currents and other waves. The possible importance of edge waves in the nearshore region was first stressed by Inman [*Bowen*, 1967]. It has been demonstrated theoretically that edge waves can be excited by waves incident from deep water [*Gallagher*, 1971; *Guza and Davis*, 1974] or by traveling atmospheric pressure fronts [*Donn and Ewing*, 1956]. It is the contention of the present work that edge waves, both directly and via their interactions with other water motions, are responsible for many cases of cuspate topography. Related ideas are discussed by *Bowen* [1967, 1969, 1972], *Bowen and Inman* [1969, 1971, 1972], *Komar* [1971, 1973], and *Inman* [1971].

The dynamics of water motion on the beach face change drastically according to whether the incident wave is strongly reflected or breaks cleanly and propagates onshore as a dissipative bore. A distinction is therefore made between 'reflective' and 'dissipative' systems. The present work deals primarily with the generation of edge waves and beach cusps in reflective systems, because the appropriate equations are analytically tractable and because beach cusps are frequently observed with strongly reflected incident waves. Reflective systems correspond to surging or collapsing incident waves according to the classification scheme of *Galvin* [1972]. Cusps also occur on clearly dissipative beaches, but the edge wave resonances theoretically predicted and observed for reflective systems are not visible on the beach face when the incident waves cleanly plunge.

EDGE WAVES ON CONSTANT SLOPE BEACHES

Coriolis forces being neglected, the dynamics of inviscid edge waves on a nonerodable beach of constant slope extending into deep water have been discussed by *Eckart* [1951], *Ursell* [1952], *Bowen and Inman* [1969, 1971], and others. The edge wave wavelength L in the longshore direction is given by *Ursell* [1952], with no assumptions about the shallowness of the water, as

$$L = (g/2\pi)T_e^2 \sin (2n + 1)\beta = 2\pi/k_y \quad (1)$$

where x and y are the offshore and longshore coordinates, respectively; T_e is the period; β is the beach slope; g is the gravitational acceleration; n is an integer known as the mode number, and k_y is the longshore wave number. A further restriction on n is $(2n + 1)\beta < \pi/2$, where β is in radians. Thus for a given edge wave period there are a series of possible wavelengths corresponding to the possible values of n. The x behavior of the Ursell solutions is algebraically quite complex. One of us (R.T.G.) has found that *Tait* [1970] made errors in computing these functions, thereby invalidating his results. It is useful to consider the simpler shallow water solutions in terms of the velocity potential ϕ; the velocity is $\nabla\phi$, and the surface elevation η is $(-1/g)\ \partial\phi/\partial t$. *Eckart* [1951] gives the shallow water velocity potential solution for edge waves on a beach sloping into deep water as

$$\phi = (a_e g/\omega) \cos k_y y \exp (-k_y x)\ L_n(2k_y x) \cos \omega t \quad (2)$$

where a_e is the wave amplitude, L_n is the Laguerre polynomial of order n, $\omega = 2\pi/T_e$ is the radian frequency, and

$$L = (g/2\pi)T_e^2 (1 - 2s) \tan \beta = 2\pi/k_y \quad (3)$$

where $-s = n = 0, 1, 2, \cdots$. The condition $(2n + 1) \tan \beta \ll 1$ is required either by comparison of (3) with the exact linear relation (1) or by the requirement that these shallow water solutions become asymptotically small while still in shallow water [*Guza and Davis*, 1974]. The profiles of the sea surface for $n = 0, 1, 2, 3$ are given in Figure 1 as functions of the dimensionless space variable χ, where $\chi = \omega^2 x/g \tan \beta$ and χ is proportional to the number of zero mode wavelengths offshore. The profile of a wave normally incident from deep water and totally reflected at the coast is also shown. Because

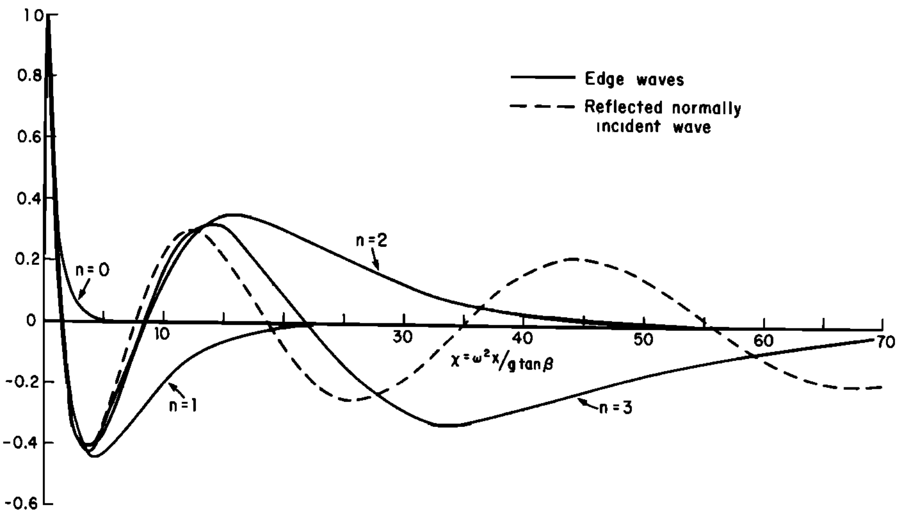

Fig. 1. Offshore dependence of profiles of Eckart-type edge waves of modes $n = 0, 1, 2, 3$ and of normally incident reflected waves, in terms of the dimensionless variable $\chi = \omega^2 x/g \tan \beta$.

of the similarities, data consisting only of onshore-offshore amplitude profiles close to shore cannot differentiate between edge waves (particularly of high mode number) and waves incident at any angle from deep water and reflected at the coast.

The laboratory experiments which have been conducted were on a beach with slope β when $x < x_0$ and then constant depth h_0 for $x > x_0$. It can be shown by using a simple matching of sea surface and onshore-offshore velocities between sloping and flat bottom shallow water solutions at $x = x_0$ that edge waves can still exist and are easily understood in terms of the Eckart-type solutions. In terms of the nondimensional beach width χ_0, suggested by *Bowen and Inman* [1969],

$$\chi_0 = \omega^2 x_0/g \tan \beta$$

the dispersion relation of edge waves will be basically unchanged (less than 1% change in the parameters s in (3)) from the Eckart form if $\chi_0 > \chi_c$, where χ_c is a critical beach width and depends on the mode number, as given in Table 1, for the first four modes. As would intuitively be expected, χ_c occurs after the last maximum of the Eckart edge wave (compare values in Table 1 with those in Figure 1): if the wave amplitude is some small value at the slope break and is already exponentially decaying, the effect of the break should be minimal.

In order for inviscid waves to be trapped and not to radiate energy away they must satisfy

$$k_y > k_f \quad (4)$$

where k_f is the wave number of ordinary gravity waves in shallow water of constant depth h_0,

$$\omega^2 h_0/g = k_f h_0 \tanh k_f h_0 \quad (5)$$

Equation (4) and the shallow water limit of (5) lead to a minimum beach width χ_{min} (a function of mode number), below which waves of mode n cannot be trapped. These are also shown in Table 1 and from Figure 1 correspond approximately to the location of the last maximum of the Eckart wave. Note that for $\chi_0 < 6.5$, only the zero mode is possible. For very 'short' sloping sections the simple matching procedure is not sensible, and so no value of χ_{min} for the zero mode is given. For values of χ,

$$\chi_{min}(n) < \chi < \chi_c(n)$$

edge waves of mode n still exist, but the parameter s in (2) changes. The wave profiles (except for the zero mode) are not changed significantly on the sloping section, but as χ approaches χ_{min}, the long exponential tail will decrease the wave amplitude for a given energy and will probably make that wave less likely to be of importance. Figure 2 shows mode 2 edge wave profiles for various χ_0. Only for values of $\chi_0 > (2n + 1)^2$ (as a rule of thumb see Table 1) can the mode n wave effectively be trapped. Unfortunately, it has not been possible to verify experimentally the effects of slope truncation in the laboratory because edge waves with significant energy on the flat section are evidently affected by coupling to the paddle; this effect has not been considered here.

The possible importance of slope truncation in natural situations is illustrated by El Moreno Beach on the northwest coast of the Gulf of California [*Bowen and Inman*, 1969]. A survey conducted on May 22, 1967, showed a steep-faced ($\beta = 6°$) beach rising above a nearly horizontal low tide terrace (slope less than 1/100). At high tide the depth at the toe of the beach was about 3 m, but at low tide the terrace was completely exposed. Table 2 gives the maximum periods (in seconds) for which mode n = 1, 2, 3 can exist for various depths. Periods given for the zero mode correspond to the edge wave period for which the distance offshore needed for the edge wave to decay to e^{-1} of its value at the shore is more than 2 times the length for the Eckart wave. The ability of this topography to trap edge waves of a given period and mode is clearly (theoretically) strongly dependent on the tidal level.

