Deuxième Partie

NOTES TECHNIQUES

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THE DRIFT CURRENT FROM OBSERVATIONS MADE ON THE BOUEE LABORATOIRE *

by

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Abstract

Wind, current and temperature data collected on the Bouée Laboratoire in the Mediterranean Sea have shown the screen effect of the thermocline on the momentum flux. The application of this hypothesis to the EKMAN's theory in an impulsive system gives a model which explains in quality and in quantity the nature of the phenomena observed during the generation of the drift current in the layer above the thermocline.

Résumé

Les observations de vent, courant et température effectuées à la Bouée Laboratoire en Méditerranée ont nettement confirmé l'effet d'écran de la thermocline vis-à-vis du transfert de la quantité de mouvement. Cette hypothèse appliquée à la théorie d'EKMAN en régime impulsionnel donne un modèle qui rend très bien compte, tant au point de vue qualitatif que quantitatif, des phénomènes observés au cours de l'établissement du courant de dérive dans la couche au-dessus de la thermocline.

INTRODUCTION : THE BASIC HYPOTHESIS

Between 1964 and 1970 numerous observations for the wind, current and temperature were collected on board the Bouée Laboratoire in the Mediterranean Sea, at both points : A, 42° 47' N and 07° 29' E, and B, 42° 14' N and 05° 35' E (Fig. 1). The data reveal two completely different systems according to whether the water is homogeneous from the surface down to the bottom (as in winter : $\Delta\rho/\rho \sim 10^{-4}$ to 10^{-6}) or stratified (as in summer : $\Delta\rho/\rho \sim 10^{-3}$).

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In winter, there is no apparent correlation between wind and current (STANISLAS, 1970 a). This can undoubtedly be explained by the fact that in an almost homogeneous ocean ($\Delta\rho / \rho \leqslant 10^{-5}$) horizontally bounded, the vertical component of the Coriolis' forces cannot be disregarded because, in such conditions, the buoyancy and Coriolis'forces are both of similar magnitude :

$$g\ \Delta\rho/\rho \sim 2\omega u .$$

On the other hand, when surface stratification occurs, the wind-current relation bears a closer ressemblance to the results given by the EKMAN's theory for a homogeneous, unbounded ocean (GONELLA, 1970). Nevertheless, even if the theory can be accepted as valid for an ocean with constant stratification, it does not account for all observed phenomena. We have in fact noticed in all seasons except winter :

- almost permanent inertial oscillations,
- the amplitude of the mean drift current decreasing,
- the current deflected at an angle of more than 45° from the wind direction at the surface and increasing with depth (GONELLA, 1968, 1969 ; STANISLAS, 1970 b),
- no coherence between currents on both sides of the thermocline (GONELLA, CREPON and MADELAIN, 1969),
- and finally, during the generation of the drift current, the importance of the inertial component that has the same magnitude throughout the whole surface layer.

The problem therefore consists in determining which basic hypothesis offers a suitable model that explains most of these phenomena. Contributions have been made by many authors completing EKMAN's theory in finite depth, with friction on the bottom or an eddy viscosity coefficient varying with depth (FJELDSTAD, 1930 ; NOMITSU, 1933). The various models do not seem applicable to the system observed at the locations of the Bouée Laboratoire when stratification occurs. The screen effect of the thermocline on the momentum flux has caused us to examine EKMAN's theory in the case of a two-layers ocean where sliding occurs without friction at the interface, at the level of the thermocline. We will treat the instance of the permanent system and the instance of the impulsive system in turn. While confronting the observations, we shall compute the eddy viscosity coefficient and the drag coefficient that intervene in the calculation of wind stress.

PERMANENT SYSTEM

In the equations of motion, undimensional quantities will be used. The inertial period T will be taken as unit of time :

