The Creep of Polycrystalline Ice
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The creep of polycrystalline ice

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Polycrystalline blocks of ice have been tested under compressive stresses in the range from 1 to 10 bars at temperatures from -13° C to the melting-point. Under these conditions ice creeps in a manner similar to that shown by metals at high temperatures; there is a transient creep component and also a continuing or quasi-viscous component. The relation between the minimum observed flow rate $\dot{\epsilon}$, the applied stress σ and the absolute temperature T is $\dot{\epsilon} = B \exp(-Q/RT)\,\sigma^n$, where R is the gas constant, and B, n and Q are constants; the value of n is about 3·2, that of Q is 32 kcal/mole, and that of B is 7×10^{24} if the stress is measured in bars and the strain rate in years^{-1}. At the higher stresses a third, accelerating stage of creep was observed; on the basis of the appearance and behaviour of sections cut from the specimens, this acceleration was attributed to recrystallization. The effect of changing the load during a test has also been studied; for large reductions creep recovery was observed.

The results of these tests are discussed in connexion with previous work on metals and ice, and also with measurements of glacier flow.

1. Introduction

Although the flow of ice has been studied by various investigators for many years, the laws determining the rate at which a mass of polycrystalline ice will deform under the action of a given stress have not yet been investigated thoroughly. Early tests on the flow of ice under bending, tensile and compressive stresses (Reusch 1864; Pfaff 1875; Koch 1885) showed that ice was capable of plastic deformation, while the more careful tests of Main (1887) and McConnel & Kidd (1888) showed that the rate of strain varied with the orientation of the grains, the temperature and the applied stress. McConnel & Kidd also showed that the rate of flow, unlike that of a simple liquid, was not proportional to the stress. Unfortunately, owing to the fact that these experiments were performed in the open air, temperature control was impossible, even during a single test, and thus no quantitative results of great value were obtained.

Since the experiments of McConnel & Kidd, many different tests have been made on various forms of ice under sundry conditions, but most of them have been in bending, torsion, extrusion or some other method of testing in which the stress is not the same at all points of the specimen, and therefore the results are difficult to interpret in terms of a direct relation between stress and strain rate.

For this reason, and because the flow law of ice is of great importance in the theory of glacier flow, experiments were designed to investigate this relation on ice which was as similar to glacier ice as possible. The ideal test would probably be a shear test, as it is believed that the shear component of the stress is primarily responsible for creep or any plastic deformation; but it is very difficult to load a specimen in uniform shear, and therefore most creep tests are performed in tension or compression. The tensile test has two disadvantages in the case of ice; the specimen may fracture, or, if any local reduction occurs in the area of the specimen, the increased stress in that section will cause it to extend more rapidly and so the irregularity will become worse. The compressive test was therefore selected for the present work. Preliminary accounts of the results obtained have already been published (Glen 1952, 1953).

2. Apparatus

A simple compression testing machine was constructed, of a suitable size to fit inside a cold chamber that had been purchased for this work from a grant provided by the Royal Society. In this machine the load from a hanging weight was applied to the top of a cylindrical specimen of ice. The stress was computed from the known load, and the strain was measured by means of a dial gauge that was mounted on the top of the machine and recorded the vertical movement of the frame which carried the weight. This dial gauge was read from outside the chamber through a double window, the space between the two panes of glass being kept free of frost by a dish of calcium chloride.

The temperature of the air inside the cold chamber was controlled by a Sunvic thermostat, which was placed near the specimen. A fan kept the air circulating within the chamber to avoid large temperature gradients. Although the thermostat was set so that the temperature at which it turned the refrigerator off was only 0·2° C below that at which it turned on, the temperature always overshot owing to a time lag between the refrigerant starting to circulate and the temperature dropping. Enormous differences in the time taken to give the same cooling on different occasions made this very difficult to compensate, and it was found to be impossible to control the air temperature to within $\pm 0{\cdot}5°$ C for any length of time. For this reason special steps were taken further to control the specimen temperature in the tests carried out close to the melting-point.

For these tests the specimen was immersed in a paraffin bath and was compressed between two plates of paxolin, a thermal insulator, the bottom plate of which served to isolate the specimen thermally from the brass pot containing the paraffin. This paraffin bath was itself immersed in a bath of ice and water, being thermally insulated from the floor of the containing bath by a further block of paxolin.

When this assembly was placed in the cold chamber with the air temperature controlled at $0 \pm 0{\cdot}5°$ C, the temperature inside the paraffin bath, measured with a Beckmann thermometer, varied by less than $\pm 0{\cdot}03°$ C; while if the chamber varied from $-1{\cdot}0$ to 0° C, the bath temperature varied by $\pm 0{\cdot}01$ about a value 0·02° C below the previous mean (assumed to be 0° C).

The paraffin bath also prevented evaporation of the ice specimen, which had proved to be a serious trouble in some of the earlier tests; for this reason later tests at temperatures below the melting-point were also made with the specimen in the paraffin bath, though without the ice and water bath.

3. Specimens

In order that the results of the tests should be comparable with the behaviour of glaciers, it is necessary that the ice used should consist of many small crystals with fairly random orientation and equiaxed shape. The ice which forms on the surface of freezing water fulfils none of these conditions, being usually in the form of columnar crystals all having their optic axes perpendicular to the water surface. It was therefore necessary to make the specimens in a way that provided randomly oriented seeds.

The method adopted was as follows. A cylindrical mould of one of the two sizes shown in figure 1 was filled with hoar-frost collected from the pipes of the cold chamber; the mould was then closed by screwing on the two end-plates, each of which carried a tube. One of these tubes was connected to a piece of rubber tubing sealed with a clamp, the other was connected to a vacuum pump with which the

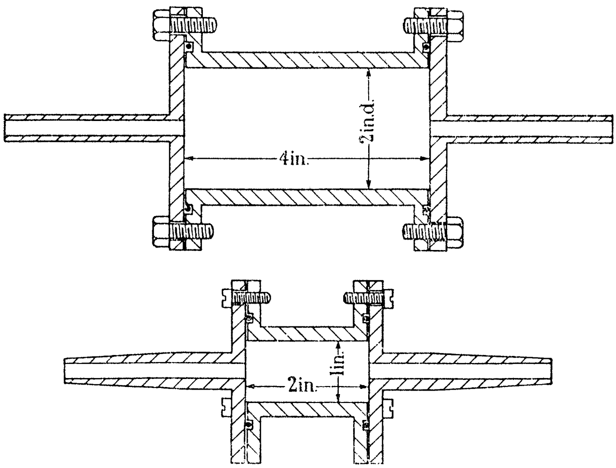

Figure 1. Moulds for making polycrystalline ice specimens.

mould was evacuated. Air-free water was prepared by corking it while it was boiling and allowing it partially to freeze under its own vapour: this water was uncorked and syphoned into the mould, which was then placed in a refrigerator and allowed to freeze. It was necessary to release one end-plate after a few minutes to avoid bursting.