TABLE 1. Minimum Beach Width χ_{min} for Existence of Mode n and Maximum Beach Width χ_c for Any Effect of Slope Truncation

Mode	χ_{min}	χ_c	$(2n+1)^2$
0		2	1
1	6.5	13	9
2	21.0	30	25
3	43.5	53	49

THEORY OF EDGE WAVE EXCITATION ON NONERODABLE BEACHES

The excitation of low-mode standing edge waves by totally reflected normally incident waves of period T_i on a beach of constant slope extending far offshore has been theoretically shown to occur via a resonant energy transfer resulting from an instability of the incident wave to edge wave perturbations [*Guza and Davis*, 1974]. The classical shallow water equations were used to show that initially small standing edge waves of various periods and low mode numbers can grow with time, extracting their energy from the incident wave field. The edge waves resonantly excited by this mechanism always have periods longer than those of the primary incident waves. Laminar viscous effects were included to show that there is a critical amplitude of the incident standing wave at the shoreline, below which a particular resonant incident wave–edge wave interaction does not lead to edge wave growth because the viscous damping of edge waves exceeds the rate of nonlinear excitation. For edge wave growth it was shown that

$$a_i^2 > \nu K(N_1, N_2)/\omega_i \quad (6)$$

where a_i is approximately the incident wave amplitude at $x = 0$; ν is the kinematic viscosity; N_1 and N_2 are the mode numbers of excited waves; ω_i is the incident wave frequency; and $K(N_1, N_2)$ is a constant. The most easily excited edge wave (requiring the smallest incident wave) is the zero mode subharmonic; $\omega_e = 0.5\omega_i$, $N_1 = N_2 = 0$, and $K(0, 0) = 5.25 \times 10^1$, where arithmetical errors of *Guza and Davis* [1974, Table 1] are corrected by dividing $K(N_1, N_2)$ by 4.0.

The minimum incident wave amplitude which can overcome viscosity and excite the zero mode subharmonic is not a function of beach slope and is shown on Figure 3 as the solid curve. For incident wave amplitudes theoretically large enough to excite other resonant edge waves the initial growth rate of the zero mode subharmonic is always at least twice that of other possible resonances, and experiments confirm that the zero mode subharmonic is preferentially excited. The other possible resonances were never observed.

The resonance theory assumes a totally reflected incident wave, and experiments [*Galvin*, 1965; *Harris*, 1967] show that when the incident wave is large and breaks by plunging, edge waves are no longer visible in the run-up. This may be explained by one or more of several possible reasons:

1. With the onset of wave breaking there is a change in the

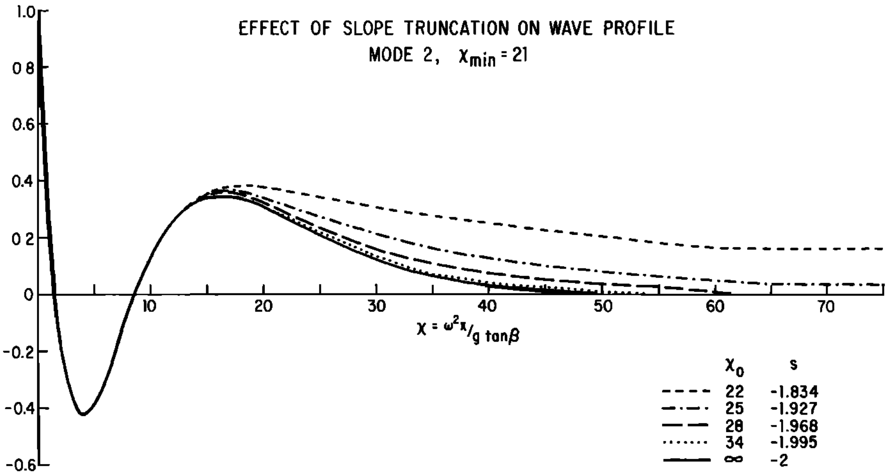

Fig. 2. Changes in mode 2 wave profile and dispersion parameter s (see equation (3)) for various dimensionless widths χ_0 of the sloping section. With $\chi_0 < \chi_{min}$, no mode 2 edge waves exist.

essential nature of the nearshore from a weakly nonlinear, weakly dissipative, potential regime to a strongly nonlinear, dissipative, turbulent one. Potential edge waves with wavelengths of the order of those of the turbulent surf zone are now irrelevant to the problem.

2. Elements of the potential theory are still valid, but the effective viscosity of the system has become so high that edge waves can no longer grow.

3. Only standing incident waves can effectively drive edge waves. As the percentage of the total incident energy reflected decreases, so does the overall importance of standing incident wave related instabilities.

Regardless of the exact reason for the cessation of subharmonic excitation an approximate range of incident wave amplitudes capable of exciting subharmonic potential edge waves can be obtained by quantifying the observation that the incident wave must be strongly reflected. *Carrier and Greenspan* [1958] used the full nonlinear, sloping bottom, shallow water equations to show that for a standing wave theoretically to be possible on a sloping beach,

$$r = a_i\omega_i^2/g \tan^2 \beta < 1 \qquad (7)$$

A matched asymptotic expansion between the Carrier and Greenspan solution and slowly varying Stokes-type solutions valid in deeper water may be used to show that the maximum a_i in (7) is approximately equivalent to the value given by *Miche* [1951] for the maximum standing wave amplitude in deep water which will not break on a slope. Existing experiments [e.g., *Ursell et al.*, 1960] have shown that the deepwater [*Miche*, 1951] equivalent of r is indeed an important parameter in determining the amount of energy reflected at a coast. If the incident waves break some distance offshore and a_i is taken as the breaker amplitude, then the parameter r (equation (7)) is important in determining breaker type [*Galvin*, 1972]. It will be assumed that the maximum size incident wave which can excite potential-type edge waves through the present mechanism depends upon the parameter r. Neither the theory nor the experiments, however, are refined enough to determine whether the amount of edge wave excitation depends on the amount of reflection in a continuous way or changes more dramatically for a specific range of r related to the onset of plunging breakers. Experiments, discussed later, do indicate that subharmonic edge wave excitation was always weak or nonexistent for $r > 2$. For simplicity, the maximum incident wave amplitude leading to subharmonic excitation is written as

$$a_i\omega_i^2/g \tan^2 \beta = 2 \qquad (8)$$

which corresponds to the transition region between surging and plunging breakers [*Galvin*, 1972]. We will use the value of r (less or greater than 2) as indicating whether the system is reflective or dissipative; reflective systems may exhibit the instabilities predicted by potential standing wave theory. The theoretical maximum incident amplitudes are indicated by the dashed curve on Figure 3 for $\beta = 6°$ and various periods and on Figure 6 for various beach slopes and $T_i = 2.7$ s. Of course, a well-defined dividing line between the two wave regimes does not exist even for the simplest incident wave conditions, and the classifications are meant to be qualitative. The existence of maximum and minimum incident wave amplitudes which can excite subharmonic edge waves is confirmed by laboratory experiments discussed in the next section, and the agreement with theory is qualitatively good.

Present and previous experiments indicate that a synchronous edge wave (same period as the incident wave) is sometimes excited by strongly reflected incident waves. These synchronous edge waves have been proposed as being a critical element in the formation of rip currents and as being related to cusp formation [e.g., *Bowen*, 1967, 1972; *Harris*, 1967; *Bowen and Inman*, 1969]. It is now shown that the same system of equations used to explain subharmonic edge wave growth [*Guza and Davis*, 1974] suggests a higher-order excitation mechanism for synchronous excitation. The nonlinear shallow water horizontal momentum equations may be vertically integrated and, by using the principle of mass conservation, combined to form a single equation for the velocity potential ϕ. In nondimensional variables (indicated by circumflex ac-

TABLE 2. Maximum Edge Wave Periods (Seconds) for Each Mode and Various Offshore Depths, With Beach Face Slope of 6°

	$h_0(m)$		
Mode	1	2	3
0	44.9	63.4	77.8
1	7.5	10.6	13.0
2	4.2	5.9	7.2
3	2.9	4.1	5.0

cent) with ϵ as a (small) expansion parameter of nonlinearity,

$$L\hat{\phi} = \epsilon Q(\hat{\phi}, \hat{\phi}) + \epsilon^2 C(\hat{\phi}, \hat{\phi}, \hat{\phi}) + O(\epsilon^3) \qquad (9)$$

where L is the usual linear long-wave differential operator and Q and C are quadratic and cubic differential operators, respectively [*Guza and Davis,* 1974]. The linear description of edge waves (equations (2) and (3)) and the forms for incident waves are obtained by neglecting terms of $O(\epsilon)$ and higher in (9). Keeping terms of $O(\epsilon)$ (i.e., the first correction of nonlinearity) shows that the primary incident wave of frequency ω_i has a first harmonic of frequency $2\omega_i$, that the mean sea surface is changed, and that certain edge waves can extract energy from the primary incident wave. The initial growth rate of edge wave amplitudes is $O(\epsilon)$. These are the resonant triads already discussed: two initially small edge waves and the incident wave interacting via the Q term to exchange energy resonantly. Generally, the lowest-order resonances (here $O(\epsilon)$) will be stronger than higher-order ones ($O(\epsilon^2)$). However, if the primary wave amplitude (or other parameters such as tank geometry) does not allow growth of the lowest-order instabilities, then higher-order instabilities may be evident. The form of (9) leads directly to the conclusion that $O(\epsilon^2)$ resonances are possible and that synchronous edge waves might be excited in this way. The calculations necessary to find growth rates and critical amplitudes and to insure that synchronous growth is theoretically predicted rather than merely possible have not yet been done. However, by analogy to the lowest-order instability it might be assumed that the zero mode synchronous wave is the most easily excited higher-order resonance, and this conclusion is verified by experiments.

