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Experimental study of the transformation of wave spectra by a uniform current

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Abstract

An experimental study is reported on the interaction of a wave field with a uniform current field, determining how the wave spectra change due to the presence of current. Wave systems with several peak periods and significant wave heights, were subjected to currents of different speeds, both in the same direction and opposing the wave propagation. Theoretical models of transformed spectra were compared with the experimental data and reasonable comparisons were obtained.

Keywords: Wave spectra; Uniform current; Wave current interaction

1. Introduction

In many applications related with the design and operation of marine structures, it is necessary to start from a spectral model capable of characterizing the frequency distribution of the wave energy. In fact the design codes for offshore platforms and for ships specify the need to use wave spectra in the assessment of wave induced loads.

However, because offshore platforms are stationary structures they also need to be designed for currents, in particular due to their mooring and riser systems. Typically, the design codes of Classification Societies or the ones from national regulatory bodies specify design conditions involving waves defined by a spectrum and currents defined by a velocity profile with depth.

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Since the early fifties many spectral models were proposed but the JONSWAP model has become the stardard in the industry, reducing to the early Pierson–Moskowitz model for a given value of its parameters. Industry is giving more attention to double peaked spectra (e.g. Ewans et al., 2004), but even in that case the model considered can be composed of two individual JONSWAP models as was suggested by Guedes Soares (1984).

Those spectral models are not valid in the presence of currents, which will in fact distort their shape. This fact is not generally taken into account and the general practice is still to supperpose the effects of a wave spectrum with the ones of a current, not accounting for the interaction between them.

This paper addresses this important problem, considering how the shape of a wave spectrum is changed by the presence of a current. Experiments are made in a offshore model basin in which different combinations of sea states and currents are generated and analysed.

Longuet-Higgins and Stewart (1961, 1964) were the first to describe the interaction between waves and currents. They introduced the radiation stress concept and proved the existence of the energy transfer between waves and currents. Later, Bretherton and Garret (1968) defined the wave action, which is important in the study of the interaction of waves and currents, as it is conserved in the absence of the wave generation or dissipation. For linear waves, the wave action is equal to the ratio between wave energy density and frequency with respect to current and its introduction leads to some simplifications in the mathematical formulations of wave–current interactions.

When waves propagate through a region with variable current, some of their characteristic parameters, such as their length, height, steepness, velocity and direction will suffer modifications. The presence of a current alters the velocity of the waves and affects the relation between the observed wave length and period. The current also produces changes in other properties of the waves, as happens with the velocity (and acceleration) of water particles. The interaction between waves and currents does not only change the waves' characteristics, but at the same time it transforms the current flow field.

Huang et al. (1972) proposed the first equations describing the change of the spectral shape due to the presence of currents. However, they did not take into account the occurrence of wave breaking induced by opposite currents, especially for the waves associated with the equilibrium limit of the spectra, that is, with the high frequency tail of the spectrum. Hedges et al. (1981, 1985) modified the theoretical model of Huang et al. so as to take that effect into account.

The basic formulations of wave–current interaction have been verified experimentally by Thomas (1981) and by Kemp and Simons (1982, 1983) in wave flumes. Hedges et al. (1985) also reported on measurements made on waves in a flume propagating without refraction into a steady almost uniform opposing current, showing how the spectra was changing.

However, the experimental validation of these spectral models has not been extensive and it was mainly done in wave flumes, which basically limit the waves to a long crested situation. Unfortunately, there does not seem to exist many full scale measurements of wave spectra and simultaneous surface current that could be used to verify the theoretical formulations of the change in spectral form. Therefore, the offshore basins that are able to reproduce directional sea states and also current are a good option to study these effects, despite the difficulty that most basins have in reproducing accurately the current fields.

Nwogu (1993) was probably the first who conducted an experimental investigation on the effect of steady currents on directional wave spectra. He conducted laboratory tests in a multi-directional wave basin using both regular and irregular waves, with different angles between the current and wave fields.

The amount of experimental data available to support the theoretical formulations of wave spectra in the presence of a current is still limited as only Hedges et al. (1985) and Nwogu (1993) presented limited experimental results on the change in wave spectra caused by currents, in deep water. This was the motivation of the work initiated by Guedes Soares et al. (2000) and continued here, which aims at providing additional experimental data on those phenomena, furthering their understanding and attempting to validate the theoretical models.

Guedes Soares et al. (2000) have confirmed that when a current meets a wave system, differences are observed in the distribution of the spectral energy of the waves. When a wave system is met by a following current, the wave spectrum decreases in terms of its energy, and the contrary happens when the current has the opposite direction. When comparing the experimental results with the theoretical models, it was found that the models proposed by Huang et al. (1972) and Hedges (1981), tend to give very similar values and to give reasonable approximations to the measured values. However in both cases of following current and opposite current, the theoretical predictions of both models underestimate the change produced by the current in the shape of the spectrum, which recommends further study.

The purpose of this study is to provide additional experimental results aiming at complementing and further clarifying the interaction of currents and wave spectra. Guedes Soares et al. (2000) conducted tests with sea states with significant wave height between 5 and 13 m, which are representative of design conditions. This paper analyses the results of a new test series using sea states with significant wave heights between 2 and 5 m. Since the same intensity of currents has been used in both test series, the present one represents situations in which the relative effect of currents is larger, and the interaction effect is more evident.

2. Interaction of water spectra and currents

The equations describing the interaction of waves and currents will be developed referring to a direction parallel to the wave direction. These equations must be modified for currents having an angle with the waves. However, for this case, there will be a complication due to the wave refraction when encountering the current.

