

the criterion of size of mean relative to standard deviation does not apply to a truncated curve, as would be the case with a complete normal curve.

The technique explained briefly in this communication offers a solution to the problem of truncated normal distributions, and is preferable to the use of regression or iterative methods employing probits, both on the score of ease of analysis as well as ease of interpretation.

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<sup>1</sup> Gaddum, J. H., *Nature*, **156**, 463 (1945).

<sup>2</sup> Spiller, D., *Nature*, **162**, 530 (1948).

<sup>3</sup> Yule, G. U., "Statistical Study of Literary Vocabulary" (Cambridge University Press, 1944).

<sup>4</sup> Bliss, C. I., *Ann. App. Biol.*, **24**, 815 (1937).

<sup>5</sup> Finney, D. J., *Nature*, **162**, 898 (1948).

### Heat-Flow Below the Oceans

PROF. HANS PETTERSSON has reported<sup>1</sup> measuring the geothermal gradient at two places in the deep-sea sediment below the Pacific Ocean. This work, done from the schooner *Albatross* of the first Swedish Deep-Sea Expedition, is apparently the first of its kind and is an important contribution to the study of the heat flowing from the interior of the earth.

As is well known, the amount of heat flowing across a plane surface in a thermal conductor is

$$h = -k \frac{\partial T}{\partial n},$$

where  $k$  is the thermal conductivity of the conductor and  $\partial T/\partial n$  is the temperature gradient normal to the surface. During the past ten years, careful work has been done on the problem of determining the heat-flow to the surface of the continents. This is accomplished by measuring both the temperature gradients and the thermal conductivities of rocks of the same locality. It is beginning to appear from this work, carried out for South Africa<sup>2</sup>, Britain<sup>3</sup>, Persia<sup>4</sup>, California<sup>5</sup> and elsewhere, that a normal heat-flow for a quiet (that is, non-volcanic) region of a continent lies in the neighbourhood of  $h = 1.2 \times 10^{-8}$  cal./cm.<sup>2</sup>-sec., with likely departures of plus or minus about 20 per cent.

However, so far nobody has measured the heat-flow below the bottom of the oceans. It has been suggested<sup>6</sup> that this may be significantly less than the continental values, owing to the supposed partial or complete absence under the oceans of the granitic layer, the radioactive content of which evidently supplies a considerable fraction of the continental heat-flow. Prof. Pettersson, in solving the difficult problem of measuring the temperature gradient in deep-sea sediments, has made it possible to begin to determine heat-flow below the oceans. His two values of 22 and 26 metres per degree Centigrade give an average geothermal gradient of about 42° C./km., which he says is unexpectedly high<sup>1</sup>.

It is true that this value would be somewhat high for a continent, but it has been shown in the past that heat-flow in the continents is considerably more uniform than either the conductivities or the geothermal gradients. For example, the mean of the heat-flows measured in Britain<sup>3</sup> is within 20 per cent of that for South Africa<sup>2</sup>, even though the British geothermal gradients on which the calculations are

based exceed those used in the determinations for South Africa by a factor of three or four. The close agreement of the heat-flows arises because the corresponding conductivities differ also by approximately this same factor.

Should the conductivity of the sediments in which Prof. Pettersson made his measurements of geothermal gradient be appreciably controlled by the conductivity of their water content, as seems not unreasonable, his heat-flows may well be less than the continental value; fresh water at 0° C. and atmospheric pressure has the low conductivity of 0.0013 cal./cm.-sec.-° C. Bridgman<sup>7</sup> has shown that a pressure of 1,000 atmospheres increases the conductivity of water by only some 6 per cent. It is, of course, unlikely that deep-sea sediments will prove to have so low a conductivity as that of water, owing to the presence of the solid material. However, a conductivity as high as about 0.003 cal./cm.-sec.-° C. would be necessary in order for the first two ocean heat-flows to be as great as the continental value. It is therefore by no means certain that the relatively high geothermal gradient of 42° C./km. also implies a high heat-flow under the oceans.

It is to be hoped that Prof. Pettersson will measure the thermal conductivities of some of his cores of sediment, collected from the same regions in which he measured the geothermal gradients, in order to find the local heat-flows. The results would be of great interest.

