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APPROXIMATE SOLUTIONS TO THE INVERSE NORMAL MODE PROBLEM

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ABSTRACT

Observed normal mode periods in the range 1.388 min to 53.90 min are compared to two approximate solutions to the inverse normal mode problem. Each solution fits the observed periods with an r.m.s. relative error less than 3.1×10^{-3} and a maximum relative error less than 1.29×10^{-2}. Each solution fits the observed mantle surface wave periods with a maximum relative error less than 3.9×10^{-3}, a phase velocity error of 0.016 km/sec. The two solutions have S velocity distributions in the upper mantle that have a mean relative difference of 1.5×10^{-2} and a maximum relative difference of 2.8×10^{-2}. The mantle surface wave periods of the two solutions have a maximum relative difference of 8.5×10^{-4}, a phase velocity difference of 0.004 km/sec. The structural details of the two solutions are significantly different in the upper mantle. Two additional solutions are presented for mantle surface wave modes in the period range 1.388 min to 6.044 min. The spectrum of each solution fits the data with a maximum relative error less than 2.2×10^{-3}, a phase velocity error of 0.009 km/sec. The spectra of the two solutions have a maximum relative difference of 4.0×10^{-4}, a phase velocity difference of 0.002 km/sec. Both solutions have a zone of low-shear velocity with a minimum at a depth of 86 km. Other features of the two solutions are dissimilar.

In a recent paper (Backus and Gilbert, 1967) we presented a procedure for obtaining solutions to the inverse normal mode problem. Several numerical experiments with raw data have been performed using that procedure and some of the results are reported here. The normal mode data used in our first two experiments are summarized in Table 1. None of the normal mode data was corrected for perturbations due to asphericity, anisotropy, rotation, or the possible misidentification of a spectral line. For every multiplet the reported datum was taken to be the line $m = 0$. In all of the numerical experiments the Earth's mean density (5517 kg/m^3) and moment of inertia ($I = 0.33089$ Ma^2) were preserved. It appears that the data can be fit within experimental error by significantly different Earth models. Therefore, we feel that no useful purpose is served by publishing detailed tables of these models.

In Table 1, the UCLA data are derived from Ness, Harrison and Slichter (1961) for the 1960 Chilean earthquake (C) and Slichter (1966) for the 1964 Alaskan earthquake (A). The CITR data are derived from Ben-Menahem and Toksöz (1962) for the 1957 Mongolian earthquake (M). The CITM data are derived from Benioff, Press and Smith (1961) for the 1960 Chilean earthquake and Smith (1966) for the 1964 Alaskan earthquake. The CTIL data are derived from Toksöz and Ben-Menahem (1963) for the 1957 Mongolian earthquake. The CITR (Rayleigh

wave) and CITL (Love wave) phase-velocity data were converted to equivalent free oscillation data by using the approximation $l(l + 1) = k^2a^2$.

In all of the numerical work reported in this paper we have assumed that the Earth has a fluid core of radius 3473 km and a solid mantle of 6371 km. Although

TABLE 1

SUMMARY OF NORMAL MODE DATA

Data	Range	Designation	Earthquake
${}_nS_l$	$0 \leqq l \leqq 32$	UCLA	A, C
${}_0S_l$	$33 \leqq l \leqq 97$	CITR	M
${}_0T_l$	$2 \leqq l \leqq 32$	CITM	A, C
${}_0T_l$	$33 \leqq l \leqq 105$	CITL	M

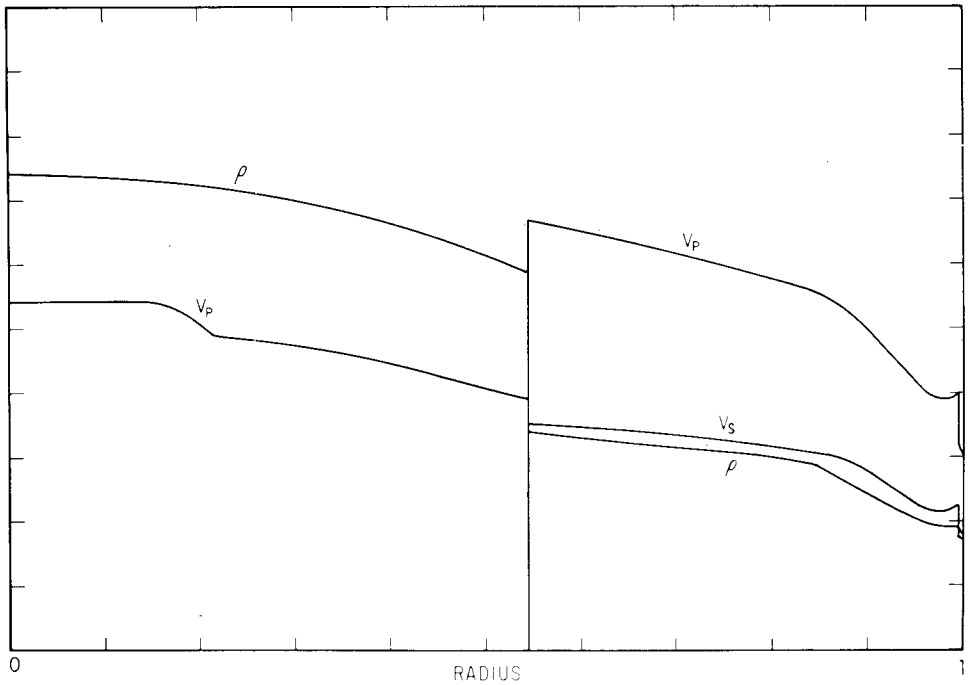

FIG. 1. The Gutenberg model used as $\mathbf{m}_G$ in the first inversion. The curve labeled ρ is the density; full scale is 16551 kg/m³. The curves labeled v_p and v_s are the P and S velocities; full scale is 20551 m/sec.

it is not difficult to investigate the effects of a solid inner core, oceanic crust, dissipation, and changes in the depth of discontinuities, we have chosen to ignore such effects in this paper.

Using the criterion min $\| \mathbf{m} - \mathbf{m}_G \|^2$ (Backus and Gilbert, 1967, sec. 5) we performed two inversions, taking as gross Earth data the Earth's mass and moment of inertia and the 16 modes ${}_0S_0$, ${}_1S_0$, ${}_3S_0$, ${}_0S_4$, ${}_0S_8$, ${}_1S_8$, ${}_0S_{16}$, ${}_0S_{32}$, ${}_0S_{64}$, ${}_0S_{97}$, ${}_0T_4$, ${}_0T_8$, ${}_0T_{16}$, ${}_0T_{32}$, ${}_0T_{64}$, ${}_0T_{105}$.

In the first inversion $\mathbf{m}_G$ was the Gutenberg model shown in Figure 1. The third iterate, designated G1, is shown in Figure 2. It fits the 16 modes with an r.m.s. relative error of 2.4×10^{-5}. The eigenfrequencies of G1 are compared with the observed data in Table 2. The r.m.s. relative error is 3.1×10^{-3} and the maximum relative error is 1.29×10^{-2} ($_0S_{22}$). The relative difference between the UCLA datum $_0S_{22}$ and the CITR datum $_0S_{22}$ is 1.88×10^{-2}. Thus, these two data, which should be identical if they belong to $m = 0$, differ by more than the maximum relative error of G1. At present, the multiplet structure of $_0S_{22}$ has not been resolved so

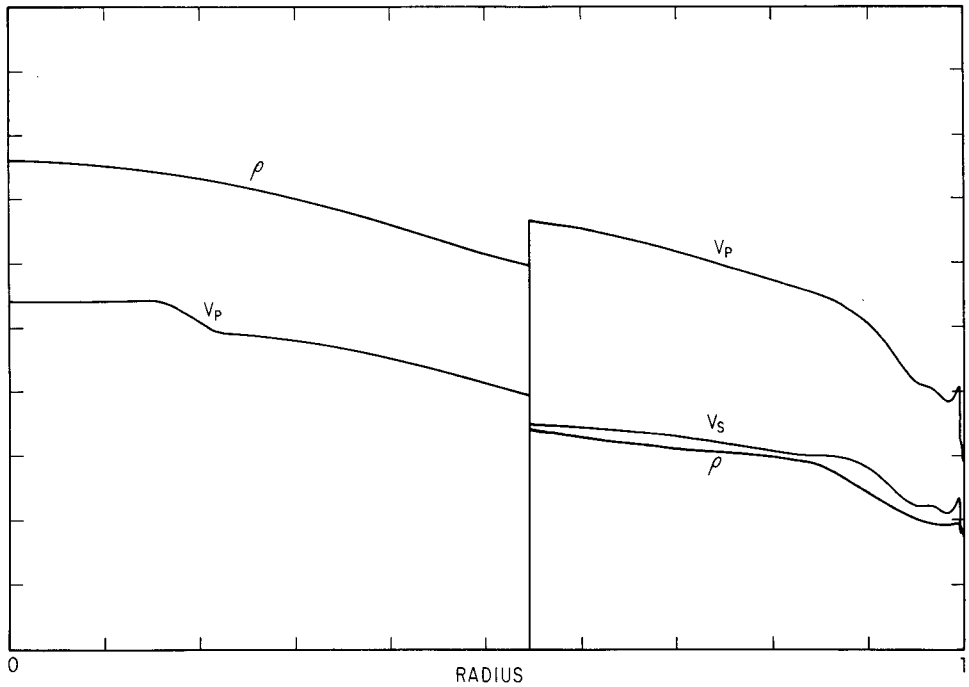

FIG. 2. Model G1.

we cannot determine where the two observed data stand in the multiplet. However, $_0S_{22}$ for G1 stands between them and is presumably within the unresolved multiplet. Only six modes of G1 differ from the data with a relative error greater than 10^{-2}. Model G1 is a good fit to the CITR and CITL data, the maximum relative error being 3.9×10^{-3} ($_0S_{71}$) which corresponds to a phase velocity error of 0.016 km/sec.

In the second inversion $\mathbf{m}_G$ was the quadratic Gutenberg model shown in Figure 3. This "starting" model is described by Backus and Gilbert (1967, sec. 6). The third iterate fits the 16 modes with an r.m.s. relative error of 3.8×10^{-4}. It is shown in Figure 4. At this stage the frequencies of the two modes $_2S_0$ and $_1S_{11}$ of G1 were added to the data. Two more iterations produced a model, designated Q1, that fits the 18 modes with an r.m.s. relative error of 1.6×10^{-5} and that also has nearly the same spectrum as G1. It is shown in Figure 5. The eigenfrequencies of Q1 are compared with the observed data in Table 3. The r.m.s. relative error is 3.0×10^{-3} and

TABLE 2

COMPARISON OF G1 SPECTRUM WITH OBSERVED DATA

The column headed ω_{obs} is the observed spectrum. The column headed ω_{com} is the G1 spectrum. Eigen-frequencies are given in radians/sec. The column headed err is $1 - \omega_{com}/\omega_{obs}$.

Mode		ω_{obs}	ω_{com}	err	Mode		ω_{obs}	ω_{com}	err
0S	0	5.1179000E-03	5.1178257E-03	1.4523E-05	0S	18	1.6797000E-02	1.6794029E-02	1.7687E-04
1S	0	1.0404000E-02	1.0404050E-02	-4.7698E-06	0S	19	1.7281000E-02	1.7451851E-02	-9.8866E-03
2S	0	1.5717000E-02	1.5885482E-02	-1.0720E-02	0S	20	1.8019000E-02	1.8095142E-02	-4.2256E-03
3S	0	2.0735000E-02	2.0735040E-02	-1.9317E-06	0S	21	1.8701000E-02	1.8725155E-02	-1.2917E-03
0S	2	1.9427000E-03	1.9454043E-03	-1.3920E-03	0S	22	1.9595000E-02	1.9343189E-02	1.2851E-02
1S	2	4.2726000E-03	4.2791861E-03	-1.5415E-03	0S	23	2.0028000E-02	1.9950587E-02	3.8652E-03
2S	2	6.8801000E-03	6.8872632E-03	-1.0412E-03	0S	24	2.0349000E-02	2.0548647E-02	-9.8112E-03
0S	3	2.9287000E-03	2.9413894E-03	-4.3328E-03	0S	25	2.1042000E-02	2.1138635E-02	-4.5925E-03
1S	3	5.9010000E-03	5.9035400E-03	-4.3043E-04	0S	26	2.1811000E-02	2.1721724E-02	4.0931E-03
2S	3	7.8156000E-03	7.8525557E-03	-4.7285E-03	0S	27	2.2150000E-02	2.2298973E-02	-6.7256E-03
0S	4	4.0614000E-03	4.0615374E-03	-3.3831E-05	0S	28	2.2880000E-02	2.2871382E-02	3.7667E-04
1S	4	7.3531000E-03	7.3658136E-03	-1.7290E-03	0S	29	2.3504000E-02	2.3439823E-02	2.7305E-03
2S	4	8.6708000E-03	8.7005927E-03	-3.4360E-03	0S	30	2.3869000E-02	2.4005069E-02	-5.7007E-03
0S	5	5.2993000E-03	5.2748807E-03	4.6080E-03	0S	31	2.4841000E-02	2.4567807E-02	1.0998E-02
1S	5	8.6272000E-03	8.6063210E-03	2.4201E-03	0S	32	2.5128000E-02	2.5128633E-02	-2.5182E-05
2S	5	9.5417000E-03	9.5534559E-03	-1.2321E-03	0S	33	2.5627000E-02	2.5688064E-02	-2.3828E-03
0S	6	6.5155000E-03	6.5152688E-03	3.5480E-05	0S	34	2.6184000E-02	2.6246539E-02	-2.3885E-03
1S	6	9.4654000E-03	9.5644037E-03	-1.0460E-02	0S	35	2.6740000E-02	2.6804438E-02	-2.4098E-03
0S	7	7.7257000E-03	7.7279237E-03	-2.8783E-04	0S	36	2.7301000E-02	2.7362082E-02	-2.2374E-03
1S	7	1.0299000E-02	1.0407059E-02	-1.0492E-02	0S	37	2.7867000E-02	2.7919742E-02	-1.8926E-03
0S	8	8.8646000E-03	8.8647609E-03	-1.8154E-05	0S	38	2.8432000E-02	2.8477643E-02	-1.6054E-03
1S	8	1.1314000E-02	1.1314163E-02	-1.4377E-05	0S	39	2.8988000E-02	2.9035974E-02	-1.6550E-03
0S	9	9.9107000E-03	9.8947958E-03	1.6048E-03	0S	40	2.9546000E-02	2.9594885E-02	-1.6545E-03
0S	10	1.0825000E-02	1.0821762E-02	2.9913E-04	0S	41	3.0107000E-02	3.0154502E-02	-1.5778E-03
0S	11	1.1713000E-02	1.1673659E-02	3.3588E-03	0S	42	3.0665000E-02	3.0714924E-02	-1.6280E-03
1S	11	1.4942000E-02	1.4770507E-02	1.1477E-02	0S	43	3.1226000E-02	3.1276227E-02	-1.6085E-03
0S	12	1.2509000E-02	1.2477237E-02	2.5392E-03	0S	44	3.1797000E-02	3.1838467E-02	-1.3041E-03
0S	13	1.3275000E-02	1.3248638E-02	1.9858E-03	0S	45	3.2365000E-02	3.2401690E-02	-1.1336E-03
0S	14	1.4004000E-02	1.3995814E-02	5.8451E-04	0S	46	3.2931000E-02	3.2965922E-02	-1.0605E-03
0S	15	1.4740000E-02	1.4722433E-02	1.1918E-03	0S	47	3.3495000E-02	3.3531181E-02	-1.0802E-03
0S	16	1.5430000E-02	1.5430310E-02	-2.0064E-05	0S	48	3.4054000E-02	3.4097475E-02	-1.2767E-03
0S	17	1.6134000E-02	1.6120520E-02	8.3553E-04	0S	49	3.4614000E-02	3.4664803E-02	-1.4677E-03