Let there be a surface wave in a general field, given by:

$$\eta(\vec{x},t) = A(\vec{x},t)\cdot \mathrm{e}^{\mathrm{i}\varphi(\vec{x},t)} \tag{1}$$

where $A(\vec{x},t)$ is the amplitude and $\varphi(\vec{x},t)$ is the phase function, $\varphi = \vec{k}\cdot\vec{x} - \omega t$, both depending on the spatial coordinates $\vec{x}$ and on time t. With this expression, the wave number $\vec{k}$ and the frequency ω can be defined as:

$$\omega^2 = gk \tag{2}$$

$$c^2 = \frac{g}{k} \tag{3}$$

The referred surface is continuous and differentiable. Combining the two expressions from Eqs. (2) and (3), one obtains the kinematic conservation law for waves, in the form:

$$\frac{\partial \vec{k}}{\partial t} + \nabla\omega = 0 \tag{4}$$

In this expression, the wave number k is defined, as usually, as $2\pi/L$, with L being the wave length. However, the frequency is not the same as for an oscilatory wave propagating in a fixed domain, but the total frequency is. For a wave propagating over a current, the total frequency ω can be divided in two, namely the relative ω_r and Doppler frequencies (White, 1999)

$$\omega = \vec{k}\vec{U}(\vec{x}, t) + \omega_r \tag{5}$$

For the steady-state one has:

$$\nabla\omega = 0 \tag{6}$$

which implies that the total wave frequency is invariant in the steady-state condition. The frequency that is apparent in a wave system propagting in a current field is given by Eq. (5) and it became clear that the relative frequency is only equal to the apparent one ($\omega_r = \omega_a$) in the absence of the current.

Neglecting the weak dynamic interaction, which is non-linear in time, one gets:

$$\omega_r^2 = gk \tag{7}$$

$$c^2 = \frac{g}{k} \tag{8}$$

where c is the phase velocity and g is the acceleration of gravity.

For a fixed vertical plane normal to the wave perpendiculars at a distance x in a fixed coordinate system and x_r from the origin of the moving coordinates system, the relation between x and x_r is written as

$$x_r = x - Ut \tag{9}$$

where t=time ($x_r = x$ for $t = 0$).

In the moving reference system, the free surface displacement η, from the mean sea level, is given in the linear wave theory by:

$$\eta = \frac{H}{2}\cos(kx_r - \omega_r t) \tag{10}$$

However, from Eqs. (5) and (9), this expression should be written in terms of apparent frequency:

$$\eta = \frac{H}{2}\cos(kx - \omega_a t) \tag{11}$$

Furthermore, in the moving reference system, the horizontal component of the velocity of the rising water particles, z, over the mean sea level is:

$$u_r = \frac{H}{2}\omega_r \frac{\cosh k(d+z)}{\sinh hd}\cos(kx_r - \omega_r t) \tag{12}$$

The horizontal component of the water particles velocity, for the fixed reference system, is:

$$u_a = U + u_r = U + \frac{H}{2}(\omega_a - kU)\frac{\cosh k(d+z)}{\sinh hd}\cos(kx - \omega_a t) \tag{13}$$

Without loosing generality, one can assume the one-dimensional case. The conservation of the energy flux between two sections localized in the region, without and with current, is given by (e.g. Clauss et al., 1992):

$$E\left(\frac{c}{2} + U\right)c = \frac{1}{2}E_0 c_0^2 = \text{constant} \tag{14}$$

Reformulating in terms of the group velocity it becomes:

$$\frac{E_0 c_{g0}}{\omega_a} = \frac{E(U + c_{gr})}{\omega_r} \tag{15}$$

and E and E_0 are the temporal variables, in the region with and without current, respectively. Again, the quantities c_{gr} and ω_r are considered with respect to the moving coordinate system, having the current velocity U. According to linear theory

$$c_{g0} = \frac{1}{2}\left(1 + \frac{2k_0 d}{\sinh 2k_0 d}\right)\frac{\omega_a}{k_0} \tag{16}$$

and

$$c_{gr} = \frac{1}{2}\left(1 + \frac{2kd}{\sinh 2kd}\right)\frac{\omega_r}{k} \tag{17}$$

With these equations, one can prove that (Phillips, 1977)

$$\frac{c}{c_0} = \frac{1}{2} + \frac{1}{2}\left(1 + \frac{AU}{c_0}\right)^{1/2} = \left(\frac{k_0}{k}\right)^{1/2} \tag{18}$$

$$\frac{A}{A_0} = \left(\frac{c}{c + 2U}\right)^{1/2} \tag{19}$$

where A is the wave amplitude without current and A_0 the amplitude in a current field.

Combining Eqs. (5), (7) and (18) one gets

$$gk = \frac{\omega^2}{\left[\frac{1}{2} + \frac{1}{2}\left(1 + \frac{4U\omega}{g}\right)^{1/2}\right]^2} \tag{20}$$

thus showing that a more general form the dispersion equation includes the influence of the current.

This formulation, although being general in what concerns the wave form, is only valid for a particular wave. It is necessary to generalize the formulation to take into consideration the effect of current in a wave field. One can begin assuming the existence a spectral representation of the wave field, for instance the Pierson–Moskowitz spectrum (1964)

$$S(\omega) = \frac{\beta g^2}{\omega^5} \exp\left[-\alpha\left(\frac{\omega_0}{\omega}\right)^4\right] \tag{21}$$

where ω is the frequency, α and β are non-dimensional constants equal to 0.74 and 0.81×10^{-2}, respectively, and $\omega_0 = g/V$, where V is the average wind velocity.

