NOTES AND CORRESPONDENCE

On the Energy Input from Wind to Surface Waves

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ABSTRACT

A basic model relating the energy dissipation in the ocean mixed layer to the energy input into the surface wave field is combined with recent measurements of turbulent kinetic energy dissipation to determine the average phase speed of the waves acquiring energy from the wind. This phase speed and the square root of the corresponding wavelength are proportional to the vertically integrated dissipation. The calculations show that the maximum energy input occurs at the high-frequency end of the wave spectrum. Dissipation data from other investigators are used to estimate the effective phase speed between 0.55 and 0.72 m s^{-1} and corresponding wavelengths varying between 0.2 and 0.33 m for the waves receiving most of the energy.

1. Introduction

Physical processes occurring in the ocean surface layer are important for the exchange of gases, heat, mass, and momentum across the air–water interface. Stewart (1961) and Richman and Garrett (1977) point out that while the rate of momentum input to the ocean must equal the wind stress in the steady state, the rate of energy input is equal to the stress times a speed. This speed may be much greater than the surface drift speed of the water if waves are generated. In fact, Kitaigorodskii and Lumley (1983) and Kitaigorodskii et al. (1983) analyzed dissipation of the turbulent kinetic energy in the surface boundary layer in the presence of the wave field and pointed out that dissipation values are higher than predicted by a law-of-the-wall scaling. At the same time, several attempts have been made to estimate the wave scale λ_{diss}, where dissipation of the wave field occurs (Kitaigorodskii 1991; Thorpe 1993; Ding and Farmer 1994; Melville 1994). Kitaigorodskii (1983, 1991) proposes that there could be separation between input and dissipation scales.

However, the scale of the energy input has remained undetermined. In this note we examine the energy budget of the surface mixed layer and use recent measurements of the dissipation rate to estimate the phase speed and corresponding wavelength of the waves receiving most of the momentum.

2. Energy balance

We set the scene by reviewing the energy budget for a steady wall-bounded layer (e.g., Soloviev et al. 1988). In this case the viscous energy dissipation ϵ_{wl} is

$$\epsilon_{wl}(z) = \frac{u_*^3}{\kappa z}, \tag{1}$$

where $\kappa = 0.4$ is von Kármán's constant. The friction velocity u_* in the water can be defined as

$$u_* = U_{10}\left(\frac{\rho_a}{\rho_w} C_D\right)^{1/2}. \tag{2}$$

Here U_{10} is the wind speed at a 10-m height, ρ_w, ρ_a are the densities of water and air, respectively, and C_D is the coefficient of surface drag.

However, recent dissipation measurements by different investigators using fast response velocimeters at fixed locations (Drennan et al. 1991; Agrawal et al. 1992), ver-

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tical profilers (Anis and Moum 1992), and a turbulence sensor mounted on a submarine (Osborn et al. 1992), provide evidence of higher dissipation rates, exceeding ϵ_{wl} by almost two orders of magnitude in the top part of the mixed layer. Drennan et al. (1991) proposed that the energy excess comes from breaking surface waves.

Momentum transfer τ from air to sea may occur via wave generation at a rate τ_w or as skin friction τ_s; that is, $\tau = \tau_w + \tau_s$. Using the wave field momentum transfer function $G_w(\omega, \theta)$, where θ is the angle between wave and wind direction, we write

$$\tau_w = \int_0^\infty \int_0^{2\pi} G_w(\omega, \theta) d\theta d\omega. \qquad (3)$$

In (3) we separate frequency and directional dependencies (Phillips 1977). Hence, the energy flux from the wind into the surface layer is

$$P_{in} = \int_0^\infty \int_0^{2\pi} G(\omega)S(\theta) \frac{c_p(\omega)}{\cos\theta} d\theta d\omega + U_s\tau_s. \qquad (4)$$

In (4), $S(\theta)$ is the directional distribution function for the generated waves, $c_p(\omega)$ is the phase speed of the waves, and U_s is the drift velocity of the sea surface where we assume that the surface drift and wind are in the same direction. Phillips (1977) shows that $S(\theta)$ is small for angles $|\theta| > 30°$ and $\cos^{-1}\theta$ varies by less than 15% for angles $|\theta| < 30°$. Therefore, we ignore the effects of directionality in (4). Integrating (4) over all frequencies then gives

$$P_{in} = \tau_w \bar{c}_p + \tau_s U_s, \qquad (5)$$

where we introduce an *effective* phase velocity $\bar{c}_p$ as a normalized mean phase speed of the wave field and define it using the momentum transfer function $G(\omega)$:

$$\bar{c}_p = \frac{\int G(\omega)c_p(\omega)d\omega}{\int G(\omega)d\omega}. \qquad (6)$$