When freezing was complete, the specimen was removed from the mould by removing both end-plates and warming the cylinder gently until the specimen slid out. One end of the specimen so formed was flat except for a small pip left where the tube joined the end-plate, and the other end was melted down to the same shape. The central pips were used to locate the specimen between the compression plates, which had small central holes to accommodate them; they also helped to prevent the specimen from slipping sideways between the plates. The dimensions of each specimen were measured before and after each test with vernier calipers.

Specimens prepared in this way were slightly cloudy, probably because of air or vacuum (water vapour) bubbles, but their density did not differ measurably from that of clear ice. The crystal shapes and orientations were determined by observing a section cut from the specimen. Such sections were prepared by cutting a thick section with a cold hacksaw and then melting it down until it was between 0·5 and 1 mm thick. The thin section was mounted by placing it on a cold slide while still damp; the sections were kept under paraffin in a cold laboratory.

When such sections were viewed between crossed polaroids, the individual grains were clearly visible. It was found that specimens initially had many small

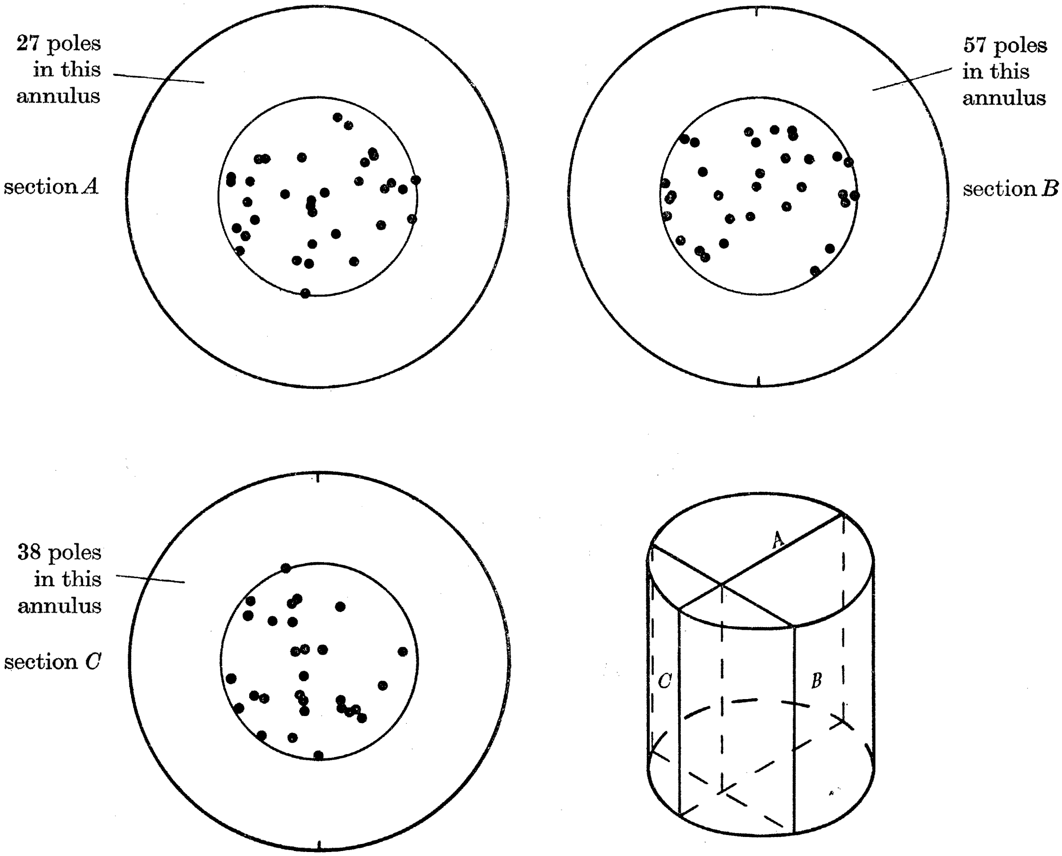

FIGURE 2. Pole figures of the orientations of grains on equal area projections. Each point represents the (0001) pole of one grain.

crystals, but that recrystallization occurred within a few days of making, presumably due to the strains induced during freezing. This recrystallization is not so serious as might be thought, as the specimens undergo recovery and recrystallization during tests, as we shall see later, and it was found that the results of tests did not vary with the time between making the specimen and starting the test.

In order to see whether any preferred orientation existed in the specimens, several were prepared and sectioned, and the orientations of the individual crystals were determined by the method described by Bader (1951). As no hemispheres were available, only those orientations in which the optic axis lay within 45° of the normal to the section could be measured exactly, but the other grains were counted. The correction for the refractive index of ice was made using tables prepared and kindly lent to me by Dr R. J. Adie.

A typical set of results is shown in figure 2. Three sections were taken from the one specimen by cutting it in half and taking a circular section from the middle (section *A* in figure 2) and the other two, a diametral and a circumferential (labelled *B* and *C* in figure 2), were cut from one of the halves. The results are presented as points on an equal area projection. This is preferable to the more usual stereographic projection, since a random distribution of points on a sphere does not give a random distribution of points on a stereographic projection, whereas it does on an equal area projection. The number of grains whose orientation could not be measured because they lay beyond the range of the apparatus without hemispheres is marked in the annuli. In the two non-circular sections the specimen axis is marked on the circumference of the projection.

TABLE 1. PROPORTION OF GRAINS WITH OPTIC AXIS WITHIN 45° TO NORMAL TO SECTION

section (see figure 2)	no. of (0001) poles within 45° to normal	no. of (0001) poles outside 45° circle	fraction of poles within 45° circle	fraction of poles if random
A	34	27	0·56	0·29
B	32	57	0·36	0·29
C	29	38	0·43	0·29

Figure 2 shows that no preferred orientation exists which is anything like as strong as that found in lake ice or glacier bands (Perutz & Seligman 1939; Adie 1954), since all orientations are present. A more rigorous test is to compare the number of points within and without the measurable circle in each section. This is done in table 1, which shows that no strong preferred orientation was present. The fact that all the ratios are above the random expectation is probably due to some systematic error in selecting grains; only those larger than a certain size (about 1 mm in diameter) could be measured, as smaller ones did not extend throughout the thickness of the section.

It is still possible that preferred orientations existed in parts of the specimens, but counts in a small region never showed a strong texture. The crystals could be seen to be roughly equiaxed, but no elaborate tests were made to see if complex grain shapes of the sort found by Bader (1951) were present. The grain boundaries were fairly smooth, not jagged as sometimes found in ice (Matsuyama 1920).