TABLE 2—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0S 50	3.5174000E-02	3.5233156E-02	-1.6818E-03	0S 86	5.6194000E-02	5.6161196E-02	5.8376E-04
0S 51	3.5732000E-02	3.5802520E-02	-1.9736E-03	0S 87	5.6776000E-02	5.6750034E-02	4.5735E-04
0S 52	3.6297000E-02	3.6372880E-02	-2.0905E-03	0S 88	5.7355000E-02	5.7339087E-02	2.7745E-04
0S 53	3.6870000E-02	3.6944211E-02	-2.0128E-03	0S 89	5.7967000E-02	5.7928350E-02	6.6677E-04
0S 54	3.7442000E-02	3.7516491E-02	-1.9895E-03	0S 90	5.8559000E-02	5.8517818E-02	7.0326E-04
0S 55	3.8027000E-02	3.8089687E-02	-1.6485E-03	0S 91	5.9146000E-02	5.9107486E-02	6.5117E-04
0S 56	3.8612000E-02	3.8663778E-02	-1.3410E-03	0S 92	5.9733000E-02	5.9697360E-02	5.9666E-04
0S 57	3.9184000E-02	3.9238732E-02	-1.3968E-03	0S 93	6.0321000E-02	6.0287412E-02	5.5682E-04
0S 58	3.9746000E-02	3.9814516E-02	-1.7239E-03	0S 94	6.0900000E-02	6.0877652E-02	3.6697E-04
0S 59	4.0308000E-02	4.0391101E-02	-2.0616E-03	0S 95	6.1479000E-02	6.1468077E-02	1.7767E-04
0S 60	4.0870000E-02	4.0968452E-02	-2.4089E-03	0S 96	6.2054000E-02	6.2058676E-02	-7.5346E-05
0S 61	4.1448000E-02	4.1546540E-02	-2.3774E-03	0S 97	6.2649000E-02	6.2649470E-02	-7.5012E-06
0S 62	4.2049000E-02	4.2125343E-02	-1.8156E-03	0T 2	2.4002000E-03	2.3780257E-03	9.2385E-03
0S 63	4.2662000E-02	4.2704777E-02	-1.0027E-03	0T 3	3.7056000E-03	3.6764108E-03	7.8770E-03
0S 64	4.3284000E-02	4.3284885E-02	-2.0441E-05	0T 4	4.8037000E-03	4.8040036E-03	-6.3191E-05
0S 65	4.3902000E-02	4.3865611E-02	8.2888E-04	0T 5	5.8307000E-03	5.8269956E-03	6.3533E-04
0S 66	4.4515000E-02	4.4446920E-02	1.5294E-03	0T 6	6.7824000E-03	6.7758287E-03	9.6887E-04
0S 67	4.5125000E-02	4.5028788E-02	2.1321E-03	0T 7	7.6447000E-03	7.6705982E-03	-3.3877E-03
0S 68	4.5735000E-02	4.5611188E-02	2.7072E-03	0T 8	8.5242000E-03	8.5250440E-03	-9.9016E-05
0S 69	4.6341000E-02	4.6194094E-02	3.1701E-03	0T 9	9.3555000E-03	9.3486581E-03	7.3133E-04
0S 70	4.6947000E-02	4.6777482E-02	3.6108E-03	0T 10	1.0164000E-02	1.0147758E-02	1.5980E-03
0S 71	4.7545000E-02	4.7361326E-02	3.8632E-03	0T 11	1.0933000E-02	1.0927205E-02	5.3002E-04
0S 72	4.8131000E-02	4.7945609E-02	3.8518E-03	0T 12	1.1683000E-02	1.1690049E-02	-6.0337E-04
0S 73	4.8714000E-02	4.8530299E-02	3.7710E-03	0T 13	1.2420000E-02	1.2439723E-02	-1.5880E-03
0S 74	4.9302000E-02	4.9115385E-02	3.7851E-03	0T 14	1.3164000E-02	1.3177627E-02	-1.0351E-03
0S 75	4.9883000E-02	4.9700843E-02	3.6517E-03	0T 15	1.3892000E-02	1.3906303E-02	-1.0296E-03
0S 76	5.0456000E-02	5.0286657E-02	3.3562E-03	0T 16	1.4626000E-02	1.4627412E-02	-9.6551E-05
0S 77	5.1023000E-02	5.0872810E-02	2.9436E-03	0T 17	1.5389000E-02	1.5341580E-02	3.0814E-03
0S 78	5.1590000E-02	5.1459287E-02	2.5337E-03	0T 18	1.6107000E-02	1.6049749E-02	3.5544E-03
0S 79	5.2138000E-02	5.2046072E-02	1.7632E-03	0T 19	1.6778000E-02	1.6754190E-02	1.4191E-03
0S 80	5.2687000E-02	5.2633153E-02	1.0220E-03	0T 20	1.7463000E-02	1.7454311E-02	4.9757E-04
0S 81	5.3255000E-02	5.3220526E-02	6.4733E-04	0T 21	1.8154000E-02	1.8151199E-02	1.5431E-04
0S 82	5.3844000E-02	5.3808162E-02	6.6559E-04	0T 22	1.8834000E-02	1.8844785E-02	-5.7261E-04
0S 83	5.4433000E-02	5.4396056E-02	6.7870E-04	0T 23	1.9519000E-02	1.9537338E-02	-9.3947E-04
0S 84	5.5005000E-02	5.4984199E-02	3.7817E-04	0T 24	2.0262000E-02	2.0227422E-02	1.7065E-03
0S 85	5.5601000E-02	5.5572582E-02	5.1111E-04	0T 25	2.1127000E-02	2.0915958E-02	9.9892E-03

TABLE 2—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0T 26	2.1681000E-02	2.1603212E-02	3.5878E-03	0T 62	4.6174000E-02	4.6177602E-02	-7.8007E-05
0T 27	2.2265000E-02	2.2288645E-02	-1.0620E-03	0T 63	4.6858000E-02	4.6859808E-02	-3.8581E-05
0T 28	2.3049000E-02	2.2973930E-02	3.2570E-03	0T 64	4.7541000E-02	4.7542074E-02	-2.2590E-05
0T 29	2.3764000E-02	2.3659337E-02	4.4043E-03	0T 65	4.8224000E-02	4.8224268E-02	-5.5654E-06
0T 30	2.4553000E-02	2.4343361E-02	8.5382E-03	0T 66	4.8905000E-02	4.8905695E-02	-1.4214E-05
0T 31	2.5224000E-02	2.5026912E-02	7.8135E-03	0T 67	4.9587000E-02	4.9587839E-02	-1.6914E-05
0T 32	2.5709000E-02	2.5710080E-02	-4.2020E-05	0T 68	5.0268000E-02	5.0269939E-02	-3.8567E-05
0T 33	2.6301000E-02	2.6392949E-02	-3.4960E-03	0T 69	5.0950000E-02	5.0952869E-02	-5.6307E-05
0T 34	2.6984000E-02	2.7074722E-02	-3.3621E-03	0T 70	5.1631000E-02	5.1634893E-02	-7.5397E-05
0T 35	2.7666000E-02	2.7757129E-02	-3.2939E-03	0T 71	5.2311000E-02	5.2316862E-02	-1.1205E-04
0T 36	2.8350000E-02	2.8440311E-02	-3.1856E-03	0T 72	5.2991000E-02	5.2998771E-02	-1.4665E-04
0T 37	2.9036000E-02	2.9122504E-02	-2.9792E-03	0T 73	5.3673000E-02	5.3680601E-02	-1.4162E-04
0T 38	2.9719000E-02	2.9804622E-02	-2.8811E-03	0T 74	5.4356000E-02	5.4362375E-02	-1.1729E-04
0T 39	3.0403000E-02	3.0486691E-02	-2.7527E-03	0T 75	5.5039000E-02	5.5044078E-02	-9.2253E-05
0T 40	3.1087000E-02	3.1168773E-02	-2.6304E-03	0T 76	5.5723000E-02	5.5725704E-02	-4.8524E-05
0T 41	3.1771000E-02	3.1850133E-02	-2.4907E-03	0T 77	5.6407000E-02	5.6407251E-02	-4.4542E-06
0T 42	3.2456000E-02	3.2532145E-02	-2.3461E-03	0T 78	5.7089000E-02	5.7088714E-02	5.0154E-06
0T 43	3.3141000E-02	3.3214964E-02	-2.2318E-03	0T 79	5.7769000E-02	5.7770085E-02	-1.8784E-05
0T 44	3.3827000E-02	3.3896926E-02	-2.0672E-03	0T 80	5.8449000E-02	5.8451367E-02	-4.0499E-05
0T 45	3.4512000E-02	3.4579004E-02	-1.9415E-03	0T 81	5.9130000E-02	5.9132554E-02	-4.3196E-05
0T 46	3.5198000E-02	3.5261107E-02	-1.7929E-03	0T 82	5.9811000E-02	5.9813643E-02	-4.4186E-05
0T 47	3.5884000E-02	3.5942745E-02	-1.6371E-03	0T 83	6.0492000E-02	6.0494629E-02	-4.3465E-05
0T 48	3.6579000E-02	3.6625403E-02	-1.2686E-03	0T 84	6.1173000E-02	6.1175510E-02	-4.1035E-05
0T 49	3.7259000E-02	3.7307387E-02	-1.2987E-03	0T 85	6.1853000E-02	6.1856282E-02	-5.3061E-05
0T 50	3.7946000E-02	3.7989546E-02	-1.1476E-03	0T 86	6.2533000E-02	6.2536942E-02	-6.3033E-05
0T 51	3.8632000E-02	3.8671896E-02	-1.0327E-03	0T 87	6.3212000E-02	6.3217486E-02	-8.6790E-05
0T 52	3.9319000E-02	3.9354230E-02	-8.9601E-04	0T 88	6.3892000E-02	6.3895626E-02	-5.6753E-05
0T 53	4.0005000E-02	4.0036524E-02	-7.8799E-04	0T 89	6.4573000E-02	6.4575880E-02	-4.4607E-05
0T 54	4.0693000E-02	4.0718944E-02	-6.3755E-04	0T 90	6.5254000E-02	6.5256010E-02	-3.0800E-05
0T 55	4.1380000E-02	4.1401271E-02	-5.1404E-04	0T 91	6.5935000E-02	6.5936010E-02	-1.5320E-05
0T 56	4.2067000E-02	4.2083269E-02	-3.8675E-04	0T 92	6.6617000E-02	6.6617777E-02	-1.1657E-05
0T 57	4.2752000E-02	4.2765599E-02	-3.1809E-04	0T 93	6.7297000E-02	6.7297546E-02	-8.1188E-06
0T 58	4.3437000E-02	4.3447933E-02	-2.5170E-04	0T 94	6.7977000E-02	6.7977182E-02	-2.6716E-06
0T 59	4.4121000E-02	4.4130652E-02	-2.1876E-04	0T 95	6.8655000E-02	6.8656674E-02	-2.4385E-05
0T 60	4.4805000E-02	4.4812991E-02	-1.7835E-04	0T 96	6.9334000E-02	6.9336025E-02	-2.9212E-05
0T 61	4.5490000E-02	4.5495289E-02	-1.1626E-04	0T 97	7.0014000E-02	7.0015232E-02	-1.7599E-05

TABLE 2—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0T 98	7.0694000E-02	7.0694292E-02	-4.1260E-06	0T102	7.3415000E-02	7.3408998E-02	8.1751E-05
0T 99	7.1376000E-02	7.1373204E-02	3.9176E-05	0T103	7.4092000E-02	7.4087280E-02	6.3702E-05
0T100	7.2057000E-02	7.2051956E-02	6.9997E-05	0T104	7.4768000E-02	7.4765402E-02	3.4744E-05
0T101	7.2736000E-02	7.2730556E-02	7.4848E-05	0T105	7.5445000E-02	7.5443355E-02	2.1807E-05

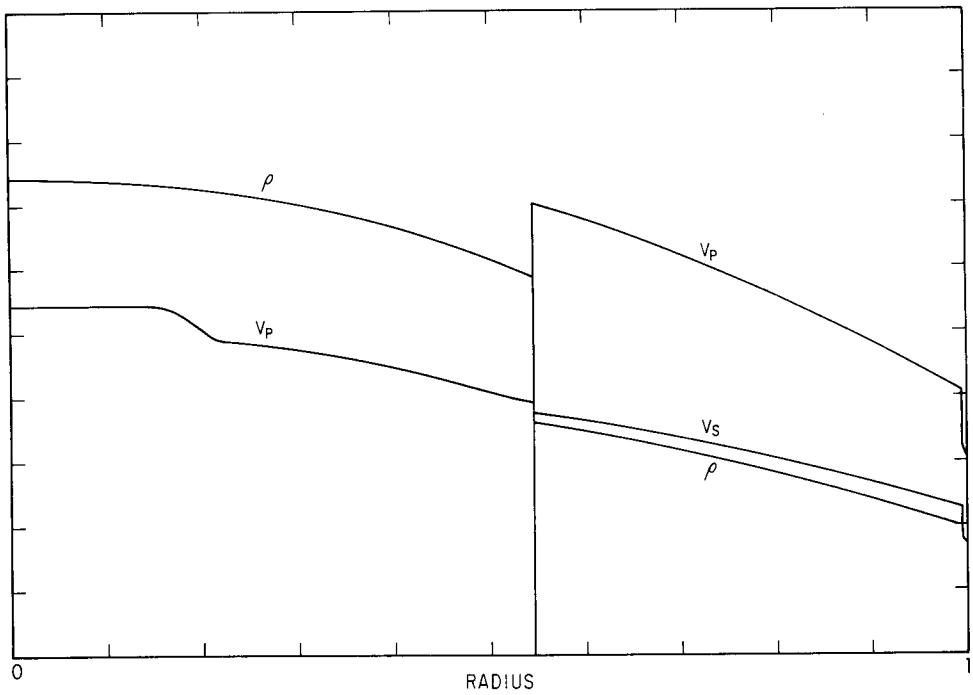

FIG. 3. The quadratic Gutenberg model used as $\mathbf{m}_G$ in the second inversion.