To analyze the energy input by the wind into the surface mixed layer we introduce a basic model, schematically presented in Fig. 1. Consider a fully developed sea with a mixed layer modeled as a box. By "fully developed" we refer to a sea state in which there are no temporal changes in the shape of the wave spectrum. The energy sources and sinks are 1) input of kinetic energy from the wind stress via wave generation P_{in}, 2) buoyancy flux J_b, 3) divergence of surface F and internal wave I energy fluxes, 4) temporal changes of the mechanical energy of the surface mixed layer dE_{ml}/dt, 5) local rate of growth of the wave energy dE_w/dt, and 6) viscous dissipation ϵ. The vertically integrated, horizontally averaged energy balance of the mixed layer of depth h is therefore

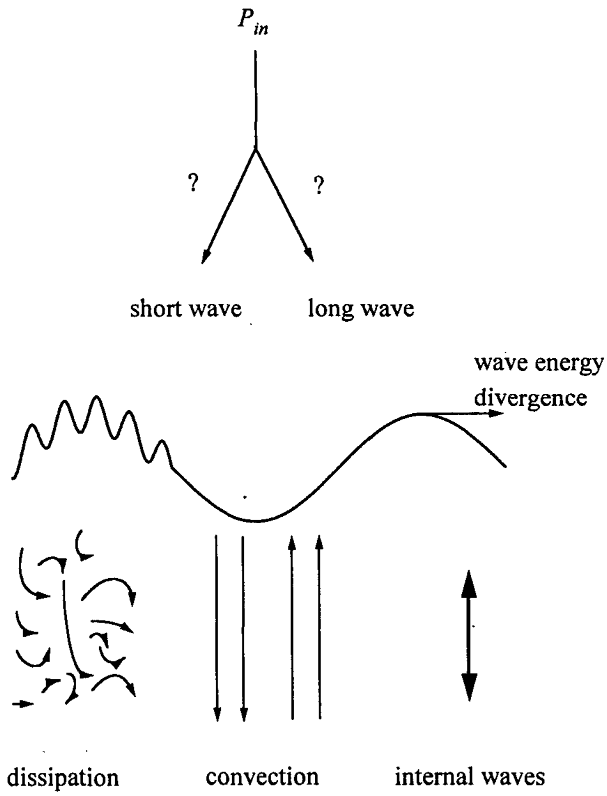

FIG. 1. Schematic of the energy balance of the mixed layer, modeled as a box, where P_{in} represents the energy input by the wind that enters the box via short or long waves. Other energy sources and sinks within the mixed layer are presented schematically and are not associated with a particular wave scale.

$$P_{in} = D - B + \frac{dE_{ml}}{dt} + \frac{dE_w}{dt} + \frac{dF}{dx} - I, \qquad (7)$$

where

$$D = \int_{-h}^0 \epsilon(z)dz, \quad B = \int_{-h}^0 J_b(z)dz.$$

As will be justified in section 3, the dominant term of the right-hand side of the balance (7) is typically D. Therefore, to estimate the effective phase speed $\bar{c}_p$ we reduce (7) to the basic balance:

$$P_{in} = D. \qquad (8)$$

For a fully rough flow ($U_{10} > 7.5$ m s^{-1}) all air–sea momentum transfer is supported by surface waves (Kinsman 1965; Donelan 1990). Hence, (8) and (5) will lead to

$$\bar{c}_p = \frac{D}{\tau}. \qquad (9)$$

From the dispersion relation for deep water waves, we obtain the corresponding wavelength