4. TESTING PROCEDURE

In each test a specimen, prepared and measured as described above, was placed on the bottom end-plate and then put into position on the machine and the top plate was lowered on. The whole apparatus was left for an hour to acquire the steady temperature required. Then, in order to apply the load, the cold-chamber door had to be opened, but, if the circulating fan was switched off, this did not seriously affect the temperature, which quickly fell again after closing the door and restarting the fan. The greatest danger was that the temperature would overshoot its correct value, and to prevent this the refrigerator motor was switched off as soon as the internal temperature had fallen to within a degree or two of its steady value.

Readings of the dial gauge and thermometer were taken at regular intervals to allow the creep curve to be plotted. Sometimes in the earlier experiments (those at $-1{\cdot}5^\circ$ C) it was necessary to reset the dial gauge, as the same gauge, calibrated in tenths of thousandths of an inch, was used in all tests. In later tests another gauge, calibrated in halves of thousandths of an inch but with a longer run, was used when large compressions were expected.

At the end of a test the specimen was removed and remeasured, and in some cases taken to a cold laboratory for sectioning; specimens were kept below their melting-point during transfer by immersing them in cold paraffin in a vacuum flask.

5. Results

The method of obtaining a testing temperature close to the melting-point has already been described. If this method was used with the cold-chamber temperature varying from $-0{\cdot}5$ to $+0{\cdot}5^\circ$ C, it was found that, below a certain stress, the creep rate to which the specimen settled down was independent of the stress. This curious result was attributed to the melting of ice by the small amount of heat that flowed in while the chamber was above 0° C. In order to avoid this effect, all subsequent tests were carried out with the cold chamber fluctuating between $-1{\cdot}0$ and 0° C, although this had the disadvantage that the ice and water bath slowly froze up and thus prevented long tests at this temperature.

A series of creep tests was carried out at stresses varying from $0{\cdot}7$ to $9{\cdot}3$ b (1 bar (b) $\equiv 10^6\,\mathrm{dyn\,cm^{-2}}$) both close to the melting-point (approximately $-0{\cdot}02^\circ$ C) and at $-1{\cdot}5 \pm 0{\cdot}2^\circ$ C. Typical creep curves are shown in figures 3 and 4 and the results are summarized in table 2. Two tests performed at $-6{\cdot}7$ and one at $-12{\cdot}8^\circ$ C are also included. In all tests an initial stage of decelerating creep was observed, and in tests at the higher stresses this was followed by a steady stage and sometimes, at the highest stresses, by an accelerating stage. Some of the earlier tests at $-1{\cdot}5^\circ$ C were performed in the open air and in these evaporation occurred, which is undoubtedly responsible for the acceleration in some of these cases, but later tests performed on specimens immersed in paraffin avoided evaporation, and in these the acceleration must be due to some other cause. The existence of an acceleration and of evaporation is noted in the remarks column in table 2. Figure 3 shows the variation of creep curves with stress, and figure 4 the variation with temperature.

The effect of changing the load during a creep test was investigated on some of the specimens during tests at $-1{\cdot}5^\circ$ C; the effect on the creep curve is summarized in table 3. When the load was increased a new transient decelerating creep was also observed, very similar to the transient observed at the beginning of a test. The subsequent behaviour of the creep was very similar to that which would have been observed if the final load had been applied from the start. When the load was reduced, rather more complex behaviour was observed; at a low stress, reducing the load to less than half its original value led to a sudden decrease in strain followed by a further slow drop (similar to the creep recovery observed if the load was removed completely); this was eventually checked by an increasing creep rate which approached a steady value asymptotically. Reducing the load somewhat less

Table 2. Results of creep tests on polycrystalline ice specimens

mean temp. (° C)	test	stress (b)	minimum strain rate (yr^{-1})	analyzed (yr^{-1})	remarks
0	β	1·56	2·76	—	
0	γ	0·70	1·77	—	tests performed with cold chamber varying through 0° C
0	γ'	1·59	1·86	—	
0	γ''	2·38	2·32	—	
0	δ	0·68	0·65	—	
−0·02	ϵ	0·91	0·29	< 0·02	had not reached steady state
−0·02	θ	1·51	0·68	0·064	
−0·02	η	2·29	1·28	0·71	
−0·02	k	3·25	4·18	—	
−0·02	j	3·60	6·85	4·9	
−0·02	h	6·06	48·9	37·0	
−0·02	i	6·20	47·5	33·4	
−0·02	a	8·54	122	—	
−0·02	b	8·67	112	—	acceleration after minimum
−0·02	d	8·67	294	—	
−0·02	f	8·67	193	—	
−0·02	g	9·00	262	—	
−0·02	c	9·20	324	—	
−1·5	xii	0·90	0·0426	—	
−1·7	xi	1·51	0·068	—	
−1·3	x	1·53	0·146	0·097	
−1·3	iv	1·68	0·086	—	acceleration due to evaporation
−1·6	vi	1·83	0·221	—	
−1·5	v	1·87	0·113	—	no transient creep
−1·1	vii	2·40	0·236	0·182	acceleration due to evaporation
−1·5	ix	2·46	0·357	—	
−2·1	viii	2·78	0·385	0·278	
−1·5	F	3·67	0·78	—	
−1·3	C	3·80	2·30	—	
−1·3	B	4·13	2·28	—	acceleration and evaporation
−2·0	A	6·32	9·40	—	acceleration after minimum
−1·2	E	6·35	7·15	—	short test
−1·4	D	9·30	21·7	—	acceleration after minimum
−6·8	1	2·22	0·103	0·041	
−6·7	2	6·00	2·59	—	acceleration after minimum
−12·7	3	5·93	0·49	0·34	

eliminated the creep recovery stage, but gave a transient increasing creep rate. Both of these effects are shown in figure 5. Reducing the load from a high stress gave different results; the curve in figure 6 shows that a drop to about half the original load gave a discontinuous decrease in creep rate, followed by a transient decreasing creep rate. These apparently contradictory results are discussed below; it is of interest to note that a small drop in load on specimen C after it had been at the high stress for a shorter time than specimen F, produced an effect similar to that observed at lower stresses (see table 3). In all cases the final rates observed, or the minimum rates in those cases which showed a deceleration followed by an acceleration, were similar to those observed at the same stresses in tests under constant stress, though in the case of the decreased stress tests where the final stress was very low the final

