Highest natural bed forms

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[1] Fluid flow interacts with sedimentary beds forming waves of different kinds, which are of considerable practical importance since they influence significantly the near-bed flow, both over and below the bed, sediment transport, and wave height attenuation. We focus here on steep bed forms capable of producing flow separation. In this case, the large-scale vorticity generated in the phenomenon of separation rules the process of friction, which appears to be practically unaffected by sediment motion. Under the crests of the bed forms, the mean shear force due to friction is balanced by the force that bed forms exert on the flow via pressure, which can be calculated from the work of *Giménez-Curto and Corniero* [2002]. Bed forms grow until they have a height such that friction at their troughs is negligible, thus ceasing the motion of fluid and sediment. This condition leads to a very simple expression for the limiting steepness, which compares favorably with existing observations on bed form geometry under steady open channel flow as well as under oscillatory flow. *INDEX TERMS:* 4558 Oceanography: Physical: Sediment transport; 4546 Oceanography: Physical: Nearshore processes; 4235 Oceanography: General: Estuarine processes; 3022 Marine Geology and Geophysics: Marine sediments—processes and transport; 1815 Hydrology: Erosion and sedimentation; *KEYWORDS:* bed forms, sediment transport, ripples, dunes, rough turbulent flow

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1. Introduction

[2] A conspicuous property of movable beds consists in their deformation under fluid flow forming quite rhythmic features that, in general, move. In nature this phenomenon occurs in water surfaces, where the wind generates waves; in sedimentary beds, where the flowing air or water builds ripples, dunes, antidunes, and bars; and also in vegetated surfaces, like grass, crop fields, forests under wind; or submerged algae fields (where the waving motion is termed "monami" [see, e.g., *Ghisalberti and Nepf*, 2002]).

[3] We are concerned here with the bed forms produced in sandy beds by water flows, a kind of interfacial waviness which ranges from small ripples (a few centimeters in wavelength and some millimeters in height) to tidal sand banks (with wavelengths of a few kilometers and tens of meters height [*Dyer and Huntley*, 1999]).

[4] From observation, we know that the bed form steepness η/λ (where η represents the height of the features, from trough to crest, and λ their spacing or wavelength) can hardly exceed 0.10 or 0.12 in steady flow. In oscillatory flow higher values can be observed, although not exceeding 0.20 or 0.25. Clearly, if the maximum steepness of the bed forms were determined by the angle of repose, ϕ, of the bed material, the limiting form would be symmetrical and its steepness would be 0.5 tan ϕ, i.e., about 0.31 for natural sand. Since figures comparable to this value have never been observed, it appears that under fluid motion there must exist some dynamic limiting condition, which until now remains unexplained.

[5] Most empirically based formulae proposed so far (see, for example, *van Rijn* [1984] for steady flow and *Mogridge et al.* [1994] for oscillatory flow) show a maximum in the bed form height trying to reflect the well-known fact that a ripple, or a dune, first increases its height, in response to an increase in the flow velocity, and finally decreases until its disappearance. Unfortunately, empirical formulae permit at the best only interpolation (and at the worse extrapolation). Explanation and prediction requires some kind of theory.

[6] From a theoretical point of view, we have gained some insight into the knowledge of the first stages of growing of the bed forms by investigating the instability of the fluid-bed interface to small disturbances [*Kennedy*, 1963; *Engelund*, 1970; *Richards*, 1980; *Blondeaux*, 1990; *Vittori and Blondeaux*, 1990]. However, little is known about the precise mechanism that limits the growing of natural bed forms.

[7] *Giménez-Curto and Corniero Lera* [1996] (hereinafter GCCI) have studied the fluid flow over irregular surfaces by introducing spatially averaged Reynolds equations, which consider the variation of the fluid domain of averaging. These equations allow the treatment of the flow between the roughness elements of the bed and are therefore particularly suitable for the investigation of the phenomena related with bed forms, especially under flow separation conditions. We shall use these equations throughout this paper as the basic theoretical framework.

