Estimation of Spectra from Speckled Images

ANDREW D. GOLDFINGER, Member, IEEE

The effect of coherent speckling on the spectra of images and other signals is investigated. A method for estimating the spectrum of the unspeckled image is developed, and the errors inherent in such an estimate are analyzed. It is found that the error decreases when number of looks, number of averaged spectra, and contrast increase, and when spectral width decreases.

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Author's address: Applied Physics Laboratory, The Johns Hopkins University, Johns Hopkins Road, Laurel, MD 20707.

I. INTRODUCTION

Images and signals produced by coherent systems are subject to the phenomenon of speckle [1]. The cause is interference between wavelets returned from the various scatterers within the resolution element of the system. The result can be looked at as a form of multiplicative noise [2-5] with standard deviation equal to its mean [1].

This noise can be particularly troublesome due to the large standard deviation, and various schemes have been proposed to deal with it, such as time or frequency diversity [1], multilook processing (incoherent averaging) [6], or various types of linear [2-5] or adaptive [7] filters. These efforts have generally been directed toward improvement of the signal in the time or image domains. However, applications exist in which the spectrum of the output is of primary interest. For example, synthetic aperture radar (SAR) images provided by the SEASAT satellite are under intensive analysis for the purpose of studying ocean wave spectra [8, 9], and even in linear or adaptive filtering aimed at image improvement, it would be useful to have a good estimate of the underlying image spectrum rather than working from an a priori assumption such as is often done [7].

Below we address the problem of estimating signal and image spectra from speckled data. We first discuss the effects of speckle and then consider the method and errors involved in making the estimate.

II. THE NATURE OF SPECKLE

A useful model of speckle is given by Goodman [1]. Based upon this model, assume that our signal z_i (defined on a discrete time domain) is subject to a multiplicative noise process v_i so that the speckled signal z'_i results

$$\boldsymbol{z}_i' = \boldsymbol{z}_i \, \boldsymbol{v}_i. \tag{1}$$

Although we will work in one dimension, our results are easily extended to two-dimensional images.

The speckle process v_i is governed by the "multilook Rayleigh" distribution; that is, it can be looked upon as the sum of *n* independent exponentially distributed variates, *n* being the number of looks. The probability distribution for each of the v_i is thus

$$P_n(\nu) = [\nu^{n-1}/(n-1)! I^n] e^{-\nu/l}$$
(2)

where $\langle v \rangle = nI$. A general expression for the moments of this distribution is

$$M_k^n = \left[(n+k-1)!/(n-1)! \right] I^k$$
(3)

where M_k^n is the k-th moment of the n-look distribution. Some important moments and the corresponding standard deviations are listed in Table I. We see that, for

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TABLE I.

	Number of Looks (n)				
	1	2	3	п	
<i>M</i> "	I	21	31	nl	
M_2^n	2 1 ²	6 1 ²	12 <i>1</i> ²	$n(n + 1)I^2$	
σ	Ι	$\sqrt{2I}$	$\sqrt{3I}$	$\sqrt{n I}$	

single look noise, the standard deviation is equal to the mean. This is, therefore, an extremely bothersome type of noise.

III. CONTRAST

We wish to estimate the spectrum of a signal present in an "image" that has been speckled by the process described above. Since images represent power, and can never be negative, they can be separated into "ac" and "dc" parts. The presence of a dc bias introduces the idea of contrast, which we define below.

Fig. 1 defines the parameters A and H for a sinusoidal signal. One definition of contrast that is often used in the synthetic aperture radar literature is

$$C \equiv A/H. \tag{4}$$

By this definition $C \le 1$ for nonnegative signals, with C = 1 corresponding to a sinusoidal signal whose minimum value is zero. Since images represent an intensity that is always nonnegative, their contrast is never greater than 1. For signals not so limited, the contrast can be greater. For example, a sinusoid of zero mean has a contrast of 2.