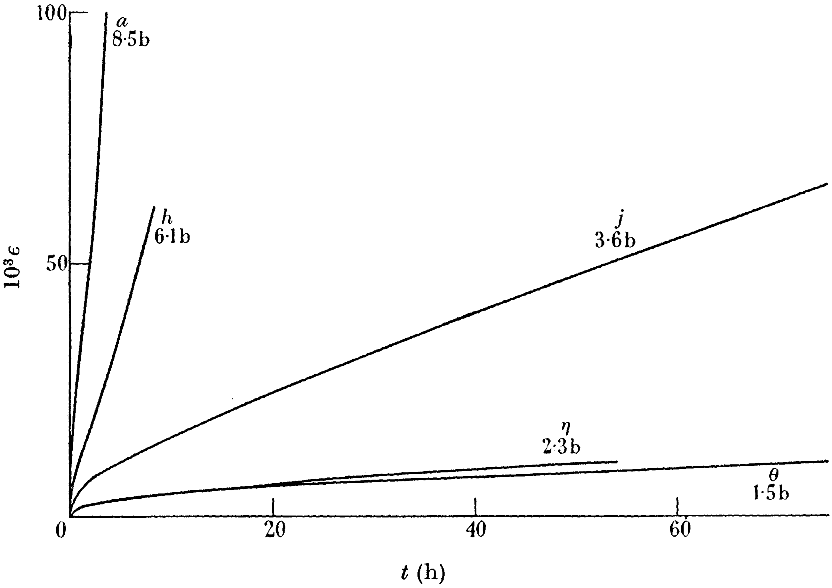

FIGURE 3. Creep curves of ice specimens at $-0{\cdot}02°$ C under various stresses.

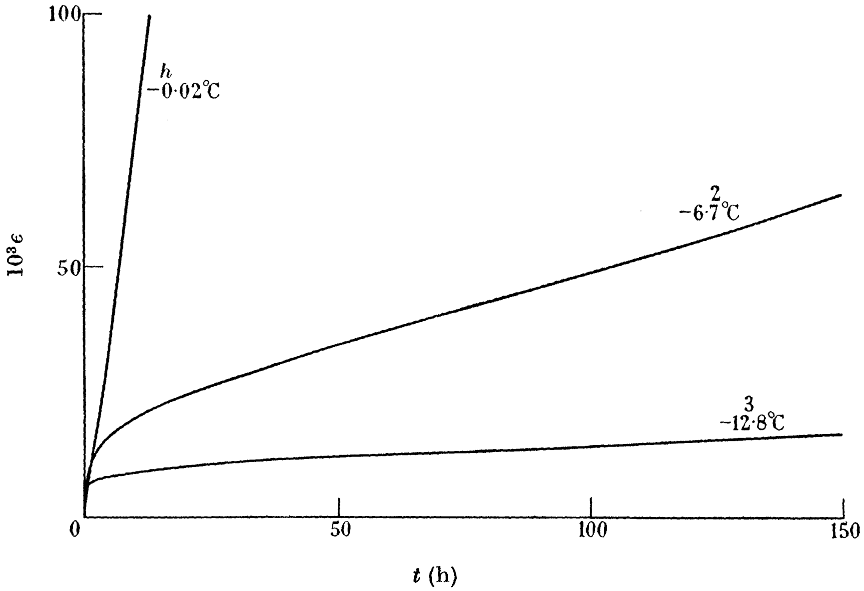

FIGURE 4. Creep curves of ice specimens under a stress of 6 b at various temperatures.

rate was less than that observed if the same load were applied directly, but was sufficiently similar to suggest that, after a long time, the rates would have become the same in the two cases. If this is so, then the reduced load tests are of great use, for they give a lower limit for the final strain rate at the lowest stresses.

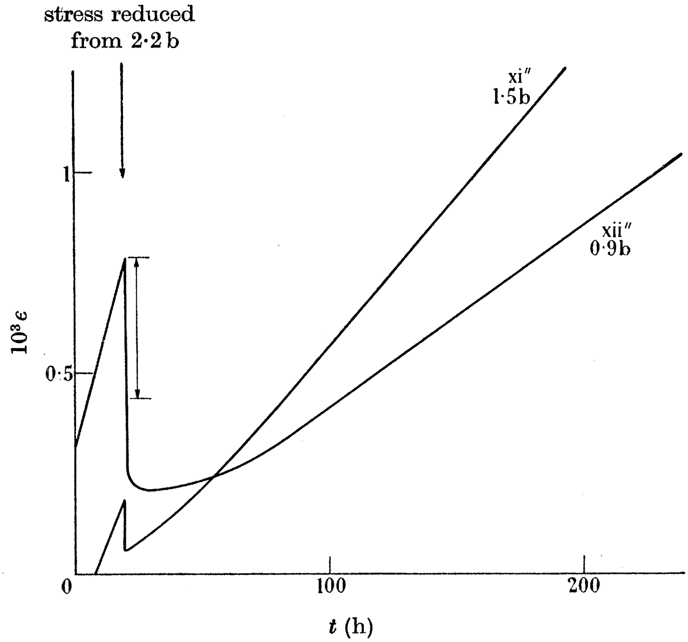

FIGURE 5. Effect of a reduction of stress during creep tests. ↕ indicates the instantaneous drop.

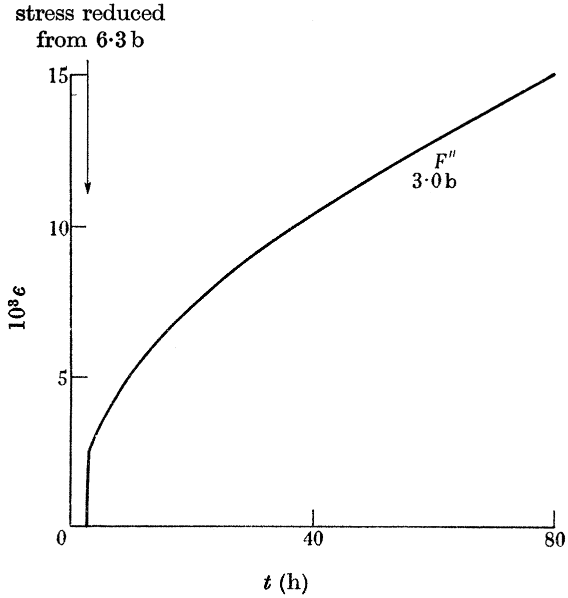

FIGURE 6. Effect of a reduction of stress during a creep test.

After they had been tested some of the specimens were sectioned and their recrystallization studied. Full details of this work will be published elsewhere, but for the purposes of this paper the results must be summarized here. The tests at the highest stress showed a very small gain size on sectioning, and the grain size increased

TABLE 3. EFFECT OF INCREASING OR DECREASING THE STRESS DURING A CREEP TEST

test	stress before (b)	stress after (b)	strain rate before (yr^{-1})	strain rate after (yr^{-1})	remarks
xii′	0·90	2·24	0·0426	0·219	increased stress gave a transient similar to original loading; final steady (or minimum) rate also similar
xi′	1·51	2·20	0·068	0·156	
F′	3·67	6·32	0·78	18·6	
C′	3·80	6·95	2·30	7·93	
C‴	4·25	10·8	2·17	63·0	
xii″	2·24	0·90	0·219	0·0425	negative creep rate immediately after reduction becomes positive and almost exactly equal to that before reduction
xi″	2·20	1·50	0·156	0·062	increasing transient creep
C″	6·95	4·25	7·93	2·17	increasing transient
F″	6·32	2·97	18·6	1·14	decreasing transient

with decreasing stress. The time taken for noticeable recrystallization of thin sections taken immediately after the end of a test at $-0{\cdot}02°$ C and kept at $-6°$ C, also increased with decreasing stress, being 22 h for 8·7 b, 10 days for 6·1 b and over 100 days for 3·6 b or below.

