Flow characteristics in the interfacial shear layer between a fluid and a granular bed

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[1] The shear layer that forms at the interface between a fluid in motion and a granular bed is shown to be tractable in the most general case, including the occurrence of flow separation from the grains, which appears to be the most common situation. The phenomenon of separation produces fundamental leading order effects, especially in relation to the mean flow between grains and also in the generation of a disturbed ensemble-averaged flow, which previous formulations were unable to detect. In this respect, we argue that some known results pertaining to the jet regime, recently introduced by the authors, can be directly extended outside that regime: in particular, the mean force that bed grains exert on the fluid, which appears to be generally applicable provided only that separation occurs. Furthermore, the maximum pressure disturbances, as well as the velocity disturbances at the outer edge of the shear layer, can be evaluated just as in the jet regime. This extension permits very general estimates of the main forces and the thickness of the shear layer, above and beneath the bed surface. In particular, the forces on the grains are shown to exhibit a spectrum. We present very simple expressions for the mean force as well as for the maximum force, which are compared with existing observations on sediment motion initiation under oscillatory flow, showing an excellent agreement. *INDEX TERMS:* 3020 Marine Geology and Geophysics: Littoral processes; 3022 Marine Geology and Geophysics: Marine sediments—processes and transport; 4546 Oceanography: Physical: Nearshore processes; 4558 Oceanography: Physical: Sediment transport; 4568 Oceanography: Physical: Turbulence, diffusion, and mixing processes; *KEYWORDS:* interfacial shear flow, sediment movement, incipient motion, porous media, granular bed

1. Introduction

[2] Although a number of problems related to the physics of sea waves can be treated, disregarding vorticity effects (within a well-known theoretical framework), there exist important phenomena in which vorticity plays a fundamental part. Commonly, this fact introduces considerable difficulties in the theoretical treatment, to such an extent that in some cases our best predictions may deviate significantly from observation. This occurs in particular in two important interfacial phenomena: wind-wave generation [*Van Duin*, 1996] and sediment movement [*Sleath*, 1984].

[3] Understanding the flow dynamics of the shear layer that forms at the interface between a fluid and a granular medium is of fundamental importance for a variety of practical problems. Among them are the effects of bed permeability on sea waves, sediment motion (and its subsequent effects like bed forms, friction, transport, etc.) in coastal regions, and also certain problems in coastal engineering practice related to porous structures and structure-foundation interaction.

[4] Perhaps one of the earlier investigations in this context was the work by *Putnam* [1949] [see also *Reid and Kajiura*, 1957] about wave-induced flow in a permeable bed and its contribution to wave damping. In that study a simple harmonic wave propagates over a horizontal porous layer; potential flow is assumed in the fluid region, whereas the classical Darcy's law governs the flow within the porous bed. The shear effects at the interface are ignored, and therefore the horizontal velocity is discontinuous across the bed surface. In this very simplified model the force driving the flow through the porous bed is the wave-induced pressure gradient, which is $O(\rho gak)$ if the depth is not too large (here, ρ is the fluid density, g is the acceleration due to gravity, a is the wave amplitude, and k is the wave number). The force that the solid skeleton exerts on the fluid, per unit volume, is modeled by Darcy's term $\mu u_s/K$ (where u_s is the seepage velocity, μ is the dynamic viscosity, and K is the intrinsic permeability of the porous medium, which is $\sim 10^{-3}$ times the square of the particle diameter for sand). By equating these two forces, the order of magnitude of the horizontal seepage velocity is obtained, which is $K\sigma/\nu$ times the typical horizontal velocity of the exterior wave flow (σ represents the angular wave frequency, and $\nu = \mu/\rho$ represents the kinematic viscosity of the fluid). It must be noted that this is a very small velocity.

[5] Since no experiment has been reported that directly measures motion within a porous bed, near its surface, the first question raised is about the validity of this simplified model, at least as a first approximation. In this respect, we have at our disposal an indirect way of measuring the force that waves exert on the bed grains of sediment, namely, the observations on sediment motion initiation on a granular bed under oscillatory flow. The observation appears to indicate that beyond a first threshold, rather poorly defined, in which some grains occasionally move (incipient motion), there exists a well-defined and reproducible transition to general motion of the uppermost particles [*Manohar*, 1955; *Chan et al.*, 1972], the so-called general motion. It is clear that when this situation is reached, the mean force on the uppermost grains due to fluid motion must be of the order of their submerged weight, which is therefore a measure of that force.

[6] *Chan et al.* [1972], after careful and extensive experimentation, with sediment diameter and specific submerged weight varying by a factor of >10 and with fluid viscosity varying by a factor of $\sim$100, were able to obtain a very simple

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expression that correlates the complete series of their observations on general motion very strongly (to within ±20%). This expression can be written as

$$\frac{a_0 \sigma^{3/2}}{(s-1)^{3/4} g^{3/4} D^{1/4}} = 0.37. \qquad (1)$$

Here, a_0 represents the amplitude of fluid oscillation, s represents the sediment density relative to that of the fluid, and D represents the particle diameter. It must be emphasized that equation (1) does not include any effect of liquid viscosity, which is a very striking fact if we take into account that many of the observations are well into the limits of the laminar flow conditions. The authors of this experimentation realized the importance of their finding and made an inconclusive attempt to explain it.

[7] *Putnam*'s [1949] model described above implies that the mean force on the grains is just the horizontal pressure gradient imposed by the exterior wave motion, which does not depend on viscosity. Unfortunately, the specific submerged weight in *Chan et al.*'s [1972] observations is typically 1 order of magnitude larger than the exterior pressure gradient, ranging between eight and forty times this force. This suggests that there must exist a mean force of larger magnitude than the pressure gradient that is not considered in this simplified model. The obvious candidate is the shear stress gradient arising at the interface. Evidently, we can hardly expect to find a correct solution for the forces on the grains, and, further, for the flow within the porous medium, without considering the shear in the porous side of the interface.