For signals more complicated than sinusoids, it is necessary to come up with a different definition of contrast. We will choose a definition that is consistent with the one above in the special case of sinusoidal signals. A sinusoid

$$f(t) = a \cos \omega t \tag{5}$$

has mean power (variance)

$$\sigma_t^2 = a^2/2 = A^2/8. \tag{6}$$

Combining (6) with (4) suggests that we define

$$C = 2\sqrt{2}\sigma_f/(\langle f \rangle + \sqrt{2}\sigma_f) \tag{7}$$

for general signals, where o_i^2 is the average ac power, and $\langle f \rangle$ is the dc signal level. For signals with zero mean (dc level), C = 2. Any signal $g_c(t)$ with contrast C can then be written in terms of another signal g_2 with contrast 2 (zero mean) as

$$g_c(t) = g_2(t) + (1/C - \frac{1}{2}) 2\sqrt{2}\sigma_g$$
 (8)

where σ_g^2 , the average ac power, is the same for g_c

and g_2 . The contrast C and contrast 2 signals differ in bias level. The relationship between their Fourier transforms is as follows

$$\widetilde{g}_{c}(k) = \begin{cases} (1/C - \frac{1}{2}) 2\sqrt{2} \sigma_{s}, & k = 0 \\ \\ \widetilde{g}_{2}(k), & k \neq 0 \end{cases}$$
(9)

where the tilde symbol indicates Fourier transform and

$$\tilde{g}_2(0) = 0.$$
 (10)

The power spectral densities are related as

$$S_{c}(k) = \begin{cases} 2[(2-C)/C]^{2} \sigma_{g}^{2}, & k = 0\\ S_{2}(k), & k \neq 0. \end{cases}$$
(11)

IV. SAMPLED SIGNAL

We define the discrete Fourier transform (DFT) as

$$\widetilde{f}_{k} \equiv (1/N) \sum_{j=0}^{N-1} f_{j} e^{-(2\pi i/N)jk}.$$
(12)

For a sampled signal, defined on a "circular" domain of length N, Parseval's theorem states that

$$(1/N) \sum_{i=0}^{N-1} |f_i|^2 = \sum_{k=0}^{N-1} |\tilde{f}_k|^2.$$
(13)

If the signal g_c of contrast C is sampled, we thus find

$$\sigma_g^2 = \langle |g_c|^2 \rangle - \langle g_c \rangle^2 \tag{14}$$

$$= \sum_{k=0}^{N-1} S_2(k) = \sum_{k=1}^{N-1} S_c(k).$$
(15)

V. EFFECT OF SPECKLING

In what follows we will be discussing three signals: g_2 , which has zero mean, g_c which has a bias added so that it has contrast C, and g'_c which is the coherently speckled version of g_c . The relationship between g_2 and g_c is given in (8); that between g_c and g'_c is

$$g_c'(i) = g_c(i) v_i \tag{16}$$

where v_i is the multilook Rayleigh process described above.

Consider the signal

$$z_i' = z_i v_i. \tag{17}$$

Transforming this we get

$$\widetilde{z}'_{k} = \sum_{p} \widetilde{z}_{k-p} \widetilde{\nu}_{p}.$$
(18)



Fig. 1. Contrast parameters.

The power spectral density of the speckled signal is then given by

$$S'_{zz}(k) \equiv \langle \tilde{z}_{k}^{*} \, \tilde{z}_{k} \rangle = \sum_{p} \sum_{q} \langle \tilde{z}_{k-p}^{*} \, \tilde{z}_{k-q} \rangle \langle \tilde{v}_{p}^{*} \tilde{v}_{q} \rangle \quad (19)$$

where the brackets indicate the ensemble average, and z and v are assumed to be independent.

With regard to (19) we note that, in general,

$$\langle \tilde{z}_k^* \tilde{z}_\ell \rangle = (1/N^2) \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \langle \tilde{z}_n^* \tilde{z}_m \rangle e^{-(2\pi i/2N) (\ell_m - k_n)}.$$
 (20)

For a stationary random process,

$$\langle z_n^* z_m \rangle = R_{ss}(n-m) \tag{21}$$

where R_{zz} is the autocorrelation function of z. Making appropriate changes of variables and noting that all arithmetic is modulo N, we can insert (21) into (20) to get

$$\langle \tilde{z}_{k}^{*} \tilde{z}_{l} \rangle = (1/N) \, \delta_{kl} \sum_{r=0}^{N-1} R_{ss}(r) \, e^{(2\pi i/N)kr}$$
(22)

$$= \delta_{k\ell} S_{zz}(k) \tag{23}$$

where $S_{iz}(k)$ is the power spectral density of z. Putting (23) into (19) we find

$$S'_{ii}(k) = \sum_{p} S_{ii}(k-p) S_{vv}(p)$$
(24)

where $S_{\nu\nu}$ is the spectral density of ν .