6. ANALYSIS OF THE CREEP CURVES

In order to show the dependence of creep rate on the applied stress and temperature, the logarithm of the minimum strain rate observed in each test has been plotted against the logarithm of the stress required to produce it. In computing the strain rate the length of the specimen at the time of the creep rate in question has been used, so that logarithmic strain rate is obtained. The resulting plot is shown in figure 7; points corresponding to different temperatures have been marked with different symbols. The minimum creep rates in increased load tests, and the final creep rates in decreased load tests, have also been marked on figure 7.

If ice obeyed the Newtonian law of viscosity, all the points would lie on straight lines (one for each temperature) of slope unity. The straight lines drawn through the points correspond to a power law between stress σ and strain rate $\dot{e}$ of the form

$$\dot{e} = k\sigma^n, \tag{1}$$

where n and k are constants. The least-square values for n obtained from the points at $-0{\cdot}02$ and $-1{\cdot}5°$ C are $3{\cdot}17 \pm 0{\cdot}2$ and $3{\cdot}17 \pm 0{\cdot}1$ respectively, so that no detectable variation of n with temperature exists to the accuracy of these experiments, and the lines at the two lower temperatures have been drawn with this slope. The value of the parameter k varies with the temperature, and is 0·17, 0·023, 0·008 and

0·0017 at 0, $-1·5$, $-6·7$ and $-13°$ C respectively if σ is measured in bars and $\dot{\epsilon}$ in years^{-1}.

In interpreting these results, the following points must be borne in mind:

(i) The slowest tests may not have reached their minimum creep rates when they had to be stopped. In some cases the results of decreased load tests can help assess the magnitude of this effect.

(ii) In practical cases of continuous flow it is not the minimum creep rate but the final creep rate that is involved; for the tests at the higher stresses this may be many times the minimum creep rate.

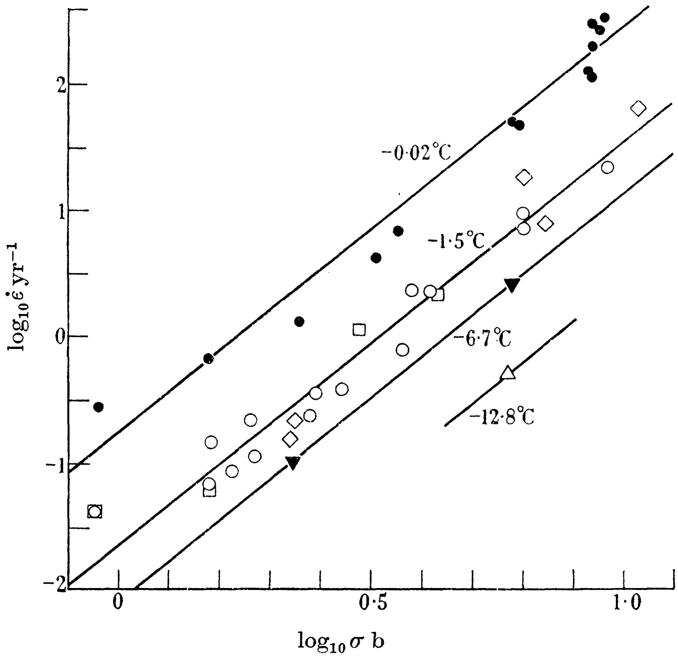

FIGURE 7. Variation of minimum creep rate $\dot{\epsilon}$ with stress σ. ●, points from tests at $-0·02°$ C; ○, $-1·5°$ C; ▼, $-6·7°$ C; △, $-12·8°$ C; ◇, $-1·5°$ C after increasing the stress; □, $-1·5°$ C after reducing the stress.

(iii) In cases where recrystallization is causing acceleration, the steady rate which would be observed without this effect might be much slower than the minimum rate.

All these remarks make the value of equation (1) uncertain, but it is noteworthy that practically observable long-time creep rates, as in a glacier, would probably depend on a higher power of the stress than the 3·2 found here, as both (i) and (ii) tend to make the relation between stress and strain rate less like the Newtonian viscous one.

To overcome the first objection, it is possible to analyze the creep curves by assuming that they fit Andrade's law (Andrade 1910, 1914)

$$l = l_0(1+\beta t^{\frac{1}{3}})\,e^{\kappa t}.$$

If this law is written in terms of the logarithmic strain, it becomes

$$\epsilon = \ln(1+\beta t^{\frac{1}{3}})+\kappa t,$$

or, if $\beta t^{\frac{1}{3}} \ll 1$,

$$\epsilon = \beta t^{\frac{1}{3}} + \kappa t. \tag{2}$$

Thus, provided the strain is small, it can be expressed as the sum of two terms, one proportional to $t^{\frac{1}{3}}$, the other proportional to t. Therefore, if the creep curve is plotted against $t^{\frac{1}{3}}$, as in figure 9, the tangent at the origin should represent the transient term. This part has been subtracted from the creep curve itself in figure 8; the remaining part, being the quasi-viscous or steady creep, should be a striaght line. As can be seen in figure 8, this line has a slope sometimes quite appreciably different from the minimum observed creep rate.

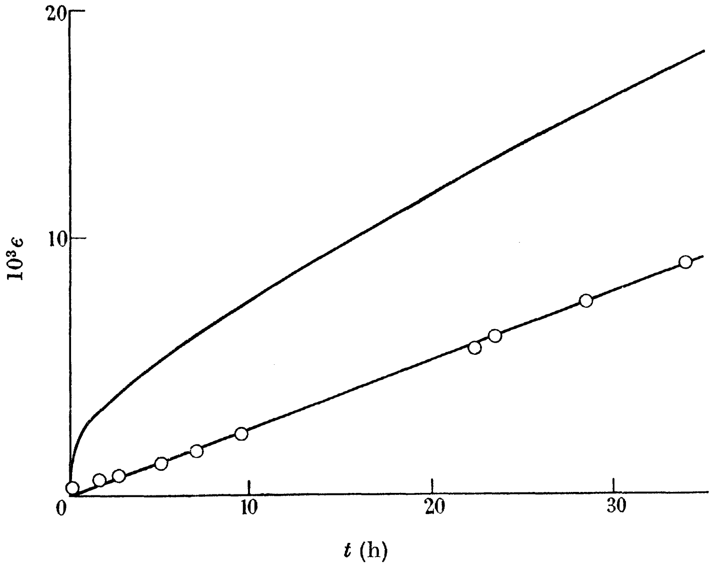

FIGURE 8. Creep curve showing analysis into transient and quasi-viscous components.