It is emphasized that the resonance formalism requires, for strict validity of the ordering procedure, that the nonlinear parameter $\epsilon \ll 1$. However,

$$\epsilon = a_i\omega_i^2/g \tan^2\beta = r$$

where r is the reflectivity parameter discussed previously and does become $O(1)$ in the experiments. This means that the assumption of weak nonlinearity and validity of the ordering procedures is breaking down. Therefore the formally higher-order synchronous edge wave resonance may be almost as strong as the lower-order subharmonic. Interestingly, the assumption of weak nonlinearity is beginning to lose validity at about the same value of ϵ (or r) for which plunging breakers begin and the visually observable edge wave resonances cease.

Gallagher [1971] considered edge wave excitation by incident wave spectra with many frequencies and angles of incidence and showed that an edge wave can be forced by the interaction of two totally progressive incident wave components with different frequencies and angles of incidence. *Gallagher*'s [1971] work concerns only dissipative beaches, but further study may show that interactions among various incident wave components can be important in exciting edge waves on reflective beaches. In the present laboratory studies, however, and for some selected field cases the modeling of the incident wave field as a unidirectional, monochromatic wave train appears to be consistent with the observed edge waves.

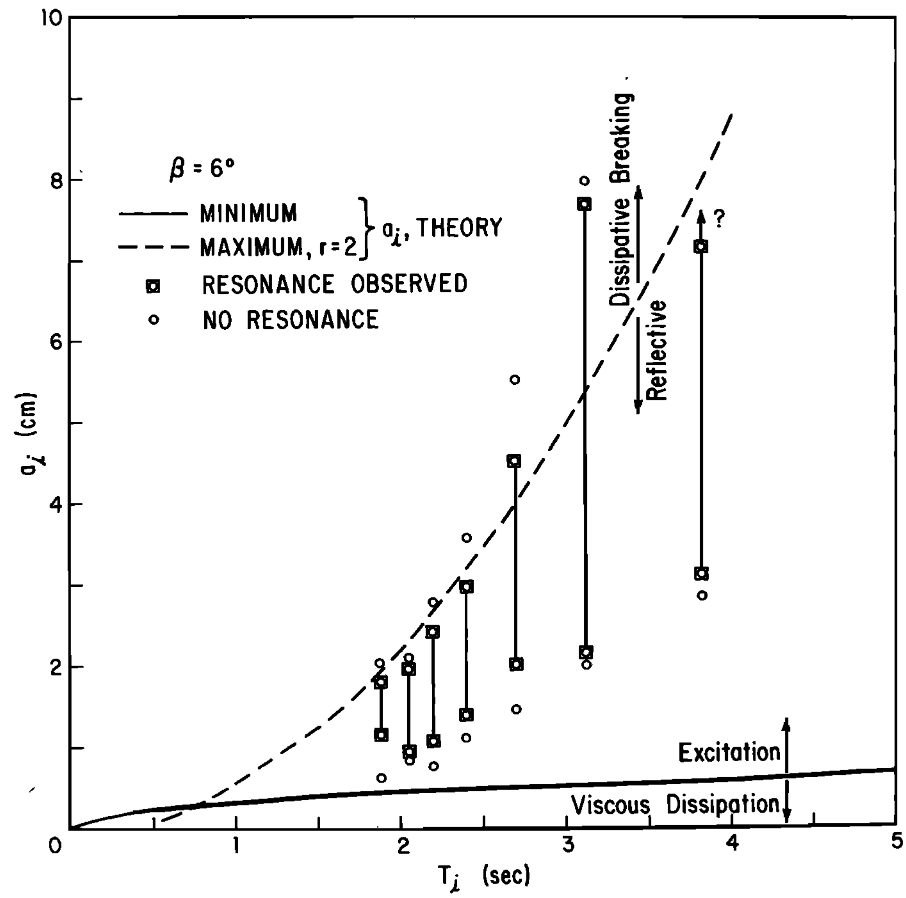

Fig. 3. Observed excitation of zero mode subharmonic edge waves as a function of incident wave period and amplitude. The dashed curve indicates the incident wave amplitude for transition from reflective to dissipative systems (equation (8)), and the solid curve indicates the amplitudes required to overcome viscous damping (equation (6)).

EXPERIMENTS ON NONERODABLE BEACHES

The experiments were conducted in the 15.2 m × 18.2 m wave basin at the Hydraulics Laboratory at Scripps Institution of Oceanography. The beach was a concrete variable sloping section extending from the shoreline end of the basin for 8.7 m, the depth being constant for the 5.1 m between the toe of the beach and the plunger-type wave maker. The sides of the basin were lined with wave absorbers, about 1 m wide and 40 cm thick, made of aluminum shavings in wire mesh containers. These absorbers were used to reduce cross waves, tank seiche, and other motions not relevant to the study. The actual working area was of width b, separated from the basin proper by solid barriers which extended offshore from above still water level to within 1 m of the wave maker, although some experiments were conducted with barriers only to the toe of the beach.

Edge wave excitation was studied by sending normally incident low-amplitude waves onto the beach and visually observing the resultant edge wave growth, if any. When subharmonic edge waves occurred, they were obvious because of the distinctive run-up pattern that they produce. Each subharmonic edge wave antinode was alternately the location of run-up maxima and minima as the edge wave alternated between constructive and destructive interference with the incident wave uprush. At any edge wave antinode the difference between run-up maxima and run-up minima along the sloping beach face, R_e, was measured with a meter stick and a crude estimate of the edge wave amplitude a_e obtained from

$$a_e = (R_e/2) \tan \beta$$

This is actually a lower limit of the actual edge wave amplitude because the edge wave may not have its maximum displacement at the same time as the incoming wave reaches its maximum run-up. The simple measurements taken, however, certainly give a reasonable estimate of the edge wave excitation at different periods and slopes relative to each other. The incident wave amplitudes at the shoreline were estimated by measuring the swash length before the edge waves grew or at an edge wave node. The incident wave amplitude occasionally was not uniform across the tank, varying by as much as 30%. An average value was used in these cases. When the incident wave surged on the beach face, there was good agreement between an amplitude determined through the run-up and a direct measurement of the surge height. Near the maximum values of incident wave amplitude studied the waves broke and plunged some small distance offshore, and the breaker amplitude was as much as 50% larger than the value calculated from the swash length. An average was used in these cases. Because of the crudeness of the measurements we regard the experimental results as being qualitative but as agreeing with the trends suggested by theory.

If standing edge waves are going to be excited on a beach of finite width b, then in order to satisfy the boundary condition of no flow through the sidewalls,

$$b = m(L/2) \qquad m = 1, 2, \cdots \tag{10}$$

is an integer indicating the number of half wavelengths in the longshore direction. By means of (3) it follows that an equivalent relation for freestanding edge waves is

$$b = (g/4\pi)T_e^2 m(1 - 2s) \tan \beta \tag{11}$$

Typically, resonant energy transfers occur most readily when the resonantly excited wave satisfies the boundary conditions for a free wave, so that various possible resonances may be studied by varying the parameters of (11). *Galvin* [1965] observed the resonantly excited zero mode subharmonic ($n = 0$, $T_e = 2T_i$) edge wave on eight plane beaches of slope varying from 6° to 20.5°. The tank width satisfied (11) with $m = 1$, but no incident wave periods were reported. *Bowen and Inman* [1969] produced the zero mode subharmonic edge wave, with low-amplitude normally incident waves, for various values of m and T_e. These edge waves, of periods up to 7.9 s and wavelengths to 7.3 m, grew so large that they occasionally overtopped the experimental beach of slope 4°. *Harris* [1967] more closely approximated an unbounded natural beach in that his wave tank was six or more longshore wavelengths wide, the exact satisfaction of (11) being less important. He observed subharmonic excitation, with a longshore wavelength close to that given by (3) with $n = 0$ and $T_e = 2T_i$ on 6° and 22° slopes. This resonance was observed for about 20 different combinations of edge wave period and beach slope; the maximum edge wave period was 3 s. *Bowen* [1967], *Harris* [1967], and *Bowen and Inman* [1969] also reported exciting synchronous edge waves, sometimes accompanied by small rip currents and circulation cells. The effects of finite offshore depth and basin length, variations in beach slope, and importance of incident wave parameters to subharmonic excitation were not considered in any detail by the above workers. The lack of edge wave amplitude data, in particular, left the actual importance of edge waves to the nearshore zone open to question. For example, *Sonu* [1972] inferred that edge waves would not have amplitudes larger than one quarter of the incident wave amplitude, although the experiments here commonly showed edge wave amplitudes 2 times larger than the incident wave amplitude.

Sets of measurements were conducted with $\beta = 4.17°$, 5.08°, 6°, and 6.83° with β/b constant. If slope truncation is not important, $s = -n$ in (11), and the same edge wave periods will then be possible resonances with the same value of m, on different slopes. The barrier spacing was 10.8 m when $\beta = 6.83°$ and was proportionally smaller when β was smaller. Except for $m = 1$ and possibly some cases with $m = 2$ the zero mode subharmonics are unaffected by slope truncation and/or the finite onshore-offshore length of the wave basin. The incident wave periods T_i were chosen so that (11) was satisfied with $T_e = 2T_i$ and $s = 0$. The maximum and minimum incident wave heights which produced visually observable zero mode subharmonic edge waves and the edge wave 'amplitudes' were measured. Figure 3 shows some results for $\beta = 6°$, the solid and dashed lines representing the theoretical minimum (equation (6)) and maximum (equation (8)) incident waves, respectively, for edge wave excitation. The circles in squares and the circles in Figure 3 represent incident wave amplitudes and periods for which edge wave excitation was and was not observed, respectively. The solid lines connecting the circles in squares indicate that excitation was always observed for an amplitude in this range. There is obviously qualitative agreement between theory and experiment. The increasing incident wave amplitudes, compared to theory, needed to excite the longer periods may be due to the paddle directly affecting the edge wave, because the longer-period incident waves do not have the profile assumed by theory or because the simple model of viscous dissipation used is not quantitatively adequate. Additional viscous damping from the sidewalls would be most important for smaller m, but a calculation shows that the bottom damping already considered is always much larger. The longshore wavelengths of the excited edge waves shown in Figure 3 vary between 2 and 10 m.