Thus, as long as z_i is stationary, the power spectral density of z'_i will be the convolution of the power spectral densities of z_i and v_i . Let us assume that a(i) is the normalized autocorrelation function of v. Then

$$\langle v_i v_j \rangle = n^2 I^2 \left[1 + (1/n) a(i-j) \right]$$
 (25)

and

$$S_{\nu\nu}(k) = n^2 I^2 \left[\delta(k) + (1/n) \tilde{a}(k) \right].$$
 (26)

Inserting this into (24) we see that, in general,

$$S'_{c}(k) = n^{2}P\left\{S_{c}(k) + (1/n)\sum_{l=0}^{N-1} S_{c}(l)\tilde{a}(k-l)\right\}.$$

We will consider three special cases.

Case 1. Uniform Target. If the target is uniform, the z process power spectrum has only a dc component

$$S_0(k) = K \,\delta(k) \tag{28}$$

where K is a constant and we have indicated by the subscript that a dc signal has zero contrast. The power spectrum of the speckled image is then

$$S'_{0}(k) = K n^{2} I^{2}[\delta(k) + (1/n) \tilde{a}(k)].$$
⁽²⁹⁾

Since this equation can be solved to give \tilde{a} in terms of S'_0 , it is possible to determine the speckle process autocorrelation function when the target is known to be uniform. The significance of this lies in the fact that the speckle correlation is often produced by the imaging system being used, and thus this property of the system can be studied by deliberately viewing a uniform target. This technique has been used in the analysis of SEASAT-A image spectra [10].

Case 2. White (Uncorrelated) Speckle. In this case

$$a(i) = \delta(i) \tag{30}$$

$$\tilde{a}(k) = 1/N \tag{31}$$

yielding

$$S'_{c}(k) = n^{2}I^{2} \{S_{c}(k) + (1/n)\overline{S}_{c}\}$$
(32)

where we have defined the average spectral power

$$\overline{S}_{c} \equiv (1/N) \sum_{l} S_{c}(l).$$
(33)

The ensemble mean power spectral density (psd) of the speckled signal is proportional to that of the unspeckled signal, but has added to it an overall bias proportional to the average psd (average taken over all frequencies). Hence, a psd that is sharply peaked, such as that shown in Fig. 2, will stand out more strongly against the bias than will one that is broader. In these figures, (32) implies that the areas are related as

$$A' = (1/n)A.$$
 (34)

In a later section we will show that the residual noise in the psd estimate is proportional to the psd itself. Therefore, the broad psd will be more corrupted by noise due to the "bias" than will be the sharp psd. The latter is thus easier to detect and measure.

Case 3. Nearest Neighbor Correlation. We assume that nearest neighbor sample points are correlated to some extent. That is, we let

$$a(i) = \delta(i) + \varepsilon \,\delta(i-1) + \varepsilon \,\delta(i+1) \tag{35}$$

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Fig. 2. Effects of speckling

so that

$$\tilde{a}(k) = (1/N) [1 + 2\varepsilon \cos(2\pi k/N)]$$
 (36)

and

$$S'_{c}(k) = n^{2}I^{2} \{S_{c}(k) + (1/n) \overline{S}_{c} + (2\epsilon/nN) \sum_{\ell=0}^{N-1} S_{c}(\ell) \cos\frac{2\pi}{N} (k-\ell)$$
(37)

a relationship much more complicated than that of (32).

It is important to note that, while uncorrelated speckle merely added a bias to the spectrum, correlated speckle adds power that is not uniform in wavenumber, thus changing the shape of the spectrum. A common practice in viewing image spectra is to use a pseudocolor display. When this is done, the choice of colors usually in effect subtracts a bias from the spectrum, so that the observer of a spectrum corrupted by uncorrelated speckle does indeed perceive the main features of it correctly. The observer of a spectrum corrupted by correlated speckle, however, may perceive features which are more characteristic of the speckle process than they are of the underlying unspeckled image spectrum.

The question of how the underlying spectrum is best to be estimated is taken up in the next section.

