Turbulent Dispersion in the Ocean

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Abstract

The mathematical framework for turbulent transport in the ocean is reasonably well established. It may be applied to large-scale fields of scalars in the ocean and to the instantaneous or continuous discharge from a point. The theory and its physical basis can also provide an interpretation of passive scalar spectra. Spatial variations in the rate of turbulent transfer can be related to the movement of the center of mass of a scalar and to a formulation in terms of entrainment. The relative dispersion of a scalar with respect to its center of mass and the streakiness of the concentration field within the relative dispersion domain need to be considered. In many of these problems it is valuable to think in terms of simple models for individual streaks, as well as overall statistical properties.

Key words: Turbulent dispersion, ocean mixing, ocean eddies.

1. Introduction

Including the effects of processes that are unresolved in models is one of the central problems in oceanography. In particular, for temperature, salinity, or some other scalar, one seeks to parameterize the eddy flux in terms of quantities that are resolved by the models. This has been much discussed, with determinations of the correct parameterization relying on a combination of deductions from the large-scale models, observations of the eddy fluxes or associated quantities, and the development of an understanding of the processes responsible for the fluxes. The key remark to make is that it is only through process studies that we can reach an understanding leading to formulae that are valid in changing conditions, rather than just having numerical values which may only be valid in present conditions.

Rather than attempt a comprehensive review, this brief article will summarize, as simply as possible, some basic ideas and results on dispersion in a turbulent flow, drawing attention in particular to results that may go beyond standard texts, such as that of Csanady (1973). Some fundamental fluid dynamical ideas and their application to the ocean will be described in Section 2. Quite apart from the importance of turbulent dispersion for the evolution of large-scale patterns in the ocean, it also determines the concentration of material released from a point source, either instantaneously or continuously. This will also be reviewed.

For an instantaneous release, it is important to consider not only the "absolute" dispersion with respect to the point of release, but also the "relative" dispersion with respect to the center of mass of the released substance (e.g. Csanady, 1973; Fischer, List, Koh, Imberger, & Brooks, 1979). This will be reviewed in Section 3. Furthermore, the "streakiness" within the domain of relative dispersion may be a matter of concern and will be discussed.

The connection between the flux of a substance, with sharp gradients ultimately disappearing by molecular diffusion, and the adiabatic stirring associated with the dispersion of marked particles, will be reviewed in Section 4. In particular, standard ideas on the connections between stirring and mixing have been generalized to allow for the treatment of a hierarchy of different turbulent motions. Other, non-turbulent, mechanisms for dispersion in the ocean will be mentioned in Section 5, though not reviewed in detail.

Although eddy fluxes of potential vorticity or other dynamical quantities are also carried by particles, this paper will be concerned only with scalars. I hope that the non-expert reader will find it a useful introduction and that the expert reader will find one or two items of interest to compensate for shortcomings.

2. Eddy fluxes

We consider an ocean in which some scalar has concentration $C = \overline{C} + C'$ where $\overline{C}$ is the ensemble average of C and C' is its fluctuation. In practice the ensemble average is replaced by an average over time or space. This requires that there be a spectral gap, i.e. a band of frequency or wavenumber with little variance, between the slowly varying mean and the rapidly varying fluctuations. This assumption may well be hard to justify; we return to it later. The equation for the evolution of the mean state $\overline{C}$ involves the eddy flux $\mathbf{F} = \overline{\mathbf{u}C'}$, where $\mathbf{u}$ is the velocity fluctuation.

It is well recognized that $\mathbf{F}$ need not be aligned with the local gradient $\nabla\overline{C}$, but may be written in tensor form as

$$F_i = -T_{ij}\frac{\partial\overline{C}}{\partial x_j}. \tag{1}$$

This is formally possible for any flux, but the connection to the local mean gradient may only make physical sense if the motions responsible for the flux have a "mixing length" that is small compared with the distance over which $\overline{C}$ varies significantly.

We may write $T_{ij} = K_{ij} + S_{ij}$ where $K_{ij} = \frac{1}{2}(T_{ij} + T_{ji})$ and $S_{ij} = \frac{1}{2}(T_{ij} - T_{ji})$. The symmetric tensor K_{ij} is diagonalizable and is likely to represent down-gradient diffusion parallel to the principal axes of the tensor. We return to this later. The antisymmetric tensor S_{ij} has an associated "skew flux" $\mathbf{F}_s$ given by

$$F_{si} = -S_{ij}\frac{\partial\overline{C}}{\partial x_j} = -(\mathbf{D}\times\nabla\overline{C})_i \tag{2}$$

where $\mathbf{D} = -(S_{23}, S_{31}, S_{12})$. This flux is perpendicular to $\nabla\overline{C}$ and may be written as

$$\mathbf{F}_s = -(\nabla\times\mathbf{D})\overline{C} + \nabla\times(\mathbf{D}\overline{C}). \tag{3}$$

The second term of this flux is non-divergent and so does not affect the evolution of $\overline{C}$. The first term represents advection of $\overline{C}$ with a velocity $\mathbf{U}_s = -(\nabla\times\mathbf{D})$ which may be written as

$$U_{si} = \frac{\partial S_{ij}}{\partial x_j}. \tag{4}$$

This standard formalism (e.g. Rhines & Holland, 1977; Moffatt, 1983; Middleton & Loder, 1989) is purely kinematic. Further insights are obtained if we write the fluctuation C' in terms of a particle displacement $\mathbf{X}$ from the position where its value of C matches the local mean value. Then $C' = -X_j\partial\overline{C}/\partial x_j$ provided that $\mathbf{X}$ is small in magnitude compared with the distance over which $\nabla\overline{C}$ varies significantly. The eddy flux becomes

$$\overline{u_iC'} = -\overline{u_iX_j}\frac{\partial\overline{C}}{\partial x_j}. \tag{5}$$