This type of analysis has been carried out for all the tests near the melting-point; the agreement with the Andrade law was not always as good as in the example shown in figures 8 and 9, but in most cases a quasi-viscous rate could be obtained. At the lowest stress the transient term was still the dominant one at the end of the test. A similar analysis could not be made for most of the creep curves obtained at $-1{\cdot}5^\circ$ C, as these tests were not so carefully conducted (being the first performed) and did not fit the law so well.

The logarithms of the resulting quasi-viscous rates at $-0{\cdot}02^\circ$ C are plotted against the logarithms of the stresses in figure 10. Considering that the lowest point is the least accurate (because the transient was still a major part of the creep at the end of the test), the closeness to a straight line is remarkable. The line corresponds to the relation

$$\kappa = 0{\cdot}017\sigma^{4{\cdot}2},$$

and has been fitted by the method of least-square error in $\log\kappa$.

The difference between these points and the corresponding points in figure 7 is only large at low stresses, and the top two points have been taken as identical, since in these tests accelerating creep complicated the analysis. The two tests taken were those which proceeded furthest before the acceleration took place.

In order to show the dependence of creep on temperature, the values for the strain rate at a stress of 6 b were taken from the lines drawn in figure 7 for each of the four

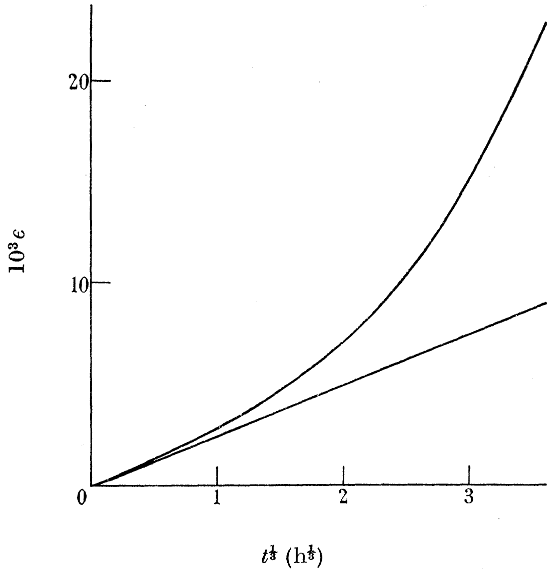

FIGURE 9. Creep curve plotted against $t^{\frac{1}{3}}$ to show method of analysis.

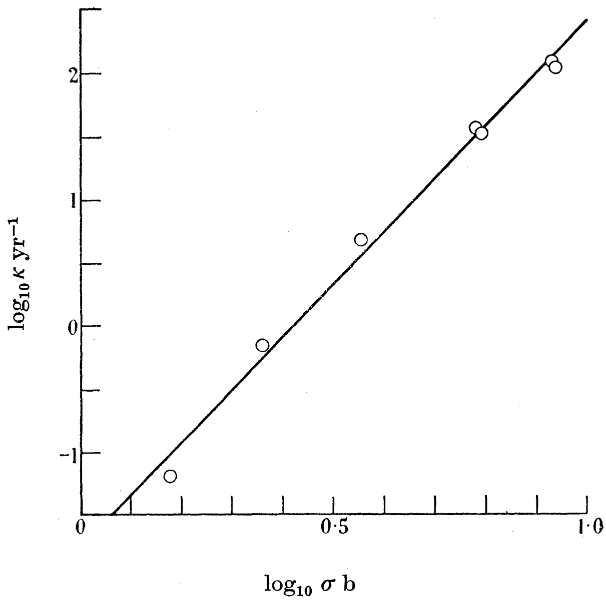

FIGURE 10. Analyzed quasi-viscous creep rate κ as a function of stress σ.

temperatures used. As the temperature range is so small, no deductions can be drawn about the functional relation between strain rate and temperature. It is reasonable, however, to plot the results so that they would give a straight line if the relation were

$$\dot{\epsilon} = A \exp(-Q/RT),$$

where A is a constant (varying with stress), R the gas constant and Q a heat of activation; this is a method of presenting data frequently used in work on the creep of metals (see, for example, Holloman & Lubahn 1947).

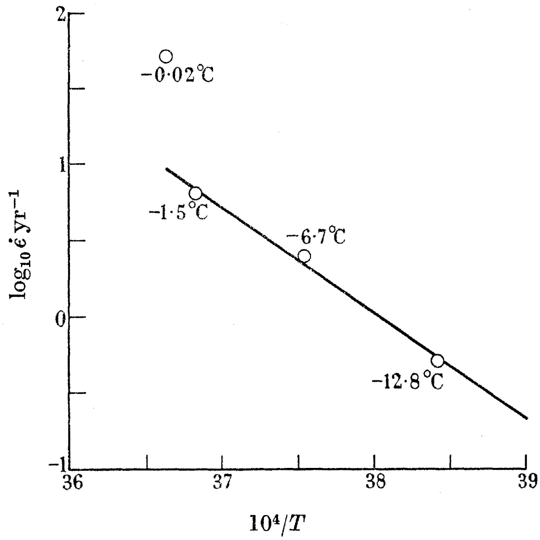

FIGURE 11. Minimum creep rate $\dot{\epsilon}$ at a stress of 6 b plotted logarithmically as a function of reciprocal of absolute temperature.

This is achieved by plotting $\log \dot{\epsilon}$ against T^{-1}, as in figure 11. As can be seen, the three points corresponding to temperatures well below the melting-point lie very close to a straight line, while the remaining rate (for $T = -0{\cdot}02^\circ$ C) is faster than would be expected from equation (3). This is a reasonable result as, so close to the melting-point, temperature fluctuations may be causing local melting on an appreciable scale, a factor that would upset the strict application of Boltzmann's law to the strain rate. An estimate of the magnitude of this partial melting has been given by Bartenev (1950).

If this interpretation is valid and a straight line is drawn through the remaining points as in figure 11, its slope gives an activation energy of 31·8 kcal/mole, or expressed in atomic units, 1·4 eV/molecule.

7. DISCUSSION

The creep curves obtained are very similar to those obtained on many other solid substances, and seem to fit Andrade's law fairly well. The discrepancies, especially those found in tests at $-1{\cdot}5^\circ$ C (the first to be made), are probably due to lack of care in fixing the end-plates. This probably also accounts for the different amounts of transient creep found from test to test, although the variation in initial structure resulting from recrystallization after specimens had been made may also be responsible. The accuracy of the tests is hardly sufficient to give a critical check of the value of the exponent $\frac{1}{3}$ in equation (2), but the general agreement is sufficient to imply that ice, along with most other substances at temperatures near their melting-points, does obey the Andrade law until accelerating creep occurs.