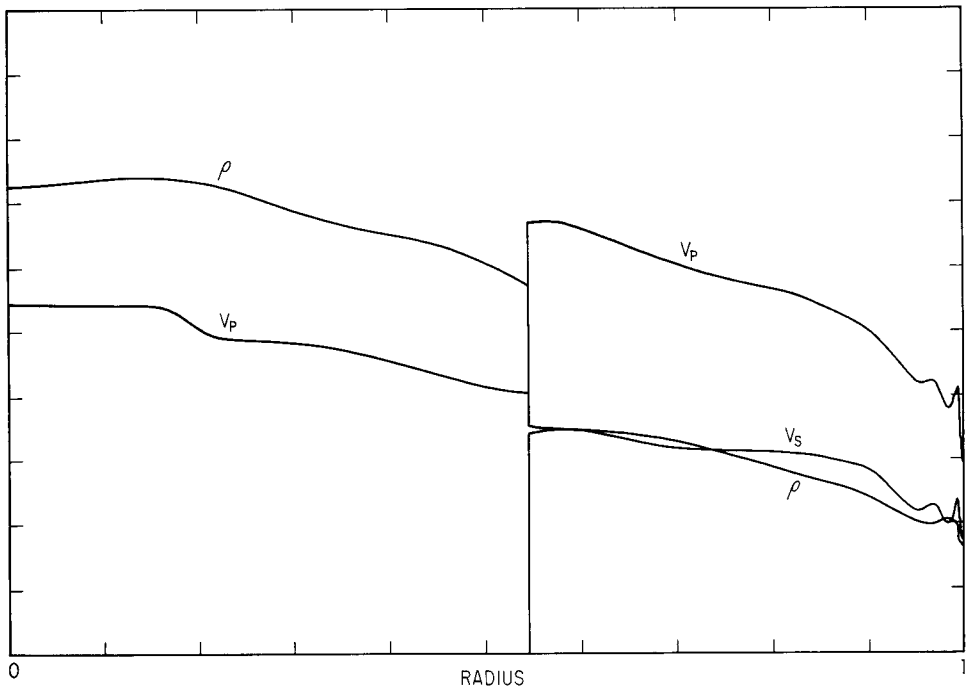

FIG. 4. The third iterate in the second inversion.

the maximum relative error is 1.27×10^{-2} ($_0S_{22}$). Only seven modes of Q1 differ from the observed data with a relative error greater than 10^{-2}. Model Q1 is also a good fit to the CITR and CITL data, the maximum relative error being 3.6×10^{-3} ($_0S_{71}$) which corresponds to a phase velocity error of 0.015 km/sec.

The eigenfrequencies of G1 and Q1 are compared in Table 4. The r.m.s. relative difference is 4.0×10^{-4} and the maximum relative difference is 2.4×10^{-3} ($_2S_5$). Only five modes have a relative difference greater than 10^{-3}. For $_0S_l$, $l > 5$, the maximum relative difference is 8.5×10^{-4} ($_0S_{46}$) or 0.004 km/sec in phase velocity.

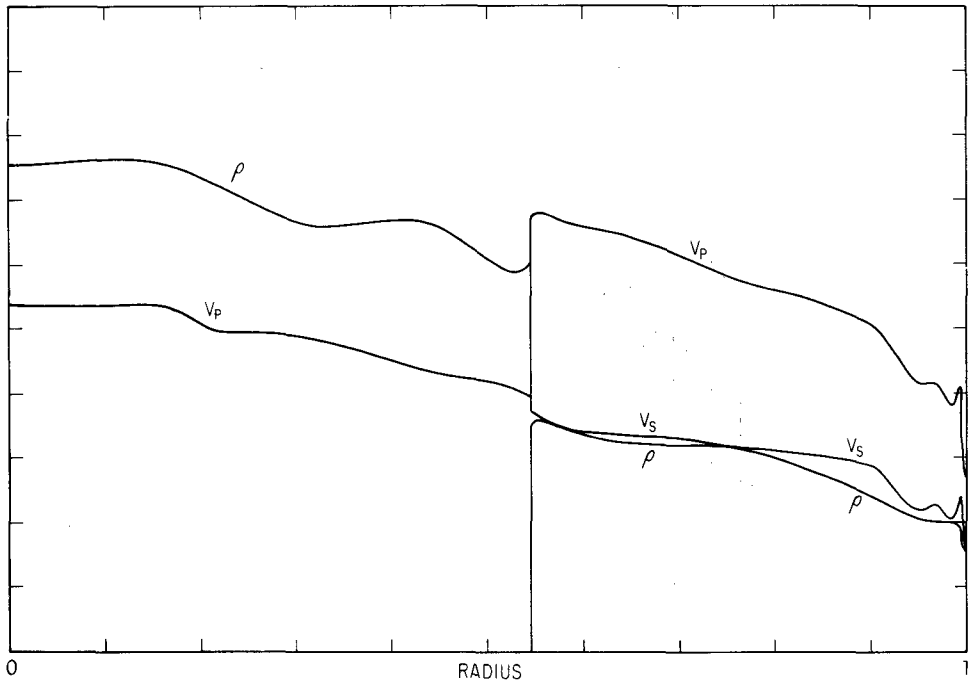

FIG. 5. Model Q1.

For $_0T_l$ the maximum relative difference is 2.2×10^{-4} ($_0T_2$) or .001 km/sec in phase velocity. Models G1 and Q1 have very nearly the same spectrum for the 216 modes used in our experiments. They are equally good fits to the observed data. In most instances the fit is better than the estimated experimental error in the observed data. Yet, models G1 and Q1 are distinctly different.

In the upper mantle we take δ as a measure of the relative difference between G1 and Q1 where

$$\delta^2 = \frac{3 \int_{r_1}^{r_2} dr r^2 \Delta^2(r)}{r_2^3 - r_1^3} \tag{1}$$

$$\Delta(r) = | 1 - v_s^{G1}(r)/v_s^{Q1}(r) |.$$

TABLE 3

COMPARISON OF Q1 SPECTRUM WITH DATA

Mode		ω_{obs}	ω_{com}	err	Mode		ω_{obs}	ω_{com}	err
0S	0	5.1179000E-03	5.1178953E-03	9.1581E-07	1S	11	1.4942000E-02	1.4770430E-02	1.1482E-02
1S	0	1.0404000E-02	1.0404031E-02	-2.9763E-06	0S	12	1.2509000E-02	1.2480811E-02	2.2535E-03
2S	0	1.5717000E-02	1.5885404E-02	-1.0715E-02	0S	13	1.3275000E-02	1.3251521E-02	1.7686E-03
3S	0	2.0735000E-02	2.0735074E-02	-3.5786E-06	0S	14	1.4004000E-02	1.3997587E-02	4.5797E-04
0S	2	1.9427000E-03	1.9434000E-03	-3.6033E-04	0S	15	1.4740000E-02	1.4723016E-02	1.1522E-03
1S	2	4.2726000E-03	4.2720281E-03	1.3384E-04	0S	16	1.5430000E-02	1.5429919E-02	5.2631E-06
2S	2	6.8801000E-03	6.8903917E-03	-1.4959E-03	0S	17	1.6134000E-02	1.6119547E-02	8.9578E-04
0S	3	2.9287000E-03	2.9399748E-03	-3.8498E-03	0S	18	1.6797000E-02	1.6792934E-02	2.4209E-04
1S	3	5.9010000E-03	5.8957612E-03	8.8778E-04	0S	19	1.7281000E-02	1.7451122E-02	-9.8444E-03
2S	3	7.8156000E-03	7.8468356E-03	-3.9966E-03	0S	20	1.8019000E-02	1.8095137E-02	-4.2254E-03
0S	4	4.0614000E-03	4.0613933E-03	1.6538E-06	0S	21	1.8701000E-02	1.8726130E-02	-1.3438E-03
1S	4	7.3531000E-03	7.3604694E-03	-1.0022E-03	0S	22	1.9595000E-02	1.9345248E-02	1.2746E-02
2S	4	8.6708000E-03	8.6840898E-03	-1.5327E-03	0S	23	2.0028000E-02	1.9953694E-02	3.7101E-03
0S	5	5.2993000E-03	5.2751085E-03	4.5650E-03	0S	24	2.0349000E-02	2.0552644E-02	-1.0008E-02
1S	5	8.6272000E-03	8.6047003E-03	2.6080E-03	0S	25	2.1042000E-02	2.1143241E-02	-4.8114E-03
2S	5	9.5417000E-03	9.5302786E-03	1.1970E-03	0S	26	2.1811000E-02	2.1726597E-02	3.8698E-03
0S	6	6.5155000E-03	6.5151361E-03	5.5851E-05	0S	27	2.2150000E-02	2.2303724E-02	-6.9402E-03
1S	6	9.4654000E-03	9.5631993E-03	-1.0332E-02	0S	28	2.2880000E-02	2.2875619E-02	1.9147E-04
0S	7	7.7257000E-03	7.7273951E-03	-2.1941E-04	0S	29	2.3504000E-02	2.3443129E-02	2.5898E-03
1S	7	1.0299000E-02	1.0406001E-02	-1.0389E-02	0S	30	2.3869000E-02	2.4007085E-02	-5.7851E-03
0S	8	8.8646000E-03	8.8645487E-03	5.7909E-06	0S	31	2.4841000E-02	2.4568204E-02	1.0982E-02
1S	8	1.1314000E-02	1.1313903E-02	8.5373E-06	0S	32	2.5128000E-02	2.5127133E-02	3.4504E-05
0S	9	9.9107000E-03	9.8958581E-03	1.4976E-03	0S	33	2.5627000E-02	2.5684447E-02	-2.2417E-03
0S	10	1.0825000E-02	1.0824357E-02	5.9422E-05	0S	34	2.6184000E-02	2.6240640E-02	-2.1632E-03
0S	11	1.1713000E-02	1.1677188E-02	3.0574E-03	0S	35	2.6740000E-02	2.6796154E-02	-2.1000E-03

TABLE 3—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0T 12	1.1683000E-02	1.1688066E-02	-4.3364E-04	0T 48	3.6579000E-02	3.6621065E-02	-1.1500E-03
0T 13	1.2420000E-02	1.2437528E-02	-1.4112E-03	0T 49	3.7259000E-02	3.7303698E-02	-1.1997E-03
0T 14	1.3164000E-02	1.3175445E-02	-8.6938E-04	0T 50	3.7946000E-02	3.7985995E-02	-1.0540E-03
0T 15	1.3892000E-02	1.3904705E-02	-9.1457E-04	0T 51	3.8632000E-02	3.8668535E-02	-9.4572E-04
0T 16	1.4626000E-02	1.4625752E-02	1.6980E-05	0T 52	3.9319000E-02	3.9350462E-02	-8.0017E-04
0T 17	1.5389000E-02	1.5339695E-02	3.2039E-03	0T 53	4.0005000E-02	4.0032883E-02	-6.9700E-04
0T 18	1.6107000E-02	1.6048551E-02	3.6288E-03	0T 54	4.0693000E-02	4.0715345E-02	-5.4910E-04
0T 19	1.6778000E-02	1.6753368E-02	1.4681E-03	0T 55	4.1380000E-02	4.1397845E-02	-4.3125E-04
0T 20	1.7463000E-02	1.7453718E-02	5.3154E-04	0T 56	4.2067000E-02	4.2080779E-02	-3.2755E-04
0T 21	1.8154000E-02	1.8150793E-02	1.7664E-04	0T 57	4.2752000E-02	4.2762462E-02	-2.4472E-04
0T 22	1.8834000E-02	1.8844407E-02	-5.5258E-04	0T 58	4.3437000E-02	4.3444606E-02	-1.7511E-04
0T 23	1.9519000E-02	1.9536375E-02	-8.9017E-04	0T 59	4.4121000E-02	4.4128072E-02	-1.6030E-04
0T 24	2.0262000E-02	2.0227243E-02	1.7154E-03	0T 60	4.4805000E-02	4.4810661E-02	-1.2634E-04
0T 25	2.1127000E-02	2.0915747E-02	9.9992E-03	0T 61	4.5490000E-02	4.5493205E-02	-7.0463E-05
0T 26	2.1681000E-02	2.1602916E-02	3.6015E-03	0T 62	4.6174000E-02	4.6174100E-02	-2.1637E-06
0T 27	2.2265000E-02	2.2288979E-02	-1.0770E-03	0T 63	4.6858000E-02	4.6856628E-02	2.9279E-05
0T 28	2.3049000E-02	2.2974131E-02	3.2483E-03	0T 64	4.7541000E-02	4.7539482E-02	3.1921E-05
0T 29	2.3764000E-02	2.3657598E-02	4.4774E-03	0T 65	4.8224000E-02	4.8221970E-02	4.2091E-05
0T 30	2.4553000E-02	2.4342330E-02	8.5802E-03	0T 66	4.8905000E-02	4.8905791E-02	-1.6167E-05
0T 31	2.5224000E-02	2.5025632E-02	7.8643E-03	0T 67	4.9587000E-02	4.9588253E-02	-2.5263E-05
0T 32	2.5709000E-02	2.5708540E-02	1.7880E-05	0T 68	5.0268000E-02	5.0270670E-02	-5.3121E-05
0T 33	2.6301000E-02	2.6391140E-02	-3.4272E-03	0T 69	5.0950000E-02	5.0953024E-02	-5.9359E-05
0T 34	2.6984000E-02	2.7073496E-02	-3.3166E-03	0T 70	5.1631000E-02	5.1635343E-02	-8.4113E-05
0T 35	2.7666000E-02	2.7755669E-02	-3.2411E-03	0T 71	5.2311000E-02	5.2317601E-02	-1.2619E-04
0T 36	2.8350000E-02	2.8437705E-02	-3.0936E-03	0T 72	5.2991000E-02	5.2999527E-02	-1.6091E-04
0T 37	2.9036000E-02	2.9118773E-02	-2.8507E-03	0T 73	5.3673000E-02	5.3681640E-02	-1.6098E-04
0T 38	2.9719000E-02	2.9801744E-02	-2.7842E-03	0T 74	5.4356000E-02	5.4363675E-02	-1.4120E-04
0T 39	3.0403000E-02	3.0483607E-02	-2.6513E-03	0T 75	5.5039000E-02	5.5045632E-02	-1.2050E-04
0T 40	3.1087000E-02	3.1165239E-02	-2.5168E-03	0T 76	5.5723000E-02	5.5727500E-02	-8.0748E-05
0T 41	3.1771000E-02	3.1847095E-02	-2.3951E-03	0T 77	5.6407000E-02	5.6409276E-02	-4.0356E-05
0T 42	3.2456000E-02	3.2528973E-02	-2.2484E-03	0T 78	5.7089000E-02	5.7090958E-02	-3.4300E-05
0T 43	3.3141000E-02	3.3210885E-02	-2.1087E-03	0T 79	5.7769000E-02	5.7772536E-02	-6.1203E-05
0T 44	3.3827000E-02	3.3892912E-02	-1.9485E-03	0T 80	5.8449000E-02	5.8454007E-02	-8.5666E-05
0T 45	3.4512000E-02	3.4575191E-02	-1.8310E-03	0T 81	5.9130000E-02	5.9135368E-02	-9.0775E-05
0T 46	3.5198000E-02	3.5257264E-02	-1.6837E-03	0T 82	5.9811000E-02	5.9816612E-02	-9.3832E-05
0T 47	3.5884000E-02	3.5938192E-02	-1.5102E-03	0T 83	6.0492000E-02	6.0497734E-02	-9.4798E-05