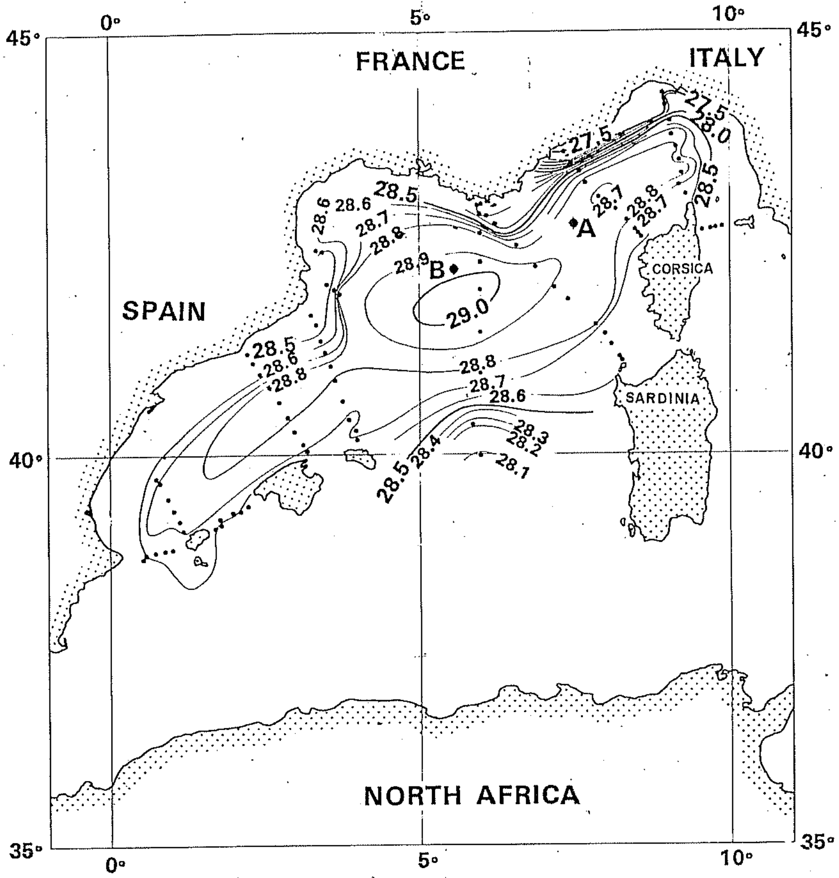

Figure 1 - Location of the "Bouée Laboratoire", positions in the Mediterranean Sea. The contours shown are surface isopycnals obtained by the research vessel "ELIE MONNIER" in February - March 1960. The "Bouée Laboratoire" was moored at position A (42° 47' N, 7° 29' E) from February 1964 to May 1968, and since it has been moored at position B (42° 14' N, 5° 34' E).

$$T = \pi/\omega \sin \Phi = 1 \qquad \text{thus} \qquad f = 2\,\omega \sin \Phi = 2\pi$$

where Φ is the latitude and ω the velocity of the earth's rotation. Additionally, the depth $D = 2\,(\pi\,\nu\,T)^{1/2}$, equal to twice the EKMAN's depth D_E will be taken as unit of length ; ν is the coefficient of eddy viscosity assumed constant (here $\nu = 1/4\pi$ with the chosen units).

Suppose we have a level ocean, horizontally unbounded, with a homogeneous upper layer, h deep, sliding without friction on the underlying layer. With EKMAN's usual hypothesis, the equation of motion for the permanent system in the Northern hemisphere is written :

$$2\,i\,\pi\,u - (1/4\,\pi)\,\partial^2 u/\partial^2 z = 0 \qquad (1)$$

where u is the complex velocity of the current.

The conditions at the boundaries are :

- on the one hand, at the surface $z = 0$ the wind stress is constant and permanent, thus $\partial u/\partial z = 4\,\pi\,\tau_o$ where τ_o is not the real stress but the rate between the wind stress by the density of the water ;

- on the other hand, a perfect sliding at the interface $z = -h$: $\partial u/\partial z = 0$.

By writting that the general solution

$$u_h(z) = A\,e^{-2\pi(1+i)|z|} + B\,e^{2\pi(1+i)|z|}$$

satisfies the conditions at the boundaries, one obtains

$$A = \frac{\sqrt{2}}{2}\,\tau_o\,e^{-i\pi/4}\,(e^{4\pi h} - e^{i4\pi h})\;/\;(\mathrm{ch}\,4\pi h - \cos 4\pi h)$$

and

$$B = \frac{\sqrt{2}}{2}\,\tau_o\,e^{-i\pi/4}\,(e^{-i4\pi h} - e^{-4\pi h})\;/\;(\mathrm{ch}\,4\pi h - \cos 4\pi h) \; .$$

In particular, for $z = 0$,

$$u_h(0) = A + B = \sqrt{2}\;\tau_o\,e^{-i\pi/4}\;\frac{\mathrm{sh}\,4\pi h - i\sin 4\pi h}{\mathrm{ch}\,4\pi h - \cos 4\pi h}$$

a graphic representation is given in Fig. 2. When h becomes small, the current becomes infinitely great at an angle of 90° to the right of the wind stress ($\sim \tau_o\,e^{-i\pi/2}/\,2\pi h$).

