On the Linear Parameterization of Drag Coefficient over Sea Surface

CHANGLONG GUAN

Physical Oceanography Laboratory, Ocean University of China, Qingdao, China, and Department of Marine, Earth and Atmospheric Sciences, North Carolina State University, Raleigh, North Carolina

LIAN XIE

Department of Marine, Earth and Atmospheric Sciences, North Carolina State University, Raleigh, North Carolina

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ABSTRACT

Combining the logarithmic law with the Charnock relation yields a drag coefficient that is a function of wind speed with the Charnock coefficient as a parameter. It is found that the function is nearly linear within the typically measured range of the drag coefficient. The slope of the linear function is dominated by the Charnock coefficient. When the Charnock relation is extended to a wave age–dependent function, the drag coefficient remains a near-linear function of wind speed after invoking the 3/2 power law. The slope of the linear function is dominated by wave steepness.

1. Introduction

Wind stress over the sea surface is the primary driving force for the upper-ocean circulation and sea surface waves. The accurate estimate of wind stress is important in modeling and forecasting atmospheric and oceanic processes, as well as for satellite remote sensing of the global wind field. Because of the central role of wind stress in understanding and modeling air–sea interaction processes, it has been studied extensively from observations and numerical models over the past several decades (Charnock 1955; Kondo 1975; Geenaert 1987; Donelan 1990; Toba et al. 1990; Rooth and Xie 1992; Xie and Rooth 1995; Xie et al. 2001).

Wind stress is usually parameterized in terms of a drag coefficient (C_d) or aerodynamic surface roughness (z_0). The drag coefficient can be expressed as a function of the 10-m wind speed (U_{10}), and the parameters in the equation are determined empirically by fitting observational data to a curve. Many of the equations concerning C_d are in the form of a linear function, especially when U_{10} is within 7–20 m s^{-1}. However, there is considerable discrepancy among the parameters in the linear parameterization proposed by different investigators (Wu 1980; Geernaert 1990). The purpose of this note is to establish a theoretical basis for the linear parameterization of C_d and to provide a unified approach to reconcile the differences among the various forms of linear parameterization.

Corresponding author address: Prof. Lian Xie, NCSU/MEAS, Box 8208, Raleigh, NC 27695-8208.
E-mail: lian_xie@ncsu.edu

2. Linear parameterization of C_d

Most of the field experiments over the past have established a statistically significant dependence of C_d on U_{10}. The general form of this linear parameterization can be expressed as

$$C_d = (a + bU_{10}) \times 10^{-3}, \tag{1}$$

where a and b are empirical parameters determined by observations. The values of a and b presented by various authors are listed in Table 1. It shows that there is considerable breadth to or scatter among the values of both a and b determined by different authors. Let

$$\mathrm{err}(x) = |[\max(x) - \min(x)]/\min(x)|. \tag{2}$$

From Table 1, it yields

$$\mathrm{err}(a) = 2.33 \quad \text{and} \tag{3}$$

$$\mathrm{err}(b) = 6.5. \tag{4}$$

Equations (3) and (4) indicate that the value of b varies more widely than that of a. The average value of a is 0.79. It is clear that the variability of C_d not explained by U_{10} is substantial.

Since sea surface roughness is due mainly to surface waves, surface waves must play an important role in air–sea momentum fluxes. Hence, there are two fundamental questions concerning the linear parameterization of C_d on U_{10}. 1) Is there a theoretical basis to fit

TABLE 1. The parameters in Eq. (1) as proposed by various authors.