TABLE 3—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0S 36	2.7301000E-02	2.7351367E-02	-1.8449E-03	0S 72	4.8131000E-02	4.7962422E-02	3.5025E-03
0S 37	2.7867000E-02	2.7906605E-02	-1.4212E-03	0S 73	4.8714000E-02	4.8549212E-02	3.3828E-03
0S 38	2.8432000E-02	2.8462141E-02	-1.0601E-03	0S 74	4.9302000E-02	4.9136292E-02	3.3611E-03
0S 39	2.8988000E-02	2.9018210E-02	-1.0422E-03	0S 75	4.9883000E-02	4.9723547E-02	3.1965E-03
0S 40	2.9546000E-02	2.9574999E-02	-9.8149E-04	0S 76	5.0456000E-02	5.0310983E-02	2.8741E-03
0S 41	3.0107000E-02	3.0132672E-02	-8.5268E-04	0S 77	5.1023000E-02	5.0898567E-02	2.4388E-03
0S 42	3.0665000E-02	3.0691350E-02	-8.5930E-04	0S 78	5.1590000E-02	5.1486269E-02	2.0107E-03
0S 43	3.1226000E-02	3.1251136E-02	-8.0499E-04	0S 79	5.2138000E-02	5.2074060E-02	1.2264E-03
0S 44	3.1797000E-02	3.1812106E-02	-4.7507E-04	0S 80	5.2687000E-02	5.2661916E-02	4.7610E-04
0S 45	3.2365000E-02	3.2374329E-02	-2.8824E-04	0S 81	5.3255000E-02	5.3249830E-02	9.7087E-05
0S 46	3.2931000E-02	3.2937803E-02	-2.0657E-04	0S 82	5.3844000E-02	5.3837738E-02	1.1630E-04
0S 47	3.3495000E-02	3.3502585E-02	-2.2645E-04	0S 83	5.4433000E-02	5.4425636E-02	1.3529E-04
0S 48	3.4054000E-02	3.4068666E-02	-4.3068E-04	0S 84	5.5005000E-02	5.5013475E-02	-1.5408E-04
0S 49	3.4614000E-02	3.4636059E-02	-6.3729E-04	0S 85	5.5601000E-02	5.5601287E-02	-5.1643E-06
0S 50	3.5174000E-02	3.5204744E-02	-8.7405E-04	0S 86	5.6194000E-02	5.6189040E-02	8.8274E-05
0S 51	3.5732000E-02	3.5774696E-02	-1.1949E-03	0S 87	5.6776000E-02	5.6776680E-02	-1.1976E-05
0S 52	3.6297000E-02	3.6345893E-02	-1.3470E-03	0S 88	5.7355000E-02	5.7364228E-02	-1.6089E-04
0S 53	3.6870000E-02	3.6918300E-02	-1.3100E-03	0S 89	5.7967000E-02	5.7951666E-02	2.6453E-04
0S 54	3.7442000E-02	3.7491877E-02	-1.3321E-03	0S 90	5.8559000E-02	5.8538969E-02	3.4206E-04
0S 55	3.8027000E-02	3.8066580E-02	-1.0408E-03	0S 91	5.9146000E-02	5.9126133E-02	3.3590E-04
0S 56	3.8612000E-02	3.8642372E-02	-7.8658E-04	0S 92	5.9733000E-02	5.9713157E-02	3.3220E-04
0S 57	3.9184000E-02	3.9219201E-02	-8.9834E-04	0S 93	6.0321000E-02	6.0300007E-02	3.4802E-04
0S 58	3.9746000E-02	3.9797019E-02	-1.2836E-03	0S 94	6.0900000E-02	6.0886685E-02	2.1863E-04
0S 59	4.0308000E-02	4.0375775E-02	-1.6814E-03	0S 95	6.1479000E-02	6.1473203E-02	9.4290E-05
0S 60	4.0870000E-02	4.0955428E-02	-2.0902E-03	0S 96	6.2054000E-02	6.2059500E-02	-8.8627E-05
0S 61	4.1448000E-02	4.1535911E-02	-2.1210E-03	0S 97	6.2649000E-02	6.2645644E-02	5.3571E-05
0S 62	4.2049000E-02	4.2117162E-02	-1.6210E-03	0T 2	2.4002000E-03	2.3775102E-03	9.4533E-03
0S 63	4.2662000E-02	4.2699165E-02	-8.7115E-04	0T 3	3.7056000E-03	3.6759124E-03	8.0115E-03
0S 64	4.3284000E-02	4.3281859E-02	4.9461E-05	0T 4	4.8037000E-03	4.8036521E-03	9.9712E-06
0S 65	4.3902000E-02	4.3865184E-02	8.3861E-04	0T 5	5.8307000E-03	5.8267423E-03	6.7876E-04
0S 66	4.4515000E-02	4.4449101E-02	1.4804E-03	0T 6	6.7824000E-03	6.7754998E-03	1.0174E-03
0S 67	4.5125000E-02	4.5033560E-02	2.0264E-03	0T 7	7.6447000E-03	7.6700318E-03	-3.3136E-03
0S 68	4.5735000E-02	4.5618514E-02	2.5470E-03	0T 8	8.5242000E-03	8.5240901E-03	1.2894E-05
0S 69	4.6341000E-02	4.6203920E-02	2.9581E-03	0T 9	9.3555000E-03	9.3472818E-03	8.7843E-04
0S 70	4.6947000E-02	4.6789735E-02	3.3498E-03	0T 10	1.0164000E-02	1.0146134E-02	1.7578E-03
0S 71	4.7545000E-02	4.7375914E-02	3.5563E-03	0T 11	1.0933000E-02	1.0925001E-02	7.3166E-04

TABLE 3—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{G1}	ω_{Q1}	diff
0T 84	6.1173000E-02	6.1178734E-02	-9.3737E-05	0T 95	6.8655000E-02	6.8661120E-02	-8.9135E-05
0T 85	6.1853000E-02	6.1859605E-02	-1.0679E-04	0T 96	6.9334000E-02	6.9340341E-02	-9.1452E-05
0T 86	6.2533000E-02	6.2540918E-02	-1.2662E-04	0T 97	7.0014000E-02	7.0019389E-02	-7.6973E-05
0T 87	6.3212000E-02	6.3221539E-02	-1.5090E-04	0T 98	7.0694000E-02	7.0698261E-02	-6.0269E-05
0T 88	6.3892000E-02	6.3902017E-02	-1.5678E-04	0T 99	7.1376000E-02	7.1376957E-02	-1.3411E-05
0T 89	6.4573000E-02	6.4582352E-02	-1.4483E-04	0T100	7.2057000E-02	7.2055426E-02	2.1839E-05
0T 90	6.5254000E-02	6.5262537E-02	-1.3082E-04	0T101	7.2736000E-02	7.2733754E-02	3.0873E-05
0T 91	6.5935000E-02	6.5942577E-02	-1.1492E-04	0T102	7.3415000E-02	7.3411896E-02	4.2284E-05
0T 92	6.6617000E-02	6.6622454E-02	-8.1874E-05	0T103	7.4092000E-02	7.4089844E-02	2.9102E-05
0T 93	6.7297000E-02	6.7302173E-02	-7.6867E-05	0T104	7.4768000E-02	7.4766643E-02	1.8153E-05
0T 94	6.7977000E-02	6.7981729E-02	-6.9568E-05	0T105	7.5445000E-02	7.5444178E-02	1.0889E-05

TABLE 4

COMPARISON OF G1 SPECTRUM WITH Q1 SPECTRUM

The column headed diff is $1 - \omega_{Q1}/\omega_{G1}$.

Mode	ω_{G1}	ω_{Q1}	diff	Mode	ω_{obs}	ω_{com}	diff
0S 0	5.1178257E-03	5.1178953E-03	-1.3600E-05	1S 6	9.5644037E-03	9.5631993E-03	1.2593E-04
1S 0	1.0404050E-02	1.0404031E-02	1.8262E-06	0S 7	7.7279237E-03	7.7273951E-03	6.8401E-05
2S 0	1.5885482E-02	1.5885404E-02	4.9101E-06	1S 7	1.0407059E-02	1.0406001E-02	1.0166E-04
3S 0	2.0735040E-02	2.0735074E-02	-1.6397E-06	0S 8	8.8647609E-03	8.8645487E-03	2.3937E-05
0S 2	1.9454043E-03	1.9434000E-03	1.0303E-03	1S 8	1.1314163E-02	1.1313903E-02	2.2980E-05
1S 2	4.2791861E-03	4.2720281E-03	1.6727E-03	0S 9	9.8947958E-03	9.8958581E-03	-1.0736E-04
2S 2	6.8872632E-03	6.8903917E-03	-4.5424E-04	0S 10	1.0821762E-02	1.0824357E-02	-2.3979E-04
0S 3	2.9413894E-03	2.9399748E-03	4.8093E-04	0S 11	1.1673659E-02	1.1677188E-02	-3.0230E-04
1S 3	5.9035400E-03	5.8957612E-03	1.3177E-03	1S 11	1.4770507E-02	1.4770430E-02	5.2131E-06
2S 3	7.8525557E-03	7.8468356E-03	7.2844E-04	0S 12	1.2477237E-02	1.2480811E-02	-2.8644E-04
0S 4	4.0615374E-03	4.0613933E-03	3.5479E-05	0S 13	1.3248638E-02	1.3251521E-02	-2.1761E-04
1S 4	7.3658136E-03	7.3604694E-03	7.2554E-04	0S 14	1.3995814E-02	1.3997587E-02	-1.2668E-04
2S 4	8.7005927E-03	8.6840898E-03	1.8968E-03	0S 15	1.4722433E-02	1.4723016E-02	-3.9599E-05
0S 5	5.2748807E-03	5.2751085E-03	-4.3186E-05	0S 16	1.5430310E-02	1.5429919E-02	2.5340E-05
1S 5	8.6063210E-03	8.6047003E-03	1.8832E-04	0S 17	1.6120520E-02	1.6119547E-02	6.0358E-05
2S 5	9.5534559E-03	9.5302786E-03	2.4261E-03	0S 18	1.6794029E-02	1.6792934E-02	6.5202E-05
0S 6	6.5152688E-03	6.5151361E-03	2.0368E-05	0S 19	1.7451851E-02.	1.7451122E-02	4.1772E-05

TABLE 4—*Continued*

Mode	ω_{G1}	ω_{Q1}	diff	Mode	ω_{G1}	ω_{Q1}	diff
0S 20	1.8095142E-02	1.8095137E-02	2.7634E-07	0S 56	3.8663778E-02	3.8642372E-02	5.5364E-04
0S 21	1.8725155E-02	1.8726130E-02	-5.2069E-05	0S 57	3.9238732E-02	3.9219201E-02	4.9775E-04
0S 22	1.9343189E-02	1.9345248E-02	-1.0645E-04	0S 58	3.9814516E-02	3.9797019E-02	4.3946E-04
0S 23	1.9950587E-02	1.9953694E-02	-1.5573E-04	0S 59	4.0391101E-02	4.0375775E-02	3.7944E-04
0S 24	2.0548647E-02	2.0552644E-02	-1.9451E-04	0S 60	4.0968452E-02	4.0955428E-02	3.1790E-04
0S 25	2.1138635E-02	2.1143241E-02	-2.1789E-04	0S 61	4.1546540E-02	4.1535911E-02	2.5583E-04
0S 26	2.1721724E-02	2.1726597E-02	-2.2434E-04	0S 62	4.2125343E-02	4.2117162E-02	1.9421E-04
0S 27	2.2298973E-02	2.2303724E-02	-2.1306E-04	0S 63	4.2704777E-02	4.2699165E-02	1.3141E-04
0S 28	2.2871382E-02	2.2875619E-02	-1.8525E-04	0S 64	4.3284885E-02	4.3281859E-02	6.9909E-05
0S 29	2.3439823E-02	2.3443129E-02	-1.4104E-04	0S 65	4.3865611E-02	4.3865184E-02	9.7343E-06
0S 30	2.4005069E-02	2.4007085E-02	-8.3982E-05	0S 66	4.4446920E-02	4.4449101E-02	-4.9070E-05
0S 31	2.4567807E-02	2.4568204E-02	-1.6159E-05	0S 67	4.5028788E-02	4.5033560E-02	-1.0598E-04
0S 32	2.5128633E-02	2.5127133E-02	5.9693E-05	0S 68	4.5611188E-02	4.5618514E-02	-1.6062E-04
0S 33	2.5688064E-02	2.5684447E-02	1.4080E-04	0S 69	4.6194094E-02	4.6203920E-02	-2.1271E-04
0S 34	2.6246539E-02	2.6240640E-02	2.2475E-04	0S 70	4.6777482E-02	4.6789735E-02	-2.6194E-04
0S 35	2.6804438E-02	2.6796154E-02	3.0905E-04	0S 71	4.7361326E-02	4.7375914E-02	-3.0802E-04
0S 36	2.7362082E-02	2.7351367E-02	3.9160E-04	0S 72	4.7945609E-02	4.7962422E-02	-3.5067E-04
0S 37	2.7919742E-02	2.7906605E-02	4.7053E-04	0S 73	4.8530299E-02	4.8549212E-02	-3.8972E-04
0S 38	2.8477643E-02	2.8462141E-02	5.4436E-04	0S 74	4.9115385E-02	4.9136292E-02	-4.2567E-04
0S 39	2.9035974E-02	2.9018210E-02	6.1179E-04	0S 75	4.9700843E-02	4.9723547E-02	-4.5681E-04
0S 40	2.9594885E-02	2.9574999E-02	6.7194E-04	0S 76	5.0286657E-02	5.0310983E-02	-4.8375E-04
0S 41	3.0154502E-02	3.0132672E-02	7.2394E-04	0S 77	5.0872810E-02	5.0898567E-02	-5.0630E-04
0S 42	3.0714924E-02	3.0691350E-02	7.6751E-04	0S 78	5.1459287E-02	5.1486269E-02	-5.2434E-04
0S 43	3.1276227E-02	3.1251136E-02	8.0224E-04	0S 79	5.2046072E-02	5.2074060E-02	-5.3775E-04
0S 44	3.1838467E-02	3.1812106E-02	8.2796E-04	0S 80	5.2633153E-02	5.2661916E-02	-5.4648E-04
0S 45	3.2401690E-02	3.2374329E-02	8.4443E-04	0S 81	5.3220526E-02	5.3249830E-02	-5.5061E-04
0S 46	3.2965922E-02	3.2937803E-02	8.5297E-04	0S 82	5.3808162E-02	5.3837738E-02	-5.4966E-04
0S 47	3.3531181E-02	3.3502585E-02	8.5282E-04	0S 83	5.4396056E-02	5.4425636E-02	-5.4379E-04
0S 48	3.4097475E-02	3.4068666E-02	8.4490E-04	0S 84	5.4984199E-02	5.5013475E-02	-5.3244E-04
0S 49	3.4664803E-02	3.4636059E-02	8.2920E-04	0S 85	5.5572582E-02	5.5601287E-02	-5.1653E-04
0S 50	3.5233156E-02	3.5204744E-02	8.0640E-04	0S 86	5.6161196E-02	5.6189040E-02	-4.9579E-04
0S 51	3.5802520E-02	3.5774696E-02	7.7715E-04	0S 87	5.6750034E-02	5.6776680E-02	-4.6953E-04
0S 52	3.6372880E-02	3.6345893E-02	7.4195E-04	0S 88	5.7339087E-02	5.7364228E-02	-4.3846E-04
0S 53	3.6944211E-02	3.6918300E-02	7.0135E-04	0S 89	5.7928350E-02	5.7951666E-02	-4.0250E-04
0S 54	3.7516491E-02	3.7491877E-02	6.5608E-04	0S 90	5.8517818E-02	5.8538969E-02	-3.6145E-04
0S 55	3.8089687E-02	3.8066580E-02	6.0665E-04	0S 91	5.9107486E-02	5.9126133E-02	-3.1548E-04