[8] Although more complete formulations have been used in the past, capable of considering nonrigid skeletons [see *Mei*, 1983] and including also a boundary layer at the fluid side of the interface [*Liu*, 1973], a study including shear stresses in the fluid phase beneath the interface, i.e., in the porous medium, has only very recently been published [*Liu et al.*, 1996]. This work introduces inertia forces and viscous forces in the fluid within the pores of the bed together with a Darcy-type model for the grain forces on the fluid, and it appears that it must produce the correct solution, at least for small Reynolds numbers such as those occurring in the experimental work by *Chan et al.* [1972]. However, Liu et al.'s model implies that the force on the grains depends on fluid viscosity, in clear conflict with the experimental results by Chan et al. We must conclude that a Darcy-type term cannot model correctly the force that bed grains exert on the fluid in the shear layer.

[9] Even though, in the specific field of sediment motion initiation, little effort has been devoted to the study of the flow structure between the bed grains, the fluid forces have traditionally been modeled in a different way. The treatment usually follows the ideas of *Shields* [1936], which originally were introduced in relation to open channel flow and later were extended to oscillatory flow [*Komar and Miller*, 1974; *Madsen and Grant*, 1975; *Sleath*, 1978] (see *Sleath* [1984] for a general review). The forces are assumed to be proportional to the square of some local velocity through a drag (and in some cases also lift) coefficient. This local velocity is assumed to be comparable and proportional to the friction velocity, and consequently, the force is again made to depend on viscosity by means of an ad hoc hypothesis.

[10] A correct evaluation of the forces on sediment grains must take into account that, as shown by *Giménez-Curto and Corniero* [2000], fluid flow will undergo separation from the uppermost grains even for rather low Reynolds numbers. This signifies that the force on the grains is due mainly to the pressure difference between their forward and rear faces, the skin friction being negligible. Furthermore, as we shall show herein, the fundamental effects of the vorticity generated at the interface by the separation phenomenon not only determine the nature and magnitude of the mean force on the grains, but are more profound and very strongly affect the disturbed flow.

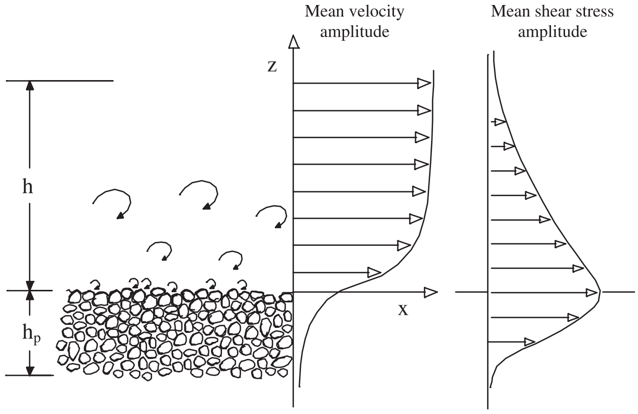

Figure 1. Schematic representation of interfacial shear layer formed near the surface of a granular bed.

2. Theoretical Framework

[11] When a progressive wave train propagates over a granular, permeable bed, the motion of water near the bed approximates an oscillatory flow. Since the water velocity within the bed is very small as compared with exterior flow velocity, there must exist a shear layer allowing transition and extending both over and beneath the bed surface. Figure 1 represents the main properties of this layer. The amplitude of the flow velocity reduces downward and is accompanied by the appearance of shear stress and vorticity. If the Reynolds number were large, as occurs commonly in the coastal region, the flow in this layer would become turbulent. Turbulent fluctuations are defined as departures from ensemble averaging, but the boundary condition at the surface of the grains obliges the ensemble-averaged values to vary strongly spatially, which introduces disturbances in the ensemble-averaged flow.

[12] An appropriate theoretical formulation of this problem must therefore include turbulence and some suitable spatial averaging. This leads to the spatially averaged Reynolds (SAR) equations. Such equations can be formulated in the entire fluid domain, outside the porous region and in the fluid region within the porous medium, thus allowing a continuous transition of the flow across the interface. *Giménez-Curto and Corniero* [1996] derived the SAR equations using plane averaging, which is a requirement for the treatment of the flow in the shear layer, and taking into consideration the variation of the fluid domain of averaging across the interface. Following this work, we assume that bed grains are fixed and form a rigid skeleton. The orientation of the interface can be defined by means of a plane that moves from the fluid side to the porous medium until a position is reached for which at least one grain can be found at distances $O(D)$ from any point of the plane. We shall adopt $z = 0$ as a reference plane parallel to the interface orientation, just resting on the uppermost crests of the bed.

[13] A spatial plane averaging (denoted by angle brackets) is defined by integration of ensemble-averaged quantities (denoted by an overbar) over the fluid portion of a fixed region of the plane $z = cte$ (parallel to the interface) centered at a generic point x_i (x, y, z) and with an area, A_w, that is large in comparison with D^2. By means of this operation we can split up the ensemble-averaged velocity and pressure into a spatial mean and a boundary disturbance (with zero spatial mean) as follows:

$$\overline{u}_i = U_i + u_{bi} \qquad (2)$$

$$\overline{p} = P + p_b, \qquad (3)$$

where $U_i = \langle \overline{u}_i \rangle$ and $P = \langle \overline{p} \rangle$ represent the mean velocity and pressure ("mean" signifies ensemble and then spatial plane averaging). Boundary disturbances u_{bi} and p_b cannot be considered as turbulence, but they are of a deterministic and repeatable nature. Of course, instantaneous velocity and pressure are obtained by adding turbulent fluctuations, u_{ti} and p_t, to the ensemble-averaged values (i.e., $u_i = \overline{u}_i + u_{ti}$ and $p = \overline{p} + p_t$). Subscript i indicates the ith Cartesian component, t indicates turbulent fluctuation, and b indicates boundary disturbance. (The usual convention, $u_i \equiv (u, v, w)$, $U_i \equiv (U, V, W)$, $u_{ti} \equiv (u_t, v_t, w_t)$, and $u_{bi} \equiv (u_b, v_b, w_b)$ will also be used.)