The existence of the accelerating stage makes it difficult to decide what is the best quantity to plot against the stress. On the one hand κ in Andrade's formula, which represents the steady rate the creep of the ice would have reached if the acceleration had not occurred, has a definite significance, while, on the other hand, if the acceleration is due to recrystallization, the ice would presumably have settled down after a long time to a steady creep rate, in which the acceleration caused by recrystallization was balanced against the work hardening produced, and this rate is the one most likely to correspond to the state of affairs in a glacier.

In the present work it was impossible to measure this second steady rate, as the specimens had not reached it even when compressed to half their height and no longer cylindrical, and from the results obtained it was impossible to estimate what this rate would be. The analysis to obtain κ was not always possible, and even if it had been, there would be an objection to presenting the results in this form, as this depends on the assumption that Andrade's law is correct, which, though probable, is not finally proved.

For these reasons the results have been presented in the form of a plot of the minimum creep rate actually observed against the stress required to produce it. In both this case and the plot of Andrade's κ the points lay much nearer to a straight line if the plot was double logarithmic than if a single logarithmic or hyperbolic sine law was used. A similar plot of the logarithm of creep rate against logarithm of stress is often used to present results obtained for metals, as, for example, by Servi & Grant (1951) for aluminium.

Direct comparison between the present work and previous work on ice is difficult for the reasons mentioned in § 1. Of the tests made by McConnel & Kidd (1888) the one most comparable with the present tests was their test 6, performed on an icicle with many, fairly randomly oriented grains. At temperatures varying between $-0\cdot7$ and $-1\cdot7°$ C under a tensile stress of $2\cdot2$ Kg cm^{-2} this gave a minimum strain rate of $1\cdot5\times10^{-5}$/h. In terms of the units used in table 2 and figure 7 this corresponds to a stress of $2\cdot1$ b and a strain rate of $0\cdot13$/yr; this test thus gives a point very close to the $-1\cdot5°$ line of figure 7.

Another experiment that is of interest for comparison was made by King (1952), who measured the indentation hardness of ice. He found that the apparent hardness decreased with time, and for an indentation taking 2 min it was about 1 Kg mm^{-2} at $-2°$ C. From comparisons between hardness and deformation tests on metals, it has been found that the Vickers hardness number is approximately three times the stress in Kg mm^{-2} for an 8 % strain. If this correlation is applied in this case to deduce a mean strain rate (a comparison first reported by King using the preliminary results of these tests), we obtain a mean strain rate of 21 000/yr. Comparing this with the (greatly extrapolated) linear relation found at $-1\cdot5°$ C, we find that the appropriate stress is 76 b, which would correspond to an indentation hardness of 2·3 compared with the observed 1. A similar calculation taking the 2 sec indentation time predicts a hardness of 8 compared with the observed 2·5. That the predicted figures are of the same order of magnitude as the observed ones is remarkable considering the very great assumptions made, and indicates that the relation deduced between stress and strain rate has some validity at much higher strain rates than those used in the present tests.

It is also possible to deduce an activation energy from King's results. The relation between strain rate, stress and temperature can be written in the form

$$\dot{\epsilon} = B \exp(-Q/RT)\, \sigma^n,$$

or, for a given strain rate,

$$\sigma = \left\{\frac{\dot{\epsilon}}{B} \exp(-Q/RT)\right\}^{1/n} = C \exp(-Q/nRT).$$

Thus if n is known, Q can be derived from hardness tests at different temperatures. An appropriate value of n can be deduced from King's curves by considering the ratio between the hardness at two different rates of indentation; in this way a value for n of 3·7 is found, which, it will be noticed, is between the values of 3·2 and 4·2 obtained for the minimum creep rate and Andrade's κ respectively. With this figure for n, Q can be found. It is 28·4 kcal/mole using the data for 2 min indentations and 26·7 using that for 2 sec indentations. These values are to be compared with 31·8 kcal/mole deduced in the present work. The activation energies at these high stresses are therefore similar but somewhat less, a result that implies a drop of activation energy with increasing stress, as found by Servi & Grant (1951) in aluminium. It is perhaps noteworthy that all these activation energies lie within the range of values (25 to 40 kcal/mole) found by Servi & Grant in aluminium.

Before turning to the comparison between the present work and results obtained on glaciers, it is pertinent to inquire what is the melting-point of a solid subjected to stress. Two equations have been proposed for this, one by Riecke (1895) and the other by Poynting (1881) and Johnston (1912). The difference between the two lies in the assumptions made about the work done by those external forces that act on the solid but not the liquid (e.g. the compressive forces in the present tests) during the melting process. If these forces do no work, then Riecke's formula is reached; if they do work by moving through the volume melted, then Johnston's formula is reached. Poynting and Johnston assume an incompatible stress system and also apply equilibrium thermodynamics to a system essentially not in equilibrium; Riecke's calculation, while still open to criticism for the second reason, corresponds to melting off the side of a compression specimen, which is a more realistic assumption, and therefore Riecke's formula for the depression of the melting-point is probably a reasonable approximation to use. It gives a depression of temperature ΔT due to a stress σ given by

$$\Delta T = -\frac{\sigma^2 V_s T}{2EL},$$

where V_s is the specific volume of the solid, T the temperature, L the latent heat and E the appropriate modulus of elasticity.

For the maximum stress used in the present tests this gives a depression of temperature of about $4{\cdot}5 \times 10^{-5}\,^{\circ}$C, which is much less than the $0{\cdot}02^{\circ}$ C by which the tests close to the melting-point were estimated to differ from the true melting-point of the unstressed solid. Poynting and Johnston's method would predict a depression of nearly $0{\cdot}9^{\circ}$ C.

A further point of difficulty in comparing the present results with those obtained from glacier measurements, or indeed any tests other than tension or compression