TABLE 4—*Continued*

Mode	ω_{G1}	ω_{Q1}	diff
0S 92	5.9697360E-02	5.9713157E-02	-2.6462E-04
0S 93	6.0287412E-02	6.0300007E-02	-2.0892E-04
0S 94	6.0877652E-02	6.0886685E-02	-1.4838E-04
0S 95	6.1468077E-02	6.1473203E-02	-8.3393E-05
0S 96	6.2058676E-02	6.2059500E-02	-1.3278E-05
0S 97	6.2649470E-02	6.2645644E-02	6.1070E-05
0T 2	2.3780257E-03	2.3775102E-03	2.1678E-04
0T 3	3.6764108E-03	3.6759124E-03	1.3557E-04
0T 4	4.8040036E-03	4.8036521E-03	7.3168E-05
0T 5	5.8269956E-03	5.8267423E-03	4.3470E-05
0T 6	6.7758287E-03	6.7754998E-03	4.8540E-05
0T 7	7.6705982E-03	7.6700318E-03	7.3840E-05
0T 8	8.5250440E-03	8.5240901E-03	1.1189E-04
0T 9	9.3486581E-03	9.3472818E-03	1.4722E-04
0T 10	1.0147758E-02	1.0146134E-02	1.6004E-04
0T 11	1.0927205E-02	1.0925001E-02	2.0170E-04
0T 12	1.1690049E-02	1.1688066E-02	1.6963E-04
0T 13	1.2439723E-02	1.2437528E-02	1.7645E-04
0T 14	1.3177627E-02	1.3175445E-02	1.6558E-04
0T 15	1.3906303E-02	1.3904705E-02	1.1491E-04
0T 16	1.4627412E-02	1.4625752E-02	1.1349E-04
0T 17	1.5341580E-02	1.5339695E-02	1.2287E-04
0T 18	1.6049749E-02	1.6048551E-02	7.4643E-05
0T 19	1.6754190E-02	1.6753368E-02	4.9062E-05
0T 20	1.7454311E-02	1.7453718E-02	3.3974E-05
0T 21	1.8151199E-02	1.8150793E-02	2.2368E-05
0T 22	1.8844785E-02	1.8844407E-02	2.0059E-05
0T 23	1.9537338E-02	1.9536375E-02	4.9290E-05
0T 24	2.0227422E-02	2.0227243E-02	8.8494E-06
0T 25	2.0915958E-02	2.0915747E-02	1.0088E-05
0T 26	2.1603212E-02	2.1602916E-02	1.3702E-05
0T 27	2.2288645E-02	2.2288979E-02	-1.4985E-05
0T 28	2.2973930E-02	2.2974131E-02	-8.7490E-06
0T 29	2.3659337E-02	2.3657598E-02	7.3502E-05
0T 30	2.4343361E-02	2.4342330E-02	4.2352E-05
0T 31	2.5026912E-02	2.5025632E-02	5.1145E-05
0T 32	2.5710080E-02	2.5708540E-02	5.9899E-05
0T 33	2.6392949E-02	2.6391140E-02	6.8541E-05
0T 34	2.7074722E-02	2.7073496E-02	4.5282E-05
0T 35	2.7757129E-02	2.7755669E-02	5.2599E-05
0T 36	2.8440311E-02	2.8437705E-02	9.1630E-05
0T 37	2.9122504E-02	2.9118773E-02	1.2811E-04
0T 38	2.9804622E-02	2.9801744E-02	9.6562E-05
0T 39	3.0486691E-02	3.0483607E-02	1.0116E-04
0T 40	3.1168773E-02	3.1165239E-02	1.1338E-04
0T 41	3.1850133E-02	3.1847095E-02	9.5384E-05
0T 42	3.2532145E-02	3.2528973E-02	9.7504E-05
0T 43	3.3214964E-02	3.3210885E-02	1.2281E-04
0T 44	3.3896926E-02	3.3892912E-02	1.1842E-04
0T 45	3.4579004E-02	3.4575191E-02	1.1027E-04
0T 46	3.5261107E-02	3.5257264E-02	1.0899E-04
0T 47	3.5942745E-02	3.5938192E-02	1.2667E-04
0T 48	3.6625403E-02	3.6621065E-02	1.1844E-04
0T 49	3.7307387E-02	3.7303698E-02	9.8881E-05
0T 50	3.7989546E-02	3.7985995E-02	9.3473E-05
0T 51	3.8671896E-02	3.8668535E-02	8.6911E-05
0T 52	3.9354230E-02	3.9350462E-02	9.5746E-05
0T 53	4.0036524E-02	4.0032883E-02	9.0942E-05
0T 54	4.0718944E-02	4.0715345E-02	8.8386E-05
0T 55	4.1401271E-02	4.1397845E-02	8.2751E-05
0T 56	4.2083269E-02	4.2080779E-02	5.9168E-05
0T 57	4.2765599E-02	4.2762462E-02	7.3353E-05
0T 58	4.3447933E-02	4.3444606E-02	7.6574E-05
0T 59	4.4130652E-02	4.4128072E-02	5.8463E-05
0T 60	4.4812991E-02	4.4810661E-02	5.1994E-05
0T 61	4.5495289E-02	4.5493205E-02	4.5807E-05
0T 62	4.6177602E-02	4.6174100E-02	7.5838E-05
0T 63	4.6859808E-02	4.6856628E-02	6.7862E-05
0T 64	4.7542074E-02	4.7539482E-02	5.4520E-05
0T 65	4.8224268E-02	4.8221970E-02	4.7652E-05
0T 66	4.8905695E-02	4.8905791E-02	-1.9630E-06
0T 67	4.9587839E-02	4.9588253E-02	-8.3488E-06

TABLE 4—*Continued*

Mode	ω_{G1}	ω_{Q1}	diff	Mode	ω_{G1}	ω_{Q1}	diff
0T 68	5.0269939E-02	5.0270670E-02	-1.4541E-05	0T 87	6.3217486E-02	6.3221539E-02	-6.4112E-05
0T 69	5.0952869E-02	5.0953024E-02	-3.0420E-06	0T 88	6.3895626E-02	6.3902017E-02	-1.0002E-04
0T 70	5.1634893E-02	5.1635343E-02	-8.7151E-06	0T 89	6.4575880E-02	6.4582352E-02	-1.0022E-04
0T 71	5.2316862E-02	5.2317601E-02	-1.4125E-05	0T 90	6.5256010E-02	6.5262537E-02	-1.0002E-04
0T 72	5.2998771E-02	5.2999527E-02	-1.4264E-05	0T 91	6.5936010E-02	6.5942577E-02	-9.9597E-05
0T 73	5.3680601E-02	5.3681640E-02	-1.9355E-05	0T 92	6.6617777E-02	6.6622454E-02	-7.0206E-05
0T 74	5.4362375E-02	5.4363675E-02	-2.3914E-05	0T 93	6.7297546E-02	6.7302173E-02	-6.8754E-05
0T 75	5.5044078E-02	5.5045632E-02	-2.8232E-05	0T 94	6.7977182E-02	6.7981729E-02	-6.6890E-05
0T 76	5.5725704E-02	5.5727500E-02	-3.2229E-05	0T 95	6.8656674E-02	6.8661120E-02	-6.4757E-05
0T 77	5.6407251E-02	5.6409276E-02	-3.5900E-05	0T 96	6.9336025E-02	6.9340341E-02	-6.2248E-05
0T 78	5.7088714E-02	5.7090958E-02	-3.9307E-05	0T 97	7.0015232E-02	7.0019389E-02	-5.9373E-05
0T 79	5.7770085E-02	5.7772536E-02	-4.2427E-05	0T 98	7.0694292E-02	7.0698261E-02	-5.6143E-05
0T 80	5.8451367E-02	5.8454007E-02	-4.5166E-05	0T 99	7.1373204E-02	7.1376957E-02	-5.2583E-05
0T 81	5.9132554E-02	5.9135368E-02	-4.7588E-05	0T100	7.2051956E-02	7.2055426E-02	-4.8160E-05
0T 82	5.9813643E-02	5.9816612E-02	-4.9637E-05	0T101	7.2730556E-02	7.2733754E-02	-4.3971E-05
0T 83	6.0494629E-02	6.0497734E-02	-5.1327E-05	0T102	7.3408998E-02	7.3411896E-02	-3.9477E-05
0T 84	6.1175510E-02	6.1178734E-02	-5.2701E-05	0T103	7.4087280E-02	7.4089844E-02	-3.4608E-05
0T 85	6.1856282E-02	6.1859605E-02	-5.3721E-05	0T104	7.4765402E-02	7.4766643E-02	-1.6599E-05
0T 86	6.2536942E-02	6.2540918E-02	-6.3578E-05	0T105	7.5443355E-02	7.5444178E-02	-1.0909E-05

For $r_1 = 5935$ km and $r_2 = 6343$ km we have

$$\delta = 1.5 \times 10^{-2}$$

$$\Delta_{\max} = 2.8 \times 10^{-2} \quad \text{at} \quad r = 6271 \text{ km}. \tag{2}$$

The relative difference between the spectra of G1 and Q1 for $l > 5$ is no greater than 8.6×10^{-4}. In other words, the mantle Rayleigh and Love wave spectra of G1

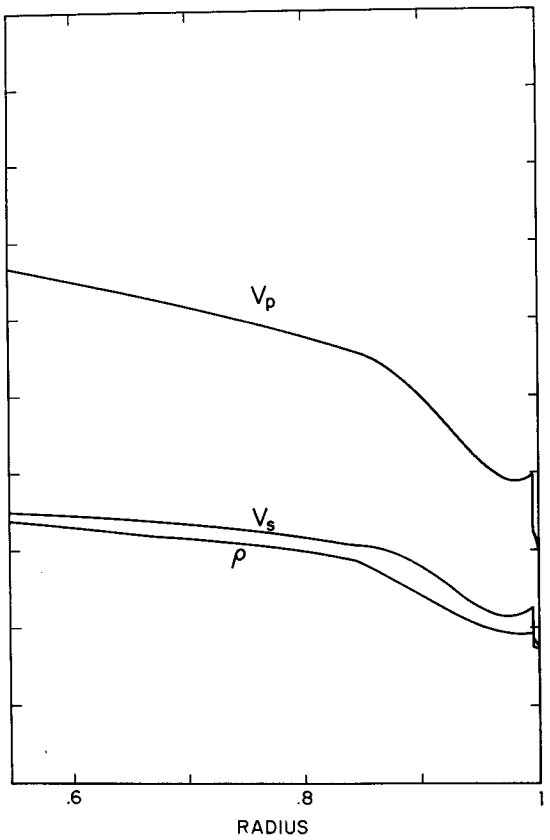

FIG. 6. $\mathbf{m}_Q$ in the first inversion of mantle surface waves.

and Q1 are practically identical, but their upper mantle structures are not. For the 218 gross Earth data that we have used, the inverse normal mode problem appears not to possess a unique solution. Whether the difference between G1 and Q1 represents the lack of sufficient resolving power in the data, or is an intrinsic non-uniqueness, is a matter that we shall pursue elsewhere.

In the two experiments just described, we used all the available data from normal modes which have appreciable energy in the deep interior. If we restrict our input data to normal modes confined to the intermediate and upper mantle, we can investigate the extent to which ordinary short period surface-wave dispersion rela-

tions determine upper-mantle structure. For this inquiry we performed two numerical experiments with data in the period range 1.388 min to 6.044 min. The CITR data were used for $19 \leqq l \leqq 97$ ($_0S_l$) and the CITL data for $21 \leqq l \leqq 105$ ($_0T_l$).

The criterion min $\| \mathbf{m} - \mathbf{m}_G \|^2$ was used to find solutions to the inverse problem. We took as gross Earth data the Earth's mass and moment of inertia and the 17

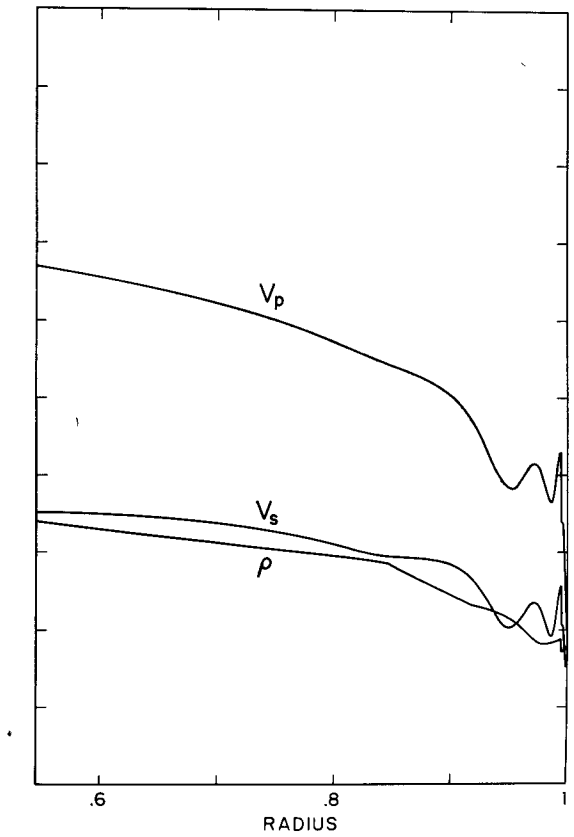

FIG. 7. Model GM.

modes $_0S_{19}$, $_0S_{23}$, $_0S_{28}$, $_0S_{35}$, $_0S_{43}$, $_0S_{52}$, $_0S_{64}$, $_0S_{78}$, $_0S_{97}$, $_0T_{23}$, $_0T_{29}$, $_0T_{36}$, $_0T_{44}$, $_0T_{55}$, $_0T_{68}$, $_0T_{84}$, $_0T_{105}$. In each inversion the iterative scheme was terminated when the r.m.s. relative error became less than 10^{-3}.