[8] Perhaps the most fundamental result in the work by *Giménez-Curto and Corniero* [2002] (hereinafter GCCII) is

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the evaluation of the amplitude of the pressure disturbances generated in the near-bed flow when it undergoes separation from the bed features. This result can be expressed simply as:

$$\hat{p}_b \sim \rho \varepsilon U_0^2, \tag{1}$$

where U_0 is a typical velocity representing the global flow, ρ is the fluid density, and

$$\varepsilon = (k/\Lambda_0)^{1/3} \tag{2}$$

represents a small parameter characterizing the phenomena associated with flow separation. Here k is the height of the bed features (η throughout this work) and $\Lambda_0 = \rho U_0^2/G_0$ represents the basic length scale of the flow, G_0 being a typical value of the driving force per unit volume (see below for a formal definition of this force). Expression (1) was obtained in the context of oscillatory flow; nevertheless, the argument is very general and can be applied directly to steady flow.

[9] Very notably, the pressure variations (equation (1)) do not either depend on viscosity or on turbulence characteristics and this result remains valid regardless of the kind of flow near the bed (whether laminar, smooth turbulent, rough turbulent or jet), provided only that flow separation from boundary features does occur. In the work of GCCII, we provided empirical evidence of the validity of equation (1) for oscillatory flow over a flat bed whose features are simply the sediment grains (in which case the height k represents the mean diameter, D). This led to the explanation of the phenomenon of incipient motion of sediment. Here we apply the fundamental result (1) to the case of bed forms under both, steady and oscillatory flow, and show that this leads to an absolute limit for their height.

2. Theoretical Considerations

[10] Let us consider the situation in Figure 1a, i.e., a near-bed flow in the x direction, which can be inclined with respect to the horizontal and varies, in general, with time. This flow carries sediment partially as suspension load with negligible relative velocity between fluid and sediment, and also as bed load with a significant relative velocity.

[11] The spatially averaged Reynolds equations introduced in the work of GCCI are valid for fixed irregular boundaries. In the case of moving contours, the problem can be treated also by means of the general method described there. The extension is straightforward and as a result some new terms emerge due to the motion of the solid boundaries. Fortunately, these new terms can be shown to be negligibly small if the motion of the boundaries produces only small-time variations in the fluid area of the averaging domain, A_w. In fact, in the problem of sediment motion near a natural bed, A_w varies much more slowly than the flow itself with time and space (except for the z direction). In this context, and neglecting also the convective terms, since we are now considering a nearly flat bed, the mean longitudinal component of the equation of motion can be written as:

$$\rho \frac{\partial U}{\partial t} = -\frac{\partial P}{\partial x} + \rho g_x + \frac{1}{A_w}\frac{\partial (A_w \tau_{xz})}{\partial z} + f_{vx} + f_{px}, \tag{3}$$

where (x, z) are the longitudinal and perpendicular to the bed coordinates; t is the time; (U, P) are the mean velocity and pressure ("mean" signifies Reynolds and then spatial plane averaging, i.e., the Reynolds averaged value integrated over the fluid region, with area A_w, of a large plane domain parallel to the bed, divided by A_w); τ_{xz} is the mean shear stress; g_x is the x component of the acceleration of gravity vector; and (f_{vx}, f_{px}) represent the x component of the mean forces that sediment grains exert on the fluid per unit of fluid volume through viscous friction and pressure.

[12] The driving force, which is given by

$$G(z,t) = -\frac{\partial P}{\partial x} + \rho g_x \tag{4}$$

will depend, in general, not only on t but also on z, since it can easily be shown, using the z component of the equation of motion, that $A_w \partial P/\partial x$ cannot vary in a direction normal to the bed. Although equation (3) describes quite general flows in the vicinity of mobile beds, at least to a first approximation, we shall consider here, to be more specific, two particular types of flow, the steady open channel flow (for which the left hand side of equation (3) as well as $\partial P/\partial x$ are zero and the z derivative becomes an ordinary one) and oscillatory, horizontal flow (in which $g_x = 0$). The appropriate values of the global flow velocity, U_0, and driving force, G_0 are as follows: (1) for open channel flow, U_0 is the mean velocity and $G_0 = \rho g_x$; (2) for the oscillatory flow, $U_0 = a\sigma$ is the maximum exterior velocity and $G_0 = \rho a \sigma^2$ is the maximum pressure gradient (in this case, a represents the amplitude of motion and σ its angular frequency).