Authors	a	b ($\mathrm{m^{-1}\,s}$)
Sheppard (1958)	0.8	0.114
Deacon and Webb (1962)	1.0	0.07
Miller (1964)	0.75	0.067
Zubkovskii and Kravchenko (1967)	0.72	0.12
Brocks and Krugermeyer (1970)	1.18	0.016
Sheppard et al. (1972)	0.36	0.1
Wieringa (1974)	0.86	0.058
Kondo (1975)	1.2	0.025
Smith and Banke (1975)	0.61	0.075
Smith (1980)	0.61	0.063
Wu (1980)	0.8	0.065
Donelan (1982)	0.96	0.041
Geernaert (1987)	0.5777	0.0847
Yelland and Taylor (1996)	0.60	0.07

the observational data to a linear function? 2) If such a basis exists, how can the scatter in the parameter values of the linear functions presented by various authors be explained? These two questions will be discussed in the next section.

3. A unified linear relationship

For neutral atmospheric stability, the wind profile in the atmospheric boundary layer can be described in terms of the logarithm of surface roughness. The vertical wind profile is generally taken as

$$U(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right), \tag{5}$$

where $U(z)$ is the wind speed measured at anemometer height z, u_* is the surface wind friction velocity and is given by $(\tau/\rho_a)^{1/2}$, τ is wind stress, ρ_a is air density, and $\kappa = 0.4$ is the von Kármán constant; u_* is related to C_d and U_{10} by

$$u_*^2 = C_d U_{10}^2. \tag{6}$$

Combining Eqs. (5) and (6) yields a unique relationship between z_0 and C_d,

$$z_0 = z_{10}/\exp(\kappa/C_d^{1/2}), \tag{7}$$

where $z_{10} = 10$ m, the 10-m height above the sea surface.

Charnock (1955) proposed, from a dimensional argument, a well-known formula known as the Charnock relationship,

$$\frac{gz_0}{u_*^2} = \alpha, \tag{8}$$

where g is the gravitational acceleration and α is the Charnock coefficient. The values of α derived from data by various authors are different from one another. For instance, $\alpha = 0.012$ by Charnock (1955), $\alpha = 0.0144$ by Garratt (1977), and $\alpha = 0.0185$ by Wu (1980).

Combining Eq. (7) and (8), Wu (1969) obtained

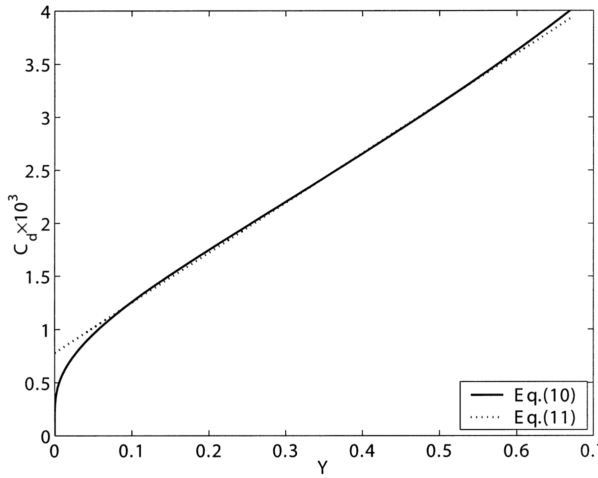

FIG. 1. Variation of drag coefficient C_d as a function of parameter Y.

$$C_d = \left(\kappa \ln^{-1} \frac{1}{\alpha C_d \tilde{U}^2}\right)^2, \tag{9}$$

where $\tilde{U} = U_{10}/\sqrt{gz_{10}}$, is in the form of Froude number. Equation (9) implies that for neutral stratification, the drag coefficient is dependent on wind speed only if the Charnock coefficient is a constant. It was reported by Wu (1982) that Eq. (9) with $\alpha = 0.0185$ is indistinguishable from Eq. (1) with the parameters corresponding to Wu (1980) listed in Table 1. Equation (9) can be rewritten in the following form,

$$Y = C_d^{-1/2} \exp\left(-\frac{\kappa}{2} C_d^{-1/2}\right), \tag{10}$$

where $Y = \alpha^{1/2}\tilde{U}$.