VI. ESTIMATION OF SPECTRUM

To estimate the spectrum S_c from the speckled spectrum S'_c , it is necessary to invert (24). Using (26) and the definition of the Fourier transform, (24) becomes

$$S'_{c}(k) = n^{2}I^{2} \Big\{ S_{c}(k) + (1/n) \sum_{j} a(j) e^{-(2\pi i/N)jk} \\ \cdot (1/N) \sum_{\ell} S_{c}(\ell) e^{(2\pi i/N)j\ell} \Big\}.$$
(38)

Defining

$$\mathcal{G}_{c}(k) \equiv (1/N) \sum_{\ell} S_{c}(\ell) e^{(2\pi i/N)j\ell}$$
(39)

(we note that \mathscr{G}_c is *neither* the Fourier transform nor

inverse transform of S_c according to our notation, hence the new definition and variable), (38) becomes

$$S'_{c}(k) = n^{2}I^{2} \{ S_{c}(k) + (1/n) \sum_{j} a(j) \mathcal{G}_{c}(j) e^{-(2\pi i/N) kj} \}.$$
(40)

Multiplying (40) by $e^{(2\pi l/N) \ell k}$ and summing over k yields

$$\mathscr{G}_{c}'(\ell) = n^{2} I^{2} \{ \mathscr{G}_{c}(\ell) + (1/n) a(\ell) \mathscr{G}_{c}(\ell) \}$$
(41)

so that

$$\mathscr{G}_{c}(\ell) = (1/n^{2}I^{2}) \{ n/[n + a(\ell)] \} \mathscr{G}_{c}'(\ell).$$
(42)

Putting (42) into (40) and solving for S_c , we obtain the desired spectral estimate:

$$S_{c}(k) = (1/n^{2}I^{2}) \left\{ S_{c}'(k) - \sum_{j=0}^{N-1} \left\{ a(j) / [n + a(j)] \right\} \mathcal{G}_{c}'(j) e^{-(2\pi i/N) k j} \right\}.$$
(43)

Equation (43) is our general result. Having used it to estimate the psd of the contrast C signal, we can then go on to estimate the psd of the zero bias signal (from (11)):

$$S_{2}(k) = \begin{cases} 0, & k = 0 \\ S_{c}(k), & k \neq 0. \end{cases}$$
(44)

We consider two special cases.

Case 1. White (Uncorrelated) Speckle. In this case $a(i) = \delta(i)$ so that (43) becomes

$$S_c(k) = (1/n^2 I^2) \{S'_c(k) - [1/(n + 1)] \mathcal{G}'_c(0)\}.$$
(45)

But

$$\mathscr{G}_{c}'(0) = (1/N) \sum_{\ell} S_{c}'(\ell) = \overline{\mathbf{S}}_{c}' \qquad (46)$$

so that (45) becomes

$$S_{c}(k) = (1/n^{2}I^{2}) \{S_{c}'(k) - [1/(n+1)] \overline{S}_{c}'\}$$
(47)

which is a result we could also have derived directly from (32).

Thus estimation of the spectrum in the case of uncorrelated speckle is accomplished by subtraction of an appropriate bias.

Case 2. Nearest Neighbor Correlation. Putting (35) for a(i) into (43) yields

$$S_{c}(k) = (1/n^{2}I^{2}) \left\{ S_{c}'(k) - [1/(n + 1)] \overline{S}_{c}' - [2\epsilon/(n + \epsilon)] (1/N) \sum_{l=0}^{N-1} S_{c}'(l) \cos(2\pi/N) (k - l) \right\}.$$

(48)

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Therefore, a priori knowledge of the speckle autocorrelation function, such as is available through the study of uniform targets (see (29) above), allows the underlying image spectrum to be estimated. The correction is, however, more complicated than merely the subtraction of a bias. For cases in which correlation exists beyond nearest neighbors, this process is easily extended, with (48) gaining added terms similar to the last one: convolutions between S'_c and cosines of various orders.

VII. ERROR IN ESTIMATION

Equation (43) gives the spectral estimate in terms of the spectrum of the speckled signal. The speckled signal is a random process, and therefore this estimate will have a noiselike character. The estimates given above will be the mean value of this random process, but what will be its standard deviation? To answer this, we will first consider the probability distribution of the $S'_c(k)$.