The diffusivity K_{ij} is now $\frac{1}{2}(\overline{u_iX_j} + \overline{u_jX_i})$ and the antisymmetric tensor S_{ij} is given by $\frac{1}{2}(\overline{u_iX_j} - \overline{u_jX_i})$. The vector $\mathbf{D}$ may be written as $\frac{1}{2}\overline{\mathbf{X}\times\mathbf{u}}$ and the advection $\mathbf{U}_s$ from (4) may be written as $U_{si} = \partial(\overline{u_iX_j} - K_{ij})/\partial x_j$.

In this expression $\mathbf{u}$, while representing the Eulerian velocity at a point, is also the Lagrangian velocity $\partial \mathbf{X}/\partial t$ of the particle carrying the anomalous value of C. Now if $\nabla \cdot \mathbf{u} = 0$, then also $\nabla \cdot \mathbf{X} = 0$ and we may write, following Middleton and Loder (1989),

$$U_{si} = \overline{\mathbf{X} \cdot \nabla u_i} + \frac{1}{2} \frac{\partial^2 \overline{X_i X_j}}{\partial x_j \partial t}. \tag{6}$$

For small amplitude motions, the first term on the right hand side of (6) is just the Stokes drift, or the difference between the Lagrangian and Eulerian mean flows. For larger displacements, the interpretation is less simple (see Plumb and Ferrari (2005) for a recent discussion and the relation of these ideas to dynamics). Moreover, the second term on the right hand side of (6) may be non-zero for motions with statistical properties that vary in space and time.

While the effects of S_{ij} on the evolution of $\overline{C}$ can be expressed as purely advective and parallel to contours of $\overline{C}$, the flux associated with K_{ij} is diffusive, but not orthogonal to contours of $\overline{C}$ unless K_{ij} is diagonal. Nonetheless, as mentioned earlier, the symmetric nature of K_{ij} means that it is diagonalizable, most likely describing large mixing rates along mean isopycnals and a very much smaller diapycnal mixing rate. Further, this diagonalization means that we may examine the problem in one dimension.

2.1 Dispersion in one dimension

As described in Taylor's (1921) famous paper, the effective diffusivity in one dimension is given by

$$K = \overline{X \frac{dX}{dt}} = \int_0^t \overline{u(t')u(t)}\, dt' \tag{7}$$

after writing $u = dX/dt$ and interchanging the integration and the averaging. This may be written as

$$K = \overline{u^2} \int_0^t R(\tau)\, d\tau \tag{8}$$

where $R(\tau) = \overline{u(t)u(t+\tau)}/\,\overline{u^2}$ is the Lagrangian velocity autocorrelation function.

Defining the Lagrangian integral time scale $T_L = \int_0^\infty R(\tau)\, d\tau$, we then have K approximately equal to $\overline{u^2}t$ for $t \ll T_L$, so that $\overline{X^2} \simeq \overline{u^2}t^2$. This is as expected; a particle's expected excursion is just its rms speed times the time since release.

For $t \gg T_L$, K tends to the constant value $\overline{u^2}T_L$ so that $\overline{X^2} \simeq 2\overline{u^2}T_L t$. This is a simple extension of the 1D "drunkard's walk" with steps $X_1, X_2, X_3, \ldots$, each of length S but randomly either forwards or backwards. In that case the speed u is S/τ_0 during each step if this lasts a time τ_0. Then $R(\tau)$ decreases linearly from 1 at $\tau = 0$ to 0 at $\tau = \tau_0$ and $T_L = \frac{1}{2}\tau_0$. Thus we have $K = \frac{1}{2}S^2/\tau_0$ and, after n steps taking a total time $n\tau_0$, $K = \overline{X dX/dt}$ gives $\overline{X^2} = 2Kn\tau_0 = nS^2$, as expected.

While it is tempting to regard this constant diffusivity regime as applying after a time that is not much greater than T_L, a long tail to $R(\tau)$ can mean that (8) does not converge quickly, and the effective diffusivity increases, albeit slowly, for a time that is very much larger than T_L.

For $K = \overline{u^2}T_L$ to be the appropriate diffusivity for use in treating the evolution of scalar fields in the ocean, we require two things:

(i) That the concentration associated with a water parcel not change much over the time scale of order T_L, or longer, for which the Lagrangian autocorrelation is significant. In fact, in the presence of molecular diffusion as well as stirring by eddies, a tracer will tend to diffuse out of filaments (in two or three dimensions) and so reduce the turbulent transport. Saffman (1960) and Bennett (1987) have reviewed this problem, with the conclusion that the corrected total diffusivity is of the form

$$K = \overline{u^2}T_L \left[1 - \mathrm{O}(Re^{-1/2})\left(\frac{\kappa}{\nu}\right)\right] + \kappa \tag{9}$$

where κ and ν are the molecular diffusivity and viscosity respectively, and Re is the Reynolds number based on the typical velocity and length scale of the eddies. The correction is likely to be small.

(ii) That the mean gradient is reasonably uniform over a distance comparable with the length scale $L = (\overline{u^2})^{1/2}T_L$ (or more if $R(\tau)$ has a long tail), the typical excursion of a particle during the time it contributes to the eddy flux. Then $K = (\overline{u^2})^{1/2}L$, which is just the rms velocity times a mixing length L.