In the first inversion $\mathbf{m}_G$ was the Gutenberg model shown in Figure 6. The final solution, denoted *GM*, is shown in Figure 7. The eigenfrequencies of *GM* are compared with the observed data in Table 5. The maximum relative error is 2.2×10^{-3} ($_0S_{72}$), a phase velocity error of .009 km/sec.

In the second inversion $\mathbf{m}_G$ was the homogeneous model shown in Figure 8. The final solution, denoted *CM*, is shown in Figure 9. The eigenfrequencies of *CM* are compared with the observed data in Table 6. The maximum relative error is 2.1 ×

TABLE 5

COMPARISON OF GM SPECTRUM WITH OBSERVED DATA

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
OS 19	1.7327000E-02	1.7324718E-02	1.3168E-04	OS 29	2.3373000E-02	2.3373088E-02	-3.7560E-06
OS 20	1.7967000E-02	1.7973701E-02	-3.7299E-04	OS 30	2.3940000E-02	2.3940544E-02	-2.2724E-05
OS 21	1.8598000E-02	1.8610612E-02	-6.7812E-04	OS 31	2.4505000E-02	2.4504653E-02	1.4176E-05
OS 22	1.9225000E-02	1.9236189E-02	-5.8202E-04	OS 32	2.5067000E-02	2.5066117E-02	3.5234E-05
OS 23	1.9842000E-02	1.9851265E-02	-4.6693E-04	OS 33	2.5625000E-02	2.5625575E-02	-2.2433E-05
OS 24	2.0451000E-02	2.0456734E-02	-2.8040E-04	OS 34	2.6184000E-02	2.6183596E-02	1.5417E-05
OS 25	2.1050000E-02	2.1053544E-02	-1.6834E-04	OS 35	2.6744000E-02	2.6740688E-02	1.2386E-04
OS 26	2.1640000E-02	2.1642645E-02	-1.2223E-04	OS 36	2.7305000E-02	2.7297293E-02	2.8226E-04
OS 27	2.2223000E-02	2.2224995E-02	-8.9789E-05	OS 37	2.7866000E-02	2.7853800E-02	4.3780E-04
OS 28	2.2801000E-02	2.2801515E-02	-2.2580E-05	OS 38	2.8427000E-02	2.8410540E-02	5.7901E-04

TABLE 5—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0S 39	2.8988000E-02	2.8967798E-02	6.9691E-04	0S 75	4.9876000E-02	4.9793614E-02	1.6518E-03
0S 40	2.9548000E-02	2.9525812E-02	7.5092E-04	0S 76	5.0451000E-02	5.0383437E-02	1.3392E-03
0S 41	3.0106000E-02	3.0084780E-02	7.0484E-04	0S 77	5.1018000E-02	5.0973064E-02	8.8078E-04
0S 42	3.0668000E-02	3.0644865E-02	7.5438E-04	0S 78	5.1579000E-02	5.1562439E-02	3.2107E-04
0S 43	3.1232000E-02	3.1206197E-02	8.2619E-04	0S 79	5.2139000E-02	5.2151511E-02	-2.3996E-04
0S 44	3.1797000E-02	3.1768872E-02	8.8461E-04	0S 80	5.2703000E-02	5.2740229E-02	-7.0639E-04
0S 45	3.2363000E-02	3.2332967E-02	9.2799E-04	0S 81	5.3271000E-02	5.3328544E-02	-1.0802E-03
0S 46	3.2928000E-02	3.2898534E-02	8.9485E-04	0S 82	5.3845000E-02	5.3916411E-02	-1.3262E-03
0S 47	3.3492000E-02	3.3465605E-02	7.8810E-04	0S 83	5.4428000E-02	5.4503786E-02	-1.3924E-03
0S 48	3.4054000E-02	3.4034188E-02	5.8178E-04	0S 84	5.5015000E-02	5.5090627E-02	-1.3747E-03
0S 49	3.4614000E-02	3.4604294E-02	2.8040E-04	0S 85	5.5602000E-02	5.5676895E-02	-1.3470E-03
0S 50	3.5174000E-02	3.5175905E-02	-5.4153E-05	0S 86	5.6186000E-02	5.6262553E-02	-1.3625E-03
0S 51	3.5738000E-02	3.5748996E-02	-3.0769E-04	0S 87	5.6779000E-02	5.6847567E-02	-1.2076E-03
0S 52	3.6303000E-02	3.6323532E-02	-5.6556E-04	0S 88	5.7370000E-02	5.7431903E-02	-1.0790E-03
0S 53	3.6874000E-02	3.6899469E-02	-6.9071E-04	0S 89	5.7961000E-02	5.8015533E-02	-9.4085E-04
0S 54	3.7450000E-02	3.7476760E-02	-7.1456E-04	0S 90	5.8552000E-02	5.8598427E-02	-7.9293E-04
0S 55	3.8027000E-02	3.8055349E-02	-7.4548E-04	0S 91	5.9145000E-02	5.9180562E-02	-6.0127E-04
0S 56	3.8602000E-02	3.8635175E-02	-8.5941E-04	0S 92	5.9732000E-02	5.9761917E-02	-5.0085E-04
0S 57	3.9175000E-02	3.9216175E-02	-1.0510E-03	0S 93	6.0316000E-02	6.0342464E-02	-4.3876E-04
0S 58	3.9744000E-02	3.9798281E-02	-1.3658E-03	0S 94	6.0897000E-02	6.0922189E-02	-4.1363E-04
0S 59	4.0311000E-02	4.0381424E-02	-1.7470E-03	0S 95	6.1481000E-02	6.1501075E-02	-3.2653E-04
0S 60	4.0884000E-02	4.0965533E-02	-1.9943E-03	0S 96	6.2061000E-02	6.2079107E-02	-2.9177E-04
0S 61	4.1467000E-02	4.1550535E-02	-2.0145E-03	0S 97	6.2641000E-02	6.2656274E-02	-2.4384E-04
0S 62	4.2063000E-02	4.2136357E-02	-1.7440E-03	0T 21	1.8119000E-02	1.8107994E-02	6.0741E-04
0S 63	4.2669000E-02	4.2722924E-02	-1.2638E-03	0T 22	1.8799000E-02	1.8801386E-02	-1.2692E-04
0S 64	4.3282000E-02	4.3310163E-02	-6.5068E-04	0T 23	1.9478000E-02	1.9492433E-02	-7.4099E-04
0S 65	4.3898000E-02	4.3898000E-02	-1.1059E-08	0T 24	2.0160000E-02	2.0181528E-02	-1.0678E-03
0S 66	4.4512000E-02	4.4486361E-02	5.7601E-04	0T 25	2.0843000E-02	2.0869005E-02	-1.2476E-03
0S 67	4.5124000E-02	4.5075172E-02	1.0821E-03	0T 26	2.1525000E-02	2.1555157E-02	-1.4010E-03
0S 68	4.5733000E-02	4.5664364E-02	1.5008E-03	0T 27	2.2208000E-02	2.2240212E-02	-1.4505E-03
0S 69	4.6339000E-02	4.6253865E-02	1.8372E-03	0T 28	2.2890000E-02	2.2924370E-02	-1.5015E-03
0S 70	4.6940000E-02	4.6843607E-02	2.0535E-03	0T 29	2.3573000E-02	2.3607832E-02	-1.4776E-03
0S 71	4.7536000E-02	4.7433522E-02	2.1558E-03	0T 30	2.4256000E-02	2.4290736E-02	-1.4321E-03
0S 72	4.8128000E-02	4.8023543E-02	2.1704E-03	0T 31	2.4938000E-02	2.4973207E-02	-1.4118E-03
0S 73	4.8715000E-02	4.8613607E-02	2.0813E-03	0T 32	2.5618000E-02	2.5655351E-02	-1.4580E-03
0S 74	4.9297000E-02	4.9203651E-02	1.8936E-03	0T 33	2.6301000E-02	2.6337258E-02	-1.3786E-03

TABLE 5—*Continued*

Mode	ω_{obs}	ω_{com}	err
0T 34	2.6984000E-02	2.7019054E-02	-1.2991E-03
0T 35	2.7666000E-02	2.7700649E-02	-1.2524E-03
0T 36	2.8350000E-02	2.8382250E-02	-1.1376E-03
0T 37	2.9036000E-02	2.9063848E-02	-9.5910E-04
0T 38	2.9719000E-02	2.9745478E-02	-8.9095E-04
0T 39	3.0403000E-02	3.0427170E-02	-7.9498E-04
0T 40	3.1087000E-02	3.1108946E-02	-7.0595E-04
0T 41	3.1771000E-02	3.1790821E-02	-6.2387E-04
0T 42	3.2456000E-02	3.2472820E-02	-5.1825E-04
0T 43	3.3141000E-02	3.3154939E-02	-4.2060E-04
0T 44	3.3827000E-02	3.3837190E-02	-3.0125E-04
0T 45	3.4512000E-02	3.4519580E-02	-2.1963E-04
0T 46	3.5198000E-02	3.5202107E-02	-1.1667E-04
0T 47	3.5884000E-02	3.5884769E-02	-2.1420E-05
0T 48	3.6579000E-02	3.6567569E-02	3.1250E-04
0T 49	3.7259000E-02	3.7250502E-02	2.2807E-04
0T 50	3.7946000E-02	3.7933564E-02	3.2772E-04
0T 51	3.8632000E-02	3.8616749E-02	3.9477E-04
0T 52	3.9319000E-02	3.9300049E-02	4.8198E-04
0T 53	4.0005000E-02	3.9983444E-02	5.3883E-04
0T 54	4.0693000E-02	4.0666957E-02	6.3999E-04
0T 55	4.1380000E-02	4.1350562E-02	7.1140E-04
0T 56	4.2067000E-02	4.2034252E-02	7.7848E-04
0T 57	4.2752000E-02	4.2718036E-02	7.9445E-04
0T 58	4.3437000E-02	4.3401868E-02	8.0881E-04
0T 59	4.4121000E-02	4.4085751E-02	7.9892E-04
0T 60	4.4805000E-02	4.4770010E-02	7.8095E-04
0T 61	4.5490000E-02	4.5453658E-02	7.9889E-04
0T 62	4.6174000E-02	4.6137658E-02	7.8707E-04
0T 63	4.6858000E-02	4.6821674E-02	7.7524E-04
0T 64	4.7541000E-02	4.7505680E-02	7.4293E-04
0T 65	4.8224000E-02	4.8189700E-02	7.1127E-04
0T 66	4.8905000E-02	4.8873724E-02	6.3952E-04
0T 67	4.9587000E-02	4.9557708E-02	5.9072E-04
0T 68	5.0268000E-02	5.0241659E-02	5.2402E-04
0T 69	5.0950000E-02	5.0925566E-02	4.7957E-04
0T 70	5.1631000E-02	5.1609407E-02	4.1822E-04
0T 71	5.2311000E-02	5.2293199E-02	3.4030E-04
0T 72	5.2991000E-02	5.2976919E-02	2.6573E-04
0T 73	5.3673000E-02	5.3660561E-02	2.3176E-04
0T 74	5.4356000E-02	5.4344103E-02	2.1888E-04
0T 75	5.5039000E-02	5.5027550E-02	2.0804E-04
0T 76	5.5723000E-02	5.5710888E-02	2.1736E-04
0T 77	5.6407000E-02	5.6394121E-02	2.2832E-04
0T 78	5.7089000E-02	5.7077214E-02	2.0644E-04
0T 79	5.7769000E-02	5.7760191E-02	1.5249E-04
0T 80	5.8449000E-02	5.8443004E-02	1.0258E-04
0T 81	5.9130000E-02	5.9125664E-02	7.3324E-05
0T 82	5.9811000E-02	5.9808163E-02	4.7433E-05
0T 83	6.0492000E-02	6.0490492E-02	2.4930E-05
0T 84	6.1173000E-02	6.1172643E-02	5.8346E-06
0T 85	6.1853000E-02	6.1854609E-02	-2.6006E-05
0T 86	6.2533000E-02	6.2536381E-02	-5.4059E-05
0T 87	6.3212000E-02	6.3217951E-02	-9.4148E-05
0T 88	6.3892000E-02	6.3899313E-02	-1.1447E-04
0T 89	6.4573000E-02	6.4580460E-02	-1.1552E-04
0T 90	6.5254000E-02	6.5261382E-02	-1.1313E-04
0T 91	6.5935000E-02	6.5942075E-02	-1.0730E-04
0T 92	6.6617000E-02	6.6622530E-02	-8.3016E-05
0T 93	6.7297000E-02	6.7302743E-02	-8.5340E-05
0T 94	6.7977000E-02	6.7982705E-02	-8.3929E-05
0T 95	6.8655000E-02	6.8662410E-02	-1.0793E-04
0T 96	6.9334000E-02	6.9341851E-02	-1.1323E-04
0T 97	7.0014000E-02	7.0021022E-02	-1.0030E-04
0T 98	7.0694000E-02	7.0699886E-02	-8.3263E-05
0T 99	7.1376000E-02	7.1378499E-02	-3.5017E-05
0T100	7.2057000E-02	7.2056825E-02	2.4285E-06
0T101	7.2736000E-02	7.2734890E-02	1.5263E-05
0T102	7.3415000E-02	7.3412624E-02	3.2362E-05
0T103	7.4092000E-02	7.4090054E-02	2.6260E-05
0T104	7.4768000E-02	7.4767176E-02	1.1026E-05
0T105	7.5445000E-02	7.5443983E-02	1.3484E-05

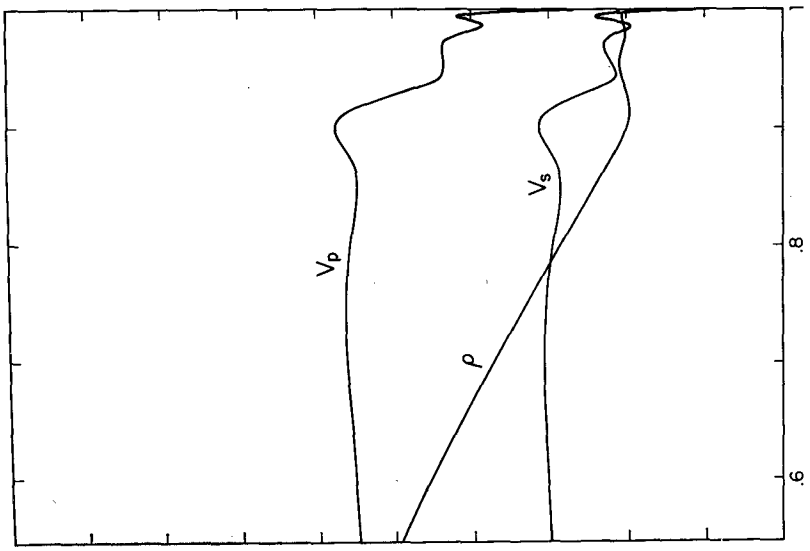

FIG. 9. Model CM.