Consider a speckled process

$$\xi_i = x_i \nu_i \tag{49}$$

in which the v_i are as described in (16) and x_i is a single realization of a real nonnegative stationary random process (the unspeckled signal). The psd is given by

$$S'(k) = \langle \widetilde{\xi}_k^* | \widetilde{\xi}_k \rangle.$$
⁽⁵⁰⁾

We begin by considering the probability distributions of the $\tilde{\xi}_{k}$. From (49)

$$\widetilde{\xi}_{k} = (1/N) \sum_{j=0}^{N-1} e^{-(2\pi i/N) kj} x_{j} v_{j}.$$
 (51)

We assume that the v_i are at most locally correlated. That is, their correlation length l is small compared to the span of the data N. Then, even if the x_i exhibit significant correlation over long distances, the right side of (51) is essentially a sum of N/l statistically independent phasors, a large number due to the smallness of lrelative to N. Since the x_i are stationary, no terms in (51) dominate the sum and the central limit theorem may be applied separately to the real and imaginary parts of $\tilde{\xi}_k$ implying that they are both normally distributed.

The real and imaginary parts of (51) are

$$R_{e} \tilde{\xi}_{k} = (1/N) \sum_{j=0}^{N-1} \cos[(2\pi/N) k_{j}] \xi_{j}$$
(52)

Im
$$\tilde{\xi}_k = (1/N) \sum_{j=0} \sin[(2\pi/N) kj] \xi_j.$$
 (53)

The expectation value of their product is

$$\langle (Re \ \tilde{\xi}_k)(Im \ \tilde{\xi}_k) \rangle = (1/N^2) \sum_{j=0}^{N-1} \sum_{l=0}^{N-1} \cos[(2\pi/N) kj] \sin[(2\pi/N) kl] h(l-j)$$
(54)

where

$$h(l-j) \equiv \langle \xi_l | \xi_j \rangle \tag{55}$$

is an even function. Algebraic manipulation shows (54) to be zero so that the real and imaginary parts of ξ_k are uncorrelated. Since they are also normal this implies that they are independent. Thus ξ_k has independent normal real and imaginary parts, and is hence a Rayleigh phasor.

The probability distribution of the squared amplitude of $\hat{\xi}_{k}$ will be

$$p(|\tilde{\xi}_k|^2) = (1/\alpha)e^{-|\tilde{\xi}_k|^2/\alpha}$$
(56)

where

$$\alpha \equiv N \langle \xi^2 \rangle / N^2 = \langle \xi^2 \rangle / N.$$
(57)

and using the moments of this distribution tabulated in Table I we find

$$S(\underline{k}) = \langle | \widetilde{\xi}_{k} |^{2} \rangle = M_{1}^{1} = \alpha$$
(58)

$$[\delta S(k)]^2 = M_2^1 - (M_1^1)^2 = \alpha^2$$
(59)

so that

$$\delta S(k) = S(k). \tag{60}$$

The cases k = 0 and k = N/2 must be treated separately since $\tilde{\xi}_0$ and $\tilde{\xi}_{N/2}$ are real. When this is done, and we combine the three cases we find

$$\delta S(k) = \begin{cases} \sqrt{2 [S(0)^2 - m_0^4]}, & k = 0\\ S(k), & k \neq 0, N/2\\ \sqrt{2} S(N/2), & k = N/2. \end{cases}$$
(61)

Equation (61) implies that the error in estimating S(k) is proportional to S(k) itself. If S(k) is estimated from vindependent averaged samples, the distribution of (56) should be replaced by the "v-look" distribution ((2)) and the error becomes (considering the k = 0 and k = N/2 cases separately)

$$\delta S(k)/S(k) = \begin{cases} \sqrt{2/\nu} & \sqrt{1 - [m_0^4/S(k)^2]}, \ k = 0, \\ \sqrt{1/\nu}, & k \neq 0, \ N/2 \\ \sqrt{2/\nu}, & k = N/2. \end{cases}$$
(62)

VIII. IMPLICATIONS

Let us consider the implications of this result for the estimation of an actual power spectral density. We assume a Gaussian shape for the psd