[13] In a general near-bed flow, the left hand side term in equation (3) is at most $O(G_0)$ far from the bed and its magnitude reduces downward as the mean velocity does. In the outer region the sediment moves as suspension load in small concentrations; therefore A_w can then be taken as nearly constant and furthermore the force exerted by the grains can be neglected. To a first approximation, equation (3) behaves then like in the case of a clean flow without sediment. This permits an estimation of the thickness of the friction layer as $h = \tau_0/G_0 = \Lambda_0 f$; where τ_0 represents the mean shear stress at the specific level and time where it is maximum, and $f = \tau_0/(\rho U_0^2)$ represents a dimensionless bed shear stress, also called friction coefficient.

[14] In the inner region of the friction layer, where bed load occurs, the mean shear stress decreases rapidly downward and the leading order balance of forces reduces to

$$\frac{1}{A_w}\frac{\partial (A_w \tau_{xz})}{\partial z} + f_{vx} + f_{px} = 0. \tag{5}$$

Here the grain force will be dominated usually by the pressure term since the grains will produce flow separation. The viscous term will enter the balance only in the case of very small grains. We observe that the left hand side sum in equation (5) is not exactly zero, but it must be $O(G_0)$. It should be pointed out that if f_{vx} is negligible, the flow just described behaves like a rough flow with an enhanced roughness, as compared to the fixed bed case, since the roughness height must be interpreted as the length scale in which A_w (or equivalently the porosity or the concentration) changes significantly normally to the bed (see Appendix A for a formal definition of roughness).

[15] If we consider now a slightly undulated bed with forms of small amplitude, the mean flow (averaging only