[14] By performing the spatial averaging of the conservation of mass and Reynolds equations governing the fluid flow, one obtains [see *Giménez-Curto and Corniero*, 1996]

$$\frac{\partial}{\partial x_j}\left(A_w U_j\right) = 0 \tag{4}$$

$$\rho\frac{\partial U_i}{\partial t} + \rho U_j\frac{\partial U_i}{\partial x_j} = -\frac{1}{A_w}\frac{\partial}{\partial x_i}(A_w P) + \rho\, g_i + \frac{1}{A_w}\frac{\partial}{\partial x_j}\left[\mu\frac{\partial}{\partial x_j}(A_w U_i) - \rho\, A_w\left(\left\langle\overline{u_{ti}u_{tj}}\right\rangle + \left\langle u_{bi}u_{bj}\right\rangle\right)\right] + f_{vi} + f_{pi}, \qquad i,j = 1,2,3, \tag{5}$$

where tensor notation has been used; $x_i \equiv (x, y, z)$, and t are Cartesian coordinates and time, respectively, g_i is the ith component of the acceleration of gravity vector, and

$$f_{vi} = \frac{1}{A_w}\int_C \mu\frac{\partial\overline{u}_i}{\partial x'_j}\frac{n_j ds'}{\sin\gamma} \tag{6}$$

$$f_{pi} = -\frac{1}{A_w}\int_C \overline{p}\,\frac{n_i ds'}{\sin\gamma} \tag{7}$$

represent the components of the mean force that the bed grains intersected by the plane of integration exert on the fluid, per unit volume, through viscous friction and pressure, respectively. In these expressions, n_i is the ith component of the unit outward vector normal to the surface of the grains, $\gamma = \arccos n_3$, and s' represents the arc length along the curve C defining the intersection of the plane domain of integration with the grains.

[15] The exact equations (4) to (7) describe slow x and y variations and arbitrarily rapid z variations of mean flow quantities. They are valid regardless of the magnitude of the disturbances, and their gradients, in comparison with mean flow values. Nevertheless, these equations are especially useful for cases in which the x and y derivatives of mean velocity and pressure are small as compared with the gradients of disturbances, as occurs when flow undergoes separation from the bed grains. We emphasize that mean forces exerted by bed grains on the fluid emerge from spatial averaging in a natural way, since the grains must be excluded from the averaging domain.

[16] The first, second, and third terms within square brackets in the right-hand side of equation (5) represent, respectively, the mean viscous stress, the Reynolds turbulent stress, and the so-called form stress, the mean momentum flux due to boundary disturbances, which requires the existence of vorticity in the disturbed motion in order to be different from zero.

[17] Over the bed surface ($z > 0$), $f_{vi} = f_{pi} = 0$ and A_w is a constant; therefore these three quantities disappear from the governing equations (4) and (5). Beneath the bed surface, A_w is proportional to a plane porosity, n, representing the ratio of the fluid area to the total area of the (fixed) averaging domain. It must be allowed to make a transition from $n = 1$ at $z = 0$ to a lesser constant value characteristic of the porous medium in distances of the order of magnitude of the grain diameter, D. This is the reason that plane averaging must be used instead of volume averaging.

[18] In this paper we assume the mean motion to be two-dimensional in the plane (x, z). This implies that there are no gradients of mean quantities nor mean velocity in the y direction.

3. Mean Flow in the Fluid Region of the Shear Layer

[19] Taking into account that the thickness of the shear layer is very small as compared with the wavelength of the exterior wave motion, the mean velocity normal to the bed, $U_3 \equiv W$, must be very small in comparison with the parallel component, $U_1 \equiv U$, when approaching the bed. This means that mean flow there can be represented very approximately by means of a uniform (in the x direction), oscillatory flow. In such a situation the mean pressure gradient plus the gravity component (if the interface were not horizontal) does not depend on z. Let us call $G(t)$ this force:

$$-\frac{\partial P}{\partial x} + \rho g_1 = G(t). \tag{8}$$

The x component of the mean momentum equation (5) is then, simply,

$$\rho\frac{\partial U}{\partial t} = G(t) + \frac{\partial\tau_{xz}}{\partial z}, \tag{9}$$

where the mean shear stress is given by

$$\tau_{xz} = \mu\frac{\partial U}{\partial z} - \rho\left\langle\overline{u_t w_t}\right\rangle - \rho\left\langle u_b w_b\right\rangle. \tag{10}$$

[20] Let U_0 and G_0 be typical values characterizing the order of magnitude of mean velocity, U, and driving force, $G(t)$. We shall use $U_0 = a_0\sigma$ and $G_0 = \rho a_0\sigma^2$ and adopt ρ, U_0, and G_0 as the basic quantities in order to define flow scales. Thus the basic length scale is

$$\Lambda_0 = \rho U_0^2/G_0 = a_0, \tag{11}$$

and the basic timescale is

$$T_0 = \rho U_0/G_0 = \sigma^{-1}. \tag{12}$$

[21] The maximum shear stress at the bed ($z = 0$), τ_0, can be given in dimensionless form by means of a friction coefficient, f, defined by

$$f = \tau_0/\left(\rho U_0^2\right). \tag{13}$$

[22] By observing that at the bed level the mean velocity is very small, we can neglect the left-hand side in equation (9) at that level, which allows an estimate of the thickness of the fluid region of the shear layer as

$$h = \tau_0/G_0. \tag{14}$$

Using equations (11) and (13), this thickness can be related with the basic scale, Λ_0,

$$h = \Lambda_0 f = a_0 f\cdot \tag{15}$$

[23] Turbulent regimes of flow are characterized by a very rapid increase of the Reynolds stress upward in the very vicinity of the bed, within the so-called inner layer. This represents a very large force that must be balanced by another shear stress variation, the viscous term in the smooth turbulent regime or the form stress term in rough turbulent flow. When the momentum flux due to turbulence is negligible, we may have a laminar regime if the viscous stress prevails, or we may have a jet regime when the form stress becomes the prevailing stress. *Giménez-Curto and Corniero* [1996, 2000] have investigated in detail this latter regime and have presented empirical evidence of its real existence in oscillatory as well as in steady flow. The jet regime exhibits one single balance above the bed level (like the laminar case) since there are no mean forces there with magnitude larger than G_0.

4. Disturbed Flow

[24] Following the arguments by *Giménez-Curto and Corniero* [2000], we expect flow separation from the crests of a granular bed if both the Reynolds number $R = U_0\Lambda_0/\nu = a_0^2\,\sigma/\nu$ and $\Lambda_0/D = a_0/D$ are large with respect to unity, which occurs practically always in nature under any flow regime. We will assume hereafter that these conditions hold and therefore will assume that separation does in fact occur.