The energy contained in an infinitesimal frequency range $\mathrm{d}\omega$ is $S(\omega)\mathrm{d}\omega$. When the waves propagate in a region without current, the energy contained in this frequency range will change due to the energy exchange between waves and current. Given the energy of the waves with current $S(\omega)_\mathrm{c}\,\mathrm{d}\omega$ and combining with Eq. (14), one gets

$$\frac{1}{2}c_0^2 S(\omega)\,\mathrm{d}\omega = \left(\frac{c}{2} + U\right) c S(\omega)_\mathrm{c}\,\mathrm{d}\omega \tag{22}$$

Since, from Eq. (6), one concludes that ω is invariant, this expression can be rewritten:

$$S(\omega)_\mathrm{c} = \frac{\frac{1}{2}c_0^2}{\left(\frac{c}{2} + U\right)c} S(\omega) \tag{23}$$

Using Eq. (18) the spectral density of the free surface displacement in the area subjected to current influence, $S(\omega_\mathrm{a}, U)$ is combined with the area value $S(\omega_\mathrm{a})$ and for zones with sufficient depth, the following expression is obtained:

$$\frac{S(\omega_\mathrm{a}, U)}{S(\omega_\mathrm{a})} = \frac{1}{\left(1 + \frac{U\omega_\mathrm{r}}{g}\right)^2 \left(1 + \frac{2U\omega_\mathrm{r}}{g}\right)} = \frac{4}{\left[1 + \sqrt{1 + \frac{4U\omega_\mathrm{a}}{g}}\right]^2 \sqrt{1 + \frac{4U\omega_\mathrm{a}}{g}}} \tag{24}$$

which is the form of the frequency spectra with the influence of current, proposed by Huang et al. (1972).

When the current velocity is negative (opposite current), the energy density shows a limit frequency, for which the denominator in Eq. (24) becomes zero. The representation is only valid for current velocities verifying the relation:

$$1 + \frac{4U\omega}{g} > 0 \tag{25}$$

Generally speaking, when propagating in opposite current, waves show the tendency of shortening and increasing the height and slope. Hence, $S(\omega_\mathrm{a}, U)$ also increases and Eq. (24) predicts that it will become infinity when $\omega_\mathrm{a} = -g/4U$, i.e. the energy of the wave components cannot propagate against the current and breaking will occur at this current limit velocity.

Hedges et al. (1985) modified the Huang et al. (1972) model, given by Eq. (24) in order to account for this effect. As there is a limit over which waves may not grow in a given frequency range (Phillips, 1977), some components traveling with the current are subjected to breaking. For deep water, this 'equilibrium limit' was given by Hedges (1981) as:

$$S_{\mathrm{ER}}(\omega_{\mathrm{a}}) = \frac{A^* g^2}{(\omega_{\mathrm{a}} - kU)^5} \frac{1}{\left[1 + \frac{2U(\omega_{\mathrm{a}} - kU)}{g}\right]} \tag{26}$$

where 'ER' refers to the equilibrium range and A^* is a numerical constant.

One can notice that in the absence of current, Eq. (26) reduces to a more usual form given by Phillips (1977) and for this case Phillips defines the value of A^* as varying in the range 0.008–0.015. Assuming that the equilibrium frontier of the spectra is associated to the water depth, then Eq. (24) applies to the calculation of the spectral densities for a region with current, provided that $S_{\mathrm{ER}}(\omega_{\mathrm{a}},U)$ is lower than $S(\omega_{\mathrm{a}},U)$ given by Eq. (24).

3. Experiment and data analysis

3.1. Experimental procedure

The data used in this study was obtained in tests performed at the DHI Water and Environment 3D offshore wave basin, which has 30 m length, 20 m width and 3 m deep. There is also a pit in the centre, whose depth can be regulated to a maximum of 12 m. The pit remains closed when not used, which was the case in these tests. To measure the spatial pattern of the wave field, several wave gauges were used but the data of the present study are referred to a gauge placed in the center of the tank, which is the reference one for all calibrations made (Fig. 1).

Two current intensities where selected in the same direction and in opposite direction of the propagation of waves, with 1.5 and 1 m/s.

The theoretical model used as input for the referred spectra was the JONSWAP spectrum:

$$S(f) = \frac{\alpha g^2}{(2\pi)^4} f^{-5} \exp\left[-\frac{5}{4}\left(\frac{f}{f_{\mathrm{p}}}\right)^{-4}\right] \gamma^{\exp\left[\frac{-(f-f_{\mathrm{p}})^2}{2\sigma^2 f_{\mathrm{p}}^2}\right]} \tag{27}$$

For $\gamma = 1$ this formulation reduces to the Pierson–Moskowitz spectrum in Eq. (19).

The tests were carried out with spectra with characteristic parameters shown in Tables 1–3. The reference sea states in Table 1 cover a range sufficient to represent sea states of wind sea and swell type.

While Guedes Soares et al. (2000) have chosen sea states with H_{s} between 5 and 13 m, which are representative of extreme conditions used for design, the present sea states all refer to Hs values lower than 5 m as can be seen in Table 1. The objective was to have

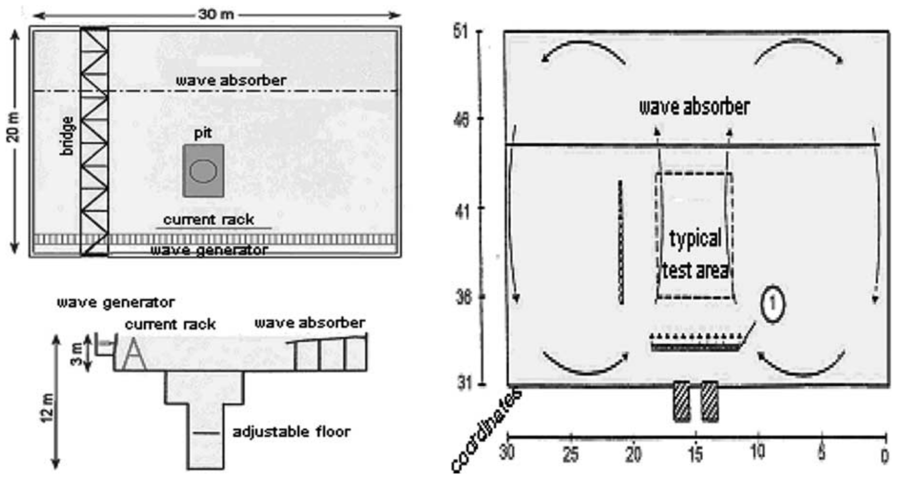


Fig. 1. The DHI 2D offshore wave basin and the system used for the creation of currents.

wave systems that would have less energy and thus would be more sensitive to the effects of currents.