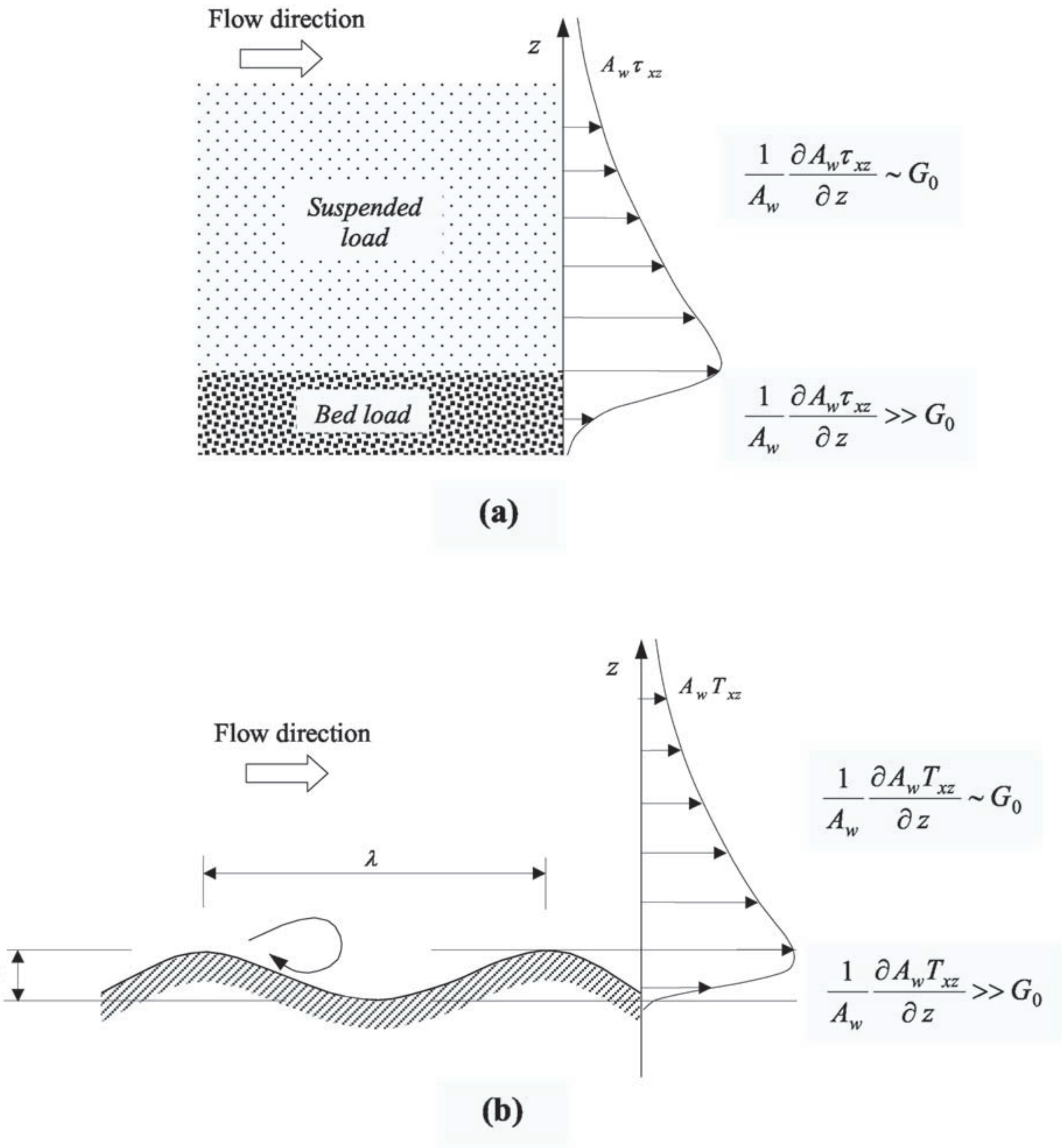

Figure 1. The total shear force (fluid area times the shear stress) in the flow over a mobile bed first increases downward but it must decrease abruptly in the lower region in order to balance the force that the bed load on a plane bed (a) or the bed forms (b) exert on the flow. The fluid area, A_w, of a fixed plain domain parallel to the bed varies with z as a consequence of the variation in the concentration or in the bed form geometry.

the disturbances introduced by the grains) becomes nonuniform. Equation (3) must then be completed with the convective terms and the longitudinal variation of the normal stress. These new terms would represent only small corrections to the fundamental flat bed balance if the steepness of the bed forms were very small. The magnitude of the main convective term, measuring the flow nonuniformity, can be estimated as $\rho U_0(\eta U_0/h)/\lambda$ which becomes comparable to the driving force, G_0, when $\eta/\lambda \sim f$. Natural bed forms, even of small amplitude, under nonseparated flow manifest the flow nonuniformity; therefore, we expect their steepness to be at least comparable to the friction coefficient.

[16] When the amplitude of the bed forms increases, the opposing force due to friction becomes eventually unable to decrease the flow velocity at the rate required by the geometry of the bed form in its lee side. *Giménez-Curto and Corniero Lera* [2000] have shown that this leads to flow separation and it occurs under the condition

$$\frac{\eta}{\lambda} > \gamma\sqrt{f_0} \tag{6}$$

in turbulent flows. Here f_0 represents the friction coefficient which would produce a uniform shear flow with the same thickness and velocity as the actual flow, running over a flat

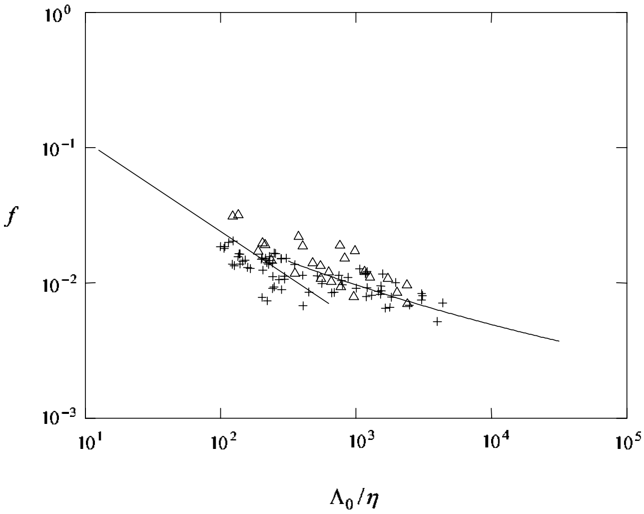

Figure 2. The friction coefficient in steady open channel flow over natural bed forms generated in the laboratory with steepness greater than 0.05. Observations by *Guy et al.* [1966] (crosses) and *Mantz* [1992] (triangles). The solid lines represent the friction for fixed bed conditions (without sediment in motion) as given by equations (9) and (10).

bed with the same characteristics; γ is a coefficient depending on the bed form geometry, which has been estimated to be between 1.0 and 2.0 for fixed, two-dimensional beds.