Figure 1 shows the values of Y plotted as a function of C_d within the range of (1.0–4.0) $\times 10^{-3}$ for C_d. By fitting the plotted data using the least squares method, we obtain

$$C_d = (0.78 + 4.7Y) \times 10^{-3}. \tag{11}$$

Figure 1 shows that within typically measured range of (1.0–2.3) $\times 10^{-3}$ for C_d, Eq. (11) is almost indistinguishable from Eq. (10). In other words, Eqs. (10) and (11) are practically identical when C_d is within (1.0–2.3) $\times 10^{-3}$. Expressing Eq. (11) in terms of U_{10} yields

$$C_d = (0.78 + 0.475\alpha^{1/2}U_{10}) \times 10^{-3}, \tag{12}$$

where the unit of U_{10} is meters per second. Comparing Eq. (12) with Eq. (1) yields

$$a = 0.78 \quad \text{and} \tag{13}$$

$$b = 0.475\alpha^{1/2}, \tag{14}$$

where the unit of b is seconds per meter. Equation (12) states that under neutral atmospheric stability the drag coefficient over the sea surface is almost linearly dependent on wind speed in the typically measured range

TABLE 2. The parameters in Eq. (16) as proposed by various authors.

Authors	A	B
Toba and Koga (1986)	0.025	1.0
Masuda and Kusaba (1987)	0.0129	−1.1
Toba et al. (1990)	0.02	0.5
Donelan (1990)	0.42	−1.03
Maat et al. (1991)	0.86	−1.01
Smith et al. (1992)	0.48	−1
Monbaliu (1994)	2.87	−1.69
Vickers and Marht (1997)	2.9	−2.0
Johnson et al. (1998)	1.89	−1.59
Sugimori et al. (2000)	0.02	0.7

of C_d. The intercept of Eq. (12) is a constant, which is nearly identical to the average value of its corresponding observations shown in Table 1. The slope in Eq. (12) is proportional to the square root of the Charnock coefficient. The drag coefficient increases with wind speed more rapidly for larger Charnock coefficient. The variations in the coefficients in Eq. (1) proposed by various authors as shown in Table 1 could be interpreted as a result of the difference in the corresponding Charnock coefficients. For $\alpha = 0.0185$, $b = 0.0646$ according to Eq. (14). Thus, with the parameters of Wu (1980), Eq. (12) is essentially identical to Eq. (1). This is consistent with the conclusion of Wu (1982), which states that with $\alpha = 0.0185$ Eq. (9) is indistinguishable from Eq. (1).

Since sea surface roughness is due mainly to surface waves, much focus in the published literature has been on the investigation of the dependence of the wind stress on wave status. Stewart (1974) proposed an extension of the Charnock relation, making the roughness an arbitrary function of wave age,

$$\frac{gz_0}{u_*^2} = f(\beta_*), \tag{15}$$

where $\beta_* = C_p/u_*$ is wave age scaled by friction velocity, and C_p is the phase velocity of the waves at the spectral peak. Equation (15) states that the Charnock coefficient is not a constant if the effect of wave status is considered. The form of the function on the right-hand side of Eq. (8) is variable. A function which has been used widely in the literature (Table 2) is

$$\frac{gz_0}{u_*^2} = A\beta_*^B, \tag{16}$$

where A and B are coefficients determined by observations. Based on field and laboratory observations, these two coefficients have been proposed by various authors as shown in Table 2. It is shown that the controversy remains over the nature of the dependence of roughness on wave age. The positive value of B means that mature waves are rougher than younger waves, while the negative value of B suggests the contrary.