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<sup>1</sup> Pettersson, H., *Nature*, **164**, 468 (1949).

<sup>2</sup> Bullard, E. C., *Proc. Roy. Soc., A*, **173**, 474 (1939).

<sup>3</sup> Benfield, A. E., *Proc. Roy. Soc., A*, **173**, 428 (1939).

<sup>4</sup> Coster, H. P., *Mon. Not. Roy. Astro. Soc., Geophys. Supp.*, **5**, 131 (1947).

<sup>5</sup> Benfield, A. E., *Amer. J. Sci.*, **245**, 1 (1947).

<sup>6</sup> Bullard, E. C., *Nature*, **156**, 35 (1945).

<sup>7</sup> Bridgman, P. W., "The Physics of High Pressures", 314 (New York: The Macmillan Co., 1931).

### Ocean Swell: its Decay and Period Increase

HITHERTO there has been a difference of opinion concerning the role of eddy viscosity in the behaviour of wind waves and swell. On one hand, Suthons<sup>1</sup> ascribes the lengthening of waves in sea and swell and part of the decay of swell to the effect of eddy viscosity. According to Suthons, waves become longer simply because viscosity tends to increase the depth of water affected by the wave motion, since it transfers kinetic energy downwards. This reasoning does not, however, seem convincing. Obviously, longer waves have greater depths of water affected by the wave motion; but this does not mean that, conversely, the waves would become longer if the depth of water having kinetic energy were increased by effects that have nothing to do with the mechanism of wave motion.

Sverdrup and Munk<sup>2</sup>, on the other hand, neglected the effect of viscosity. They ascribe the decay and the period increase of swell entirely to the effect of air resistance, and Sverdrup<sup>3</sup> has presented

theoretical computations in order to show that this effect gives rise to a selective damping, which acts in favour of the longer components of a wave spectrum.

The present communication gives a preliminary account of some computations and considerations which have led us to believe that eddy viscosity is of much more importance for the theory of ocean swell (and also for wind waves) than would appear from the work of Sverdrup and Munk. It is even possible to explain the decay and lengthening of ocean swell by means of selective damping by eddy viscosity alone, and the values of the viscosity coefficient which must then be used in order to obtain a satisfactory agreement with observed rates of decay and of period increase are quite reasonable and even rather small.

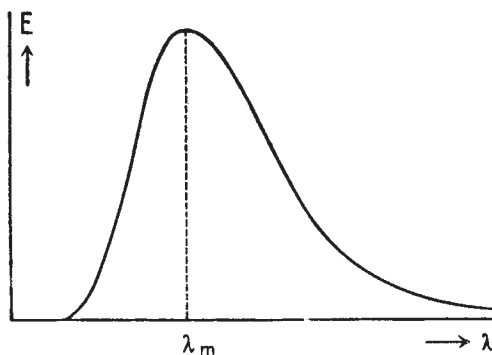
To show this, we must bear in mind that the eddy viscosity coefficient to be used depends on the size of the systems under consideration and is therefore larger for long waves than for short ones. According to Richardson's empirical results<sup>4,7</sup> and von Weizsäcker's<sup>5</sup> recent theoretical deduction, the eddy viscosity  $\mu$  is proportional to the four-thirds power of the linear dimensions of the smallest details considered, so that we may write:  $\mu(L) = mL^{4/3}$ , where  $L$  is the wave-length. Now let  $E_L$  be the energy (per unit area) of a plane monochromatic wave component of length  $L$ . When examining a swell, which is at any fixed place supposed to be quasi-stationary (because of continuous supply of wave energy from the generating area), we have for the rate of variation of  $E_L$  with the distance of decay  $x$  on account of viscosity:

$$\frac{C}{2} \frac{dE_L}{dx} = - \frac{16\pi^2 \mu E_L}{\rho L^2} = - \frac{16\pi^2 m E_L}{\rho L^{2/3}},$$

where  $C$  is phase velocity,  $\rho$  is density. Since, in deep water,  $C^2 = Lg/2\pi$ , we have:

$$E_L = E_L(0) \exp - \alpha x/L^{7/6}, \quad (1)$$

where  $\alpha = 8(2\pi)^{5/2} m \rho^{-1} g^{-1/2}$ .