Experiments on the other slopes, with the same incident wave periods, gave minimum amplitudes very similar to those shown in Figure 3. This is as is expected, since the critical amplitudes (equation (6)) do not depend on β. Maximum incident wave amplitudes with subharmonic excitation are shown in Figure 4 for all slopes and clearly demonstrate the importance of the parameter r (equation (8)).

The minimum subharmonic edge wave periods which could visibly be excited varied from 2.18 s when $\beta = 6.83°$ to 4.8 s when $\beta = 4.17°$. The minimum-period incident wave which can resonantly excite edge waves is theoretically predicted through the amplitude limits of (6) and (8). As is shown in Figure 3 for short-period waves, increasing the incident wave amplitude cannot overcome viscous damping before wave plunging takes place. The minimum excitable period is shown by the intersection of the maximum (a function of β) and minimum amplitude lines. The theoretical and observed minimum incident wave periods for zero mode subharmonic excitation are shown in Figure 5. An aluminum beach, made of thin, smooth aluminum sheets, was installed on the 6° slope, and the minimum period dropped from 1.40 to 1.12 s. *Harris* [1967] observed a minimum incident period of about 0.95 s on a 6° slope of concrete which was specially coated to insure smoothness. As is expected, a reduction of viscous damping allows higher-frequency waves to be excited. The minimum excitable period might vary considerably according to the roughness (and perhaps the porosity) of the bed. The assumption that swash length can be simply related to wave amplitude also breaks down on porous beds.

Thus far, only minimum and maximum incident wave amplitudes for edge wave excitation have been given without consideration of edge wave amplitudes. No theory presently exists which predicts edge wave amplitudes for prescribed beach and incident wave conditions, but some trends are indicated by the data. The subharmonic edge wave response to incident waves of fixed period (2.7 s) and constant m was measured on four slopes. The incident wave amplitude varied over the range which excited edge waves, and the edge wave induced run-up variation R_e was determined. On Figure 6 the theoretical minimum incident amplitude (not a function of slope) is indicated by a dotted line, and the theoretical maximum for the four slopes by dashed lines. Solid lines, showing trends in the data, indicate that the excited edge wave amplitudes grow with increasing incident wave amplitude until the incident wave amplitude is near the theoretical maximum. The edge waves visible in the run-up then decrease in amplitude as the incident waves begin to plunge cleanly rather than surge. Other experiments, not shown here, indicate that the decrease in edge wave amplitude with increasing breaker size can be rather sharp in some cases, a small increase in incident wave amplitude causing a well-developed edge wave system almost to disappear.

With incident wave amplitudes near the upper limit for excitation the edge wave response occasionally was not stable. Large edge waves would sometimes grow and then rapidly disappear, only to reappear again. Generally, however, once the edge waves grew to their equilibrium amplitude, no significant changes occurred. When the incident amplitudes are chosen to develop maximum edge wave amplitudes, the edge wave induced swash variations are generally at least equal to the inci-

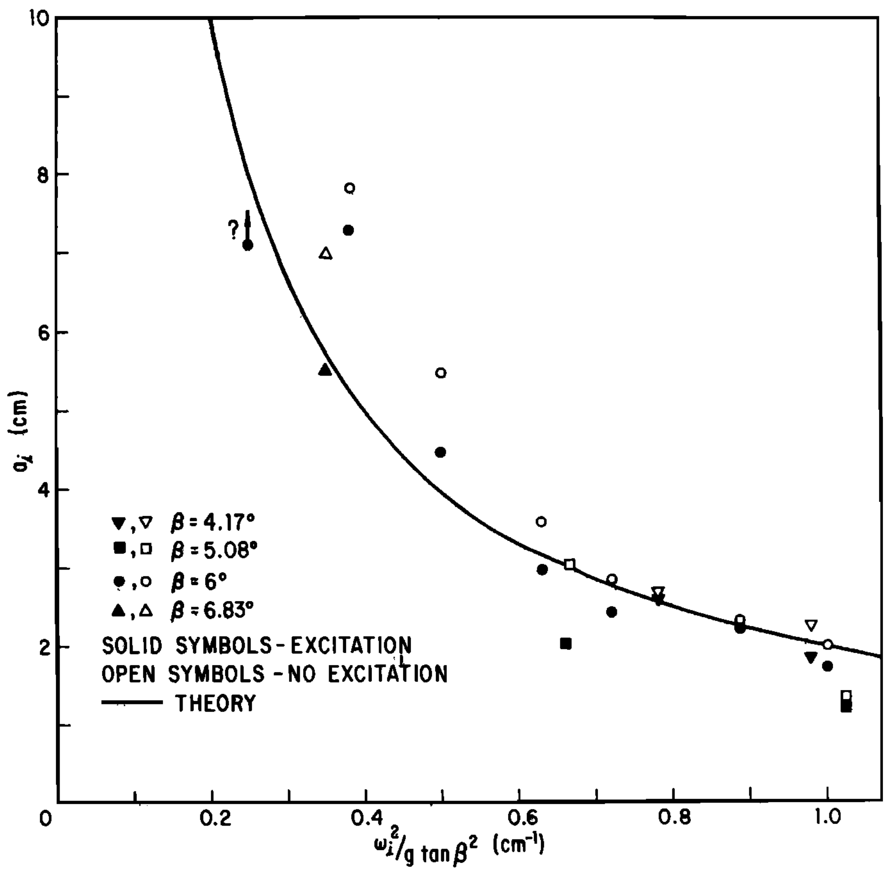

Fig. 4. Maximum incident wave amplitudes at the shoreline for zero mode subharmonic edge wave excitation (theory from (8)).

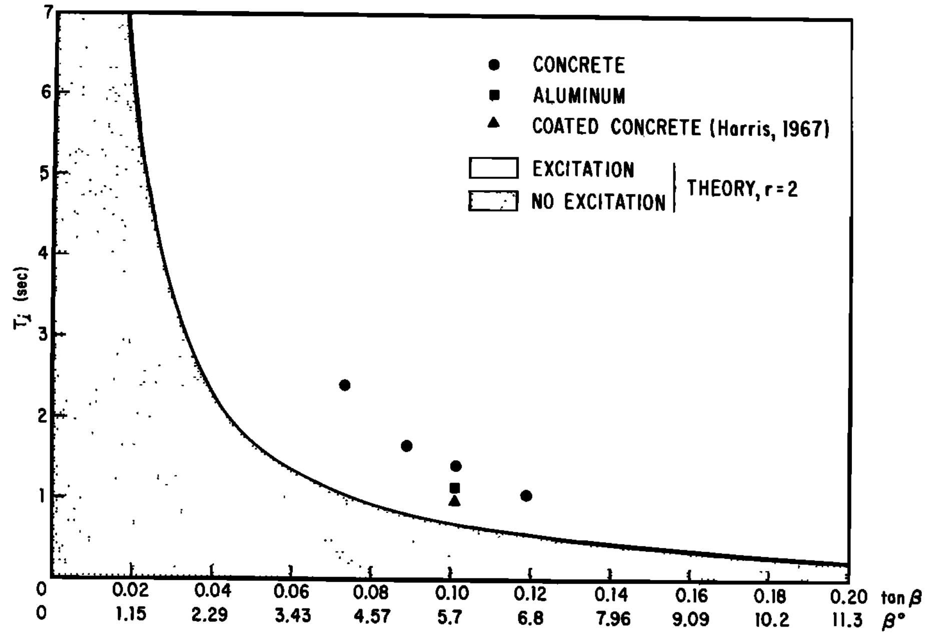

Fig. 5. Observed minimum incident wave periods (solid symbols) for $n = 0$ subharmonic excitation on beaches of various slopes and smoothness (theory from (6) and (8)).

dent wave swash length, frequently more than twice as large. The maximum amplitude edge wave, of a given period, is much smaller on gentler slopes than on steeper ones. Viewed simplistically, the gentle slopes cannot reflect sufficient incident wave energy to produce large incident standing waves which can drive large edge waves. Similarly, on a fixed slope, long-period incident waves can generate larger edge waves than shorter-period incident waves because larger long-period incident waves are reflected. With $T_i = 3.82$ s, $\beta = 6°$, and the incident wave swash length 135 cm ($a_i = 6.75$ cm) a subharmonic edge wave developed which had a 220-cm difference in run-up between adjacent antinodes ($a_e = 11$ cm). The surging breaker amplitude varied along the shore from about 5 cm where the edge wave was out of phase with the incident wave to 10 cm where it was in phase. The maximum shoreward incursion of water was more than 1 m greater with subharmonics than without.