It is also important to recognize that it is the Lagrangian integral time scale that is needed in Taylor's (1921) formula, requiring data from drifters rather than from current meters, even though the mean square velocity can be estimated from either. Middleton (1985) investigated the difference between T_L and the Eulerian integral time scale T_E, defined as the integral of the autocorrelation of the velocity measured at a fixed point for zero mean flow, or moving with the mean flow if it is not zero. He argued that T_L/T_E is less than 1 and depends on the size of the eddies compared with the distance $L = (\overline{u^2})^{1/2}T_L$. If the eddies are large compared with L then the Eulerian and Lagrangian velocities are very similar over a correlation time and so $T_L \simeq T_E$. On the other hand, if the eddies are not large compared with L, the Lagrangian velocity changes more quickly as a water parcel is carried into a different eddy and so $T_L < T_E$.

Finally, it is worth pointing out that the energy spectrum of the Lagrangian velocity is proportional to the Fourier transform of $R(\tau)$, with the long-time diffusivity, from (8) with $t \to \infty$, related to the spectrum at zero frequency. Thus, with data of finite length, one can judge whether (8) is converging by seeing whether the spectrum seems to be levelling off at low frequency and tending to a finite limit at zero frequency (e.g. Rupolo, Hua, Provenzale, & Artale, 1996)

2.2 Oceanic values

Freeland, Rhines, & Rossby (1975) used SOFAR floats at mid-depths in the North Atlantic in a pioneering application of Taylor's (1921) approach. They found $T_L \simeq 11$ days, with a mean square velocity of about 7×10^{-4} m^2 s^{-2} in both zonal and meridional directions. Hence $K \simeq 700$ m^2 s^{-1}. Similar values of both velocity variance and Lagrangian integral time scales have been found in many other locations in the open ocean (Ferrari and Polzin (2004) summarize some Atlantic data). On the other hand, a similar value of K, about $1,100$ m^2 s^{-1}, came from much a higher velocity variance, about 2×10^{-2} m^2 s^{-2}, but a much shorter time scale, $T_L \simeq 15$ h, in a strongly eddying region of the Labrador shelf (Garrett, Middleton, Hazen, & Majaess, 1985). The Lagrangian autocorrelation functions in two directions are shown in Figure 1. The near equality of these, combined with the smallness of the lagged cross-correlation (not shown) of the two velocity components, implies nearly isotropic dispersion with diffusivities as implied by R_u and R_v. The antisymmetric part of the full correlation tensor, with the cross correlations as the off diagonal elements, gives a skew flux, but this has zero divergence if the eddy field is statistically homogeneous.

The study by Garrett et al. (1985) relied on the trajectories of icebergs tracked by radar from oil company drillships; constant reseeding of the area with fresh "drifters" permitted the calculation of the Eulerian, as well as the Lagrangian, statistics of the eddy field. Assuming isotropy, the cross-correlation functions were evaluated for velocity components separated by a distance r as well as lagged by time τ. These functions were found to be different for velocity components longitudinal and transverse to the line joining the two points, but were reasonably separable in time and space as $f(r)F(\tau)$ and $g(r)G(\tau)$ for the longitudinal and transverse components, respectively. The spatial functions $f(r)$ and $g(r)$ were found to be different, as expected for isotropic turbulence (see Garrett et al. (1985) for details). The time dependent functions were somewhat different, but an average of F and G from Figure 1 gives an Eulerian integral timescale of approximately 40 h, considerably greater than T_L, and qualitatively in line with Middleton's (1985) argument, mentioned above, that $T_L < T_E$. Finally, note the presence of inertial oscillations in all the functions shown in Figure 1.

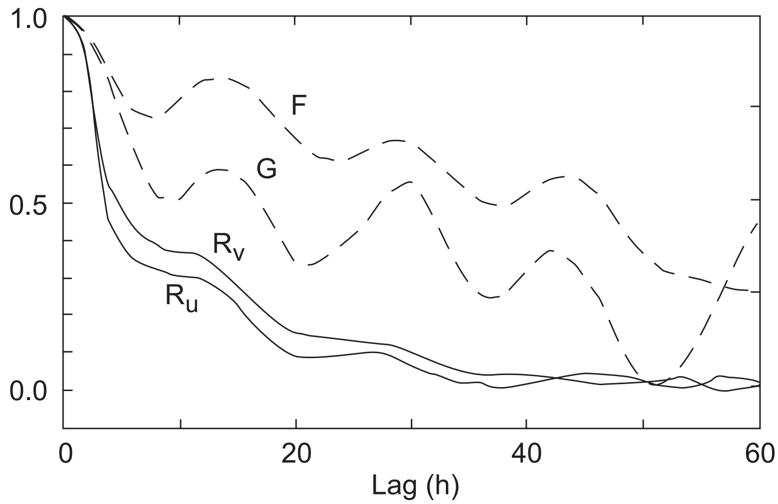

Fig. 1. The Lagrangian autocorrelation functions R_u and R_v for the eastward and northward velocity components u, v respectively, of iceberg trajectories off Labrador. Also shown are the time-dependent parts F and G of Eulerian longitudinal and transverse correlation functions expressed as functions of spatial separation and time lag. (Redrawn from Garrett et al. (1985).)