A uniform current of about 1.5 m/s was applied to the 10 sea states with following waves. Out of the 10 referred sea states 4 were subjected to a current of about 1 m/s, as shown in Table 2. As can be checked in Table 2, which shows the values obtained at the tests with following current, in most of them there exists a non-negligible difference between the intended and measured velocity. However, since there was no particular reason to define the values of the current velocities chosen, it was decided to use the values that were produced in the tank, thus avoiding spending excessive time in calibration of the sea state and current so that a larger number of tests could be performed.

Finally, the reference sea states were subjected to opposite current and the differences between desired and measured velocity were also observed. These tests are shown in Table 3.

Table 1
Wave systems without current

Test N	H_s (m)	T_p (s)	Wave dir. (deg.)	Curr. dir. (deg.)	Curr. vel. (m/s)	Obs vel. real
1	3.6	7	0	–	–	–
2	3.6	10	0	–	–	–
3	3.6	14	0	–	–	–
4	3.6	20	0	–	–	–
5	4.6	7	0	–	–	–
6	4.6	14	0	–	–	–
7	4.6	20	0	–	–	–
8	2.3	7	0	–	–	–
9	2.3	14	0	–	–	–
10	2.3	20	0	–	–	–

Table 2
Wave systems with following current indicating the nominal and observed current velocity

Test N	H_s (m)	T_p (s)	Wave dir. (deg.)	Curr. dir. (deg.)	Curr. vel. (m/s)	Obs. vel. real
15	3.6	7	0	0	1.5	1.52
16	3.6	10	0	0	1.5	1.42
17	3.6	14	0	0	1.5	1.53
18	3.6	20	0	0	1.5	1.53
19	4.6	7	0	0	1.5	1.55
20	4.6	14	0	0	1.5	1.38
21	4.6	20	0	0	1.5	1.54
22	2.3	7	0	0	1.5	1.55
23	2.3	14	0	0	1.5	1.52
24	2.3	20	0	0	1.5	1.56
43	3.6	7	0	0	1	0.96
44	3.6	14	0	0	1	0.88
45	4.6	14	0	0	1	0.86
46	2.3	14	0	0	1	0.90

3.2. Spectral analysis

The time series recorded at the central wave gauge were subjected to a spectral analysis resulting in the experimental spectra that were compared with the theoretical models previously described.

It was noticed that when an old or a swell sea state (large period) is analyzed, the behavior of the spectra is smoother than a young wind sea state (short period), that is, as the peak period increases the spectra appears to be smoother, emphasizing what is noticed through direct observation: at low frequencies the sea state is more regular than at high frequencies, as shown in Fig. 2.

Table 3
Wave systems with opposite current indicating the nominal and observed current velocity

Test N	H_s (m)	T_p (s)	Wave dir. (deg.)	Curr. dir. (deg.)	Curr. vel. (m/s)	Obs. vel. real
25	3.6	7	0	180	1.5	−1.33
26	3.6	10	0	180	1.5	−1.25
27	3.6	14	0	180	1.5	−1.35
28	3.6	20	0	180	1.5	−1.32
29	4.6	7	0	180	1.5	−1.33
30	4.6	14	0	180	1.5	−1.27
31	4.6	20	0	180	1.5	−1.23
32	2.3	7	0	180	1.5	−1.30
33	2.3	14	0	180	1.5	−1.35
34	2.3	20	0	180	1.5	−1.30
47	3.6	7	0	180	1	−0.89
48	3.6	14	0	180	1	−0.86
49	4.6	14	0	180	1	−0.88
50	2.3	14	0	180	1	−0.96

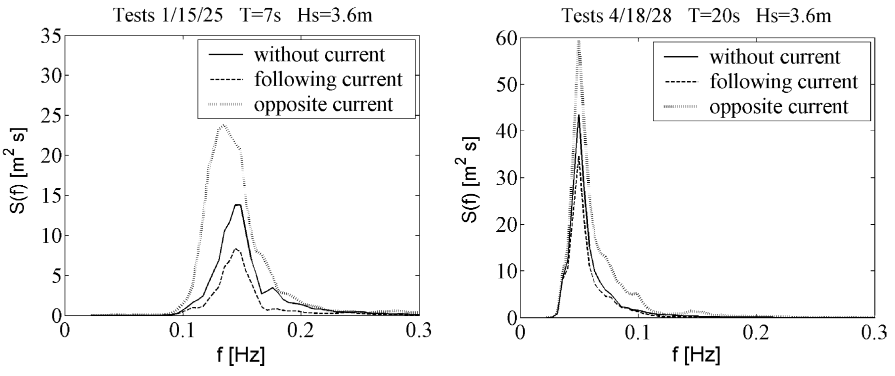


Fig. 2. Frequency spectra with and without current. $U = 1.5$ m/s.

As shown in Figs. 2 and 3, peak frequency conservation exists between the spectra with and without current, whatever the direction of the current. Taking into consideration that the figures have at the abscissa the apparent frequency, f_a, i.e. the one measured by the gauge, the current affects the significant height of the reference sea state, being increased when subjected to opposite current and decreased when influenced by following current. This results in an increase or decrease of the free surface spectral energy, for opposite or following current, respectively.

In Fig. 3a, for 1 m/s current, the maximum values are very similar, although it is also noted the tendency of a relative increase of the spectral peak with opposite current when the significant wave height decreases (Fig. 3b).