Experiments ($\beta = 6°$, $b = 10.8$ m) were made to determine the importance of exactly satisfying the condition (equation (11) with $s = 0$ and $T_e = 2T_i$) that the excited subharmonics have the longshore wave number-frequency relation of free edge waves (equation (3)). If T_i is chosen so that (with $T_e = 2T_i$) equation (11) is not satisfied, then the longshore wave numbers of excited subharmonics do not satisfy (3). The inci-

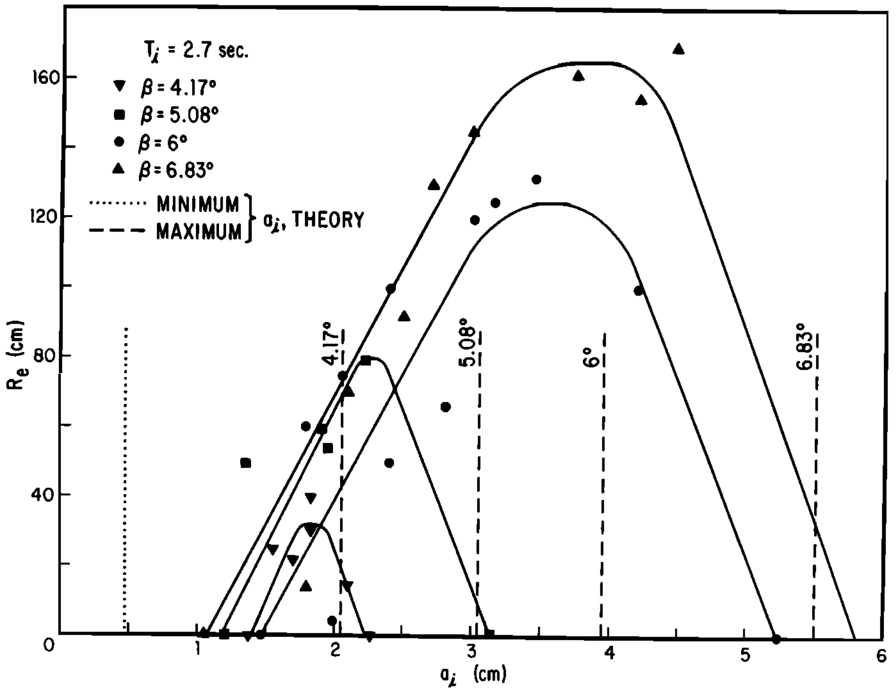

Fig. 6. Edge wave run-up amplitudes versus incident wave amplitudes for various beach slopes. The dashed lines are maximum incident wave amplitudes given by equation (8) and also correspond to the onset of plunging breakers.

dent wave periods covered the range 1–4 s. As was expected, edge wave excitation was greatest when (11) was satisfied. For all periods except $m = 1$ the effects of slope truncation and finite onshore-offshore tank length were negligible. For $m = 1$, $T_e = 11.5$ s, and the change in the dispersion relation (3) is small ($s = 0.02$), but the edge wave is so long that it would theoretically extend to the paddle still having 10% of its amplitude at the shoreline. This wave could not be excited, indicating that in this case the coupling to the paddle and the effect of energy leakage to the wave absorbers prevented resonant growth. The shortest edge wave visibly excited was at $m = 16$ and $T_e = 2.8$ s. Figure 7 shows the edge wave response observed, as a function of incident wave period. At each m the incident wave amplitude was approximately constant and corresponded to surging incident waves. The edge wave amplitudes generally decreased with increasing wave frequency and with $m > 10$ were too small to measure accurately. When the tank width is of the order of a subharmonic edge wave wavelength ($m = 2, 3, 4$), the possible free edge wave periods are far apart, and incident wave periods may be chosen that will not excite subharmonic edge waves. When the tank is wide in comparison with a wavelength ($m > 4$), the possible resonant periods are so close together that any incident wave period between 1.4 and 3.1 s of proper amplitude will excite a subharmonic edge wave. The fluctuations of edge wave amplitude within a given m also become small. Thus the narrow band of incident wave periods for subharmonic excitation observed by *Galvin* [1965] and the continuous resonant response observed by *Harris* [1967] are functions of the values of m appropriate to those experiments. With $m > 5$ the number of edge wave antinodes sometimes alternated between adjacent m, but the edge wave amplitude remained approximately constant. No cases were observed with visible simultaneous excitation of different values of m.

When the incident wave period was chosen so that the tank geometry excluded subharmonic resonances, synchronous edge waves were sometimes clearly visible in the run-up. For example, with the tank geometry of Figure 7 and $T_i = 3.64$ s, a synchronous edge wave with $m = 10$ and $s = 0$ satisfied the boundary conditions for free edge waves (equation (11)) and was observed. Synchronous edge wave amplitudes were generally much smaller than subharmonic amplitudes observed with similar incident wave amplitudes and periods. With incident wave periods which allowed subharmonic excitation but with amplitudes too small to overcome viscous damping of subharmonics, small synchronous waves were usually observed. With larger incident waves of the same period the synchronous wave appeared almost as soon as the incident waves impinged on the beach, but the subharmonic soon grew to large amplitudes and clearly dominated the system. Synchronous edge waves were also observed at periods too short (Figure 5) for subharmonic excitation.

Incident wave periods were studied for which both mode 0 and mode 1 synchronous waves were possible but the mode 0 subharmonic was not. With $\beta = 6.83°$, $b = 10.8$ m, and $T_i = 4.4$ s the zero mode subharmonic response was in the 'dead zone' between $m = 1$ and $m = 2$ (analogous to Figure 7). Very gentle swash clearly excited only the zero mode synchronous wave, but with increasing incident amplitudes, mode 1 became progressively more important and eventually dominated the longshore periodicity. *Bowen and Inman* [1969] also found that larger incident waves excited higher-mode synchronous waves, resulting in larger rip current spacings, and they experimentally studied the nondimensional surf zone width for transition from mode 0 to mode 1. They further reported that synchronous edge waves were not visible in the surf with cleanly plunging breakers. We reached a similar conclusion, finding that both synchronous excitation and subharmonic excitation became much less apparent when r (equation (7)) was greater than about 2. Thus according to the present classification the experiments of *Bowen and Inman* [1969], with well-defined longshore periodic circulation cells and visible synchronous edge wave excitation, were reflective systems.

On reflective laboratory beaches, synchronous edge waves had smaller amplitudes than subharmonics, occurred best when the tank geometry excluded the subharmonic, and were observed even with very small nonbreaking incident waves. These observations support the present hypothesis that the synchronous edge wave resonance on nonerodable reflective beaches is weaker than the subharmonic and can be explained without reference to rip currents and other phenomena associated with wave breaking.

A few experiments on dissipative beaches showed less well defined longshore periodicities than those on reflective beaches and a general trend toward increasing rip current spacing with increasing amplitudes. *Harris'* [1967] work and the present ex-

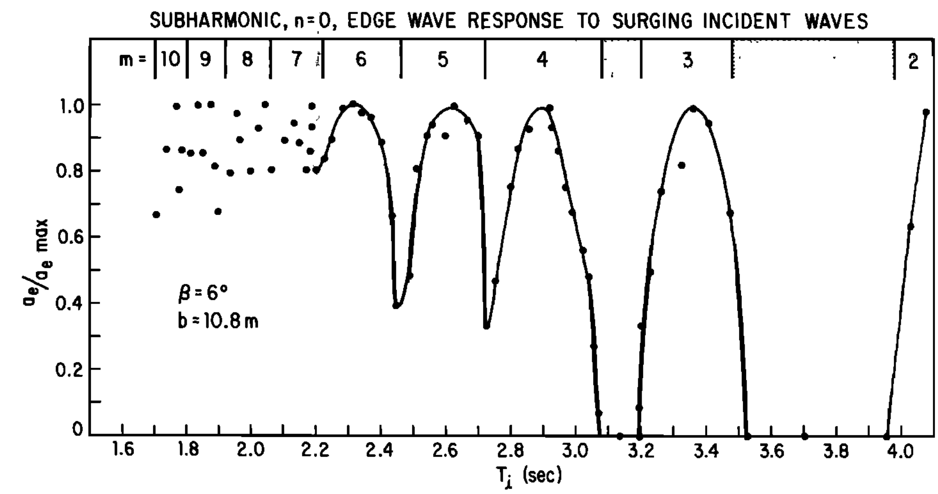

Fig. 7. Normalized edge wave response to surging incident waves of various periods. The vertical axis indicates the edge wave amplitude normalized by the maximum amplitude observed at a particular m. Synchronous edge waves were observed in the stippled region.

periments show that the dominant longshore spacing of rip currents on nonerodable dissipative beaches exceeds the maximum possible longshore wavelength of synchronous edge waves (equations (4) and (5)). This indicates that the rip currents are related to a longshore periodic instability other than the classical synchronous edge wave mechanism operative with collapsing incident waves [*Bowen and Inman*, 1969]. *LeBlond and Tang* [1974] have proposed a criterion for rip current spacing based on the minimization of relative energy dissipation rates, but the results do not compare well with observations. *Hino* [1972, 1973] has suggested that the set-up sea surface associated with breaking waves is unstable and that the instabilities are rip currents. This model does not require the presence of edge waves or other time periodic intermediaries. *Bowen and Inman* [1969] suggest that, in the field, long-period edge waves may interact with a two-dimensional long-period 'beat wave' to give circulations with very long wavelengths. They also speculate that edge wave-like motions might primarily exist seaward of the surf zone, since considerable reflection and/or scattering of the incident wave occurs at the break point. There is also the possibility that circulations on the smallest scale are related to edge wave instabilities and that the largest-scale rip currents are driven by the smaller ones, energy cascading to a final longshore wavelength determined by the surf zone width. Observations on dissipative beaches often show small-scale circulation cells within a larger, stronger circulation cell [e.g., *Harris*, 1967].