[25] It appears that vorticity of scale D generated in the flow separation from a specific bed grain will interact with that generated at neighboring grains and with mean flow, which has a strong velocity gradient very near the bed. This results in an intensification of vorticity, which cannot remain restricted to distances of the bed $O(D)$, but extends even beyond the upper edge of the mean shear layer. Thereby, disturbances are created in the flow covering a wave number spectrum with corresponding wavelengths ranging from $O(D)$ to at least $O(h)$, as has already been shown to occur in the jet regime [*Giménez-Curto and Corniero*, 1996].

[26] The phenomenon of separation is accompanied by the singular appearance of strong pressure differences between the front and the rear faces of the grains, which generate high adverse pressure gradients ($\partial p_b/\partial x$) in the fluid between grains. This occurs because the continuous force due to skin friction can no longer balance the large convection term in the direction of motion imposed by the boundary geometry [*Giménez-Curto and Corniero*, 2000]. The singular nature of the pressure gradients, which arise in order to produce equilibrium, indicates that they cannot depend on viscosity nor on turbulence characteristics. Therefore they will depend exclusively on the bed geometry characterized by the grain diameter, D, the fluid density, ρ, and the characteristics of the exterior flow driving the shear layer, U_0 and G_0, regardless of the particular manner in which viscosity effects build that layer. This means that if we were able to calculate the mean disturbed pressure gradient $\langle\partial p_b/\partial x\rangle$ between the grains of the bed surface as a function of these parameters under some specific conditions, the result must apply also under any other conditions, being therefore generally valid, provided only that flow separation occurs. Fortunately, we can calculate this quantity in the jet regime, on the basis of the work by *Giménez-Curto and Corniero* [1996], from which we know that the order of magnitude of the force $f_{p1} = -\langle\partial p_b/\partial x_1\rangle$ is $\rho\varepsilon^2 U_0^2/D$, where ε is a small parameter, which can be expressed as

$$\varepsilon = (D/\Lambda_0)^{1/3}. \tag{16}$$

Therefore the mean pressure difference (jump) between the front and the rear faces of the grains of the bed surface is

$$\langle\Delta p_b\rangle = \beta\rho\varepsilon^2 U_0^2, \tag{17}$$

where β is an $O(1)$ coefficient.

[27] The previous argument, that when flow separation occurs, the mean pressure gradient can only depend on ρ, U_0, G_0, and D, leads to the conclusion that equation (17) must be valid regardless of the kind of flow (whether laminar, smooth turbulent, rough turbulent, or jet). This is a very simple and powerful result, which, as we shall show below, appears to be clearly confirmed by the observations of *Chan et al.* [1972].

[28] By subtracting the mean equation (5) from the Reynolds equations governing the ensemble-averaged local motion, we obtain the momentum equations for the boundary disturbed flow, which, for the fluid region over the bed surface, can be expressed as

$$\rho\frac{\partial u_{bi}}{\partial t} + \rho U\frac{\partial u_{bi}}{\partial x_1} + \rho\delta_{i1}u_{b3}\frac{\partial U}{\partial x_3} = -\frac{\partial p_b}{\partial x_i} + \rho\frac{\partial}{\partial x_j}\cdot\left[\nu\frac{\partial u_{bi}}{\partial x_j} - u_{bi}u_{bj} + \langle u_{bi}u_{bj}\rangle - \overline{u_{ti}u_{tj}} + \langle\overline{u_{ti}u_{tj}}\rangle\right] \quad i,j = 1,2,3, \tag{18}$$

with δ_{ij} being the Kronecker delta. Equation (18) reduces, in the outer edge of the shear layer, $z \approx h$, to the leading order balance

$$\rho U\frac{\partial u_{bi}}{\partial x_1} = -\frac{\partial p_b}{\partial x_i}, \tag{19}$$

on the assumption that, as will be examined below, boundary disturbances remain significant there. Equation (19), which must hold for the entire spectrum of disturbances, indicates that in this far region the order of magnitude of u_{bi} and p_b can only depend on the grain diameter (which represents the length scale of variation introduced by the bed), the fluid density, and the outer flow parameters, U_0 and G_0, without any direct influence of viscosity and turbulence, just as occurred for the mean pressure jump, $\langle\Delta p_b\rangle$, at the bed. As in that case, we know the order of magnitude of u_{bi} in the jet regime, which is εU_0. Equation (19) then indicates that, due to interaction with mean flow, the magnitude of pressure disturbances raises to

$$p_b \sim \rho\varepsilon U_0^2, \tag{20}$$

i.e., 1 order (in ε) larger than the mean pressure difference introduced by the bed grains $\langle\Delta p_b\rangle$, which is given by equation (17). We emphasize that these estimates must remain correct outside the jet regime, provided that flow separation occurs.

[29] It only remains for us to check that the largest disturbances reach the highest levels of the shear layer. Indeed, equation (19) dictates that a disturbance of length scale $\Lambda \approx h$ would produce pressure gradients $O(\rho\varepsilon U_0^2/h)$, which, using equations (13) and (14), can also be expressed as $O(\varepsilon G_0/f)$. Mean forces are $O(G_0)$; consequently, they are smaller than disturbed forces if ε is larger than f. It appears that flow disturbances will grow, in order to decrease their gradients, until they produce forces smaller than those produced by the mean flow, i.e., until the fundamental convective term $\rho U\partial u_{bi}/\partial x$ decreases to $O(G_0)$, which occurs for disturbances of length scale $O(\varepsilon\Lambda_0)$. Therefore flow disturbances will reach the outer edge of the shear layer if $\varepsilon > f$, a condition that would require extremely small grains or very small Reynolds numbers to be violated. In the case that ε were less than f, flow separation effects would likely be negligible.