[17] The dynamical situation changes substantially when flow separation from the bed forms occurs. In such a case, which we represent in Figure 1b, it is convenient to perform a new spatial averaging of the equations of motion to obtain a new mean, uniform flow. This can be done, following the method described in the work of GCCI, by integrating over plain domains that, as shown in Appendix A, must have a large longitudinal dimension as compared with $\varepsilon\Lambda_0$. As a result, there emerge new large mean forces in the flow region between the crest and the trough of the forms, where the leading order balance is

$$\frac{1}{A_w}\frac{\partial(A_w T_{xz})}{\partial z} + F_{px} = 0. \quad (7)$$

Here F_{px} represents the mean force that bed forms exert on the flow via pressure (the mean force due to the grains, i.e., the mean value of $f_{vx} + f_{px}$ must be small when compared with F_{px} as it indicates the very occurrence of separation); T_{xz} is the new averaged shear stress, and A_w represents now the area of the fluid region of the new averaging domain. The total mean stress T_{xz} has a viscous component, which usually will be negligible, a turbulent component, and the so-called form stress, the mean momentum flux due to velocity disturbances introduced by the bed forms (the flow disturbances associated with the grains are small).

[18] Clearly, the force balance (7) represents a conventional (free of sediments) rough flow, whether rough turbulent or jet, with roughness height η (see Appendix A). This provides justification to the fact that when the flow undergoes separation from the bed forms it can be considered as a flow over a fixed bed, ignoring the direct effects of sediment grains in motion within the fluid.

[19] We have presented empirical evidence of this result in the work of GCCI for oscillatory flow, where the common situation corresponds with the so-called jet regime. In this case, the total friction can be calculated by means of the following friction coefficient:

$$f = 0.36\varepsilon^2 \quad (8)$$

for fixed artificial as well as for naturally formed rippled beds.

[20] It is worthwhile to provide also empirical evidence for steady open channel flow, because the flow scale Λ_0 is usually much longer than in oscillatory flow, and therefore the common situation represents a rough turbulent flow. In this respect, we consider herein the 232 laboratory observations on bed forms collected by *Guy et al.* [1966] and the 38 experiments by *Mantz* [1992] with very fine sediments. In order to select the observations with flow separation, we first disregard those having a Froude number greater than unity, which very likely correspond to antidunes and do not produce separation. Condition (6) was derived for fixed bed. Although we expect this condition to be formally valid also for mobile bed, unfortunately neither do we have information about γ nor can we estimate with accuracy the friction coefficient f_0 for such bed conditions. Therefore since $\sqrt{f_0}$ must exhibit a rather small variation, we will adopt a constant and quite conservative value of 0.05 as the minimum steepness capable of producing flow separation from the bed forms.

[21] Figure 2 represents the friction coefficient f as a function of Λ_0/η for the 73 observations by *Guy et al.* [1966] and 25 by *Mantz* [1992] with bed form steepness greater than 0.05 and Froude number less than unity. In this figure, we have represented also the friction coefficient corresponding to fixed bed with forms of height η, see the work of *Giménez-Curto and Corniero Lera* [2000], which is given by

$$f = 0.52\varepsilon^2 \quad (9)$$

for the jet regime (very large roughness), and the following expression for fully rough turbulent flow:

$$\frac{1}{\sqrt{f}} = \frac{1}{\kappa}\ln\left(\frac{h}{k_s}\right) + B, \quad (10)$$

where $k_s \equiv \eta$ represents the equivalent sand roughness; κ = 0.40 is the Von Karman's constant and B = 4.5 Clearly, Figure 2 provides empirical support for our above argument that when flow separation occurs the friction coefficient can be calculated as if the bed were fixed, using η as the roughness height.

3. The Maximum Bed Form Height

[22] We first note that during the evolution of a specific bed form, it is possible that its steepness never attains the condition of separation (equation (6)). The complete growing, decay, and disappearance of the bed form under a gradual increase of flow velocity may occur entirely in the absence of separation. Nevertheless, it is clear that for a

given wavelength, the absolute maximum possible bed form height will be reached under the occurrence of flow separation. Under such conditions, we have just argued that the maximum total shear stress can be calculated by means of the friction coefficient f given by formula (8) or (9) for oscillatory and steady flows, respectively, in the jet regime (Λ_0/η less than 100–200), and by equation (10) in the case of rough turbulent flow.