We shall use  $L^{7/6} = \lambda$  as the independent variable and describe a continuous wave spectrum by means of an energy function  $E(\lambda, x)$ , such that  $E(\lambda, x)d\lambda$  is the wave-energy contained in the part of the spectrum from  $\lambda$  to  $\lambda + d\lambda$ . Now  $E(\lambda, 0)$  will, as a function of  $\lambda$ , have a form of the general type shown in the accompanying diagram. Accordingly, we tried, as an analytical representation of the initial spectrum, the following form:

$$E(\lambda, 0) = A\lambda^{-a} \exp - b\lambda^{-1},$$

where  $a$  and  $b$  are as yet unknown positive constants. From (1) it follows that

$$E(\lambda, x) = A\lambda^{-a} \exp - (b + \alpha x)\lambda^{-1}. \quad (2)$$

The maximum of  $E(\lambda, x)$  is found for  $\lambda = a^{-1}(b + \alpha x) = \lambda_m$ , which we might take to be the dominant or significant value of  $\lambda$ . Hence:

$$\lambda_m(x)/\lambda_m(0) = 1 + \alpha x/b. \quad (3)$$

The total energy is found by integration to be

$$E_{\text{total}} = E_0(1 + \alpha x/b)^{1-a},$$

provided that  $a > 1$ . If we now take this energy to be proportional to the square of the significant wave-height  $H$ , we have apparently

$$H(x)/H(0) = (1 + \alpha x/b)^{(1-a)/2}. \quad (4)$$

It is found that (3) and (4) are in accordance with the semi-empirical relationships found by Sverdrup and Munk<sup>2</sup> if we take  $a = 3.26$ , whereas  $b$ , which has the dimension of  $\lambda$ , depends on the initial significant wave-length, that is, the wave-length at the end of the fetch. The value of  $\alpha$  which fits most of the observations is  $\alpha = 0.85 \times 10^{-3} \text{ (metre)}^{1/6}$ , so that  $m$  is found to be:  $m = 3.5 \times 10^{-3} \text{ kgm.m.}^{-7/6} \text{ sec.}^{-1}$ . This yields for a wave-length of 100 m. a value  $\mu = 1.6 \text{ kgm. m.}^{-1} \text{ sec.}^{-1} = 16 \text{ gm. cm.}^{-1} \text{ sec.}^{-1}$ . Since, according to the theory of wind-driven ocean currents, any wind of 2.5 m./sec. would already be sufficient to maintain such a value of the eddy viscosity, our result seems very reasonable.

The assumption, which has been used in the above deduction, of a stationary state is, of course, an over-simplification. Especially if the duration of the wind in the generating area is not so very long (although long enough to reach a quasi-stationary state within the fetch), a stationary state of the swell in remote places will not be reached, because the long waves will already have died out there before the short ones arrive. Thus, in those cases the energy of the swell will, for sufficiently large values of  $x$ , be smaller than in the quasi-stationary case, so that even smaller values of the eddy viscosity will account for the observed decay.

But how about the air resistance? Sverdrup and Munk<sup>2</sup> ascribed all the energy-loss of swell to air resistance, and their computations based on this assumption seem to support it. There is, however, one more assumption that is essential for those computations, namely, that the value of the so-called 'sheltering coefficient'<sup>6</sup> or 'stream-lining coefficient', which is a measure of the pressure-asymmetry along the wave profile, is the same for swell as for wind waves. This is, however, rather improbable. Indeed, it makes a great difference whether (as in the case of swell in calm air) the air moves only with respect to the wave profile and (practically) not with respect to the water; or, whether it moves also with respect to the water. Since in the first case the resistance due to friction between the air and the water is very much smaller than in the latter case, the pressure-resistance will also be much smaller, especially if we have to do with a long swell of small steepness. It may easily be seen that without friction and viscosity there would even be no pressure-resistance at all. Computation of the pressure-resistance of swell is very difficult; but it may be anticipated that it is, in general, of the same order of magnitude as the resistance due to the (small) friction between the air and the water.

Although we are aware of the fact that our presentation of a computation of swell characteristics on the basis of eddy viscosity considerations is, in more than one respect, very much over-simplified, our



conclusion is that eddy viscosity is the principal factor determining the decay and effective period increase of ocean swell.