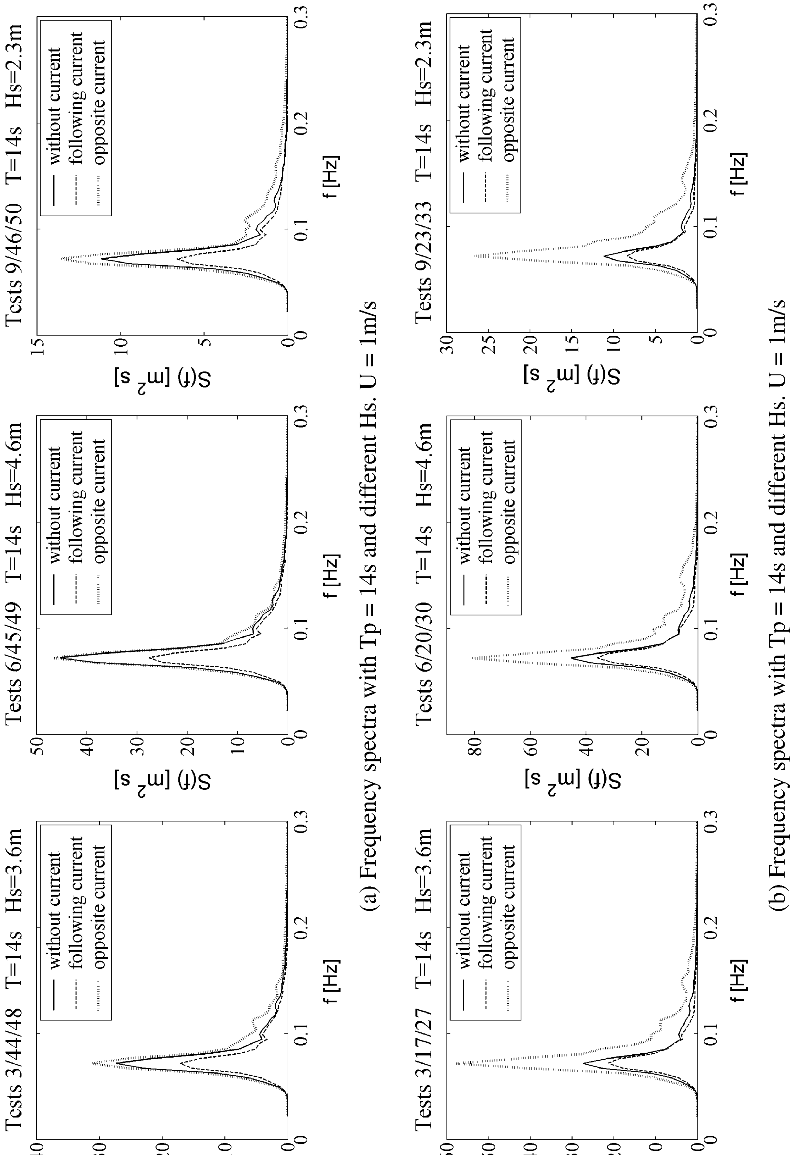
In all cases more energy is observed at the tail of the spectra with opposite current than with following current. Differences are higher when the current is higher and the significant height is smaller. This increase of energy at the peak and tail of the spectra can only be explained by an energy transfer from the current to the wave system.

3.3. Wave number spectra

While the study of the wave spectra in terms of apparent frequency did not show any change in the peak frequency, the analysis of the wave number spectra shows a shift in peak wave number in the presence of currents. This wave number refers to the intrinsic wave frequency, that is, the wave number in the moving reference with a velocity equal to the one of the current.

The number of waves is constant in time, $\partial k/\partial t = 0$, as there is no generation in the domain, i.e. the waves produced by the wave generator and the ones absorbed on the beach are the same (from the point of view of the quantity) but the wave number is not constant in space.

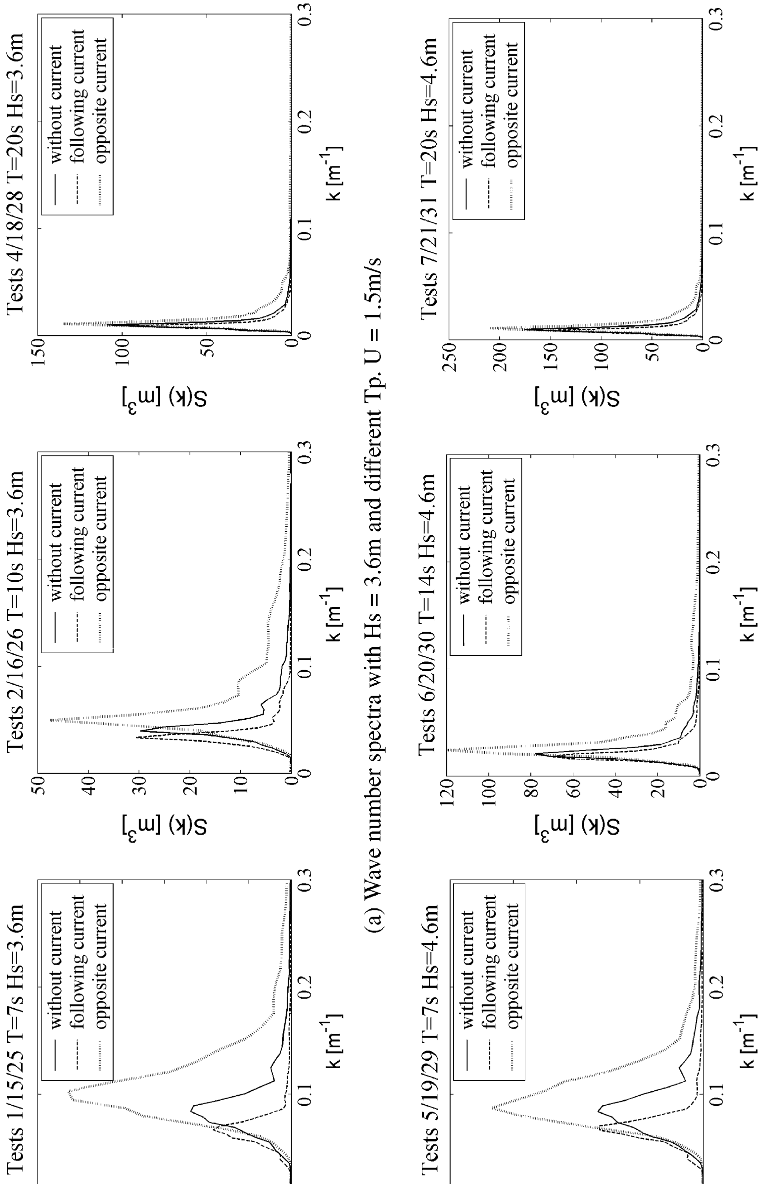
What is happening is that with opposite current the wave height increases, waves become shorter, while exactly the opposite happens with following current. If the current hypothetically ceases to exist, they move anyway in a medium in opposite movement, therefore the effects are cancelled. Hence, it is clear that the wave number depicted in Fig. 4 is related to the case with current, namely to the new group velocity characterizing the modified sea state.