[30] In the following, we assume that $\varepsilon > f$. In this case, we have, in summary, that pressure disturbances caused by separation range from $O(\rho\varepsilon^2 U_0^2)$ to $O(\rho\varepsilon U_0^2)$ and that velocity disturbances at the outer edge of the shear layer are $O(\varepsilon U_0)$. However, in lower levels of this layer, velocity disturbances may be larger, especially in turbulent regimes, since the boundary condition at the grain surface requires that $u_{bi} = -U_i$. Equation (18) indicates that a strong relation between

turbulence and boundary disturbances may arise in the very vicinity of the bed surface.

[31] A very important property of the class of disturbances we are dealing with is that, unlike turbulent fluctuations, they are always very nearly steady (they pertain to the ensemble-averaged flow, and therefore its timescale of variation must correspond with that of external driving flow, which implies that the time derivative in equation (18) is always very small). A fundamental consequence of this fact is that boundary disturbances are not transported by mean flow.

5. Mean Flow in the Porous Region of the Shear Layer

[32] Let us consider the magnitude of the terms in equation (5). From the arguments in section 4, it follows that for any kind of flow, the order of magnitude of f_{p1} is

$$f_{p1} \sim \rho\varepsilon^2 U_0^2/D \sim G_0/\varepsilon, \tag{21}$$

where we have used equations (11) and (16). The mean local inertial term is $O(G_0)$ in the outer region of the shear layer, and the mean convective term is even smaller. At the bed level both terms are much smaller; therefore, within the porous region, they are negligible as compared with f_{p1}. The mean pressure term plus the gravity term are, together, $O(G_0)$. By also neglecting f_{v1} because of the occurrence of flow separation, and by substituting the plane porosity, n, for A_w, the x component of the mean momentum equation (5), as applied to the porous region of the shear layer, reduces to

$$f_{p1} + \frac{1}{n}\frac{\partial}{\partial z}(n\tau_{xz}) = 0, \tag{22}$$

where the mean shear stress, τ_{xz}, is given by

$$\tau_{xz} = \frac{\mu}{n}\frac{\partial}{\partial z}(nU) - \rho\langle\overline{u_t w_t}\rangle - \rho\langle u_b w_b\rangle. \tag{23}$$

[33] Thus the shear stress gradient arises as the actual driving force for the flow through the porous bed. The stress in this region is $O(\tau_0)$. Equation (22), with the aid of equations (14) and (21), produces an estimate for the thickness of the porous region of the shear layer as

$$h_p = \varepsilon h. \tag{24}$$

[34] Regarding now the mean velocity in this region, we must consider the boundary condition at the surface of the grains, which requires that $U = -u_b$. *Giménez-Curto and Corniero* [1996] have shown that in the jet regime the mean velocity in the porous shear layer is $U_p = O(\varepsilon U_0)$. It can easily be verified that this estimate remains valid for laminar flow. However, turbulence obliges the mean velocity at the interface, and therefore the boundary disturbances, to be larger than εU_0, up to $O(U_*)(U_* = U_0\sqrt{f}$ represents the so-called friction velocity). It appears that in turbulent flows mean velocity, turbulent fluctuations and boundary disturbances at the interface must all have the same order of magnitude. Very recently, *Nikora et al.* [2001] have measured flow velocity between bed irregularities of the spherical segment type under open channel rough turbulent flow within the framework introduced by the authors. Despite the fact that these observations do not represent a correct spatial averaging, since this would have required a domain of averaging including many segments, *Nikora et al.* [2001, Figure 8] show that u_b has the same magnitude as the friction velocity. These experiments also confirm the validity of equation (22), as explicitly stated by the investigators in their conclusions.

[35] Finally, we consider the mean flow well within the porous bed, far from the interface, at distances that are large in comparison with h_p. In this region of flow the shear stresses become negligible, and the mean momentum equation (5) reduces to an equilibrium between pressure gradient plus gravity, which again becomes the driving force, and the force that grains exert on the fluid:

$$-\frac{\partial P}{\partial x} + \rho g_1 + f_{v1} + f_{p1} = 0. \tag{25}$$

[36] For a strictly uniform flow, it can easily be shown (by differentiating the z component of the momentum equation with respect to x) that $n\partial P/\partial x$ does not depend on z. As the wave-generated shear layer is very approximately uniform, we have that $n\partial P/\partial x$ does not depend significantly on z. Therefore the order of magnitude of the pressure gradient remains as G_0 traversing the shear layer, and it drives the fluid below. Moreover, the porosity can be considered as a constant, and volume averaging can be used instead of plane averaging since the length scale of variation of mean flow in the z direction becomes large in comparison with the grain diameter. We remark that in the porous region the mean flow can never be considered as essentially unsteady. Beneath the shear layer, flow separation effects can be neglected because local convective terms in the pores, giving a forward force on the fluid, can always be equilibrated by means of an increase in the pressure at the front face of the next grain. Equation (25) simply represents Darcy's typical equilibrium with time-modulated pressure gradient, if the grains are small enough. In the case of large bed grains, such that local Reynolds numbers (UD/ν) were much greater than unity, f_{v1} would become negligible, and f_{p1} could be modeled by a Forchheimer-type term.

6. Experimental Evidence of Mean Flow Characteristics: Justification of Chan et al.'s Results

[37] In this section, we present a first comparison with observation of the previous theoretical results. We focus our interest here on the particular situation described by experimenters as "general motion" of sediment under oscillatory flow. In such a situation all of the uppermost particles on the bed appear to move at the instants of maximum velocity.

[38] While the grains remain fixed, the flow has no mean velocity relative to the grains in the z direction. Moreover, any z asymmetry in the pressure distribution, giving rise to a mean lift force, must be small as compared with the main x asymmetry generated in flow separation. Therefore, besides the buoyancy effects, the mean fluid force on the grains reduces to the mean force in the x direction to a first approximation. This force is equal to $-f_{p1}$, under the assumption that flow separates from the bed grains, and is due to the mean pressure difference between their front and rear faces (drag force). Its magnitude is given by equation (21), irrespective of the flow regime.