OBSERVATIONS ON ERODABLE REFLECTIVE BEACHES

The effect of edge waves on nearshore morphologies was modeled with sand tracers. A thin veneer (2 cm) of fine sand was spread on a 6° concrete slope, and low-amplitude waves were sent normally incident onto the beach. After edge waves developed, more and more sand was randomly scattered into the swash zone, and the resulting accumulations were noted. It was not possible to begin with a large quantity of sand on the slope because the incident waves very quickly reworked the sand into an entirely different topography than the simple plane beach configuration under consideration. Edge waves can develop on complex equilibrium beach profiles, but the longshore wavelengths and incident wave conditions needed for excitation are not known. Even though it was not practical to produce true cusps on initially fully erodable equilibrium beaches, the features produced have much in common with naturally observed cusps.

Well-developed subharmonic edge waves resulted in a cusp-like configuration, as shown in Figure 8; here T_e = 4.8 s, and the cusp spacing is about 1.8 m. Initially, the sand veneer (dyed black for contrast) was eroded by the swash and deposited in accumulations seaward of cusp bays and at the cusp horns. Since the run-up is maximum at antinodes, cusp horns correspond to edge wave nodes and are spaced one-half wavelength apart. Erosion at antinodes may occur because the fine sand is not in equilibrium with the relatively steep 6° slope, and once it is suspended by wave action, it moves offshore. The steady drift velocities associated with edge waves are offshore on the beach face [*Bowen and Inman*, 1971], and this would complement erosional processes. On an accreting beach, depositional processes would be important, and location of maximum run-up (antinodes) might develop into cusp horns, as has been observed in the laboratory by *Galvin* [1965] and in the field by *Russell and McIntire* [1965].

The horns were observed to prograde seaward several centimeters and to grow several centimeters thicker, indicating accretion, not merely an absence of erosion by the swash. The 'offshore deltas' near the surge line appear to be analogous to those associated with natural cusps [e.g., *Johnson*, 1919; *Kuenen*, 1948]. With more sand added to the system, accumulations would occasionally occur at an edge wave antinode, forming a very distinct ridge down the center of the bay and resulting in two smaller embayments. A series of cusps would then have intermittent spacings half as large as the dominant spacing. Such 'compound cusps' have been observed in the field [*Johnson*, 1919, p. 484; *Russell and McIntire*, 1965]. The cusp horns and deltas became increasingly well developed and usually connected, forming crescentic bars with the same wavelengths as the cusps. With the continued addition of sand the morphologies interfered with continued edge wave excitation. The edge waves now died away, the incident wave field reverted to a two-dimensional state, and the cusps began to lose their definition. Edge wave amplitudes could be significantly reduced by placing a long metal bar about 5 cm high on the beach face at the edge wave node. Naturally, because there are maximum longshore velocities at nodes, the partial barrier inhibits edge wave growth. Perhaps well-developed cusp horns act in a similar way. Because of the problems with establishing simple equilibrium beach profiles it was not possible to determine if the edge waves could reform after the cusps were reworked into shapes more conducive to edge wave excitation. *Flemming* [1964] conducted experiments on an initially very steep beach (β = 30°) with small incident waves. Cusps generally appeared but were continuously destroyed and reformed. This may demonstrate a cycle of edge wave excitation, cusp growth, cessation of excitation, cusp destruction, etc., but may also reflect the overall instability of the entire beach profile.

The swash circulations with small amounts of sand (Figure 8) were basically similar to the circulations with edge waves on the nonerodable beach. With the retreat of each incident wave surge, water would flow from an in-phase antinode (a bay with maximum run-up) around the nearest horn to neighboring bays, where it would rush up the beach face with the next incident wave. There was very little longshore transport across antinodes, as would be expected, since edge wave theory (equation (2)) predicts only onshore-offshore motion there. With cusps and deltas developed to the point that edge wave excitation was somewhat inhibited but was not stopped completely the flow of water with each retreating surge tended to become channelized between the horns and the deltas, concentrating flow nearer the horns than the bay centers.

Longshore periodic morphologies of two different wavelengths could be generated with synchronous edge waves, but these were not nearly as well developed as cusps due to the larger subharmonics. With gently surging waves no rip currents developed, but synchronous edge waves were clearly visible in the run-up. *Bowen* [1972] discusses theoretically how synchronous edge waves (with no rip currents present) can produce cusps every half edge wavelength, possibly with alternate cusps of differing size. The present observations with gently surging incident waves often showed half-wavelength cusps of equal size which, following *Bowen* [1972], suggests a phase difference of $\pi/2$ between incident and synchronous edge waves. Minimum run-up occurred at edge wave nodes, where very small horns developed.

As the incident wave amplitude increased, synchronous edge waves were less apparent in the run-up, and rip currents developed at alternate antinodes, as was predicted by *Bowen and Inman* [1969]. Cusps were not well defined, and horns

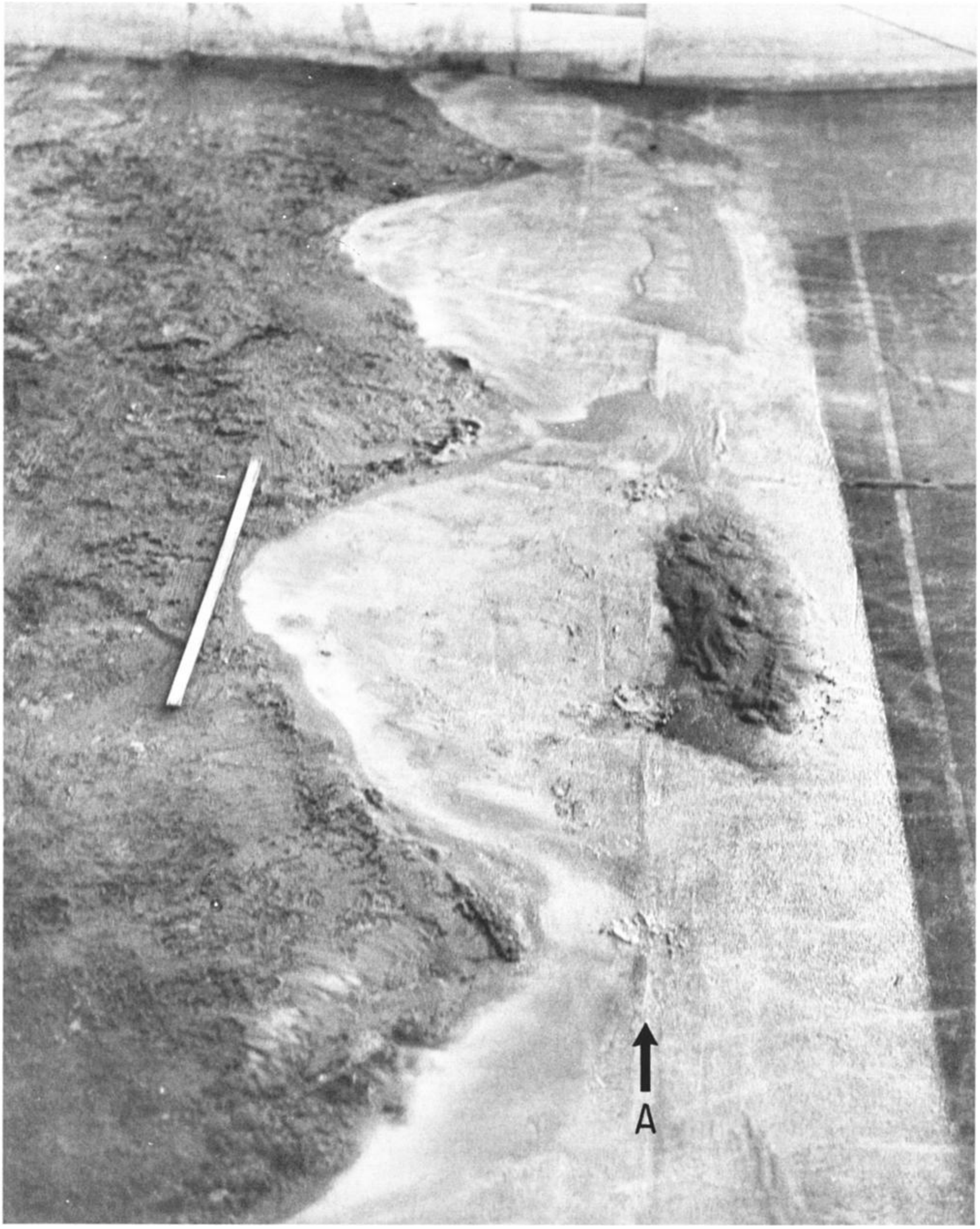

Fig. 8. Cuspate morphologies produced by subharmonic edge waves; water has been drained below still level (marked 'A') to increase visibility. Black is sand, white is exposed concrete, and a meter stick is shown for scale.

sometimes occurred onshore of rips, and at other times, in between rips. As was true on the nonerodable slopes, synchronous edge waves generally would not dominate the longshore periodicity if conditions allowed subharmonic excitation. With $T_i = 2.7$ s and $a_i = 2.5$ cm, synchronous waves appeared first and formed small cusps spaced about 1 m apart. With the onset of subharmonic edge waves a few moments later the surge greatly increased in strength, rapidly washed out the synchronous cusps, and began forming cusps spaced 2.25 cm apart. However, once in an apparently identical experiment the synchronous waves formed cusps sufficiently well developed to prevent growth of the subharmonic! Whether this is an isolated case and whether similar topographic feedback is important in the field are not known.