(a) Frequency spectra with Tp = 14s and different Hs. U = 1m/s

(b) Frequency spectra with Tp = 14s and different Hs. U = 1m/s

Fig. 3. Frequency spectra with and without current.



(a) Wave number spectra with Hs = 3.6m and different Tp. U = 1.5m/s

(b) Wave number spectra with Hs = 3.6m and different Tp. U = -1.5m/s

Fig. 4. Wave number spectra.

3.4. Analysis of the wave spectra transformation

Measured spectra are compared with the theoretical spectral model of Huang et al. (1972), described earlier. In the previous experimental study, Guedes Soares et al. (2000) have shown that although the effect of the current was correctly represented by the theoretical model, there was not a perfect match in all cases. Therefore, in this work the current velocity used in the theoretical model was allowed to vary so as to produce the best possible match between the experimental and theoretical spectra. The hypothetical current velocity that should exist in the wave–current field so that the model adjustment became more precise is denoted as 'convergence velocity' and its comparison with the measured velocity provides an indication of the lack of adjustment of the spectral model when compared with the existing velocity.

In order to estimate the quality of the obtained approximation, a performance index, I_a, is used (Guedes Soares et al., 2000):

$$I_a = 1 - \frac{\sum_{i=1}^{N} (P_i - O_i)^n}{\sum_{i=1}^{N} \left(\left| P_i - \bar{O} \right| + \left| O_i - \bar{O} \right| \right)^n}$$

where $\bar{O}$ is the mean of the observed values and O_i and P_i are the predicted values of the spectral ordinates. The values of I_a vary between 0 and 1, where 1.0 indicates a perfect agreement between observations and predictions and 0 is the opposite case. In this study a value of $n=2$ was adopted.

Fig. 5 shows that the model adapts very well when compared to the tests with following current and maximum velocity of 1.5 m/s. The adjustment is perfect for the tests with 14 s or higher periods.

When the tests with following current, of about 1 m/s velocity are analyzed (Fig. 6), the maximum spectral density expected by the theoretical model is larger than the one obtained in the tests. In fact, the convergence velocity estimated from the model exceeds

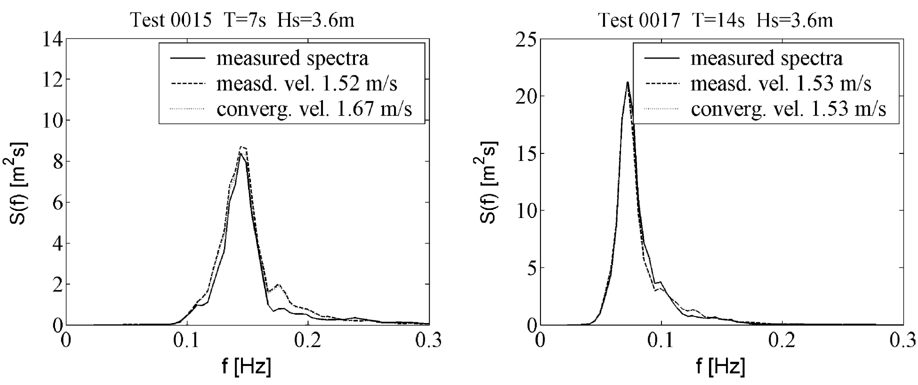


Fig. 5. Comparison of the Huang et al. model for tests with $H_s=3.6$ m and $U=1.5$ m/s.

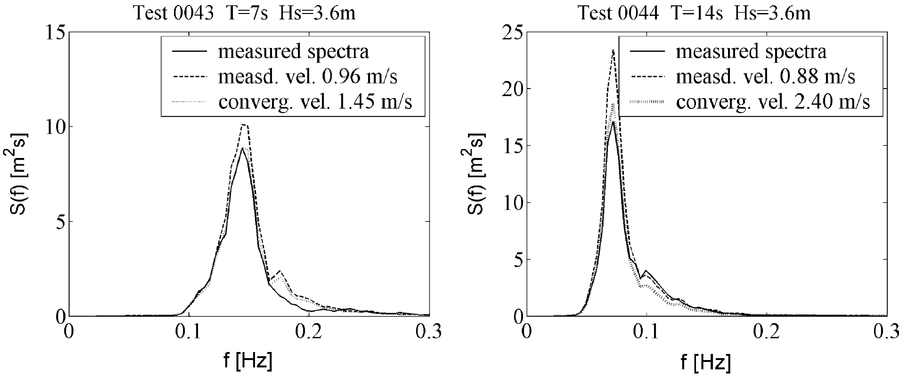


Fig. 6. Comparison of the Huang et al. model for tests with $H_s=3.6$ m and $U=1$ m/s.

twice the one measured in reality. In this case, the estimated velocity is higher as the sea states period increases.