Rupolo et al. (1996) examined float data from 700 m depth in the North Atlantic in terms of Lagrangian spectra, finding spectral shapes corresponding to a long tail to $R(\tau)$. They found an intermediate dispersion regime for $T_L < t < 10T_L$ in which displacement variance grew more slowly than the t^2 behavior at short time, but more quickly than the t behavior at long time. They attributed this to particle trapping in coherent structures in the flow; similar behavior, with a 5/4 power law, has been found in numerical simulations of 2D and quasigeostrophic turbulence (Bracco, von Hardenberg, Provenzale, Weiss, & McWilliams, 2004). Also, Rupolo et al. (1996) found that convergence of (8) at long time, as deduced from the shape of the Lagrangian frequency spectrum, was not found for all the trajectories, presumably as a consequence of gradients in the mean flow. The complexity of possible and observed oceanic motions clearly calls for caution in applying the basic theory of turbulent dispersion.

It should be also be stressed that the averaging required in Taylor's (1921) model is properly that over an ensemble of realizations. Replacing the ensemble average with a space average means that equal areas need to be given equal weight (e.g. Davis, 1991). An average over the drifters can give undue weight to regions of previous convergence. In fact, Middleton and Garrett (1986) found that, while low frequency currents measured by the icebergs tended to rotate clockwise if the averaging was over data weighted according to their density in space, the sense of rotation reversed if the averaging was just over the icebergs. This was because drifters tend to congregate in regions of previous convergence and these regions tend to have acquired a cyclonic rotation if the eddies are quasi-geostrophic!

The values of diffusivity discussed above apply to ensemble average "absolute" dispersion by mesoscale eddies, with scales of order 100 km in the open ocean, and perhaps a tenth of this on the continental shelf. Smaller diffusivities are often derived from examination of the spread of a patch in a single realization, but this is usually with respect to the center of mass of the patch, in a process termed "relative" dispersion. (For example, Martin, Richards, Law, and Liddicoat (2001) found a lateral mixing rate of $22 \pm 10\ \mathrm{m^2\,s^{-1}}$ for a patch size of 10 km or so.) The distinction is important and will be discussed later.

2.3 Relation to the spectrum of a passive scalar

We have a mental image of a homogeneous field of turbulent eddies stirring a passive scalar with a large-scale mean gradient which may be treated as locally uniform. We expect that the rms fluctuation in the scalar concentration will be at least as large as an eddy scale times the mean gradient. It may be larger, however, if wisps of high or low concentration can survive for a while before they are thin enough to diffuse away through molecular processes.

For homogeneous, isotropic, 3D turbulence, this may be discussed further by considering standard results for the spectrum of a passive scalar. This spectrum $E_C(k)$, as a function of wavenumber k, is proportional to $\chi\epsilon^{-1/3}k^{-5/3}$ from some low wavenumber k_0 inversely proportional to the scale of the largest eddies. Here, ϵ is the turbulent kinetic energy dissipation rate per unit mass and χ is the dissipation rate of the scalar fluctuations, given by $2\kappa\overline{\nabla C' \cdot \nabla C'}$, with κ the molecular diffusivity of the scalar. If the Prandtl number $Pr = \nu/\kappa$, with ν the kinematic viscosity, is much less than 1, the $-5/3$ spectral form is cut off rapidly near $k = (\epsilon/\kappa^3)^{1/4}$. If, on the other hand, $Pr \gg 1$, the $-5/3$ spectrum applies up to the Kolmogorov wavenumber $k_\nu = (\epsilon/\nu^3)^{1/4}$ beyond which viscosity cuts off velocity fluctuations. For higher wavenumbers, the scalar spectrum has a high wavenumber tail proportional to $\chi\tau k^{-1}$ out to the diffusive cutoff wavenumber $k_c = (\epsilon/\nu\kappa^2)^{1/4}$ (Batchelor, 1959). Here,

$\tau = (\nu/\epsilon)^{1/2}$ is a timescale inversely proportional to the rms strain rate.

Given these spectra, the mean square concentration fluctuation is given by

$$\overline{C'^2} = \mathrm{O}(\chi\epsilon^{-1/3}k_0^{-2/3}) + \mathrm{O}(\chi\tau\ln[k_c/k_\nu]). \tag{10}$$

Garrett (1989) showed how this may be written as

$$\overline{C'^2} = [\mathrm{O}(k_0^{-2}) + l^2](d\overline{C}/dx)^2 \tag{11}$$

where $l^2 = \mathrm{O}(K\tau\ln[k_c/k_\nu])$, with K the eddy diffusivity proportional to the rms current speed u_0 times the eddy scale of order k_0^{-1}. The derivation of (11) simply uses the representation of χ as $2K(d\overline{C}/dx)^2$ and $u_0^2 \propto \epsilon^{2/3}k_0^{-2/3}$ from the integral of the Kolmogorov spectrum for velocity.

The rms scalar fluctuation may thus be interpreted as coming from a mixing length, times the mean scalar gradient, where there are two contributions to this length. The first contribution is the expected one of the eddy scale. The second corresponds to the rms distance that a particle is dispersed, according to Taylor's (1921) theory described earlier, in the time $\tau\ln[k_c/k_\nu]$ that it would take for the rms strain rate of order τ^{-1} to have the smallest dimension of a streak reduced from the viscous cutoff scale k_ν to the diffusive cutoff scale k_c. (We are assuming here that this time is greater than the Lagrangian integral time scale of order $k_0^{-1}u_0^{-1}$, an assumption equivalent to $l > k_0^{-1}$ so that the second term in (11) is important.)