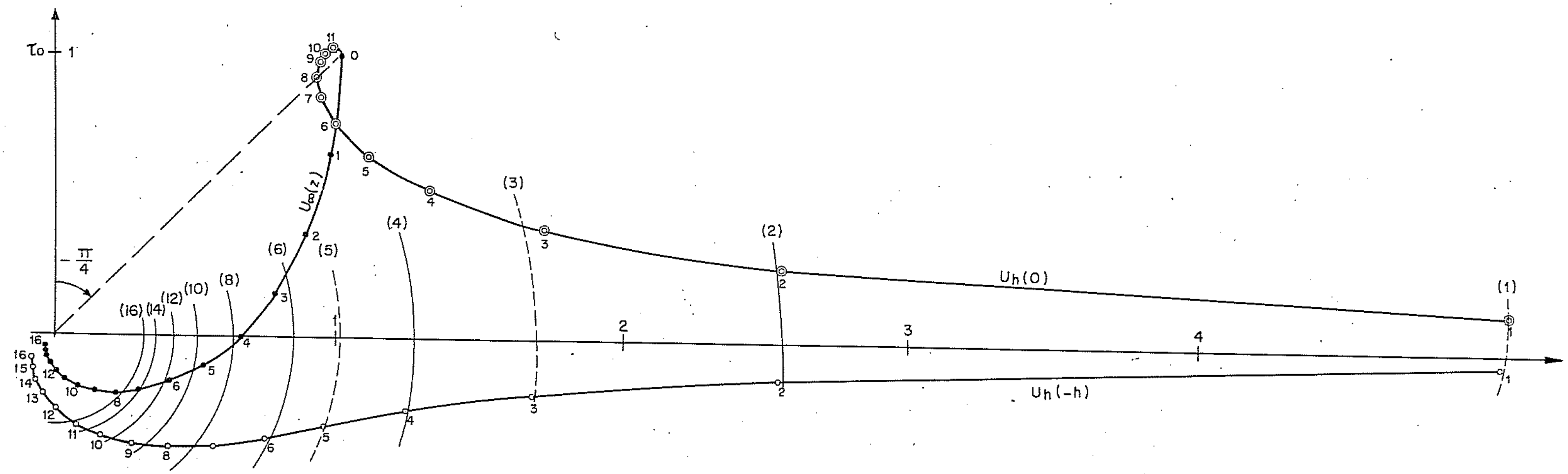

Figure 2 - The wind stress τ_o is taken as being equal to one. The number n is proportional to the depth of the thermocline $h = n\ D/32$. The curve $u_\infty(z)$ is the horizontal projection of the EKMAN's spiral. The curve $u_h(o)$ showns the variations of the surface drift current as a function of n . The curve $u_h(-h)$ gives the amplitude of the drift current at the level of the thermocline as a function of its depth. For a given depth of the thermocline (given n), the radius of the corresponding circle gives the amplitude of the inertial component.

On the other hand, when h increases, $u_h(0)$ moves rapidly towards the limiting value of the EKMAN's spiral ($\sim \sqrt{2}\ \tau_o\ e^{-i\pi/4}$).

For $z = -h$ we have :

$$u_h(-h) = \sqrt{2}\ \tau_o\ e^{-i\pi/4}\ \frac{e^{2\pi(1-i)h} - e^{-2\pi(1-i)h}}{\text{ch}\ 4\pi h - \cos 4\pi h}$$

The graphic representation is also given in Fig. 2. When h moves towards zero, $u_h(-h)$ becomes infinitely great, like $u_h(0)$:

$$u_h(-h) \sim u_h(0) \sim \tau_o\ e^{-i\pi/2} / 2\pi h$$

On the other hand, when h becomes greater,

$$u_h(-h) \sim 2\ u_\infty(-h) \qquad \text{where} \qquad u_\infty(z) = \sqrt{2}\ \tau_o\ e^{-i\pi/4}\ e^{-2\pi(1+i)|z|}$$

is EKMAN's solution for an infinitely deep ocean ; in other words, at the level of the thermocline, the current would be roughly equal to twice the current at the same depth h of the EKMAN's spiral.