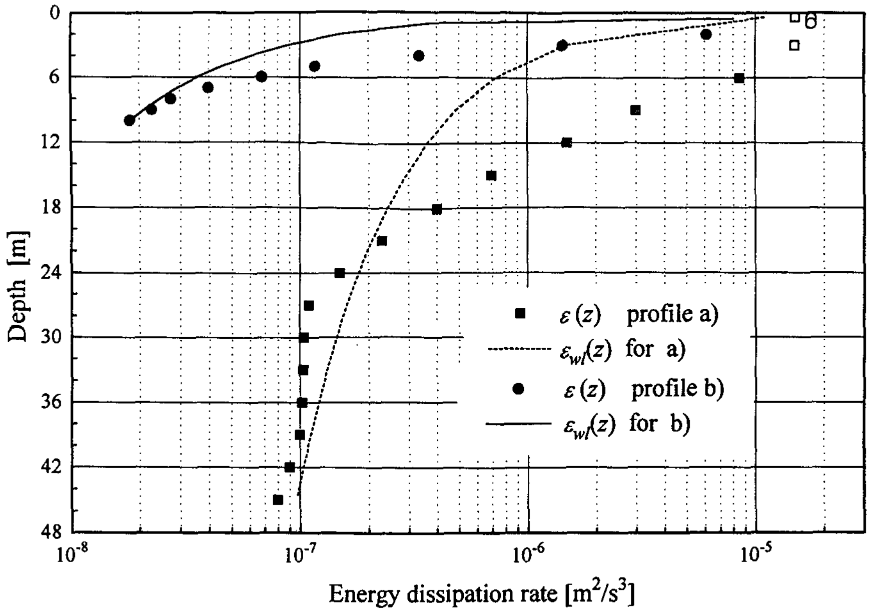

FIG. 2. Measured dissipation profiles $\epsilon(z)$ for two datasets [(a) Anis and Moum (1992) and (b) Osborn et al. (1992)] and wall-layer dissipation $\epsilon_{wz}(z)$ for average wind conditions, assuming neutral atmospheric stability and drag coefficient $C_D = 1.5 \times 10^{-3}$. The profile from Anis and Moum (1992) (solid squares) was extrapolated up to 3-m depth using the z^{-4} parameterization (open squares). For both profiles we assume a constant dissipation layer in the top of the mixed layer (Drennan et al. 1991) (open symbols). The surface buoyancy fluxes are 1.1×10^{-8} m^2 s^{-3} for (a) and 1.2×10^{-7} m^2 s^{-3} for (b).

$$\bar{\lambda} = \frac{2\pi \bar{c}_p^2}{g}, \qquad (10)$$

where g is the gravitational acceleration.

3. Results and discussion

As already mentioned, measurements by Anis and Moum (1992), Osborn et al. (1992), and Drennan et al. (1991) indicate a higher dissipation rate in the top few meters of the ocean surface layer than can be explained by the wall layer scaling. It has been suggested (Drennan et al. 1991; Agrawal et al. 1992) that breaking waves are the source of this anomalously high dissipation. Moreover, results of Drennan et al. (1991) and Osborn et al. (1992) clearly show the existence of a constant dissipation layer (depth of order 1–2 m) extending from the surface. Drennan et al. (1991) suggested that the thickness d_{cd} of this constant dissipation layer can be scaled with the significant wave height H_s as

$$d_{cd} = \beta H_s, \qquad (11)$$

where β is a scaling factor depending on the wave age (β = 1.2 for fully developed seas, $\beta \approx 1.6$ for "young" seas).

The averaged dissipation profiles $\epsilon(z)$ from Anis and Moum (1992), extrapolated up to 3-m depth with a z^{-4} profile and matched to a constant dissipation layer above that level consistent with Drennan et al. (1991) together with an upper bound in the dissipation profile consistent with Osborn et al. (1992, their Fig. 9b), are shown in Fig. 2. We also include the theoretical dissipation rates for a wall-bounded layer for the prevailing wind speed during the measurements. At the top of the mixed layer the observed dissipation rates exceed values predicted by a law-of-the-wall scaling by more than one order of magnitude. The profiles were numerically integrated over the depth of the ocean mixed layer to obtain the total dissipation D. The vertically integrated buoyancy flux B is negligible for the datasets of Anis and Moum (1992) ($B = 9 \times 10^{-6}$ m^3 s^{-3} vs $D = 1.04 \times 10^{-4}$ m^3 s^{-3}) and of Osborn et al. (1992) ($B = 1 \times 10^{-7}$ m^3 s^{-3} vs $D = 4.2 \times 10^{-5}$ m^3 s^{-3}). Drennan et al. (1991) give their dissipation profile in a parameterized form that could be integrated analytically with the assumption that the constant dissipation

TABLE 1. Summary of the calculation wave parameters, where $\bar{c}_p$ is the *effective* wave phase speed and $\bar{\lambda}$ the wavelength, corresponding to the deep water dispersion relation; U_{10} is the average wind speed during each measurement.

U_{10} (m s^{-1})	$\bar{c}_p$ (m s^{-1})	$\bar{\lambda}$ (m)	Data source
9.0	0.72	0.33	Anis and Moum (1992)
5.8	0.66	0.28	Osborn et al. (1992)
10.0	0.55	0.20	Drennan et al. (1991)

layer at the top extends to a depth $d_{cd} = 1.2H_s$. Here we used $\beta = 1.2$ in (11) for the fully developed sea. The resultant values vary from 0.4×10^{-4} to 1.1×10^{-4} m^3 s^3. These dissipation values together with (9) and (10) are used to calculate the effective wave phase speed and wavelength (Table 1). The momentum flux τ was calculated from the bulk formula $\tau = \rho_a C_D U_{10}^2$ for the mean wind speed using a drag coefficient $C_D = 1.5 \times 10^{-3}$. The effective length of waves responsible for the energy transfer from the wind into the wave field is approximately 0.27 m.