[23] On the other hand, we assume that bed forms adjust to the pressure disturbances generated by themselves. From the work of GCCII, we know the magnitude of these disturbances to be given by equation (1), with $k \equiv \eta$ and this permits us to calculate an estimate for the mean force that bed forms exert upon the fluid flow as:

$$F_{px} \sim \rho \frac{\varepsilon U_0^2}{\lambda}. \tag{11}$$

Balance (7) indicates then that the decrease in total shear force that is produced between the level of the bed form crests and that of their troughs is

$$(A_w T_{xz}) \sim \rho \varepsilon U_0^2 \frac{\eta}{\lambda} A_w. \tag{12}$$

[24] Consider now naturally formed undulations on a granular bed producing flow separation. The fluid flow over the crests of the bed forms exerts a total shear force $A_w \tau_0$ upon the lower region, therefore expression (12) is bounded by this value. If the decrease in the shear force approached this bound, the force at the troughs level would become insignificant, the flow there would cease, and the grains cannot be moved. This sets up an absolute limit for the growing of natural bed forms, which can be obtained easily by equating expression (12) to $A_w \tau_0$. Since $\tau_0 = \rho f U_0^2$ the limiting steepness must be $O(f/\varepsilon)$, and realizing that this order of magnitude estimate cannot depend on any other parameter, we can write:

$$\left(\frac{\eta}{\lambda}\right)_{\lim} = \beta \frac{f}{\varepsilon}, \tag{13}$$

where β is an $O(1)$ constant.

4. Comparison With Observation

4.1. Steady Flow

[25] Taking into account that when the flow undergoes separation from bed forms, the friction coefficient (9) or (10) only depends on η/Λ_0 and that $\varepsilon = (\eta/\Lambda_0)^{1/3}$ expression (13) represents a single line on axes (η/Λ_0, λ/Λ_0). Therefore if we represent the observations on bed form geometry on those axes, this line must define a limit so that no observations with η greater than that given by equation (13) must exist.

[26] In Figure 3, we show the entire set of observations by *Guy et al.* [1966] and *Mantz* [1992] on axes (η/Λ_0, λ/Λ_0), where $\Lambda_0 = U_0^2/g_x$ appropriate to steady, uniform open channel flow, together with expression (13) with $\beta = 1$ and f as given by equation (10). Clearly, the line that represents equation (13) defines almost perfectly a limiting condition for all the bed forms. It gives the maximum height for given Λ_0 and λ, or conversely, the minimum length for given Λ_0 and η.

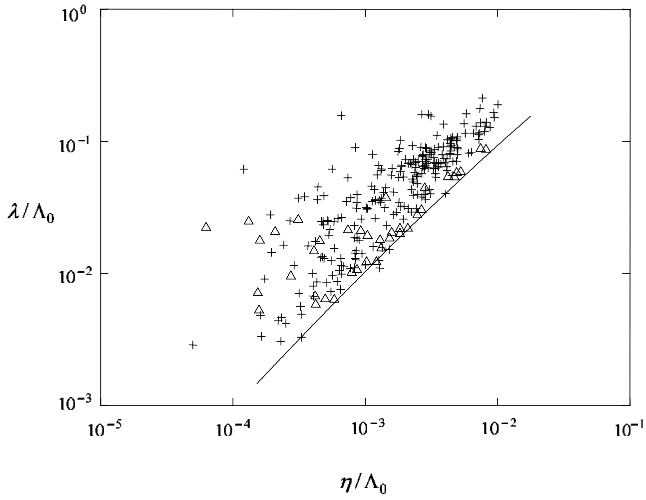

Figure 3. Dimensionless height and length of the bed forms under steady open channel flow. The complete series of observations by *Guy et al.* [1966] (crosses) and *Mantz* [1992] (triangles) as compared with the limiting height given by expression (13) with $\beta = 1$ and f from equation (10).

4.2. Oscillatory Flow

[27] In the lower region of flow, where bed forms occur, the local acceleration term is small. This is the reason why problems like friction, bed forms, and transport can be treated for oscillatory flow much in the same way as for steady flow. The basic length scale Λ_0 corresponds now with the amplitude of motion, $\Lambda_0 = a$ which in general has a value much less than in steady open channel flow. This means that unlike steady flow, which exhibits commonly a rough turbulent regime, in oscillatory flow the near-bed flow under separation will be usually in the so-called jet regime and the friction coefficient will be given by expression (8) as shown in the work of GCCI.