Our physical picture is thus that eddies first stir a scalar, producing scales of variation down to that of the smallest eddy. The resultant blobs and streaks are then strained by the velocity field until they are thin enough to diffuse away. This will happen very quickly if the Prandtl number is not large so that diffusion is already acting at the scale of the turbulent eddies. However, for large Prandtl number, it takes a finite time for the eddy straining to produce the small scales at which diffusion acts, and in that time the blobs and streaks disperse farther, giving rise to further fluctuations in scalar concentration.

Ross, Garrett, and Lueck (2004) have assumed that this scenario also applies in a situation in which two passive scalars with different diffusivities are present. In that case it seems reasonable to assume that the fluctuations of the two scalars will remain coherent at small scales, even though one will diffuse more quickly than the other. This model has novel implications for the scattering of high-frequency sound by small-scale fluctuations of density and sound speed. These fluctuations are induced by fluctuations in both temperature and salinity, so it is important to know how the small-scale structures in temperature and salinity are related to each other.

2.4 A point source

We have, so far, discussed the application of Taylor's (1921) theory to the dispersion of a scalar with a large-scale gradient, and have argued that the "long time" limit of Taylor's formula is relevant. This is not appropriate for the evaluation of the concentration expected from the release of a scalar at a point, either instantaneously or steadily in time. For an instantaneous release, the ensemble-averaged concentration field is assumed to be Gaussian (though this is not proven (Csanady, 1973)), with a spread σ_x given by integrating (7), with $\sigma_x^2 = \overline{X^2}$, and similarly in two or three dimensions as appropriate.

A continuous release at a fixed point at rate Q is just a succession of instantaneous point releases (Csanady, 1973), so that the resulting average concentration field in three dimensions is given by

$$\overline{C} = Q(2\pi)^{-3/2} \int_0^\infty \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2}\right)\right] (\sigma_x\sigma_y\sigma_z)^{-1}\, dt. \tag{12}$$

In the simplest case of isotropy, we can write $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \sigma^2$ and $r^2 = x^2 + y^2 + z^2$ to obtain

$$\overline{C} = Q(2\pi)^{-3/2} \int_0^\infty e^{-r^2/(2\sigma^2)} \sigma^{-3}\, dt. \tag{13}$$

For molecular diffusion with diffusivity κ, $\sigma^2 = 2\kappa t$ and then (13) is easily integrated to give $\overline{C} = Q(4\pi\kappa r)^{-1}$, the standard solution for a continuous point source in 3D. In the turbulent case, $\sigma^2 = u_0^2 t^2$ for $t \ll T_L$, where we have denoted $u_1^2 = u_2^2 = u_3^2$ by u_0^2, and $\sigma^2 = 2u_0^2 T_L t$ for $t \gg T_L$. For large values of r compared with $u_0 T_L$, the integral in (13) is dominated by contributions from values of t greater than T_L and so $\overline{C} \simeq Q(4\pi u_0^2 T_L r)^{-1}$, as for constant diffusivity $u_0^2 T_L$. On the other hand, for $r \ll u_0 T_L$, the integral in (13) is dominated by contributions from values of t much less than T_L and hence

$$\overline{C} \simeq Q(2\pi)^{-3/2} u_0^{-1} r^{-2}. \tag{14}$$

Thus a stronger singularity near a source arises for turbulent diffusion than for constant diffusion, with the average concentration proportional to r^{-2} rather than r^{-1}. This can be thought of as a consequence of a smaller effective diffusivity near the source.

As discussed by Csanady (1973), who also evaluated the concentration field for a point source in a steady mean flow, this anomalous behavior near the source is important in marine and atmospheric pollution problems. Garrett and Shepherd (1987) considered an anisotropic situation, with turbulent diffusion in both horizontal directions, but with a constant diffusivity in the vertical, as appropriate for the much smaller eddies and shorter timescales in the vertical. After scaling each coordinate by the square root of the diffusivity in that direction, they found that the near-source concentration was proportional to $r^{-3/2}$. The singularity is weaker than before, but still stronger than for constant diffusion in all three directions.

The discussion here has concerned only the ensemble-averaged concentration to be expected. We consider later the fluctuations that are to be expected in various situations, but next discuss briefly the consequences of a spatially variable eddy diffusivity.

2.5 Variable eddy diffusion

The discussion so far has assumed statistical homogeneity. If this does not apply, the situation becomes considerably more complicated (Davis, 1987). If, however, the statistics of the eddy field vary over a scale much larger than that of the eddies themselves, it is possible to represent their effect in terms of a spatially variable eddy diffusivity, say $K(x)$ in one dimension. Thus, without any mean flow, the mean scalar concentration $\overline{C}$ satisfies the equation

$$\frac{\partial \overline{C}}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial \overline{C}}{\partial x}\right). \tag{15}$$

We can write this as

$$\frac{\partial \overline{C}}{\partial t} - \frac{dK}{dx}\frac{\partial \overline{C}}{\partial x} = K\frac{\partial^2 \overline{C}}{\partial x^2} \tag{16}$$

in which the right hand side of the equation has K outside the derivative, as we are used to seeing if K is constant. This form of the equation makes it appear that the mean concentration field is being advected at a speed $-dK/dx$ at the same time as it spreads by diffusion. On the other hand, if we define the center of mass of the scalar as $\overline{x} = \int_{-\infty}^{\infty} x\overline{C}\,dx$, assuming that $\int_{-\infty}^{\infty} \overline{C}\,dx = 1$, then

$$\frac{d\overline{x}}{dt} = \int_{-\infty}^{\infty} x\frac{\partial \overline{C}}{\partial t}\,dx \tag{17}$$

may be written, using (14), integrating by parts, and assuming $\overline{C} \to 0$ for $x \to \pm\infty$, as

$$\frac{d\overline{x}}{dt} = \int_{-\infty}^{\infty} \frac{dK}{dx} \overline{C}\, dx. \tag{18}$$

Thus the center of mass moves with a weighted average of $+dK/dx$, i.e. up, rather than down, the gradient of K (Freeland et al., 1975; Davis, 1987). This makes good physical sense, of course; the scalar is stirred towards regions of high diffusivity.