Now consider the underestimation of P_{in} due to the neglect of all terms other than D in the energy balance. Combining the effective phase velocity derived above with a mixed-layer turbulent closure model (Pollard et al. 1982), we estimate the rate of change of the mechanical energy in the mixed layer dE_{ml}/dt to be 10%–20% of the above estimates for energy input by the wind P_{in}. The local rate of change of the wave energy was estimated by Richman and Garrett (1977) to be a small fraction of the wave energy flux divergence dF/dx. They suggested that the wave energy flux divergence is fetch dependent. For moderate fetch, wave energy flux divergence is insignificant, but for large fetch it could be as much as 50% of P_{in}. For the Anis and Moum (1992) data the fetch is unknown; therefore, dF/dx cannot be determined. For Osborn et al. (1992) the fetch is of order 70 km and the wave energy flux divergence is about 10% of D. For the varying fetch conditions for the Drennan et al. (1991) dataset it is estimated on average to be less than 10% of D. Characteristic values of the open ocean internal wave flux (Olbers 1983) are 0.5%–5% of the above values of D. Hence, D may represent only 25%–90% of P_{in} for the Anis and Moum (1992) data, 65%–80% for the Osborn et al. (1992) data, and 65%–80% for the Drennan et al. (1991) data. Correction for these uncertainties could increase the effective wavelength by 0.05–2 m for the Anis and Moum (1992) data, 0.16–0.38 m for the Osborn et al. (1992) data, and 0.11–0.27 m for the Drennan et al. (1991) data.

We consider separately the uncertainties in the values of wind stress τ and vertically integrated dissipation D. Our potential error in estimation of wind stress is ±15% due to uncertainties in drag coefficient C_D and the wind speed U_{10}. It is more difficult to estimate uncertainties in the vertically integrated dissipation. Osborn et al. (1992) give a factor of 2 for the uncertainty, but Drennan et al. (1991) provide no error bounds for their dissipation estimates. Anis and Moum (1992) give uncertainties of ±20% for their dissipation estimates below 7 m. However, most of the dissipation occurs at depths between 0 and 7 m where, as mentioned before, we used an extrapolation procedure. The error due to this technique is unknown, but it may be as large as or exceed that for the Osborn et al. (1992) data. Therefore, uncertainty in τ and D suggests that the uncertainty in the effective wavelength is a factor of 4 (0.06–1 m). However, given that the data were from different experiments and collected with different equipment, the consistency of the results suggests that this uncertainty in effective wavelength is conservative.

We now compare the energy dissipation wave scale with the predominant wave scale at which wind energy enters the wave field. Kitaigorodskii (1983) suggests that the dissipation of wave energy due to breaking waves occurs at length scales much smaller than the dominant wavelength. In three recent papers an attempt was made to quantify the dissipation scale: Kitaigorodskii (1991) gives for the wavelength $\lambda_{\rm diss}$ where the transition toward the dissipation regime ($\lambda < \lambda_{\rm diss}$) occurs,

$$\lambda_{\rm diss} = \frac{2\pi E_0^{2/3}}{Ag}, \quad A = 1.6 \times 10^{-3}, \qquad (12)$$

where E_0 is the energy flux from the region of energy input through the nondissipative region of the wave spectrum toward the dissipation subrange.

According to our model this energy flux is equal to the total dissipation: $E_0 = D$. Given the dissipation values used above for the calculation of the scale of the energy input we obtain 0.6–1.2 m for the transitional wavelength $\lambda_{\rm diss}$. Thorpe (1993) combined observations of the frequency of wave breaking with laboratory results of the energy loss of a single breaking wave to give the ratio of the phase speed of the dominant waves and the dissipating waves as $c_{\rm peak}/c_{\rm diss} = 4$. In the case of developed wave fields and the given wind speeds this translates into a dissipation length scale of the order of 2 m. Ding and Farmer (1994) acoustically tracked individual breaking waves and found the normalized phase speed of breaking waves $c_N = c_{\rm br}/c_{\rm peak}$ in open ocean conditions to be a function of the phase speed of the dominant waves: $c_N = f(c_{\rm peak})$. Here $c_{\rm br}$ is the phase speed of waves that break. This gives a dissipation length scale about one order of magnitude larger than the scale of energy input; however, the dissipation scale remains smaller than the dominant wave scale.

Our result for the length scale of waves contributing to the turbulent kinetic energy of the surface layer implies that the maximum energy input occurs toward the high-frequency end of the wave spectrum, over a range from the capillary–gravity transition (17 mm) up to a length of 0.5–1 m. This differs by two to three orders of magnitude from the length of the dominant waves (about 60 m for unlimited fetch conditions and 10 m s^{-1} wind speed). This result implies that no more than 7% of the energy input can enter the wave field via the dominant waves. While the observed dissipation is much higher than one would expect for a wall-bounded layer, it is still one order of magnitude less than it would be if the energy entered through the dominant waves. Comparison with estimates for the dissipation subrange indicate that for a fully developed sea the energy input occurs at a slightly smaller scale than wave dissipation.

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