The lengthening of wind waves forms a different problem; but it seems probable that here also a selective loss of energy due to turbulence acts in favour of the longer waves.

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<sup>5</sup> von Weizsäcker, C. F., *Z. Phys.*, **124**, 614 (1948).

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<sup>7</sup> Richardson, L. F., and Stommel, H., *J. Meteorol.*, **5**, 238 (1948).

### Spacings of Retained Austenite in Quenched White Cast Iron

LITTLE data has, as yet, been published concerning the spacings of retained austenite in iron-carbon alloys when the quenched sample contains the quantity of carbon ascribed to the saturation point *E* (1.776 per cent carbon at 1,145° C.) shown in the iron-carbon diagram. The only data as yet found refer to samples of plain carbon steel (1 mm. in diameter) containing 1.74 per cent of carbon and quenched at 1,130° C., for which the authors<sup>1</sup> give  $a = 3.613$  and  $3.624$  Å. The first of the samples, according to the authors, was decarburized.

The difficulty in preventing loss of carbon when heating thin rods of high-carbon steel is obvious. Consequently, to provide a comparison between the estimated parameters of retained austenite in samples of high-carbon steel with those of samples that have, after heating, retained sufficient quantity of carbon, a white cast iron was chosen in which the total amount of alloying elements (manganese, silicon, phosphorus and sulphur) was less than 1 per cent, that is, similar in quality and quantity to those in mild plain carbon steels. Samples in the form of flat lumps (a few grams in weight) were quenched from a series of *A*-cementite temperatures. The samples were then polished and examined both microscopically and by X-rays (low-angle glancing reflexion method, cobalt  $K\alpha$  radiation). In certain cases the X-ray photogram was repeated and a pure tungsten powder was imposed on the surface for checking the positions of the lines of retained austenite.

For control of the position of the 220 line of austenite, the 211 line of tungsten ( $\theta = 43^\circ 52'$ ) was used. An additional X-ray photogram was taken with pure powders of tungsten and copper, mixed together with tragacanth to give control by the combined

substance method<sup>2</sup>. The position of the 220 line of copper ( $\theta = 44^\circ 26'$ ) corresponds to  $a = 3.61$  Å.

The use of this method is demonstrated in Fig. 1. It shows a white cast iron quenched at 1,132° C. in brine. The right-hand bottom part of Fig. 1 shows the sample under examination; the right-hand upper part is for the mixture of copper and tungsten powders, while the left-hand part shows the pure copper and pure tungsten photograms imposed on the copper and tungsten mixture X-ray photogram for the identification of the lines. The positions of the 220 line of copper ( $a = 3.61$  Å.) and that of the retained austenite line ( $a = 3.62$  Å.) are easily compared, the latter equalling in parameter that given by Honda and Nishiyama<sup>1</sup> for plain carbon steel quenched from the same temperature (1,130° C.). The quantity of carbon dissolved in both cases, owing to the temperature of quenching<sup>3</sup>, results in the same parameter of retained austenite. Consequently, the assumption that the character of retained austenite in both cases is the same appears to be justified.

One of the samples of white cast iron, when quenched from the semi-liquid state (about 1,180° C.), shows the retained austenite in the form of dots, owing to the large crystals produced during melting (Fig. 2); only two dots represent the 220 line. The X-ray photogram repeated with the tungsten powder deposit shows the 220 line of retained austenite also represented by two dots only (Fig. 3), which overlap with the 211 line of tungsten. The position of the line of retained austenite has moved to the position of the 211 line of tungsten ( $\theta = 43^\circ 52'$ ), giving the parameter of the retained austenite as  $a = 3.64$  Å., that is, more than the value calculated from the formula given by me<sup>3</sup> ( $a = 3.6281$  for 1.776 per cent carbon). The micro-examination of the sample shows that the grains of retained austenite, owing to incipient fusion, have not only grown into large crystals but also are filled with needles of primary cementite. The dots indicating the 220 line of retained austenite are blurred, revealing the lattice distortion of retained austenite. As there is close linkage between the iron atoms in the interfaces between retained austenite and cementite, this distortion and the increase of parameter must be expected.