Incident waves of period 1.02 s did not produce visible subharmonic excitation on the 6° concrete slope. If, however, a small berm (~3 cm high) was constructed just seaward of the maximum uprush so that water would pond on the flatter slope behind the berm, the return flow was in continuous narrow streams which cut evenly spaced channels in the berm and deposited eroded material offshore. These flows were very narrow and were evenly spaced at 30 cm, compared with a half subharmonic wavelength of 31 cm, but no edge waves were visible in the run-up. The channels gradually widened slightly, and so the berm was alternately high and low, but the high points did not taper seaward into noticeably cuspate forms. Onshore transport of water by incident surges occurred over the broad high points, converged, and returned seaward in the narrower bays. These circulations resemble the flows most often observed on natural cusps [e.g., *Bagnold,* 1940; *Russell and McIntire,* 1965]. Note that even though subharmonic edge waves are presumably responsible for the longshore periodicities, the circulation is much different from that with large, well-developed subharmonics. With artificial berms of proper size and location, high-frequency surging incident waves of other periods which excited no visible subharmonics on the concrete slope produced similar narrow flows at half subharmonic wavelength spacing, with occasionally a whole wavelength or irregular intervals.

Evans' [1938, 1945] observations of 'ideal cusps' on various lakes are consistent with a model hypothesis that small edge waves provide an initial periodic longshore perturbation which is rapidly accentuated by ridge breeching and channeling of the return flow. Evans describes formation of a small beach berm by a small, very regular series of waves locally generated by a gentle steady breeze. With a slight increase in wind velocity and wave height the beach berm was simultaneously breeched at many places by a wave train with some regular longshore amplitude variation. In less than 100 s a perfect system of cusps was formed. With a further increase in wind velocity and wave height the cusps were completely obliterated, leaving the beach smooth. Repeated observations showed that for cusp formation, close adjustment is required between amplitude and uniformity of incident waves and size and shape of the berm. The experiments on nonerodable beaches show that with high-frequency incident waves, as would occur in lakes, only a very narrow range of incident wave amplitudes will excite edge waves (Figure 3) and that these will be of small amplitude. Without the exact tuning of the incident waves, edge waves will not occur; without the positive feedback of berm breeching and channeling the edge waves might not effect the topography; and *Evans* [1938, 1945] actually concluded that breeching of sand or seaweed ridges was required for cusp formation. Subsequent observations have shown that breeching of berms or ridges is not essential for cusp formation, even with high-frequency waves, but some positive correlation has been noted [*Mii,* 1959; *Otvos,* 1964; *Williams,* 1973].

Komar [1973] observed small cusps at Mono Lake, best formed with surging incident waves and disappearing when the incident wave energy increased so as to produce appreciable wave breaking. If the beach slope is taken as the average of bay and cusp slopes, 7 of the 9 reported spacings are reasonably close to synchronous edge wave wavelengths (as suggested by Komar), or half a zero mode subharmonic wavelength (Figure 9). Six of these 7 cases have average beach slopes greater than 5° and incident wave periods close to the theoretical minimum (Figure 5) for subharmonic excitation. One point (β = 3.25°, T_i = 1 s) is distinctly below the minimum incident period line, and this is the only case which gives better agreement with a synchronous than with a subharmonic spacing. No longshore variation in run-up was evident, and so the presence of synchronous or subharmonic edge waves could not be confirmed visually. *Longuet-Higgins and Parkin* [1962] and *Williams* [1973] found that cusp spacing on dissipative beaches increased with increasing swash length, but no such trend appears in Komar's data. The incident wave conditions for the two anomalous cusp spacings should excite subharmonics by the criterion used for all the other points, but the cusp spacings are very short, about half a zero mode synchronous wavelength. *King* [1965] reported cusps formed by 'low' regular swell waves of period 9.5 s normally incident on Trahane Beach in Ireland (β = 2.6°). The average of 36 cusp spacings was 14.7 m, compared with the 12.7 m predicted for cusps formed by zero mode subharmonics. On nearby Cruet Island Beach, which is approximately twice as steep, the cusp spacing was also doubled, in accord with theory.

Kuenen [1948] described in detail some cusps and associated circulations on the Isle of Wight. Figure 10 [from *Kuenen,* 1948] shows cusps formed under surging wave conditions. Notice in particular the longshore periodic highs (H) and lows (L) of the swash line. Kuenen noted that a bay surge appeared to alternate (in time) between being stronger and being weaker than a horn surge. This description and this photograph are completely consistent with the presence of a subharmonic edge wave with bays at each antinode. The surge at out-of-phase antinodes (corresponding to points of minimum run-up on a nonerodable beach) is lower than that at in-phase antinodes and appears to be retarded in relation to a horn. For the next incident surge the locations of in- and out-of-phase antinodes are reversed, and the 'alternating action' of the surge at a given bay and horn is explained. The retreating surge was concentrated along the horns, as was true in the laboratory with well-developed subharmonic edge waves and in the field with 'gentle swash' [*Williams,* 1973].

OBSERVATIONS ON ERODABLE DISSIPATIVE BEACHES

Komar [1971] produced cuspate features with waves of period 1.46 s breaking normally incident on a granular coal

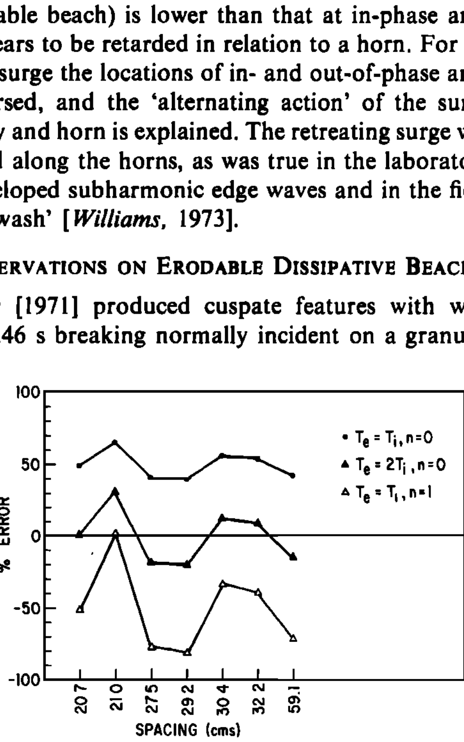

Fig. 9. Seven of nine cusp spacings observed by *Komar* [1973], compared with models hypothesizing formation by synchronous and subharmonic edge waves producing cusps with spacings equal to and half of the edge wave wavelength, respectively.

beach. The observed wavelength was about 6 m, although the maximum potential synchronous edge wave wavelength for Komar's experiment is 228 cm (equations (4) and (5)). Contrary to Komar's assertion the cusp spacing is therefore not due to rip currents associated with synchronous edge waves of the usual potential type. The cusp spacing may be the result of large-scale circulations of unknown genesis, like those observed by *Harris* [1967] on nonerodable dissipative beaches.

It is also conceivable that the cusp spacing is somehow related to cross waves. Cross waves are standing waves with crests at angles (usually 90°) to the wave maker (their energy source), just as edge waves have crests at right angles to normally incident waves. Longshore periodicities observed on laboratory beaches may be due to cross waves driven at the wave maker and extending down the entire tank but are not due to edge waves. *Garrett* [1970] has theoretically demonstrated that the lowest-order cross-wave instability is a subharmonic of the paddle frequency and that a higher-order resonance may be responsible for driving synchronous cross waves, exactly as we have suggested is the case with edge waves. *McGoldrick* [1968] used progressive primary waves to generate synchronous and subharmonic cross waves having the longshore wave number of free gravity waves with crests perpendicular to the wave maker, k_f in (5). Subharmonic cross waves were easier to excite and of larger amplitude than synchronous waves. *Garrett* [1970] theoretically showed that with standing primary waves, additional cross-wave resonances may occur with crests at angles other than 90° to the paddle and longshore wave numbers $k_y < k_f$. These instabilities have been observed experimentally but have not been studied in any detail [*McGoldrick*, 1968]. Regardless of the exact cross-wave instability they will always satisfy $k_y \leq k_f$, while inviscid edge waves obey $k_y > k_f$ (equation (4)).

Escher [1937] produced cusps with cross waves, although he understood these oscillations only as standing waves with crests at right angles to the primary progressive waves, not distinguishing between cross-waves and edge waves. The observed transverse oscillations were synchronous, and the number of antinodes reported can be shown to be generally consistent with $k_y = k_f$ (equation (5)). In some of the experiments [*Escher*, 1937, p. 86] a board was placed between the flat and the sloping basin sections, and the primary waves were prevented from reaching the beach until good transverse oscillations had developed. The board was then removed, and the waves were allowed to rework the beach. This mode of generation rules out the possibility that the transverse oscillations were high-mode edge waves. *Komar* [1973] believed *Escher*'s [1937] oscillations to be edge waves, but both observations may represent the effects of cross waves. The frequencies and paddle strokes for which cross waves will be excited are a complicated function of paddle configurations and tank geometry [*Garrett*, 1970]; thus no simple rule exists for determining when cross waves will occur. Low-steepness primary waves, however, generally cannot initiate the cross-wave instability, so that cross-wave contamination will be most serious in experiments on dissipative beaches.

DISCUSSION

Under the very simplified conditions of the laboratory the distinction between reflective and dissipative systems is qualitative but conceptually clear. On real beaches, however, the complications of irregular topography and incident wave spectra make the application of this classification somewhat ambiguous. Waves on real beaches may break on a series of bars, reform as smaller potential waves, and eventually reflect off the beach face. Thus on the largest scale, with a wide surf zone, the nearshore system is dissipative but may behave as a reflective system on the much smaller scale of the beach face. Reflective beach cusps with spacings of tens of meters may appear on very high energy dissipative beaches with surf zone widths and rip spacings of hundreds of meters [*Inman*, 1971].