It should be pointed out that the function $u_h(z)$ defined in the interval $(-h, +h)$ by taking its symmetric about $z = 0$ can be written in Fouri serie :

$$u_h(z) = \sum_{n=-\infty}^{+\infty} a_n\ e^{in\pi z/h}$$

with :

$$a_n = (1/2h) \int_{-h}^{+h} u_h(z)\ e^{-in\pi z/h}\ dz$$

thus

$$a_n = (\tau_o/2\pi h) / (i + n^2/8h^2) \ .$$

This other expression of $u_h(z)$ would have been directly obtained by treating the equation (1) at the sense of distributions, by inserting into the right side of the equation the stress distribution obtained from the "image" theory : $2\,\tau_o \sum_{n=-\infty}^{+\infty} \delta(z - 2nh)$ where δ is the DIRAC delta-function.

With the two different expressions of $u_h(z)$ one can easily check that the mass transport

$$R = \int_{-h}^{o} u_h(z)\,dz = \tau_o\, e^{-i\pi/2} / 2\pi$$

which is identical to the result obtained for an ocean of infinite depth.

IMPULSIVE SYSTEM

1. Equation of motion and general solution

The drift current phenomenum in situ, far from being permanent is rather, transient and impulsive. Taking the same hypothesis as above, the wind stress is assumed to vary as the time, and at initial time the ocean to be at rest. The equation of motion, treated in the sense of distributions, can be written :

$$\partial u/\partial t + 2i\pi u - (1/4\pi)\,\partial^2 u/\partial z^2 = 2\tau(t) \sum_{n=-\infty}^{+\infty} \delta(z-2nh) \qquad (2).$$

It has been shown elsewhere (GONELLA, 1970) that the elementary solution corresponding to the distribution $\delta(z)\,\delta(t)$ is :

$$E_\infty(z,t) = Y(t)\, t^{-1/2}\, e^{-\pi z^2/t}\, e^{-2i\pi t}$$

where $Y(+)$ is the Heaviside function ; in other terms, it is the response of an ocean of infinite depth to a wind stress impulse 1/2 on the surface at zero time. The elementary solution for (2) is directly derived from E_∞ by convolution with

$$\sum_{n=-\infty}^{+\infty} \delta(z - 2nh) = (1/2h) \sum_{n=-\infty}^{+\infty} \delta(z/2h - n)$$

thus
$$E_h(z,t) = (1/2h)\ E_\infty(z,t) \underset{(z)}{*} \sum_n \delta(z/2h - n)$$

where the symbol $*$ is the convolution product. Now, a property of the serie $\sum_n \delta(z/2h - n)$ where n is an integer (+ or -) is that is equals $\sum_n e^{i2\pi nz/2h}$; one can also write

$$E_h(z,t) = (1/2h)\ E_\infty(z,t) \underset{(z)}{*} \sum_n e^{in\pi z/h} ,$$

and since :

$$t^{-1/2}\, e^{-\pi z^2/t} \underset{(z)}{*} e^{in\pi z/h} = \int_{-\infty}^{+\infty} e^{-\pi J^2/t}\, e^{in\pi(z-J)/h}\, t^{-1/2}\, dJ$$

$$= e^{-\pi tn^2/4h^2}\, e^{in\pi z/h} ,$$

we have for $E_h(z,t)$ the expression in Fourier serie :

$$E_h(z,t) = \frac{Y(t)\ e^{-2i\pi t}}{2h} \sum_{n=-\infty}^{+\infty} e^{-\pi tn^2/4h^2}\, e^{inz/h} .$$

Finally, the solution that corresponds to the original problem is derived from the elementary solution $E_h(z,t)$ by convolution with the stress $2\tau(t)$ thus :

$$u_h(z,t) = 2\tau(t) \underset{(t)}{*} E_h(z,t)$$

Similarly, it can be shown that, when the case of an initial velocity distribution $u_h(z,0)$ in the upper layer occurs, the general solution equals :

$$u_h(z,t) = 2\,\tau(t) \underset{(t)}{*} E_h(z,t) + u_h(z,0) \underset{(z)}{*} E_h(z,t)$$