[28] Figure 4 represents the complete series of the most widely cited observations on bed form height and length under oscillatory flow: *Manohar* [1955] (319 obs.), *Kennedy and Falcon* [1965] (25 obs.), *Horikawa and Watanabe* [1967] (27 obs.), *Carstens et al.* [1969] (42 obs.), *Mogridge and Kamphuis* [1972] (161 obs.), *Nielsen* [1979] (90 obs.), *Lofquist* [1980] (10 obs.), *Ribberink and Al-Salem* [1994] (25 obs.), *Rankin and Hires* [2000] (10 obs.), *Faraci and Foti* [2001] (36 obs.), and *O'Donoghue and Clubb* [2001] (35 obs.). It can be appreciated that expression (13), with $\beta = 1$ and f as given by equation (8), represents again a very good approximation for the maximum bed form height.

[29] A small number of observations in Figure 4 appear to exhibit bed form steepness greater than that given by the limiting condition (13). At first glance, this fact seems to indicate a higher value for β in equation (13), however, further analysis of these observations reveals that they correspond with flume experiments in which the flow is not a pure oscillatory one, but it exhibits harmonic components of large amplitude or significant mean flows. Although the theoretical arguments that lead to equation

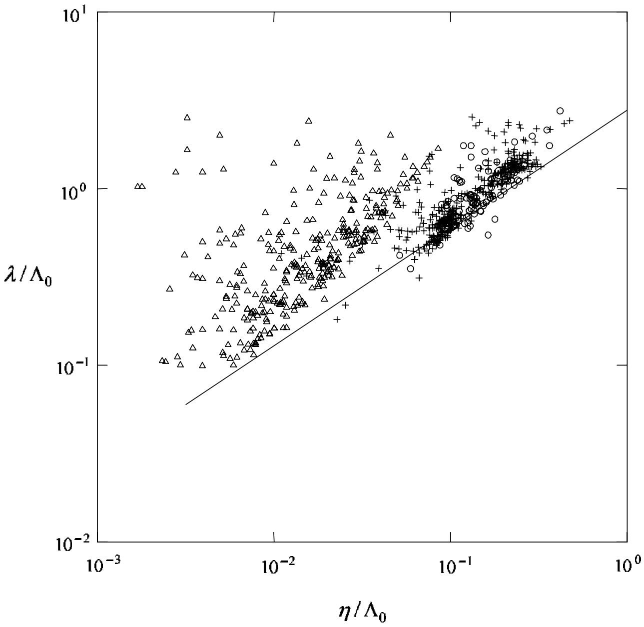

Figure 4. Dimensionless height and length of the bed forms under oscillatory flow. Observations by *Manohar* [1955] (triangles); *Kennedy and Falcon* [1965], *Horikawa and Watanabe* [1967], *Carstens et al.* [1969], *Mogridge and Kamphuis* [1972], *Nielsen* [1979], and *Lofquist* [1980] (crosses); *Ribberink and Al-Salem* [1994], *Rankin and Hires* [2000], *Faraci and Foti* [2001], and *O'Donoghue and Clubb* [2001] (circles). The solid line represents the limiting height as given by equation (13) with β = 1 and f from equation (8).

(13) remain valid in such cases, the estimate of the representative velocity, U_0, and driving force, G_0, which produces the length Λ_0, could be in error. This is explicitly recognized by *Kennedy and Falcon* [1965] for their three observations with fine sand and large values of the maximum bed velocity and orbital diameter, which correspond precisely with the three crosses with smaller λ/Λ_0 appearing on the right side of the line which represents equation (13) in Figure 4.

5. Conclusions

[30] We have investigated the dynamics of the flow over steep sand bed forms producing flow separation. Our theoretical arguments lead to the following main conclusions:

1. When separation occurs, the large-scale vorticity associated with this phenomenon becomes the most important mechanism generating friction. In this respect, other mechanisms like skin friction, small-scale roughness, and sediment transport, albeit present, appear to be of minor importance.

2. The mean force balance under the crests of the bed forms, equation (7) or (A1), introduces formally, in a natural way, an important parameter which in past treatments entered the problem in an ad hoc manner, the roughness height.

3. This fundamental balance expresses that the mean shear force due to friction is equilibrated by the force that bed forms exert on the fluid via pressure. The bed forms grow until they have a height such that friction at their troughs becomes negligible, since in that situation the motion of fluid and sediment at the troughs would cease.