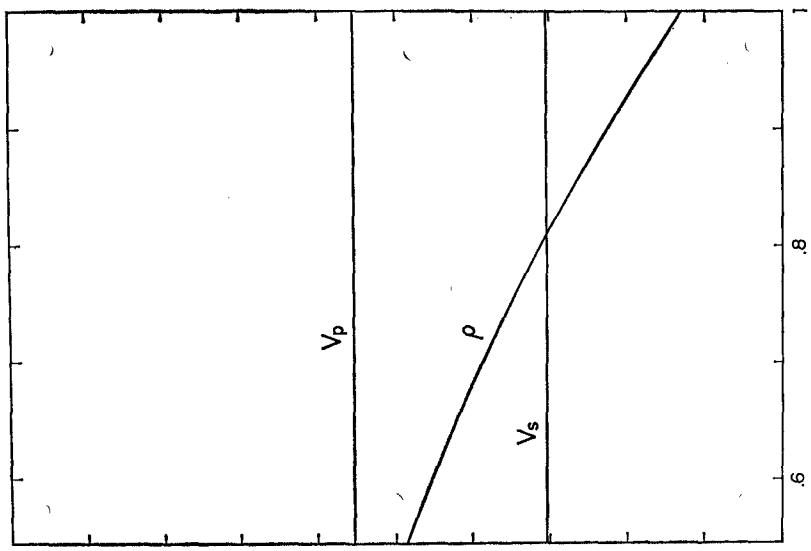

FIG. 8. $\mathbf{m}_G$ in the second inversion of mantle surface waves.

TABLE 6

COMPARISON OF CM SPECTRUM WITH DATA

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0S 19	1.7327000E-02	1.7324244E-02	1.5908E-04	0S 53	3.6874000E-02	3.6895285E-02	-5.7723E-04
0S 20	1.7967000E-02	1.7971897E-02	-2.7255E-04	0S 54	3.7450000E-02	3.7473340E-02	-6.2322E-04
0S 21	1.8598000E-02	1.8608676E-02	-5.7406E-04	0S 55	3.8027000E-02	3.8052696E-02	-6.7572E-04
0S 22	1.9225000E-02	1.9234900E-02	-5.1495E-04	0S 56	3.8602000E-02	3.8633291E-02	-8.1060E-04
0S 23	1.9842000E-02	1.9851070E-02	-4.5710E-04	0S 57	3.9175000E-02	3.9215041E-02	-1.0221E-03
0S 24	2.0451000E-02	2.0457801E-02	-3.3256E-04	0S 58	3.9744000E-02	3.9797867E-02	-1.3554E-03
0S 25	2.1050000E-02	2.1055817E-02	-2.7634E-04	0S 59	4.0311000E-02	4.0381686E-02	-1.7535E-03
0S 26	2.1640000E-02	2.1645923E-02	-2.7373E-04	0S 60	4.0884000E-02	4.0966430E-02	-2.0162E-03
0S 27	2.2223000E-02	2.2228975E-02	-2.6886E-04	0S 61	4.1467000E-02	4.1552000E-02	-2.0498E-03
0S 28	2.2801000E-02	2.2805858E-02	-2.1306E-04	0S 62	4.2063000E-02	4.2138312E-02	-1.7904E-03
0S 29	2.3373000E-02	2.3377428E-02	-1.8946E-04	0S 63	4.2669000E-02	4.2725321E-02	-1.3199E-03
0S 30	2.3940000E-02	2.3944551E-02	-1.9010E-04	0S 64	4.3282000E-02	4.3312892E-02	-7.1373E-04
0S 31	2.4505000E-02	2.4508030E-02	-1.2365E-04	0S 65	4.3898000E-02	4.3900970E-02	-6.7667E-05
0S 32	2.5067000E-02	2.5068633E-02	-6.5126E-05	0S 66	4.4512000E-02	4.4489492E-02	5.0567E-04
0S 33	2.5625000E-02	2.5627065E-02	-8.0567E-05	0S 67	4.5124000E-02	4.5078373E-02	1.0112E-03
0S 34	2.6184000E-02	2.6183936E-02	2.4260E-06	0S 68	4.5733000E-02	4.5667543E-02	1.4313E-03
0S 35	2.6744000E-02	2.6739830E-02	1.5593E-04	0S 69	4.6339000E-02	4.6256934E-02	1.7710E-03
0S 36	2.7305000E-02	2.7295214E-02	3.5840E-04	0S 70	4.6940000E-02	4.6846478E-02	1.9924E-03
0S 37	2.7866000E-02	2.7850554E-02	5.5431E-04	0S 71	4.7536000E-02	4.7436111E-02	2.1013E-03
0S 38	2.8427000E-02	2.8406207E-02	7.3146E-04	0S 72	4.8128000E-02	4.8025769E-02	2.1241E-03
0S 39	2.8988000E-02	2.8962491E-02	8.7999E-04	0S 73	4.8715000E-02	4.8615394E-02	2.0447E-03
0S 40	2.9548000E-02	2.9519660E-02	9.5913E-04	0S 74	4.9297000E-02	4.9204933E-02	1.8676E-03
0S 41	3.0106000E-02	3.0077939E-02	9.3206E-04	0S 75	4.9876000E-02	4.9794327E-02	1.6375E-03
0S 42	3.0668000E-02	3.0637495E-02	9.9470E-04	0S 76	5.0451000E-02	5.0383527E-02	1.3374E-03
0S 43	3.1232000E-02	3.1198459E-02	1.0739E-03	0S 77	5.1018000E-02	5.0972484E-02	8.9215E-04
0S 44	3.1797000E-02	3.1760932E-02	1.1343E-03	0S 78	5.1579000E-02	5.1561156E-02	3.4596E-04
0S 45	3.2363000E-02	3.2324981E-02	1.1748E-03	0S 79	5.2139000E-02	5.2149495E-02	-2.0130E-04
0S 46	3.2928000E-02	3.2890646E-02	1.1344E-03	0S 80	5.2703000E-02	5.2737469E-02	-6.5403E-04
0S 47	3.3492000E-02	3.3457953E-02	1.0166E-03	0S 81	5.3271000E-02	5.3325049E-02	-1.0146E-03
0S 48	3.4054000E-02	3.4026904E-02	7.9568E-04	0S 82	5.3845000E-02	5.3912175E-02	-1.2476E-03
0S 49	3.4614000E-02	3.4597480E-02	4.7726E-04	0S 83	5.4428000E-02	5.4498827E-02	-1.3013E-03
0S 50	3.5174000E-02	3.5169664E-02	1.2327E-04	0S 84	5.5015000E-02	5.5084978E-02	-1.2720E-03
0S 51	3.5738000E-02	3.5743377E-02	-1.5046E-04	0S 85	5.5602000E-02	5.5670601E-02	-1.2338E-03
0S 52	3.6303000E-02	3.6318609E-02	-4.2998E-04	0S 86	5.6186000E-02	5.6255672E-02	-1.2400E-03

TABLE 6—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0S 87	5.6779000E-02	5.6840168E-02	-1.0773E-03	0T 46	3.5198000E-02	3.5208576E-02	-3.0047E-04
0S 88	5.7370000E-02	5.7424074E-02	-9.4254E-04	0T 47	3.5884000E-02	3.5890554E-02	-1.8265E-04
0S 89	5.7961000E-02	5.8007370E-02	-8.0002E-04	0T 48	3.6579000E-02	3.6573103E-02	1.6121E-04
0S 90	5.8552000E-02	5.8590044E-02	-6.4974E-04	0T 49	3.7259000E-02	3.7255903E-02	8.3119E-05
0S 91	5.9145000E-02	5.9172083E-02	-4.5790E-04	0T 50	3.7946000E-02	3.7938448E-02	1.9902E-04
0S 92	5.9732000E-02	5.9753525E-02	-3.6036E-04	0T 51	3.8632000E-02	3.8620713E-02	2.9217E-04
0S 93	6.0316000E-02	6.0334254E-02	-3.0264E-04	0T 52	3.9319000E-02	3.9303471E-02	3.9495E-04
0S 94	6.0897000E-02	6.0914335E-02	-2.8467E-04	0T 53	4.0005000E-02	3.9986333E-02	4.6661E-04
0S 95	6.1481000E-02	6.1493749E-02	-2.0737E-04	0T 54	4.0693000E-02	4.0669293E-02	5.8259E-04
0S 96	6.2061000E-02	6.2072506E-02	-1.8541E-04	0T 55	4.1380000E-02	4.1352717E-02	6.5932E-04
0S 97	6.2641000E-02	6.2650601E-02	-1.5327E-04	0T 56	4.2067000E-02	4.2035819E-02	7.4121E-04
0T 21	1.8119000E-02	1.8101073E-02	9.8941E-04	0T 57	4.2752000E-02	4.2717495E-02	8.0711E-04
0T 22	1.8799000E-02	1.8799025E-02	-1.3050E-06	0T 58	4.3437000E-02	4.3400713E-02	8.3539E-04
0T 23	1.9478000E-02	1.9492609E-02	-7.5001E-04	0T 59	4.4121000E-02	4.4083992E-02	8.3878E-04
0T 24	2.0160000E-02	2.0183869E-02	-1.1840E-03	0T 60	4.4805000E-02	4.4767318E-02	8.4102E-04
0T 25	2.0843000E-02	2.0873173E-02	-1.4476E-03	0T 61	4.5490000E-02	4.5448641E-02	9.0918E-04
0T 26	2.1525000E-02	2.1560839E-02	-1.6650E-03	0T 62	4.6174000E-02	4.6135237E-02	8.3950E-04
0T 27	2.2208000E-02	2.2247141E-02	-1.7625E-03	0T 63	4.6858000E-02	4.6819024E-02	8.3179E-04
0T 28	2.2890000E-02	2.2932315E-02	-1.8486E-03	0T 64	4.7541000E-02	4.7502503E-02	8.0976E-04
0T 29	2.3573000E-02	2.3616565E-02	-1.8481E-03	0T 65	4.8224000E-02	4.8185626E-02	7.9574E-04
0T 30	2.4256000E-02	2.4300067E-02	-1.8167E-03	0T 66	4.8905000E-02	4.8868949E-02	7.3716E-04
0T 31	2.4938000E-02	2.4982969E-02	-1.8032E-03	0T 67	4.9587000E-02	4.9552400E-02	6.9777E-04
0T 32	2.5618000E-02	2.5665403E-02	-1.8504E-03	0T 68	5.0268000E-02	5.0235825E-02	6.4007E-04
0T 33	2.6301000E-02	2.6347474E-02	-1.7670E-03	0T 69	5.0950000E-02	5.0919215E-02	6.0421E-04
0T 34	2.6984000E-02	2.7029275E-02	-1.6779E-03	0T 70	5.1631000E-02	5.1602566E-02	5.5071E-04
0T 35	2.7666000E-02	2.7710885E-02	-1.6224E-03	0T 71	5.2311000E-02	5.2285869E-02	4.8041E-04
0T 36	2.8350000E-02	2.8393620E-02	-1.5386E-03	0T 72	5.2991000E-02	5.2968551E-02	4.2363E-04
0T 37	2.9036000E-02	2.9071115E-02	-1.2094E-03	0T 73	5.3673000E-02	5.3652305E-02	3.8557E-04
0T 38	2.9719000E-02	2.9756032E-02	-1.2461E-03	0T 74	5.4356000E-02	5.4335419E-02	3.7863E-04
0T 39	3.0403000E-02	3.0437463E-02	-1.1335E-03	0T 75	5.5039000E-02	5.5018467E-02	3.7305E-04
0T 40	3.1087000E-02	3.1118968E-02	-1.0284E-03	0T 76	5.5723000E-02	5.5700944E-02	3.9581E-04
0T 41	3.1771000E-02	3.1799729E-02	-9.0424E-04	0T 77	5.6407000E-02	5.6383787E-02	4.1152E-04
0T 42	3.2456000E-02	3.2481332E-02	-7.8051E-04	0T 78	5.7089000E-02	5.7066532E-02	3.9356E-04
0T 43	3.3141000E-02	3.3163032E-02	-6.6479E-04	0T 79	5.7769000E-02	5.7749171E-02	3.4324E-04
0T 44	3.3827000E-02	3.3844624E-02	-5.2099E-04	0T 80	5.8449000E-02	5.8431702E-02	2.9595E-04
0T 45	3.4512000E-02	3.4526550E-02	-4.2159E-04	0T 81	5.9130000E-02	5.9114107E-02	2.6878E-04

TABLE 6—*Continued*

Mode	ω_{obs}	ω_{com}	err	Mode	ω_{obs}	ω_{com}	err
0T 82	5.9811000E-02	5.9796394E-02	2.4420E-04	0T 94	6.7977000E-02	6.7972847E-02	6.1096E-05
0T 83	6.0492000E-02	6.0478552E-02	2.2231E-04	0T 95	6.8655000E-02	6.8653110E-02	2.7532E-05
0T 84	6.1173000E-02	6.1160576E-02	2.0310E-04	0T 96	6.9334000E-02	6.9333183E-02	1.1782E-05
0T 85	6.1853000E-02	6.1842459E-02	1.7041E-04	0T 97	7.0014000E-02	7.0013072E-02	1.3259E-05
0T 86	6.2533000E-02	6.2524530E-02	1.3545E-04	0T 98	7.0694000E-02	7.0692771E-02	1.7391E-05
0T 87	6.3212000E-02	6.3206133E-02	9.2822E-05	0T 99	7.1376000E-02	7.1372274E-02	5.2202E-05
0T 88	6.3892000E-02	6.3887604E-02	6.8796E-05	0T100	7.2057000E-02	7.2051584E-02	7.5160E-05
0T 89	6.4573000E-02	6.4568896E-02	6.3556E-05	0T101	7.2736000E-02	7.2730697E-02	7.2914E-05
0T 90	6.5254000E-02	6.5250025E-02	6.0915E-05	0T102	7.3415000E-02	7.3409609E-02	7.3434E-05
0T 91	6.5935000E-02	6.5930995E-02	6.0736E-05	0T103	7.4092000E-02	7.4088313E-02	4.9760E-05
0T 92	6.6617000E-02	6.6611788E-02	7.8241E-05	0T104	7.4768000E-02	7.4766821E-02	1.5772E-05
0T 93	6.7297000E-02	6.7292406E-02	6.8262E-05	0T105	7.5445000E-02	7.5445118E-02	-1.5649E-06