2.6 Diffusion or entrainment?

In many situations in the ocean we represent dispersion and mixing through the use of eddy diffusivities, but in other situations, particularly for turbulent boundary layers at the surface and bottom of the ocean, it is common to represent the effects of strong mixing by the use of an entrainment velocity. The connection between mixing gradients and the entrainment rate has been discussed by Csanady (1990). In particular, considering a 1D situation with vertical density profile $\rho(z)$ and a vertical mixing rate that may be represented by $K(z)$, the density equation (ignoring a mean vertical velocity) is

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial \rho}{\partial z}\right). \tag{19}$$

This may be used to describe the change in depth $z(\rho, t)$ of a particular isopycnal, using $0 = (\partial\rho/\partial z)_t(\partial z/\partial t)_\rho + (\partial\rho/\partial t)_z$ for the change of density on an isopycnal. Hence

$$\left.\frac{\partial z}{\partial t}\right|_{\rho} = -\left(\frac{\partial \rho}{\partial z}\right)^{-1} \frac{\partial}{\partial z}\left(K \frac{\partial \rho}{\partial z}\right) \tag{20}$$

$$= -\frac{dK}{dz} - K\left(\frac{\partial \rho}{\partial z}\right)^{-1} \frac{\partial^2 \rho}{\partial z^2} \tag{21}$$

describes the evolution of $z(\rho, t)$. The first form of this, in (20), is discussed more completely by Pelegrí and Csanady (1994) and by McDougall (1987) who also allows for nonlinearity in the equation of state. Csanady's (1990) form, (21) here, shows that the vertical motion of the isopycnal at a point of inflection of the density profile is given simply by dK/dz, the vertical gradient of the diffusivity. In practice, the separation between layers might be taken at a different location. Moreover, with sharp gradients, the eddies causing the mixing might have a larger scale than that over which the density gradient changes significantly. It would not then be appropriate to represent mass transfer as a local process.

3. Relative dispersion

So far we have focused on the ensemble-averaged concentration field for a dispersing scalar, though in Section 2.3 we also discussed the fluctuations to be expected at a point. This topic warrants further discussion, with the issues being simple and clear if we consider an instantaneous point release. After a finite time there are really three domains of occupation for the scalar which need to be considered, as illustrated in Figure 2.

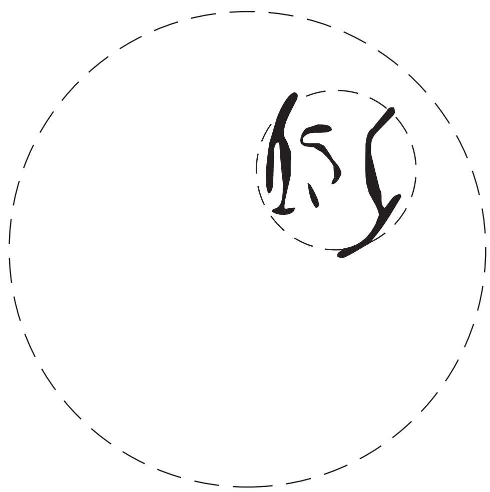

Fig. 2. The three domains of occupation for a scalar, at a fixed time after an instantaneous release. The large circle applies to the ensemble average over many releases. The smaller circle shows the expected area or volume occupied by floats seeded in the scalar after a short initial diffusion time. The shaded streaks show the actual domain of high concentration in a single release.

Here, the larger circle, for 2D dispersion, or sphere for 3D dispersion, represents the area or volume occupied by the scalar on the basis of what is known as "absolute" dispersion. It might be taken as the domain containing some fraction of the assumed Gaussian distribution of the scalar, with the concentration given by Taylor's (1921) formula. (If the stirring is anisotropic, the domain will be an ellipse or ellipsoid, becoming a circle or sphere in coordinates scaled with the square roots of the eddy diffusivities in each direction. We ignore this simple extension.)

In any given release, however, an initial blob at the origin will be carried off in some particular direction at the same time as it is spread by molecular diffusion, or some small-scale stirring and mixing process, and torn apart by the larger eddies. The domain occupied by the scalar in a particular release, shown as the smaller circle in Figure 2, thus refers to the dispersion of the scalar elements with respect to the center of mass and can be estimated by considering the "relative" dispersion of particles seeded into the patch at the end of an initial diffusive phase, just as the patch begins to be torn apart by the eddies.

Even within this domain defined by relative dispersion, however, the scalar may be concentrated in thin streaks, shown by the dark patches separated by clear water in Figure 2. Alternatively, and perhaps after sufficient time, these streaks may merge, so that the scalar distribution is more uniform within the domain defined by relative dispersion, though clearly still somewhat patchy. Also, after sufficient time, the relative dispersion domain may grow to exceed the size of the largest eddies. After this the relative dispersion rate is the same as that for absolute, one-particle, dispersion.