4. Previous calculation by the authors, GCCII, of the pressure difference between the front and the rear faces of the bed irregularities when the flow undergoes separation, permits the prediction of the absolute limit for the growing of natural bed forms. This limit is given by equation (13), which is compared with observations in Figures 3 and 4 showing an excellent behavior for steady open channel flow as well as oscillatory flow.

Appendix A: On the Structure of Rough Flows

[31] Rough flows can be defined as those flows near an irregular boundary in which friction does not depend on viscosity, but depends primarily on the geometrical properties of the boundary irregularities from which the flow separates. In such flows, the viscous stress is negligible and the actual frictional stress may have a turbulent nature (in the so-called rough turbulent flows) or it may be a form stress, in cases with very high roughness. In this latter case (see the work of GCCI), there is one single layer in the region over the crests of the irregularities, representing an equilibrium between the driving force, G, and the form stress variation, entering also the mean local acceleration. In the case of a rough turbulent flow, the well known three-layer structure can be observed: an outer layer in which G is balanced by the Reynolds stress variation (plus local acceleration); an overlap layer in which the Reynolds stress is nearly constant, the form stress term balancing approximately the driving force; and an inner layer in which the Reynolds stress decreases abruptly downward, this force being balanced by an equivalent increase in the form stress.

[32] Although in the works of GCCI and GCCII, we have investigated mainly the flow over flat granular beds, the method of spatially averaging is generally applicable, and has already been used for the study of friction in rippled beds in GCCI and in the work of *Giménez-Curto and Corniero Lera* [2000]. If there exist bed forms capable of producing flow separation, we perform a spatial averaging of the governing Reynolds averaged equations. As shown in the work of GCCII, the largest scales of the flow disturbances introduced by the irregularities are $O(\varepsilon\Lambda_0)$ which represents also the magnitude of the length scale in which we expect correlation between disturbances. As the smallest disturbances are $O(\eta)$, the domain of averaging must have a large linear dimension as compared with η. On the other hand, its longitudinal length scale must be large as compared with $\varepsilon\Lambda_0$ in order to smooth out the entire spectrum of disturbances. Clearly, as long as λ is greater than η and smaller than $\varepsilon\Lambda_0$, it does not affect significantly the mean flow properties over the crests of the bed forms. This is the reason why the friction coefficient depends only on η/Λ_0 as shown in the works of GCCI and *Giménez-Curto and Corniero Lera* [2000] and in Figure 2. However, under the crests of the bed forms, the operation of averaging itself introduces a dependence on λ.

[33] *Nikora et al.* [2001] have distinguished some layers in order to clarify the structure of the mean flow in the context of the spatially averaged equations introduced in the

work of GCCI. However, as they did not know the magnitude of the fundamental term due to the force that bed irregularities exert on the flow, the flow structure under the crests of the bed could not be considered fully understood. In the work of GCCII, we have presented a very simple and general estimation of that force, which permits the understanding of the flow region of the shear layer between the roughness elements. Here we complete the investigation of this flow region by considering the variation of the fluid area A_w separately. Equation (7) can be written in the form:

$$\frac{1}{A_w}\frac{\partial A_w}{\partial z}T_{xz}+\frac{\partial T_{xz}}{\partial z}+F_{px}=0 \qquad \text{(A1)}$$

making explicit a well-known parameter of rough flows, their roughness height, k_s, which can be defined as

$$k_s=\left[\left(\frac{1}{A_w}\frac{\partial A_w}{\partial z}\right)^{-1}\right]_0. \qquad \text{(A2)}$$

This represents the length scale, normal to the bed, in which the fluid area, A_w, varies significantly (the subscript 0 indicates a specific representative value).

[34] The first term in equation (A1) is $O(\tau_0/k_s)$, which is usually greater than F_{px}. Therefore an "interfacial layer" can be distinguished, with a thickness $O(k_s)$, downward from the crests of the bed irregularities. In this layer, the shear force due to the variation of A_w, i.e., the first term in equation (A1), must be balanced by an increase downward of the shear stress. Consequently, in a rippled bed, for which $k_s = O(\eta)$ although the total shear force, A_wT_{xz}, must always decrease, the mean stress T_{xz} increases downward from the crest level unless the bed forms are of maximum height. In the limiting condition (13), the magnitude of the pressure force, given by equation (11), becomes comparable to τ_0/η, which permits the reduction of the shear stress.

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