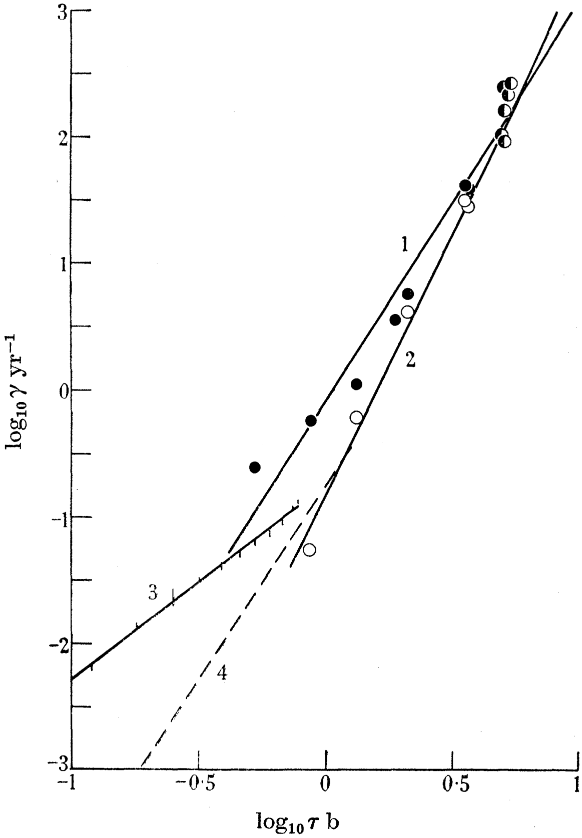

FIGURE 12. Double logarithmic plot of octahedral shear strain rate $\dot{\gamma}$ against octahedral shear stress τ for various experiments. ●, points from the plot of minimum observed creep rate (figure 7); ○, points from the plot of quasi-viscous creep rate (figure 10); |, points derived from the Jungfraujoch pipe experiment of Gerrard *et al.* (1952); broken line, law deduced by Nye (1953) from the tunnel experiments.

tests, is that the present results can by themselves give no information on the full relation between stress and strain in a general system. In order to make such comparisons further assumptions have to be made, the simplest set probably being those assumed by Nye (1953) in a paper in which he has already compared the preliminary results of the present tests with the tunnel contraction experiments of Haefeli (Haefeli 1952; Haefeli & Kasser 1951) and McCall (1952) and with the vertical velocity profile of a glacier found by Gerrard, Perutz & Roch (1952).

Now that fuller results of the laboratory tests are available, especially the results close to the melting-point, it seems desirable to repeat this correlation. Analyzing the various results in the way described by Nye, we arrive at figure 12, in which the results close to the melting-point have been taken from both figure 7 and figure 10. The agreement between the present results, especially in the analyzed form, and the tunnel experiments is very striking, and certainly within the experimental error of the present tests, and it is interesting that the relation derived from the experiment of Gerrard *et al.*, although of different slope, reaches values very similar to those obtained from the laboratory and tunnel experiments at the high stress end.

Two reasons can be advanced for the discrepancies at lower stresses. First, there may be a bend in the stress-strain rate relation at about this point; this is hinted both by the upper points on the glacier flow curve and by the bottom points on the laboratory minimum creep-rate curve; a curvature in this sense has been observed in aluminium by Servi & Grant (1951) and was attributed by them to a transition from grain boundary to intra-granular slip as the primary mechanism of creep. However, this explanation fails to account for the tunnel closing law, which continues with a slope of 3·07 compared with 1·5 for the law deduced by Gerrard *et al.*

The second possible explanation is that the laboratory tests and the tunnel experiments give the actual flow law, and that the data of Gerrard *et al.* have been wrongly interpreted when deducing the points plotted in figure 12. In the calculation of these points from the experimental data (which are the shear rates of the ice at various depths), it is assumed that the ice is at all points shearing on planes parallel to the bed under the action of a shear stress alone. This corresponds to the 'plug-flow' case considered by Nye (1951) in his plastic theory of glacier flow, that is to say, a case in which a block is sliding down a slope as a rigid body, the yield stress being just reached in its bottom layer. If in fact there exist stresses in the ice tending to extend or compress it, then the rate of flow as a function of depth will undoubtedly be modified. The only calculation which has yet been published which can give any idea of the extent of these modifications is the calculation of the pure plastic case by Nye (1951). This showed that two other flow states are possible in a purely plastic glacier, one giving an overall extension of the material, the other an overall compression. The variation of velocity with depth is of the same form in both these cases, an elliptical curve with a superposed constant addition. This is to be compared with the constant velocity at all depths in 'plug flow'. In fact these other solutions can be thought of as formed by the addition of a plug-flow component to the elliptical extending or compressing flow. For a material with a more complex relation between stress and strain rate, such as the power law used in this paper, similar effects are to be expected, and it is possible that the effect referred to as adding the elliptical velocity and plug flow will give a series of velocity profiles differing in form, as the analogues of both the added components now give flow throughout the material. If this argument is true, then results deduced by neglecting the extending or compressing component of flow will be most accurate near the bed, for it is here that the other (plug flow) component gives its largest contribution; in the purely plastic case the stress-strain rate law deduced in this way is correct at the bottom and only there, and thus it is to be expected that the law deduced on the false assumption that no longitudinal extension or compression is occurring will be most nearly true at the bottom of the glacier, that is to say, at the highest stresses.

Thus if such a longitudinal extension or compression is occurring in the Jungfraufirn, the result might well be to displace the apparent relation between shear stress and shear strain rate to faster strain rates at the lower stresses, precisely as observed. A direct check of this interpretation is possible since it postulates an extension or compression at the glacier surface, which should change the separation of two stakes in the surface along a line of flow. Such measurements are in progress.

A comparison is also possible between the results of the present work and the relation between the maximum velocity of glaciers and their depths and surface slopes. A graph showing this comparison has been published by Perutz (1954); it gives reasonable agreement.

The acceleration which occurred in some of the creep curves can reasonably be attributed to recrystallization. It was observed that sections taken from such specimens recrystallized, even in thin section, in times of the same order as the duration of the tests; and the fact that the grain size was a function of the stress under which a specimen had been strained also implies that recrystallization had occurred.

When a material is undergoing creep, its grains can be either slipping rigidly past each other, with consequent boundary changes (grain boundary creep), or else accumulating strain inside the grains. In the latter case, the probability of recrystallization, i.e. of both nucleation of new grains and of their subsequent growth, will rise as the internal strain rises, and new recrystallized material will be continually appearing, which can be considered as free from internal strain at the instant of its formation. After a long time a steady state will be reached in which the hardening caused by the rise in internal strain is balanced by the softening caused by recrystallization; at a higher strain rate there will be a higher mean internal strain. A simple dimensional argument can be used to relate this strain to the grain size, if the functional relation of both nucleation and growth rate are known as a function of strain. In metals the nucleation depends on a higher power of the strain than the growth rate, and if this is the case also in ice, the grain size should be less the larger the strain rate, that is to say, the larger the stress. This is the variation observed.

This theory can also be used to account for the behaviour of ice in the reduced load test in which a decelerating creep occurred after a reduction of stress from a high value. If the strain rate is reduced, the rate of recrystallization begins to drop, and hence it will initially contain more soft material than later. If the specimen had not been at the higher stress long enough for much recrystallization to have occurred, no soft material will be present, and the only effect will be the normal one of a material initially too hard for its reduced stress.

The lack of an accelerated stage in the creep curves at lower stresses may be due to the fact that these tests were not continued to anything like the same total strain as the other tests, and would have developed acceleration later, or it may be due to recrystallization occurring sufficiently slowly for the final steady state to be reached without a period of acceleration, or even not occurring at all, the whole process being accomplished by grain-boundary slip and migration.

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