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$$P(k) = a e^{-(k-k_0)^2/w^2}$$
(63)

a contrast of C, and an *n*-look image with white speckle. Finally, we assume that v independent power spectra are averaged. From Parseval's theorem we see that the mean power of the process g is

$$\sigma_{\varepsilon}^{2} = \sum_{k} P(k) \approx a \ w \sqrt{\pi}$$
 (64)

so that the power spectral density of the contrast C signal is (from (11))

$$P_{c}(k) = \begin{cases} 2[(2-C)/C]^{2} a w \sqrt{\pi}, & k = 0 \\ P(k), & k \neq 0. \end{cases}$$
(65)

The averaged psd will be

$$\overline{P}_{c} = (1/N) \sum_{\ell} P_{c}(\ell)$$

$$= (a w) \sqrt{\pi} / N \{1 + 2 [(2 - C)/C]^{2}\}$$
(66)
(66)

so that the psd of the speckled process is, by (32),

$$P'_{c}(k) = \begin{cases} n^{2}I^{2} aw \sqrt{\pi}[2(1 + 1/nN)[(2 - C)/C]^{2} + 1/nN], & k = 0 \\ n^{2}I^{2}(P(k) + (aw \sqrt{\pi}/N)\{(1/n) + (2/n)[(2 - C)/C]^{2}\}), & k \neq 0. \end{cases}$$

Let us consider only the estimate of the magnitude of the spectral peak, which (63) implies should be

$$P(k_0) = a. \tag{69}$$

From (68)

$$P'_{c}(k_{0}) = a n^{2} I^{2} (1 + \sqrt{\pi} (w/N) \{(1/n) + (2/n) [(2-C)/C]^{2}\}).$$
(70)

Equation (62) implies the error in the estimate of this will be

$$\delta P'_{c}(k_{0}) = (1/\sqrt{\nu}) P'_{0}(k_{0}). \tag{71}$$

From the estimate of the speckled spectrum, we subtract the correction term given in (47). Of course, there will be some error in estimation of the correction term \overline{P}'_c , but since a large number of terms are averaged to compute it, we will assume that this error is negligible compared to (71). That is, the correction is applied perfectly so that the final estimate has the correct mean value (circumflex here implies an estimate)

$$\langle \hat{P}(k_0) \rangle = a \tag{72}$$

but the error in this estimate has standard deviation



Fig. 3. Results for n = 1.

$$\delta \dot{P}(k_0) = \delta P'_0(k_0)/n^2 I^2 = (a/\sqrt{\nu}) \left(1 + \sqrt{\pi} (w/N) + (1/n) + (2/n) \left[(2-C)/C\right]^2\right)$$
(73)

and the fractional measurement error is

$$\sigma \hat{P}(k_0) / \langle \hat{P}(k_0) \rangle = (1/\sqrt{\nu}) \left(1 + \sqrt{\pi} (w/N) \left\{ (1/n) + (2/n) \left[(2-C)/C \right]^2 \right\} \right).$$
(74)

Clearly, then, the measurement error decreases when

- 1) number of looks *n* increases,
- 2) contrast C increases,
- 3) width w decreases,
- 4) number of averaged spectra v increases.

The fractional error for various values of contrast, number of looks, and width: N ratio is shown for v = 1in Figs. 3 and 4. Values for C > 1 are shown as dotted lines since images must have $C \le 1$. We observe that the minimal value of the fractional error is 1. This is to be expected since, in the absence of other effects that increase noise, the distribution of \hat{P} is exponential with standard deviation equal to the mean. Clearly, some averaging of spectra (v > 1) is necessary in all cases for reasonable estimates to be obtained. This can be effected by either averaging independent estimates of the spectra, or by low pass filtering of the spectrum to smooth it.



Fig. 4. Results for n = 4.

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Andrew D. Goldfinger (M'78) was born in New York, N.Y., on March 12, 1945. He received the B.S. degree in physics from Rensselaer Polytechnic Institute in 1965 and, following a year of study at Cambridge University, the Ph.D. degree in physics from Brandeis University in 1972. He is currently working toward an M.S. degree in applied behavioral counseling at The Johns Hopkins University.

He is now a member of the Principal Professional Staff at the Applied Physics Laboratory of the Johns Hopkins University. He has worked in various areas of aerospace technology, including orbital dynamics, remote sensing, and image processing, in addition to making contributions in the field of lechimetry. His hobbies include religious engineering and the study of Babylonian texts.