2. The particular case in which $\tau(+) = \tau_o\, Y(t)$

A wind with constant direction and amplitude blows over an ocean initially at rest. The solution is written :

$$u_h(z,t) = 2\,\tau_o \int_o^t E_h(z,\theta)\, d\theta$$

hence $$u_h(z,t) = \frac{\tau_o}{2\pi h}\left[\sum_{n=-\infty}^{+\infty} \frac{e^{-b(n)2\pi\theta}}{-\,b(n)}\, e^{in\pi z/h}\right]_o^t$$

with $$b(n) = i + n^2/8h^2$$

and further : $$u_h(z,t) = u_h(z) - i_h(z,t)$$

where $$u_h(z) = \frac{\tau_o}{2\pi h} \sum_n e^{in\pi z/h} / b(n)$$

is the solution for the permanent system and,

$$i_h(z,t) = \frac{\tau_o}{2\pi h}\, e^{-2i\pi t} \sum_n \frac{e^{-\pi t n^2/4h^2}}{b(n)}\, e^{in\pi z/h}$$

the inertial component (term in $e^{-2i\pi t}$). It should be noticed that $i_h(z,t)$ can also be written :

$$i_h(z,t) = \frac{\tau_o\, e^{-i\pi/2}}{2\pi\, h}\, e^{-2i\pi t} \left(1 + 2i \sum_{n=1}^{+\infty} \frac{e^{-\pi t n^2/4h^2}}{b(n)} \cos n\pi z/h\right).$$

When t is sufficiently great : $i_h(z,t) \sim (R/h)\, e^{-2i\pi t}$

and
$$u_h(z,t) \rightarrow u_h(z) - (R/h)\, e^{-2i\pi t}$$

When a wind starts blowing over a two-layer ocean with interface sliding, a drift current is generated in the upper layer comprising :

- a mean current that decreases and is deflected with depth ($u_h(z)$: the solution for the permanent system).

- and an inertial component - $i_h(z,t)$ which tends to acquire the same value at every level.

It should be noted by integrating the velocities in the surface layer that the result is the same as that obtained by CREPON (1967) ;

$$R(t) = R\,(1 - e^{-2i\pi t}) .$$

However, if, at initial time, there was an initial velocity distribution $u_h(z,0)$ the following component would be added to the inertial component :

$$u_h(z,0) \underset{(z)}{*} E_h(z,t) \sim \frac{R(0)}{h}\, e^{-2i\pi t}$$

when t is sufficiently great, with $R(0) = \int_{-h}^{0} u_h(z,0)\, dz$.

CONCLUSION - COMPARISON WITH OBSERVATIONS

Here our interest lies more with the generation of the drift current itself than with the behaviour of the inertial current after the gale. A study of the latter would have to take into account the spatial limitation of the wind field and introduce "dissipation factor" caused by the "radiation" of the inertial oscillation (CREPON, 1967 a, b ; POLLARD and MILLARD, 1970).

A serie of relatively complete data for the wind, currents and temperatures at differents levels were obtained during two particular periods : october 26th - 27th 1966 (GONELLA, CREPON, MADELAIN, 1969) and July 29th - 30th (STANISLAS, 1970, b). The meteorological and oceanographical observed conditions are summarized in tables I and II.

From these elements we can ascertain that the proposed hypothesis corroborates the observations, especially for :

- the amplitude of the inertial current that has the same magnitude throughout the surface layer,

- the attenuation of the mean current as a function of the depth and a closely marked deflection to the right from the wind direction.

It can be noticed that the inertial amplitude is distinctly higher than the mean drift current ; this can be explained by the presence of a velocity distribution $u_h(z,o)$ that is not at zero at the onset of the wind.

The observed angular deviations enable us to compute the "eddy viscosity coefficient" ν . In October 1966, we have

$$\text{Arg } u_h(-h) - \text{Arg } u_h(o) \sim 50°$$

which gives a $h \sim (5/32)\,D$ and further a depth $D \sim 190$ m and a eddy viscosity coefficient $\nu = D^2 f/8\pi^2 \sim 500$ CGS. In July, an angular deviation of 70° between the surface and the depth $h \sim 20$ m give a $D \sim 110$ m and a $\nu \sim 120$ CGS. The turbulence in the surface layer is greater in October than in July. This difference may be accounted for by the fact that the homogeneity, in the upper layer is greater in October than in July because of the greater amount of mixing and cooling (sea tables I and II). Thus, the angular deviation between two levels can be used to represent the turbulence in the layer under study.