TABLE 7

COMPARISON OF GM SPECTRUM WITH CM SPECTRUM

Mode	ω_{GM}	ω_{CM}	diff	Mode	ω_{GM}	ω_{CM}	diff
0S 19	1.7324718E-02	1.7324244E-02	2.7360E-05	0S 36	2.7297293E-02	2.7295214E-02	7.6161E-05
0S 20	1.7973701E-02	1.7971897E-02	1.0037E-04	0S 37	2.7853800E-02	2.7850554E-02	1.1654E-04
0S 21	1.8610612E-02	1.8608676E-02	1.0403E-04	0S 38	2.8410540E-02	2.8406207E-02	1.5251E-04
0S 22	1.9236189E-02	1.9234900E-02	6.7009E-05	0S 39	2.8967798E-02	2.8962491E-02	1.8320E-04
0S 23	1.9851265E-02	1.9851070E-02	9.8230E-06	0S 40	2.9525812E-02	2.9519660E-02	2.0836E-04
0S 24	2.0456734E-02	2.0457801E-02	-5.2159E-05	0S 41	3.0084780E-02	3.0077939E-02	2.2739E-04
0S 25	2.1053544E-02	2.1055817E-02	-1.0796E-04	0S 42	3.0644865E-02	3.0637495E-02	2.4050E-04
0S 26	2.1642645E-02	2.1645923E-02	-1.5146E-04	0S 43	3.1206197E-02	3.1198459E-02	2.4796E-04
0S 27	2.2224995E-02	2.2228975E-02	-1.7908E-04	0S 44	3.1768872E-02	3.1760932E-02	2.4993E-04
0S 28	2.2801515E-02	2.2805858E-02	-1.9047E-04	0S 45	3.2332967E-02	3.2324981E-02	2.4699E-04
0S 29	2.3373088E-02	2.3377428E-02	-1.8568E-04	0S 46	3.2898534E-02	3.2890646E-02	2.3977E-04
0S 30	2.3940544E-02	2.3944551E-02	-1.6737E-04	0S 47	3.3465605E-02	3.3457953E-02	2.2865E-04
0S 31	2.4504653E-02	2.4508030E-02	-1.3781E-04	0S 48	3.4034188E-02	3.4026904E-02	2.1402E-04
0S 32	2.5066117E-02	2.5068633E-02	-1.0037E-04	0S 49	3.4604294E-02	3.4597480E-02	1.9691E-04
0S 33	2.5625575E-02	2.5627065E-02	-5.8145E-05	0S 50	3.5175905E-02	3.5169664E-02	1.7742E-04
0S 34	2.6183596E-02	2.6183936E-02	-1.2985E-05	0S 51	3.5748996E-02	3.5743377E-02	1.5718E-04
0S 35	2.6740688E-02	2.6739830E-02	3.2086E-05	0S 52	3.6323532E-02	3.6318609E-02	1.3553E-04

TABLE 7—*Continued*

Mode	ω_{GM}	ω_{CM}	diff	Mode	ω_{GM}	ω_{CM}	diff
0S 53	3.6899469E-02	3.6895285E-02	1.1339E-04	0S 89	5.8015533E-02	5.8007370E-02	1.4070E-04
0S 54	3.7476760E-02	3.7473340E-02	9.1257E-05	0S 90	5.8598427E-02	5.8590044E-02	1.4306E-04
0S 55	3.8055349E-02	3.8052696E-02	6.9714E-05	0S 91	5.9180562E-02	5.9172083E-02	1.4327E-04
0S 56	3.8635175E-02	3.8633291E-02	4.8764E-05	0S 92	5.9761917E-02	5.9753525E-02	1.4042E-04
0S 57	3.9216175E-02	3.9215041E-02	2.8917E-05	0S 93	6.0342464E-02	6.0334254E-02	1.3606E-04
0S 58	3.9798281E-02	3.9797867E-02	1.0402E-05	0S 94	6.0922189E-02	6.0914335E-02	1.2892E-04
0S 59	4.0381424E-02	4.0381686E-02	-6.4881E-06	0S 95	6.1501075E-02	6.1493749E-02	1.1912E-04
0S 60	4.0965533E-02	4.0966430E-02	-2.1896E-05	0S 96	6.2079107E-02	6.2072506E-02	1.0633E-04
0S 61	4.1550535E-02	4.1552000E-02	-3.5258E-05	0S 97	6.2656274E-02	6.2650601E-02	9.0542E-05
0S 62	4.2136357E-02	4.2138312E-02	-4.6397E-05	0T 21	1.8107994E-02	1.8101073E-02	3.8221E-04
0S 63	4.2722924E-02	4.2725321E-02	-5.6106E-05	0T 22	1.8801386E-02	1.8799025E-02	1.2558E-04
0S 64	4.3310163E-02	4.3312892E-02	-6.3011E-05	0T 23	1.9492433E-02	1.9492609E-02	-9.0291E-06
0S 65	4.3898000E-02	4.3900970E-02	-6.7657E-05	0T 24	2.0181528E-02	2.0183869E-02	-1.1600E-04
0S 66	4.4486361E-02	4.4489492E-02	-7.0381E-05	0T 25	2.0869005E-02	2.0873173E-02	-1.9972E-04
0S 67	4.5075172E-02	4.5078373E-02	-7.1015E-05	0T 26	2.1555157E-02	2.1560839E-02	-2.6360E-04
0S 68	4.5664364E-02	4.5667543E-02	-6.9617E-05	0T 27	2.2240212E-02	2.2247141E-02	-3.1155E-04
0S 69	4.6253865E-02	4.6256934E-02	-6.6351E-05	0T 28	2.2924370E-02	2.2932315E-02	-3.4657E-04
0S 70	4.6843607E-02	4.6846478E-02	-6.1289E-05	0T 29	2.3607832E-02	2.3616565E-02	-3.6992E-04
0S 71	4.7433522E-02	4.7436111E-02	-5.4582E-05	0T 30	2.4290736E-02	2.4300067E-02	-3.8414E-04
0S 72	4.8023543E-02	4.8025769E-02	-4.6352E-05	0T 31	2.4973207E-02	2.4982969E-02	-3.9090E-04
0S 73	4.8613607E-02	4.8615394E-02	-3.6759E-05	0T 32	2.5655351E-02	2.5665403E-02	-3.9181E-04
0S 74	4.9203651E-02	4.9204933E-02	-2.6055E-05	0T 33	2.6337258E-02	2.6347474E-02	-3.8789E-04
0S 75	4.9793614E-02	4.9794327E-02	-1.4319E-05	0T 34	2.7019054E-02	2.7029275E-02	-3.7829E-04
0S 76	5.0383437E-02	5.0383527E-02	-1.7863E-06	0T 35	2.7700649E-02	2.7710885E-02	-3.6952E-04
0S 77	5.0973064E-02	5.0972484E-02	1.1379E-05	0T 36	2.8382250E-02	2.8393620E-02	-4.0060E-04
0S 78	5.1562439E-02	5.1561156E-02	2.4882E-05	0T 37	2.9063848E-02	2.9071115E-02	-2.5004E-04
0S 79	5.2151511E-02	5.2149495E-02	3.8657E-05	0T 38	2.9745478E-02	2.9756032E-02	-3.5481E-04
0S 80	5.2740229E-02	5.2737469E-02	5.2332E-05	0T 39	3.0427170E-02	3.0437463E-02	-3.3828E-04
0S 81	5.3328544E-02	5.3325049E-02	6.5537E-05	0T 40	3.1108946E-02	3.1118968E-02	-3.2216E-04
0S 82	5.3916411E-02	5.3912175E-02	7.8566E-05	0T 41	3.1790821E-02	3.1799729E-02	-2.8021E-04
0S 83	5.4503786E-02	5.4498827E-02	9.0984E-05	0T 42	3.2472820E-02	3.2481332E-02	-2.6213E-04
0S 84	5.5090627E-02	5.5084978E-02	1.0254E-04	0T 43	3.3154939E-02	3.3163032E-02	-2.4410E-04
0S 85	5.5676895E-02	5.5670601E-02	1.1305E-04	0T 44	3.3837190E-02	3.3844624E-02	-2.1970E-04
0S 86	5.6262553E-02	5.6255672E-02	1.2230E-04	0T 45	3.4519580E-02	3.4526550E-02	-2.0191E-04
0S 87	5.6847567E-02	5.6840168E-02	1.3016E-04	0T 46	3.5202107E-02	3.5208576E-02	-1.8377E-04
0S 88	5.7431903E-02	5.7424074E-02	1.3632E-04	0T 47	3.5884769E-02	3.5890554E-02	-1.6121E-04

TABLE 7—*Continued*

Mode	ω_{GM}	ω_{CM}	diff	Mode	ω_{GM}	ω_{CM}	diff
0T 48	3.6567569E-02	3.6573103E-02	-1.5134E-04	0T 77	5.6394121E-02	5.6383787E-02	1.8325E-04
0T 49	3.7250502E-02	3.7255903E-02	-1.4499E-04	0T 78	5.7077214E-02	5.7066532E-02	1.8715E-04
0T 50	3.7933564E-02	3.7938448E-02	-1.2875E-04	0T 79	5.7760191E-02	5.7749171E-02	1.9079E-04
0T 51	3.8616749E-02	3.8620713E-02	-1.0265E-04	0T 80	5.8443004E-02	5.8431702E-02	1.9338E-04
0T 52	3.9300049E-02	3.9303471E-02	-8.7074E-05	0T 81	5.9125664E-02	5.9114107E-02	1.9547E-04
0T 53	3.9983444E-02	3.9986333E-02	-7.2255E-05	0T 82	5.9808163E-02	5.9796394E-02	1.9678E-04
0T 54	4.0666957E-02	4.0669293E-02	-5.7442E-05	0T 83	6.0490492E-02	6.0478552E-02	1.9739E-04
0T 55	4.1350562E-02	4.1352717E-02	-5.2115E-05	0T 84	6.1172643E-02	6.1160576E-02	1.9726E-04
0T 56	4.2034252E-02	4.2035819E-02	-3.7279E-05	0T 85	6.1854609E-02	6.1842459E-02	1.9643E-04
0T 57	4.2718036E-02	4.2717495E-02	1.2664E-05	0T 86	6.2536381E-02	6.2524530E-02	1.8951E-04
0T 58	4.3401868E-02	4.3400713E-02	2.6612E-05	0T 87	6.3217951E-02	6.3206133E-02	1.8694E-04
0T 59	4.4085751E-02	4.4083992E-02	3.9900E-05	0T 88	6.3899313E-02	6.3887604E-02	1.8324E-04
0T 60	4.4770010E-02	4.4767318E-02	6.0130E-05	0T 89	6.4580460E-02	6.4568896E-02	1.7906E-04
0T 61	4.5453658E-02	4.5448641E-02	1.1038E-04	0T 90	6.5261382E-02	6.5250025E-02	1.7402E-04
0T 62	4.6137658E-02	4.6135237E-02	5.2473E-05	0T 91	6.5942075E-02	6.5930995E-02	1.6803E-04
0T 63	4.6821674E-02	4.6819024E-02	5.6598E-05	0T 92	6.6622530E-02	6.6611788E-02	1.6124E-04
0T 64	4.7505680E-02	4.7502503E-02	6.6876E-05	0T 93	6.7302743E-02	6.7292406E-02	1.5359E-04
0T 65	4.8189700E-02	4.8185626E-02	8.4541E-05	0T 94	6.7982705E-02	6.7972847E-02	1.4501E-04
0T 66	4.8873724E-02	4.8868949E-02	9.7701E-05	0T 95	6.8662410E-02	6.8653110E-02	1.3545E-04
0T 67	4.9557708E-02	4.9552400E-02	1.0711E-04	0T 96	6.9341851E-02	6.9333183E-02	1.2500E-04
0T 68	5.0241659E-02	5.0235825E-02	1.1612E-04	0T 97	7.0021022E-02	7.0013072E-02	1.1354E-04
0T 69	5.0925566E-02	5.0919215E-02	1.2471E-04	0T 98	7.0699886E-02	7.0692771E-02	1.0064E-04
0T 70	5.1609407E-02	5.1602566E-02	1.3255E-04	0T 99	7.1378499E-02	7.1372274E-02	8.7211E-05
0T 71	5.2293199E-02	5.2285869E-02	1.4017E-04	0T100	7.2056825E-02	7.2051584E-02	7.2734E-05
0T 72	5.2976919E-02	5.2968551E-02	1.5796E-04	0T101	7.2734890E-02	7.2730697E-02	5.7648E-05
0T 73	5.3660561E-02	5.3652305E-02	1.5386E-04	0T102	7.3412624E-02	7.3409609E-02	4.1069E-05
0T 74	5.4344103E-02	5.4335419E-02	1.5980E-04	0T103	7.4090054E-02	7.4088313E-02	2.3498E-05
0T 75	5.5027550E-02	5.5018467E-02	1.6506E-04	0T104	7.4767176E-02	7.4766821E-02	4.7481E-06
0T 76	5.5710888E-02	5.5700944E-02	1.7849E-04	0T105	7.5443983E-02	7.5445118E-02	-1.5044E-05

$10^{-3}({}_0S_{72})$, a phase velocity error of .009 km/sec. The eigenfrequencies of *GM* and *CM* are compared in Table 7. The maximum relative difference is $4.0 \times 10^{-4}({}_0T_{36})$, a phase velocity difference of .002 km/sec.

The mantle surface wave spectra of *GM* and *CM* are nearly identical, but the mantle structures of *GM* and *CM* are not. However, there is one common feature of *GM* and *CM*; a zone of low-shear velocity with a minimum at a depth of 86 km. Gutenberg (1953) hypothesized such a zone and estimated the depth of the minimum to be about 75 km. Other workers [see the reviews of Nuttli (1963) and Anderson (1967)] have shown that such a feature is compatible with the surface-wave and travel-time data. We have no proof that the low-velocity zone is a feature of the real earth, but it is a feature of all solutions, that we have so far obtained, to the inverse problem for the Mongolian surface wave data regardless of the $\mathbf{m}_G$'s used in our numerical experiments.

A second zone of low-shear velocity with a minimum at a depth of 320–340 km is compatible with the observed data, but the detailed features of *GM* and *CM* are sufficiently different to make a more positive statement unwarranted.

The shallower low-velocity zone is also evident in the *P* velocity structure of *GM* and *CM*, but the deeper zone is absent in *CM*.

The density structures of *GM* and *CM* are quite obviously distinctly different.

These results are certainly in coarse qualitative agreement with those reviewed by Anderson (1967) although it is disappointing how little we can learn about the structure of the upper mantle *from surface waves alone.* Of course, the situation could change very much for the better if travel time data and normal mode overtones (leaking modes) were also used as constraints on the solution (the subject of a subsequent paper). Even then the outlook for learning much about the density distribution from surface waves is not bright.

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