[39] Clearly, the force exerted on the grains by the flow must overcome the stabilizing effect of the grains' weight in order to move them. Therefore, to set in motion most of the grains on the bed surface (general motion), it is required that the mean force upon them have the same order of magnitude as their submerged weight; that is,

$$\rho\varepsilon^2 U_0^2/D \sim \rho g(s-1). \tag{26}$$

[40] This relation must remain valid for any value of the involved parameters, which requires the ratio between the two

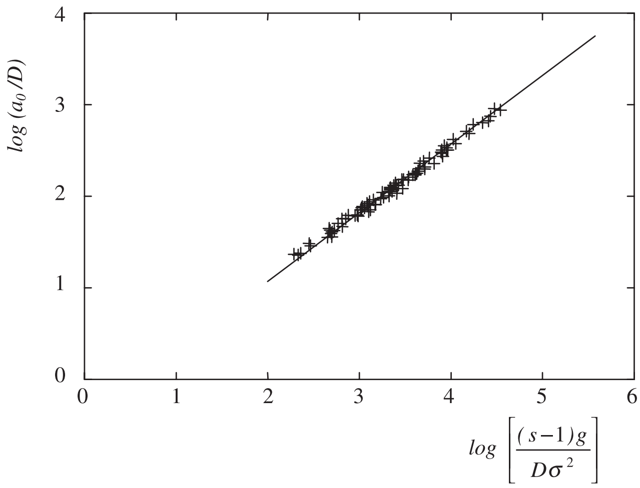

Figure 2. Comparison of equation (28) to experimental observations on general motion of sediment under oscillatory flow by *Chan et al.* [1972].

terms in equation (26) to be a constant. This leads to the conclusion that the condition for general motion of sediment can be expressed formally as

$$\varepsilon^2 U_0^2 = \beta_g g D(s-1), \tag{27}$$

where β_g is a coefficient $O(1)$. Taking into account that $U_0 = a_0\sigma$ and using equations (11) and (16), equation (27) exactly produces equation (1), obtained empirically by *Chan et al.* [1972]. The corresponding value for β_g is 0.27. This expression can also be written as

$$\frac{a_0}{D} = 0.37\left[\frac{(s-1)g}{D\sigma^2}\right]^{3/4}. \tag{28}$$

[41] Figure 2 represents the 83 observations on general motion by *Chan et al.* [1972] together with equation (28). Figure 2 exhibits a really impressive agreement between them, which must be interpreted as first evidence supporting the theory introduced herein, especially if it is realized that most observations were made under laminar flow conditions. This can be seen by plotting them on a graph, with axes R and Λ_0/D, where the limits of the different flow regimes have been drawn. Figure 3 shows such a graph including these limits, which must be considered as rough estimates. The limits of the jet regime have been discussed by *Giménez-Curto and Corniero* [1996]; the border between laminar and smooth turbulent regimes can be estimated to be at some Reynolds number $R = O(10^5)$ from the measurements by *Kamphuis* [1975], and the transition between smooth turbulent and rough turbulent flows will occur when $R_* = U_* D/\nu$ is $O(10)$ [see, e.g., *Monin and Yaglom*, 1971]. We note that in all experiments, flow separation very likely occurs, since the Reynolds number ranges from 26 to 8.4×10^4, and a_0/D varies between 23 and 9.1×10^2. Furthermore, it can easily be verified that $\varepsilon > f$ in all cases.

[42] Figure 2 not only provides strong empirical evidence that the mean force on the grains is, in fact, given by equation (21) irrespective of the flow regime, but also confirms experimentally the general validity of equation (22) for the porous region of the shear layer, as well as our estimate of its thickness, given by equation (24). Indeed, it is evident that the total force exerted by the flow over the bed upon the porous region (including both grains and the fluid between them) per unit area is τ_0. Clearly, the solid skeleton must absorb a significant part of this force, which must occur within a thickness $O(\varepsilon h)$ so that that the mean force acting on the grains has the correct magnitude.

7. Experimental Evidence in Relation to Disturbed Flow Characteristics

[43] Whereas the general motion of the uppermost particles of the bed must be related to mean forces, the threshold of incipient motion gives us information about the maximum local forces caused by disturbed flow. Observation indicates that long before reaching the general motion condition, some grains are occasionally dislodged from the bed and set in motion [*Manohar*, 1955; *Chan et al.*, 1972]. This fact clearly requires the existence of a disturbed flow capable of generating local forces even larger than f_{p1}.

[44] We have argued in section 4 that separation from the bed grains generates spatial disturbances in the ensemble-averaged flow covering a wide spectrum of length scales and affecting the flow beyond the mean shear layer. Equation (20) gives the largest magnitude of the pressure disturbances, which could be associated locally with the smallest length scale of variation, giving rise to absolute maximum local forces

$$\left(\frac{\partial p_b}{\partial x}\right)_{\max} \sim \rho\varepsilon U_0^2/D, \tag{29}$$

i.e., 1 order of magnitude larger than mean forces.

[45] This estimate of the maximum forces generated by disturbed flow, which must be valid irrespective of the flow regime, allows us to define formally an incipient motion condition. Indeed, if the maximum possible force were small in comparison with the submerged weight of sediment, no bed grain could be moved. However, when the magnitude given by equation (29) is comparable to the specific submerged weight of the grains, it becomes possible to move individual particles in some points of the bed surface, which must correspond with what the experimenters have called "incipient motion." Such a condition can be expressed as

$$\frac{\rho\varepsilon U_0^2}{D} = \beta_i \rho g(s-1), \tag{30}$$

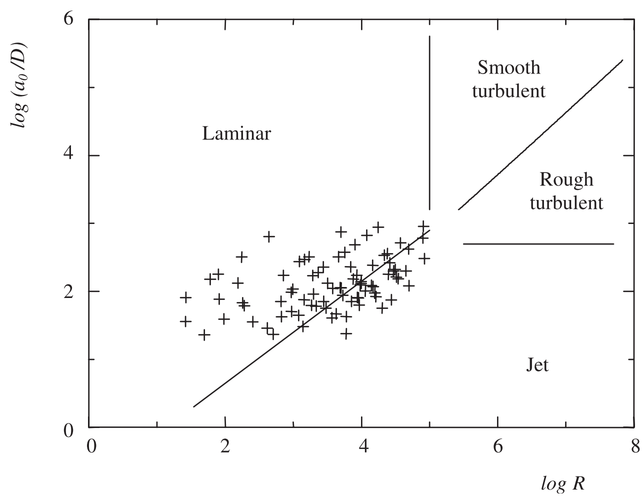

Figure 3. Flow conditions corresponding to observations on general motion of sediment by *Chan et al.* [1972] (crosses). Justification for limits between different regimes can be found in the text.