The situation is familiar to anyone who has stirred cream into coffee, and was discussed in a pioneering and illuminating way be Eckart (1948). Further studies have included simple estimates by Garrett (1983), who claimed that streaks would merge very soon after the scalar release in 3D turbulence, but that streakiness would tend to persist for a significant time for 2D stirring. This expectation was reinforced by the numerical simulations of Haidvogel and Keffer (1984) and has been borne out in tracer release experiments in the ocean (Ledwell, Watson, & Law, 1993). Sundermeyer and Price (1998) have conducted more sophisticated analysis of the basic fluid dynamics.

To discuss this further, we start with a summary of the basic arguments for relative dispersion of a cloud of particles. If they have a concentration $C(r)$ with respect to their center of mass, we define their spread by s where $s^2 = \int Cr^2\, dV / \int C\, dV$, where the integration is over area for 2D or volume for 3D and includes all of the particles. For 3D turbulence characterized by dissipation rate ϵ, Richardson (1926) and Batchelor (1952) argued that the relative diffusivity, defined as $\frac{1}{2} ds^2/dt$, can only depend on ϵ and on s itself. Hence, on dimensional grounds, the diffusivity must be proportional to $\epsilon^{1/3} s^{4/3}$, the famous "four-thirds" law. Okubo (1971) found that much oceanographic data on relative dispersion does seem to obey such a law, but this must be for other reasons, as the assumption of homogeneous 3D turbulence is hardly valid over the large scales analyzed.

The physics of this result is worth discussing. As reviewed by Bennett (1987), two particles with separation s are, on average, separated by the strain $\gamma_s =$

$s^{-1} ds/dt$ of eddies larger than s where

$$\gamma_s \propto \left(\int_0^{s^{-1}} k^2 \epsilon^{2/3} k^{-5/3} \, dk \right)^{1/2} \propto \epsilon^{1/3} s^{-2/3} \tag{22}$$

if we assume a standard Kolmogorov energy spectrum, with an energy spectrum $\propto k^{-5/3}$, and multiply it by k^2 to obtain the strain spectrum. Now, since $ds/dt \propto \gamma_s s$, we obtain $\frac{1}{2} ds^2/dt \propto \epsilon^{1/3} s^{4/3}$ as before. This is dimensionally inevitable, but the derivation is interesting as it shows that small eddies are less important as the patch grows, rather than the large eddies becoming more important. The patch growth has $s^2 \propto t^3$ and should really be regarded as slower than the exponential growth that would apply if the strain were independent of patch size, rather than faster than the linear growth appropriate for a constant diffusivity.

This physical interpretation of relative dispersion in 3D turbulence is valuable in providing a framework for the 2D case. We might expect (22) to still apply, but note that if the energy spectrum is steeper than k^{-3}, as seems to be the case, the strain is dominated by the lowest wavenumber in the spectrum, provided it is smaller than s^{-1}, i.e. by the largest eddies. The strain is thus independent of size, giving exponential separation of a particle pair, and thus exponential growth of the radius of the relative dispersion domain. It is clear, though, that within this circle, the scalar in a particular realization of the flow will be confined to a thin streak, with the streak length growing like $\exp(\gamma t)$ for a strain γ. The width of the streak is readily determined by an advective-diffusive balance in the cross-streak direction to be proportional to $(\kappa/\gamma)^{1/2}$, where κ is the mixing rate associated with small-scale processes.

This simple physical scenario was exploited by Ledwell et al. (1993) in their interpretation of the remarkable lateral dispersion of a tracer released in the North Atlantic. After six months or so, the tracer was found to be largely in a streak that was not straight but had an overall length consistent with stretching by a strain rate of $3 \times 10^{-7}\,\mathrm{s}^{-1}$. Combined with a streak width of about 3 km, this implied a lateral diffusivity of about 3 $\mathrm{m}^2\,\mathrm{s}^{-1}$. This is much greater than the molecular value, implying the presence of some small-scale mixing process. A first guess might be that this was shear dispersion, a process in which oscillatory vertical shear of horizontal currents, associated with internal waves, combines with the vertical mixing caused by the waves to give lateral mixing. Young, Rhines, and Garrett (1982) estimated that this lateral mixing would have $\kappa \simeq (N/f)^2 K_v$, where N, f are the buoyancy and Coriolis frequencies, respectively, and K_v is the vertical mixing rate. However, this only seems to give a value of order $10^{-2}\mathrm{m}^2\,\mathrm{s}^{-1}$ for κ, much less than required. A plausible explanation for the larger observed value is that it was associated with stirring by small-scale "vortical modes" (Polzin & Ferrari, 2004). These vortical modes, which may arise from the collapse and geostrophic adjustment of vertically mixed patches (Sundermeyer, Ledwell, Oakey, & Greenan, 2005), can be thought of as an extra part of the mesoscale eddy spectrum, giving it a bump at high wavenumbers which rises above the background spectrum that is steeper than k^{-3}. This small-scale stirring most likely feeds directly into molecular diffusion without causing significant extra streakiness, in the same way that in 3D turbulence it seems that streaks rapidly become convoluted and merge within the relative dispersion domain (Garrett, 1983).

This discussion has been for an instantaneous release. The fluctuation problem for a continuous release is much more difficult, but Csanady (1983) presented a useful discussion by regarding the discharge as made up of "old puffs" which contribute a background concentration field, and "new puffs" which emerge as a plume. The location of this plume depends on the current direction at the time and so gives rise to fluctuations.

Finally, it is worth remarking that the issue of streakiness may be important if the impact of the dispersing tracer is nonlinear. For example, if the response of, say, fish to a pollutant is a nonlinear function of the concentration, there is clearly a difference between the effects of a pollutant which is uniformly dispersed and one which is confined to streaks, even if the average concentration is the same; a population of 90% healthy fish and 10% dead fish in the latter case is not the same as 100% slightly sick fish in the former!