For the tests with opposite current, especially the ones with 7 and 10 s periods, a breakdown of the Huang model is noticed, as starting from this frequency the model does not converge, becoming infinity. This also happens in tests with peak periods higher than 10 s, but with a significant height of 2.3 m, Fig. 7. This means that at those wave frequencies the blockage of wave propagation is originated by the current, as discussed earlier.

Table 4 depicts all the results obtained for each of the tests. The difference between estimated and real velocities is a quality indicator of the adjustment of the theoretical model.

Generally, if one compares the adjustment between the theoretical model and real data, it can be said that the model has a better adjustment when the wave field is subjected to higher intensity current and has the same propagation direction as the waves. Likewise, the worst adjustment was found in the tests with opposite current and these ones with highest intensity.

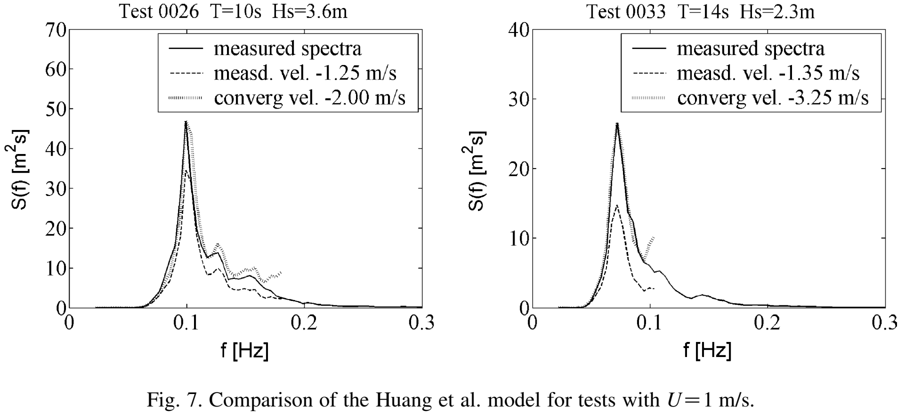


Fig. 7. Comparison of the Huang et al. model for tests with $U=1$ m/s.

Table 4
Measured and convergence velocities for the Huang et al. model for each of the tests

Test	H_s	T_p	Measd vel.	Convg. vel.	Convg./ meads	Test	H_s	T_p	Measd vel.	Convg. vel.	Convg./ measd
$U=-1.5$ m/s. Tests with following current						$U=-1.5$ m/s. Tests with opposite current					
15	3.6	7	1.52	1.63	1.07	25	3.6	7	−1.33	−1.14	0.86
16	3.6	10	1.42	0.64	0.45	26	3.6	10	−1.25	−2.00	1.60
17	3.6	14	1.53	1.53	1.00	27	3.6	14	−1.35	−2.94	2.18
18	3.6	20	1.53	1.53	1.00	28	3.6	20	−1.32	−2.12	1.61
19	4.6	7	1.55	0.71	0.46	29	4.6	7	−1.33	−1.57	1.18
20	4.6	14	1.38	1.38	1.00	30	4.6	14	−1.27	−2.44	1.92
21	4.6	20	1.54	1.54	1.00	31	4.6	20	−1.23	−1.59	1.29
22	2.3	7	1.55	1.60	1.03	32	2.3	7	−1.30	−3.25	2.50
23	2.3	14	1.52	1.52	1.00	33	2.3	14	−1.35	−2.10	1.56
24	2.3	20	1.56	1.56	1.00	34	2.3	20	−1.30	−0.77	0.59
$U=1$ m/s. Tests with following current						$U=-1$ m/s. Tests with opposite current					
43	3.6	7	0.96	1.41	1.47	47	3.6	7	−0.89	−0.77	0.87
44	3.6	14	0.88	2.4	2.73	48	3.6	14	−0.86	−0.86	1.00
45	4.6	14	0.86	2.62	3.05	49	4.6	14	−0.88	−0.88	0.73
46	2.3	14	0.90	2.94	3.27	50	2.3	14	−0.96	−0.96	1.00

3.5. Analysis of the high frequency range of the spectra

To better explain the cases in which the formulation of Huang et al. (1972) did not provide results at the high frequency end of the spectrum it is worthwhile to study in more detail the behaviour of the high frequency range of the spectra in the presence of currents.

Starting from the original idea of Phillips, Hedges et al. (1985) introduced the effect of the equilibrium limit in the spectrum, by using Eq. (26). It should be emphasized that $S_{ER}(\omega_a)$ in that equation is an approximation which basically makes it easier to understand the idea of the limit of wave increase, that is, a delimitation from which the wave behavior cannot be explained using linear theory.

Fig. 8 shows the equilibrium limit for different tests with the same significant wave height and with different periods. These tests also include the influence of a following or opposite current. It should be emphasized that the equilibrium frontier depends on the current direction and magnitude, indicating an increase or decrease of the wave breaking probability.

With the following current, the referred limit shifts to higher frequencies, hence the probability of finding the entire frequency range describing the wave spectra is higher, resulting in a smaller tendency to wave breaking and the contrary for opposite current.

Fig. 8 shows that when the test is carried with the following current all sea states are under the equilibrium limit. In Fig. 8b, corresponding to pure sea states, the only wave components with high breaking probability correspond to the test with a period of 7 s.

If the significant wave height characterizing the sea state were higher, that is, if there had been another sea state with a period of 7 s and significant height higher than 3.6 m, then the frequency range outside the equilibrium frontier would be higher, as with an increase in height, the wave energy increases and therefore these will have a higher probability to breaking.