Furthermore, the same observations enable us to compute the drag coefficient. The observations made in 1968 enabled us to measure a mean current that was proportional to wind velocity squared (GONELLA, 1969). It would be fair to say that $\tau^{(1)} = \rho_{air}\, C_z\, V^2(z)$ when C_z is the drag coefficient of the wind $V(z)$ measured at the level z . And since $\tau = \rho_{eau}\, Rf$ one obtains $C_z = \rho_{eau}\, Rf/\rho_{air} V^2$. In the two instances studied above, this give us a $C_{14} \sim 11.10^{-4}$ which corresponds to a coefficient $C_{10} \sim 12.10^{-4}$; the incertitude of this determination corresponds

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(1) Here the usual scalar dimensions are used.

TABLE I

Gale of October 26th - 27th, 1966 (Point A).

Average values taken over one inertial period (T ∿ 17 h 30)

Beginning 26.10 - 1966, 12 h 00

	Level	Wind (m/s) or mean current (cm/s)	Direction in which wind or current is heading	Angular deviation from wind direction	Mean amplitude inertial current (cm/s)	Mean tempe-rature (°C)
AIR	+ 14	17	80°	0°	-	15,4
WATER	- 5	17,5	165°	85°	38	19,22
	- 15	12,5	190°	110°	37	19,21
	- 20	12,0	190°	110°	37	19,20
	- 25	11,0	200°	120°	36	X
	- 30	X	X	X	X	15,50
	- 40	X	X	X	X	13,58

X No observation
- Mean level of the thermocline : 30 m
- Mean velocity in the surface layer : 12,5 cm/s heading to 170°
- General current weak both before and after the gale.

to the incertitude of the thermocline depth. Here it must be pointed out that the drag coefficient determined from the May 1966 observations (C_{10} ∿ 30.10^{-4}; GONELLA 1968) did not take into account the screen effect of the stratification, which, though weak, was nevertheless present in that season ; the resultant value must be reduced by at least half, and one then obtains a magnitude of C_{10} which corroborates the above values and the value found by wind profiles (C_{10} ∿ 12.10^{-4} LACOMBE, GONELLA, ESKENAZI, 1966).

Thus the hypothesis of sliding layers at the level of the thermocline gives us model to enable us to understand and compute the behaviour of the curren

TABLE II

Gale of July 29th - 30th, 1969 (Point B)

Average values taken over two periods beginning 29.7.1969

10 h 00

	Level (m)	Wind (m/s) or mean current (cm/s)	Direction in which wind or current is heading	Angular deviation from wind direction	Mean amplitude inertial current (cm/s)	Mean temperature (2) (°C)
AIR	+ 14,5	14,5	140°	0°	-	20,7
WATER	- 5	19	225°	85°	50	20,2
	- 10	18	240°	100°	45	19,7
	- 15	16,5	270°	130°	40	19,4
	- 30 (1)	∿ 4	0°	220°	< 10	15,2
	- 60	∿ 4	330°	190°	< 2	13,1

- Mean level of the thermocline : 20 m
- Mean velocity in the surface layer : 16 cm/s heading towards 230°
- General current weak (< 4 cm/s) before the gale ; no observation after the gale.
- (1) The mean current at 30 m was computed during the first inertial period in the gale ; during the second period the level rises occasionally into the surface layer (internal wave).
- (2) Here, the accuracy is only ± 0,1°C.

at the "Bouée Laboratoire" during an atmospheric disturbance. However, the comparison of the drag coefficients obtained from the air and water profiles methods raises a problem with regard to the momentum flux. The C_{10} values obtained by both methods (air and water) have in fact approximately the same magnitude, and most of the energy from this momentum transfert would therefore be transformed into current. The amount of energy thereby released to the waves by the tangential wind stress at the surface would have approximately the same magnitude as the incertitude on the determination of the drag coefficient. It seems henceforth established that a more detailed study of the genesis of surface currents should incorporate wave genesis.

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