It has been pointed out by Smith et al. (1992) that a spurious self-correlation exists in Eq. (16) by scaling z_0 with u_*. Most recently Mahrt et al. (2003) reported that spurious self-correlation in the Charnock formulation explains more variance than actual physical relationships, even after eliminating weak wind cases. The uncertainty in u_* is often larger than that of U_{10}, C_p, and H, where H is the significant wave height. Thus, it is more appropriate to scale z_0 with U_{10} instead of u_* in Eq. (16), and then combine it with Eq. (7) to yield

$$A^{1/2}\beta^{B/2}\tilde{U} = C_d^{-1/2+B/4}\exp\left(-\frac{\kappa}{2}C_d^{-1/2}\right), \tag{17}$$

where $\beta = C_p/U_{10}$ is the wave age scaled with wind speed. With the aid of the dispersion relation for deep water, the wave age can be expressed as $\beta = g/U_{10}\omega_p$, where ω_p is the frequency of the wave spectral peak. Equation (17) indicates that the drag coefficient could be parameterized using wind speed and wave age. However, wave age is dependent on wind speed by way of its definition. A pure wave parameter measuring wave status—for instance, wave steepness—is more proper than wave age. In fact, Hsu (1974) suggested that the Charnock coefficient is more properly a function of wave steepness. Most recently, Taylor and Yelland (2001) proposed that z_0 should be parameterized by the height and steepness of waves.

A 3/2 power law was proposed by Toba (1972) for growing wind waves. It was expressed as

$$H_* = B_*T_*^{3/2}, \tag{18}$$

where $B_* = 0.062$, $H_* = gH/u_*^2$, $T_* = gT/u_*$, and T is the significant wave period. The 3/2 power law is verified by Kawai et al. (1977) and Ebuchi et al. (1992) with field data. Most recently it is shown that most of the wind-wave growth relations support the 3/2 power law (Guan and Sun 2001). With the dispersion relation for deep water and $T = 0.91 \times 2\pi/\omega_p$ (Wen et al. 1989; Goda and Nagai 1974), Eq. (18) can be rewritten as

$$\delta\beta^{1/2} = 0.085C_d^{1/4}, \tag{19}$$

where $\delta = H\omega_p^2/g$ is the wave steepness. Combining Eqs. (17) and (19) yields

$$f(\delta)\tilde{U} = C_d^{-1/2}\exp\left(-\frac{\kappa}{2}C_d^{-1/2}\right), \tag{20}$$

where

$$f(\delta) = 0.85^B A^{1/2}\delta^{-B}. \tag{21}$$

Note that the right-hand side of Eq. (20) is identical to that of Eq. (10). Therefore, we have

$$C_d = [0.78 + 4.7f(\delta)\tilde{U}] \times 10^{-3}. \tag{22}$$

Then, in terms of U_{10}, a unified linear parameterization for C_d can be written as

$$C_d = [0.78 + 0.475f(\delta)U_{10}] \times 10^{-3}. \tag{23}$$

Equation (23) shows that even when the Charnock relation is extended to the form of Eq. (16), which explicitly includes the effect of wind waves, the drag coefficient over the sea surface is still nearly linearly dependent on wind speed. The slope of the linear function is governed by the wave steepness. A positive value of B implies that the drag coefficient increases with wind speed more rapidly for mature waves than for younger waves, while a negative value of B implies the contrary. So far most field measurements suggest that B is negative.

4. Conclusions

For neutral atmospheric stability, combining the logarithmic law of wind profile and the Charnock relation yields a drag coefficient that is a function of the wind speed only with the Charnock coefficient as a parameter. It is found that the function is nearly linear within the usually measured range of drag coefficient of (1.0–2.3) $\times 10^{-3}$. The slope of the linear function is proportional to the square root of the Charnock coefficient. In fact the Charnock coefficients given by various authors are different, and, as a result, the linear functions corresponding to the different Charnock coefficients from different investigators vary across a wide range.

By fitting the observational data, the Charnock relation is extended so that the Charnock coefficient is a function of wave age rather than a constant. The extended Charnock relation is in the form of a power function of wave age. Combining the logarithmic law and the extended Charnock relation with the aid of the 3/2 power law proposed by Toba (1972), it is shown that the drag coefficient is still nearly a linear function of wind speed for the usually measured range of drag coefficient of (1.0–2.3) $\times 10^{-3}$. The slope of the linear function is dominated by the wave steepness.

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