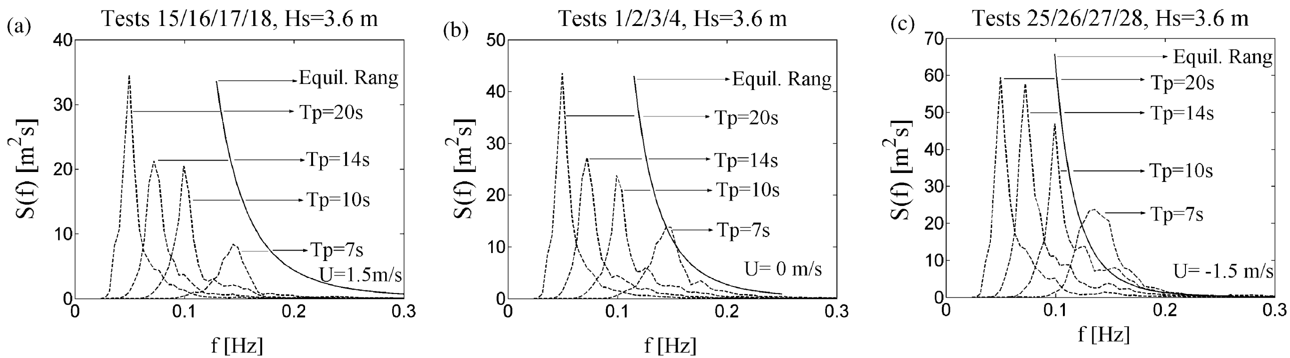


Fig. 8. Equilibrium limit with following current (a), without current (b), and opposite current (c).

Fig. 8c shows that with an opposing current of 1.5 m/s several spectra would have their high frequency tail intersecting the equilibrium range line indicating that the original formulation of Huang et al. (1972) would need to have the modification of Hedges et al. (1985) to properly represent that part of the spectrum.

4. Conclusions

This study has shown that the theoretical model of Huang et al. (1972), provides a good description of the change in spectral shape in the cases of high intensity currents following the wave direction. However, it underestimates the effects of the current when its velocity is smaller or equal to 1 m/s.

When the current opposes the wave propagation the model of Huang et al. (1972) underestimates the effect of the current, which has been observed earlier by Guedes Soares et al. (2000) for higher sea states. To obtain good agreement between measured spectra and model, the current considered in the theoretical model would need to have a velocity almost twice the measured one.

With opposing current the waves absorb the energy up to a certain limit, above which it is impossible to continue the growth, resulting in their breaking. The model of Huang et al. (1972) does not cover this situation but the modification of Hedges et al. (1985) is then applicable.

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References

Bretherton, F.P., Garrett, C.J.R., 1968. Wave trains in inhomogeneous moving media. Proc. R. Soc. Lond., Ser. A 302, 529–554.

Clauss, G., Lehman, E., Östergaard, C., 1992. Offshore Structures Volume I: Conceptual Design and Hydromechanics. Springler–Verlag, Germany.

Ewans, K.C., Bitner-Gregersen, E., Guedes Soares, C., 2004. Estimation of wind-sea and swell components in a bimodal sea state. Proceedings of the OMAE Specialty Conference on Integrity of Floating Production, Storage and Offloading (FPSO) Systems; ASME. Paper OMAE-FPSO'04-0045.

Guedes Soares, C., 1984. Representation of double-peaked sea wave spectra. Ocean Eng. 11, 185–207.

Guedes Soares, C., Rodriguez, G.R., Cavaco, P., Ferrer, L., 2000. Experimental study on the interaction of wave spectra and currents Proceedings of the ETCE/OMAE Joint Conference Energy for the New Millennium, ASME, paper OMAE 2000/S&R-6133.

Hedges, T.S., 1981. Some Effects of Currents on Wave Spectra Proceedings of the First Indian Conference in Ocean Engineering, vol. 1. Indian Institute of Technology, Madras pp. 30–35.

Hedges, S.T., Anastasiou, K., Gabriel, D., 1985. Interaction of random waves and currents. J. Waterway, Port, Coastal Ocean Eng., ASCE 111 (2), 275–288.

Huang, N.E., Chen, D.T., Tung, C.C., Smith, J.R., 1972. Interactions between steady non-uniform currents and gravity waves with application for current instruments. J. Phys. Oceanogr. 2, 420–431.

Kemp, P.H., Simons, R.R., 1982. The interaction between waves and a turbulent current: waves propagating with the current. J. Fluid Mech. 116, 227–250.

Kemp, P.H., Simons, R.R., 1983. The interaction of waves and a turbulent current: waves propagating against the current. J. Fluid Mech. 130, 73–89.

Longuet-Higgins, M.S., Stewart, R.W., 1961. The changes in amplitude of short gravity waves on a steady non-uniform current. J. Fluid Mech. 10, 529–549.

Longuet-Higgins, M.S., Stewart, R.W., 1964. Radiation stress in water waves; a physical discussion with applications. Deep-Sea Res. 11, 529–562.

Nwogu, O., 1993. Effect of Steady Currents on Directional Wave Spectra Proceedings of the 12th International Conference on Offshore Mechanics and Arctic Engineering (OMAE), ASME, vol. 1 1993 pp. 25–32.

Phillips, O.M., 1977. The Dynamics of Upper Ocean. Cambridge University Press, Cambridge.

Pierson, J.R., Moskowitz, L., 1964. A proposed spectral form for fully developed wind sea based on the similarity theory of S.A. Kitaigorodskii. J. Geophys. Res. 69 (24), 5181–5190.

Thomas, G.P., 1981. Wave–current interactions: an experimental and numerical study. Part 1. Linear waves. J. Fluid Mech. 110, 457–474.

White, B.S., 1999. Wave action on currents with vorticity. J. Fluid Mech. 386, 329–344.