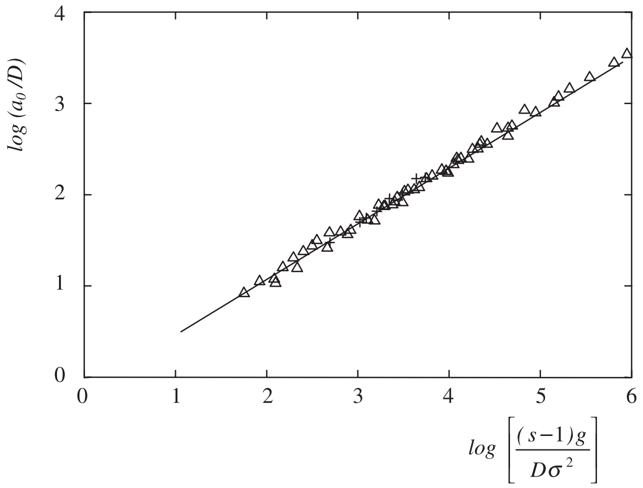

Figure 4. Comparison of equation (31) to experimental observations on incipient motion of sediment under oscillatory flow by *Bagnold* [1946] (triangles) and *Chan et al.* [1972] (crosses). Mean value obtained for B is 0.82.

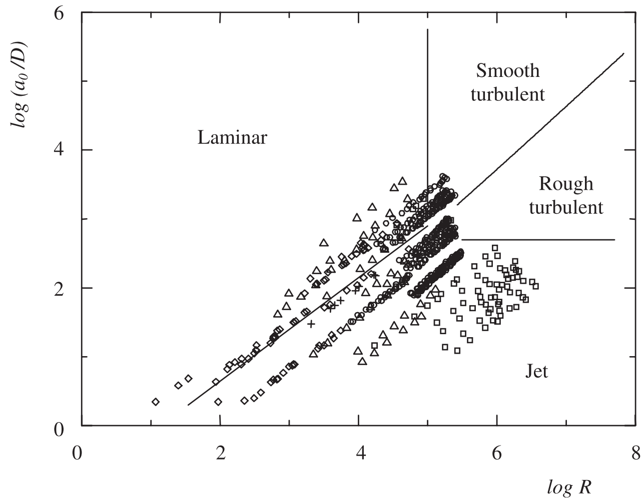

Figure 5. Flow conditions in experiments on initiation of sediment motion by *Bagnold* [1946] (triangles), *Manohar* [1955] (circles), *Goddet* [1960] (rhombs), *Rance and Warren* [1968] (squares), and *Chan et al.* [1972] (crosses).

where β_i is a coefficient $O(1)$. Substituting $U_0 = a_0\sigma$ and taking into account the definition of ε, equation (30) transforms to

$$\frac{a_0}{D} = B\left[\frac{(s-1)g}{D\sigma^2}\right]^{3/5}, \tag{31}$$

where $B = \beta_i^{3/5}$.

[46] *Chan et al.* [1972] recognize that incipient motion is not as well defined as general motion; however, they gave eight observations corresponding to that threshold and pointed out that the observations by *Bagnold* [1946] very likely corresponded to incipient motion. All of these experimental results (eight by Chan et al. and 56 by Bagnold) are represented in Figure 4, together with equation (31). It appears to be an almost perfect agreement, which strongly supports the description of the disturbed flow introduced herein. We emphasize that Bagnold's experiments cover a wide range of submerged relative densities (from 0.30 to 6.90) and particle diameters (from 0.09 mm to 8 mm). The mean value obtained for B from all 64 observations in Figure 4 is B = 0.82 (which corresponds with β_i = 0.72).

[47] In order to present a more complete comparison with observation regarding disturbed flow, we consider now the extensive experimental results on sediment motion initiation under oscillatory flow by *Manohar* [1955], who reported 333 observations using a flat oscillating tray (just like *Bagnold* [1946]), *Goddet* [1960], who gave 77 observations on a wave flume, and *Rance and Warren* [1968], who used an oscillating water tunnel (the same kind of apparatus as that used by *Chan et al.* [1972]) and also reported 77 observations. Figure 5 shows the flow conditions covered by the observations considered in this section. It is apparent that they correspond to different flow regimes, many of them representing complex transitional flows. Separation occurs probably in all experiments, since the Reynolds number varies between 12 and 3.6×10^6, and a_0/D ranges from 2.2 to 4.15×10^3. Moreover, it can be verified that $\varepsilon > f$ for all experiments.

[48] When all of these observations are represented in axes $[(s-1)g/(D\sigma^2), a_0/D]$, as suggested by previous analysis, together with equations (31) and (28), corresponding to incipient motion and general motion, respectively, as we have done in Figure 6, one can see that almost all of them are between both limits. This provides further support for the theoretical description of the disturbed flow that we have introduced here. Indeed, since no observation on sediment motion initiation appears to exist well beyond the previously obtained limits, it seems that even the subjective factor that accompanies this kind of experiment, with regard to the particular criterion used by each experimenter, can be convincingly explained. Clearly, whichever criterion about initiation of motion is adopted, it must range between the movement of some grains occasionally and the movement of all the grains. Figure 6 explains the existence of such a degree of subjectivity, which is but a consequence of the existence of a spectrum of pressure disturbances with length scales starting from D.