4. Stirring to mixing

Much of the above discussion has referred to the interplay between stirring and mixing. Stirring tends to sharpen gradients to the point where molecular diffusion takes over and provides the ultimate mixing.

In a statistically steady state, one can calculate the flux of a scalar as either (i) the sum of the eddy flux and a much smaller molecular flux across the contours of the mean scalar field, or (ii) the average molecular flux alone across the highly convoluted instantaneous contour of the scalar. This equivalence, and the latter approach, have been discussed by Nakamura (1996) and Winters and D'Asaro (1996).

Further to this approach, the "eddy" field itself may be made up of large-scale mesoscale eddies and much smaller-scale turbulence. The former, which we refer to just as eddies, may really just be stirring the scalar, with the link to molecular dissipation being provided by the turbulence. A "triple decomposition" of the scalar field into mean, eddies, and turbulence has been discussed by Joyce (1977), Davis (1994), and Garrett (2001) and is summarized here in

Figure 3. The eddies and the turbulence both create scalar variance by stirring the mean field. The eddy-scale fluctuations in the scalar are then acted upon by the turbulence to produce scalar fluctuations which, along with those produced by the turbulence acting on the mean field, are then cascaded to small enough scales that they can be dissipated by molecular processes. If the upper pathway in Figure 3 is the dominant one, then the eddies are the rate-limiting process and the turbulence just does what it has to, much like molecular diffusivity itself in ordinary high Reynolds number 3D turbulence. On the other hand, if the lower pathway is dominant, it is the turbulence that is the rate-limiting process.

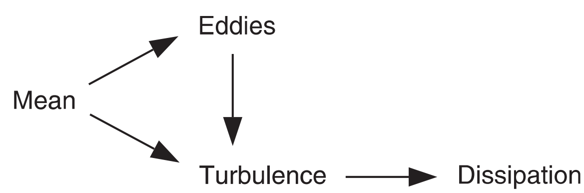

Fig. 3. A schematic showing the possible ways in which scalar concentration variance is produced from a mean field and carried to dissipation scales.

A very nice example of this distinction has been found by Ferrari and Polzin (2004). They discuss the origin of observed dissipation of temperature fluctuations at a site in the North Atlantic. At one range of depths the dissipation is associated just with turbulence acting on the mean vertical gradient, as in the lower path in Figure 3, whereas at greater depths the upper path is followed, with the production of temperature variance being dominated by isopycnal stirring of lateral temperature gradients.

Other aspects of the connection between stirring and mixing have been discussed in the workshop proceedings of Müller and Henderson (2001) and summarized by Müller and Garrett (2002). Mahadevan (2001), Richards, Brentnall, McLeod, and Martin (2001), Young (2001), and Seuront (2001) discuss the interesting patterns which arise if the scalar being stirred is non-conservative, as for growing phytoplankton. A comprehensive review has been presented by Martin (2003). While numerical simulations and rigorous statistical treatments are clearly essential, it is possible that further insight can be gained by simple models focussing on the evolution of the scalar in the thin streaks produced by eddy stirring. The lateral compression of the streaks is assumed to be balanced by lateral diffusion, as in the earlier discussion for conservative scalars.

5. Discussion

We tend to think of dispersion as requiring a turbulent flow field. In recent years, however, it has been recognized that even simple "deterministic" flow fields can lead to the tearing apart of a patch of scalar (e.g. Aref, 1984). In a nice oceanographic example, Ridderinkhof and Zimmerman (1992) built on earlier analyses to show how dispersion can arise in a flow field consisting of oscillatory tidal currents, with large spatial scales, superimposed on small-scale residual eddies. These eddies can themselves be generated by rectification of the tidal currents over small-scale features in the bottom topography. Not all of the domain is dispersive; there tend to be regions from which particles do not escape without the addition of extra physical processes. Parameterizing these effects in a model that does not resolve the small eddies presents a major challenge.

Particle separation can occur in a variety of spatially variable and time-dependent flow fields which one would not characterize as turbulent. The methods of dynamical systems theory has been helpful in analyzing these situations (e.g. Kirwan, Toner, & Lipphardt, 2001). Of course, if the spatial and temporal scales of the flow are resolved in a model, then the dispersion will be accomplished by the model without the need for the parameterization that is required for processes not resolved in space and time.

There is still much to be learned about the actual physical processes responsible for dispersion in the ocean. It is important to remain aware of conceptual problems and weak assumptions. One prominent issue concerns the "spectral gap" that is assumed to exist between the mean field and the eddies. In the triple decomposition discussed above, two spectral gaps are assumed to exist, one between the mean and the mesoscale eddies and one between the mesoscale eddies and the breaking internal waves or whatever else is responsible for the turbulent fluxes. The absence of a spectral gap reminds us of conventional 3D turbulence where one cannot reliably describe the effect of unresolved small eddies on individual large eddies. Statistically, however, the small eddies may just be absorbing variance produced at larger scales. In this case, one might have confidence in Large Eddy Simulation (e.g. Metais, 1998), which rests on the assumption that it is only necessary to resolve the start of the inertial subrange, with the parameterization of smaller-scale eddies being unimportant to the overall dynamics. On the other hand, if there is a reverse cascade from small scales to large scales, failure to resolve the small scales will inevitably lead to errors at large scales. Finally, it bears repeating that, in any investigation, it is valuable to think not just in terms of statistics and parameterization, but also in simple physical ways about the Lagrangian evolution of individual patches of scalar.

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