Different frequency components of an incident wave spectrum may undergo quite different amounts of reflection [*Suhayda*, 1972]. It is not known if a strongly reflected long wave can exhibit reflective edge wave instabilities in the presence of a dissipative wave breaking at higher frequencies. Further work is also needed to investigate the possibility of many incident frequency bands, each satisfying the reflective resonance conditions, acting independently of each other, and exciting edge waves of many frequencies and wave numbers. Incident wave frequency and directional spectra also allow edge wave excitation through the interaction of different incident wave components [*Gallagher*, 1971] as well as through the instability of a single incident wave component. *Gallagher*'s [1971] work suggests that 'surf beat,' the low-frequency oscillations characteristic of dissipative beaches, is, in fact, low-mode edge waves. The frequencies of these edge waves have no simple relationship to the frequency of the principal incident wave component; they may be progressive or standing and, for typical incident wave conditions, may have approximately equal energy in the three lowest modes. These edge waves would typically (T_e = 25 s, β = 6°, n = 2) have longshore wavelengths of several hundred (325) meters. *Inman and Bowen* [1967] have measured spectra which suggest the presence of such waves. Much more theoretical and experimental work is needed to determine the relative importance of the different possible edge wave excitation mechanisms for various incident wave conditions. Furthermore, *Kenyon* [1970] has shown theoretically that edge waves can transfer energy to other edge waves via nonlinear resonances. This means that significant edge wave energy might occur at frequencies and wave numbers not directly driven by the incident wave field. Whatever the edge wave energy source, if many edge wave frequencies and wave numbers occur simultaneously, no single periodicity may be obvious in the run-up and/or morphologies [*Bowen*, 1972]. The present work has stressed the edge wave and the edge wave induced topographic response to single-frequency, normally incident, strongly reflected, incident waves. Only selected, special field cases satisfy this assumption, and so direct application to cusp formation on all beaches is not appropriate.

Some confusion has arisen concerning the presence of standing edge waves on beaches not bounded in the longshore direction by headlands or groins. *Sonu* [1973] stated that 'on a long straight beach an edge wave is progressive,' and *LeBlond and Tang* [1974] agreed. *Huntley and Bowen* [1973], however, measured velocities on a reflective ($r \approx 1$) beach consistent with the presence of the standing zero mode subharmonic edge waves. The beach ran in a gentle curve for more than 5 km, and the edge wave had a period of 10 s and a theoretical wavelength of 32 m, so that end effects were indeed negligible. Standing edge waves (of longer period) have been observed in the open waters of the California shelf [*Munk et al.*, 1964].

It is also true that beach cusps, taken as indicating the presence of standing edge waves, seem to occur more frequently near jetties, sharp curves in the beach, etc., than on very long straight beaches. The standing edge wave excitation theory of *Guza and Davis* [1974] assumed a beach which was

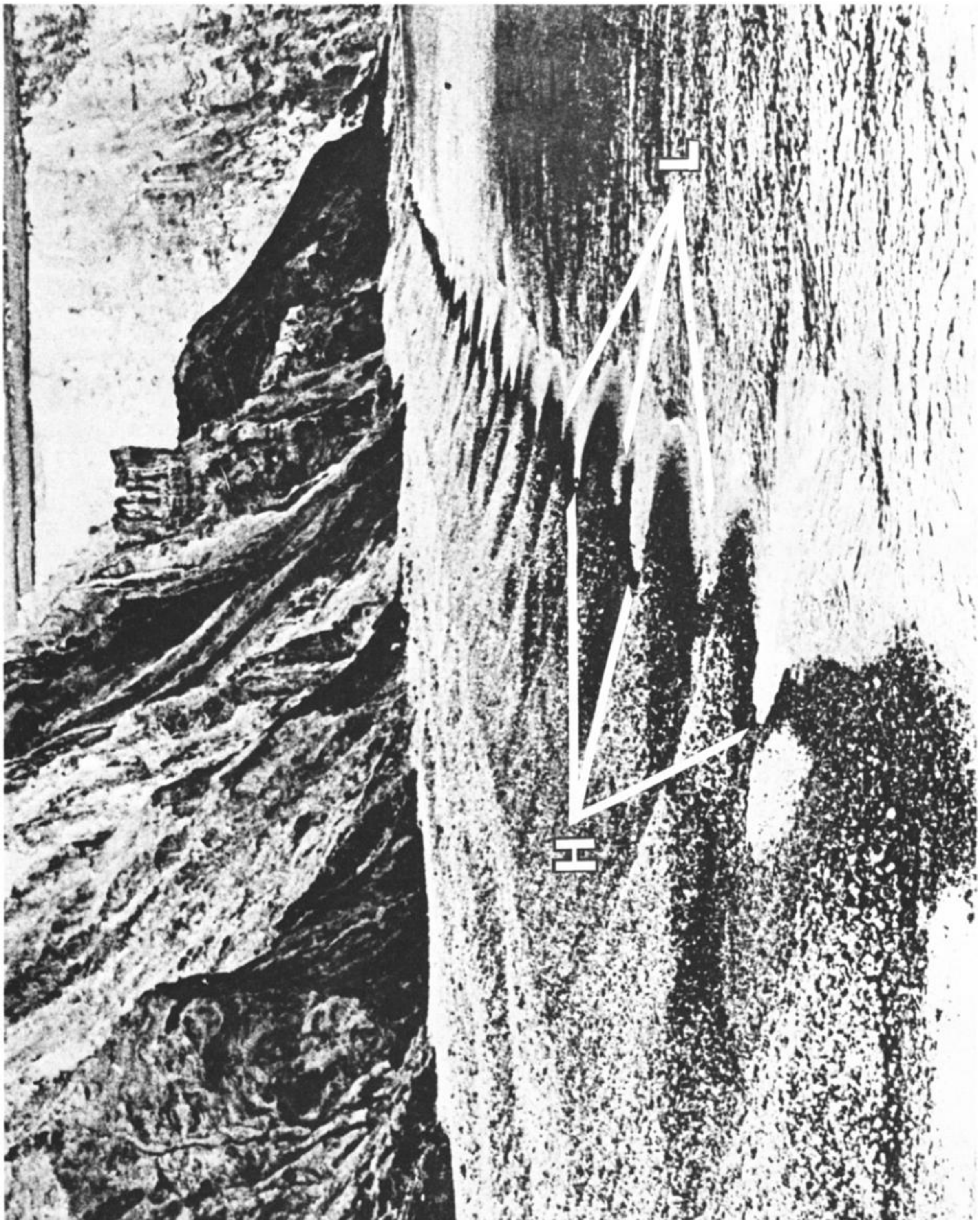

Fig. 10. Subharmonic edge waves playing on beach cusps [from *Kuenen*, 1948]; highs (H) and lows (L) occur at alternate antinodes.

unbounded in the longshore direction and an incident standing wave which was completely coherent and of uniform amplitude alongshore. Edge wave growth occurs because the input of edge wave energy via nonlinear forcing exceeds the energy loss of viscous damping, with no requirement for onshore-offshore barriers. On real beaches, however, the incident waves may not be phase coherent for large longshore distances, and this must reduce the strength of the resonance. Furthermore, if the edge wave resonance stops at some longshore location, the resonant part of the system will radiate edge wave energy along the shore into the nonresonant zone. These radiative losses could greatly decrease the amplitudes of excited edge waves. A headland, a groin, or even a gently curving shoreline may provide end effects which limit the spatial extent over which resonance must occur, resulting in localized edge wave excitation and/or cusp formation. When the incident wave field is very long crested and of uniform amplitude alongshore, there is no need for such end effects because any radiative energy losses out the 'ends' of the system are only a small fraction of the total nonlinear energy input. Many authors [e.g., *Longuet-Higgins and Parkin*, 1962; *Worrall*, 1969] have noted a correlation between regularity of the incident wave field and cusp formation.

No theoretical work on edge wave generation has considered the importance of topographic feedback. *Hino* [1972, 1973] and *Noda* [1972] presented bottom interactive theories for dissipative systems, but the averaging procedures used do not allow any edge wave resonances. Resonantly excited subharmonic edge waves with nonerodable beds and reflective incident waves often had amplitudes larger than those of the incident waves, but the experiments with sand tracers on plane beaches indicate that edge wave induced topographic changes provide negative feedback to the edge wave excitation process. On the other hand, ridges properly located and of appropriate size provide strong positive feedback to edge wave induced longshore perturbations in the topography. Thus edge waves which are too small to be readily apparent in the run-up can exert some influence in forming periodic morphologies. The fact that the alternating surge strengths, which characterize subharmonic edge waves, have rarely been observed on natural cusps emphasizes the importance of considering the coupled (fluid-bed) system. The synchronous edge wave resonance, although it is weaker than the subharmonic resonance on nonerodable beds, may be more important with movable beds. It is also conceivable that topographic feedback to edge wave excitation is so strongly negative that there is no important topographic response. We believe this to be highly unlikely. Edge waves most probably provide an initial longshore periodic perturbation in the topography, the further evolution of cusps being due to positive coupling between the primary incident wave train and the perturbed bed form, berm breeching being an extreme example. This contention is supported by observations [e.g., *Russell and McIntire*, 1965; *Worrall*, 1969; *Williams*, 1973] that regular longshore variations in swash, topography, and/or circulation are necessary for cusp development.

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