8. Practical Comments

[49] The sedimentary nature of actual marine beds and the fact that the maximum length scale of boundary disturbances, $\varepsilon\Lambda_0$, is very small as compared with the wavelength of natural waves permit the treatment of near-bed flow under waves as a nearly two-dimensional, uniform, oscillatory flow over a plane bed. This fact allows us to apply directly the results obtained herein, which can

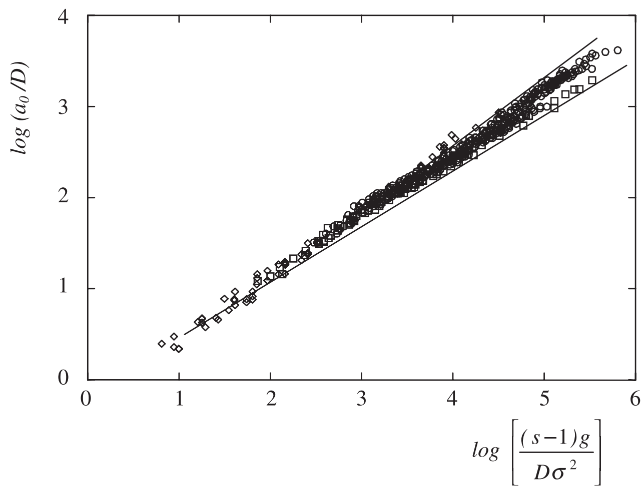

Figure 6. Threshold of sediment motion under oscillatory flow since observations by *Manohar* [1955] (circles), *Goddet* [1960] (rhombs), and *Rance and Warren* [1968] (squares). Solid lines represent equations (28) and (31).

easily be done by observing that for a wave of height $H = 2a$ and period $T = 2\pi/\sigma$ (wave number k) in water depth d, the linear wave theory gives the near-bed amplitude of motion as $a_0 = a/\sinh(kd)$, which produces immediately the values of $U_0 = a_0\sigma$, $G_0 = \rho a_0 \sigma^2$, and also $\Lambda_0 = a_0$.

[50] The arguments used herein tacitly assume nearly spherical grains with uniform size distribution. In general, natural sediments do not fit these characteristics. However, we observe that the diameter of the grains influences the fundamental parameter ε only as $D^{1/3}$. Therefore, in applying the formulae obtained herein to a natural setting, we expect only small deviations in the empirical constants, unless the shape of the grains differs significantly from the spherical or unless the sediment exhibits a very wide range of sizes.

[51] The basic balance of the disturbed flow (equation (19)) as well as that of the mean flow between bed grains (equation (22)) does not contain time derivatives. This means that this kind of variation in the exterior driving flow simply modulates the disturbed flow characteristics and also modulates the forces on the grains. The time variation within a single wave and further on, along successive waves, even of a random nature, merely causes a time modulation in the external velocity and pressure gradient. This reduces the statistical analysis of the events of sediment motion under natural waves to an analysis of what happens with all the individual waves. Thus, for example, the percent of waves that produces motion of sediment under given (short term) conditions can be obtained simply by integrating the joint probability density function of wave heights and periods [see, e.g., *Longuet-Higgins*, 1983] over the region of the plane (H, T) defined as H greater than the value obtained from equations (28) or (31), which is a function of T for a given sediment. This kind of analysis, which remains valid if there exist weak currents superimposed on the waves, allows, for instance, the calculation of the seaward limit of significant sediment movement.

[52] Further practical applications regarding the modeling of natural flows near sand beds, especially with the aim of studying sediment transport processes, require not only the modeling of the mean flow over the bed [$O(G_0)$] and beneath it [$O(G_0/\varepsilon)$], but also require that especial attention be paid to the treatment of the fundamental disturbed flow. It has a nonturbulent nature and produces forces of larger magnitude than mean flow. Moreover, its effects can be felt well beyond the mean shear layer, up to distances $O(\varepsilon\Lambda_0)$ from the bed.

9. Conclusions

[53] Previous work by the authors allows the theoretical treatment of flows over and between irregular boundaries, a kind of flow that depends fundamentally on the occurrence or nonoccurrence of the separation phenomenon. The results herein extend the significance of the parameter ε, equation (16), introduced in the investigation of the jet regime properties, beyond the limits of that regime of flow. This parameter appears to determine quantitatively the fundamental physics of flows over irregular surfaces when they undergo separation.

[54] In the case of granular beds, we expect the occurrence of separation if both the Reynolds number $U_0\Lambda_0/\nu$ and $\Lambda_0/D = \varepsilon^{-3}$ are large as compared with unity, a circumstance that, even when coupled with the requirement that $f < \varepsilon$, occurs practically always in nature.

[55] We argue that under the occurrence of separation the mean force that the grains of the bed surface exert on the fluid is due to the mean pressure difference between the front and the rear faces of the grains, which is given by equation (17), irrespective of the kind of flow (that is, it is $O(\varepsilon^2)$ when made dimensionless with ρU_0^2). This result appears to be clearly confirmed by experimental observations on general motion of sediment by *Chan et al.* [1972].

[56] Vorticity generated by flow separation from the bed grains interacts with itself and with mean flow, thus producing a complex rotational disturbed flow giving rise to very large local forces. Analysis of disturbed flow leads to the conclusion that maximum pressure disturbances are $O(\varepsilon)$, as indicated by equation (20). This is 1 order of magnitude larger than mean forces at the bed level. Observations on incipient motion of sediment by *Bagnold* [1946] and *Chan et al.* [1972] confirm the real existence of such strong local forces. Velocity disturbances in the outer region of the shear layer are $O(\varepsilon U_0)$, whereas very near the bed surface, they could be larger, up to $O(U_*)$ in the case of turbulent flows.

[57] Although mean flow over the bed surface can be correctly described to a first approximation by well-known standard flow models, these models cannot account for the disturbed flow, which is, in fact, the leading order flow and extends beyond the edge of the mean shear layer. This is perhaps the reason that phenomena related with local flow disturbances, like incipient motion or the formation of ripples, have not been fully understood as yet.

[58] Previous theoretical formulations are unable to represent realistically the mean flow in the shear layer beneath the bed surface, not even as a first approximation. This is because the force that grains exert on the fluid in the shear layer had not been correctly modeled. We have shown that this force, which is due to the pressure differences generated in separation, must be mainly balanced by the shear stress gradient, the inertial terms being completely negligible and mean pressure gradient also being small. The thickness of the porous region of the shear layer is ε times that of the outer region. Beneath this layer the mean pressure gradient is greater than over the bed, because $n\partial P/\partial x$ must remain constant across it. This must be taken into account in order to correctly evaluate the seepage velocity through the porous bed below the shear layer, where mean pressure gradient again becomes of leading order, and a Darcy-type equilibrium, as proposed by *Putnam* [1949], can represent the actual mean flow.

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