

dynamics of atmospheres and oceans

Dynamics of Atmospheres and Oceans 23 (1996) 19-34

# Processes in the surface mixed layer of the ocean

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Received 29 June 1994; revised 6 March 1995; accepted 10 March 1995

#### Abstract

Models for the evolution of the surface mixed layer need to be improved to include dominant processes such as Langmuir circulation. It is shown that the wave forcing in Langmuir circulation models is much stronger than that due to a surface buoyancy loss, and studies of the erosion by the cells of a pre-existing stratification are described. Mixed layer models will also need to allow for horizontal inhomogeneity. It is shown, for example, that the horizontal buoyancy gradient that may be left behind after a storm produces restratification that can be significant. The nonlinearity of the equation of state is another real-world factor; it gives rise to an annual average surface buoyancy that is misleading as it is compensated by interior cabbeling. Current work linking the mixed layer to water mass formation is also introduced.

#### **1. Introduction**

The upper ocean typically exhibits a surface mixed layer, with a thickness of a few metres to several hundred metres, in which the density stratification is weak because of turbulent mixing that is driven by surface fluxes of momentum and buoyancy and by shear across the base of the layer. The physics of this surface mixed layer presents a variety of fascinating fluid dynamical problems, and is of great importance for a wide variety of problems arising in studies of climate, biological productivity and marine pollution.

In questions of climate, an elementary and common remark is that the heat capacity of the top 2.5 m of the ocean equals that of the whole column of air above it, so that, with turbulent mixing typically extending to a depth many times this in both summer and winter, the ocean essentially acts as a 'thermal flywheel' for the climate system, smoothing out both temporal (seasonal) temperature changes and, through advection, meridional gradients. Of course, heat exchange between ocean

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and atmosphere involves more than just the turbulent near surface layer of the ocean, with the subduction of surface layer water into the interior being a topic of great current interest (e.g. Marshall et al., 1993).

Sea surface and mixed layer processes also play a vital tole in the exchange between the atmosphere and the ocean of gases such as oxygen and carbon dioxide. The exchange occurs by diffusion across the sea surface and across the surfaces of bubbles created by breaking waves (e.g. Wanninkhof, 1992). Carbon dioxide is then taken up in the surface layers of the ocean by growing phytoplankton, with the growth rate depending on the availability of nutrients that are entrained from below the base of the surface mixed layer. The average light intensity to which the phytoplankton are exposed is also critical and this becomes insufficient for further growth if the depth over which the phytoplankton are mixed becomes too great (e.g. Denman and Gargett, 1995).

Many models for the evolution of the surface mixed layer have been developed. These are usually one-dimensional (although possibly allowing for local convergence by prescription of a vertical velocity at the base of the layer). The simplest models parameterize the changes in average properties of the layer, and of its depth, in terms of the surface buoyancy flux and wind stress and/or the differences between the average buoyancy and average velocity of the mixed layer and the buoyancy and velocity of the water just below it. These models are based on plausible physical arguments and can, to some extent, be tuned to match data, but they are clearly incomplete. Various more complicated models, based on turbulence closure schemes at different levels, have thus been developed. These are often expected to be effective in view of the success of similar schemes in other boundary layer situations, though weaknesses in closure assumptions in the strongly stratified region at the base of the layer are admitted and sometimes compensated for by assumptions as ad hoc as those of the bulk models.

These models will be reviewed briefly in Section 2, but 1 believe that their most important shortcoming is that they ignore some of the real physical processes that are unique to the oceanic surface mixed layer. Breaking surface waves are clearly one phenomenon of great importance for the generation of turbulence near the surface, but the most important process appears to be Langmuir circulation. This refers to the helical circulation cells in the upper ocean, with axes generally oriented downwind, which appear to be generated by a mechanism involving the Stokes drift of surface waves; the cells are thus very different from coherent longitudinal vortices that may occur in other boundary layers. The physics of Langmuir circulation, and beginning studies of the role it plays in eroding the stratification at the base of the mixed layer and so deepening it, will be discussed in Section 3.

Increasing attention is also being paid to the effect on the surface mixed layer of horizontal gradients in its properties. Simple advection of these properties by a prescribed current is frequently allowed for in locally one-dimensional models, if only to balance the heat budget, but recent work points also to the possible importance of dynamical effects of a horizontal buoyancy gradient. This will be reviewed in Section 4.

Interesting physical effects can sometimes arise from the nonlinearity of the equation of state. One example, an annual average buoyancy flux in spite of zero annual average heat flux, will be described in Section 5. Recent consideration of water mass formation associated with a horizontal gradient of surface buoyancy flux will be mentioned in Section 6. The paper, which thus emphasizes my own current interests and biases, concludes in Section 7 with a discussion.

#### 2. Existing models

# 2.1. Forcing functions

The surface of the ocean is forced by wind stress  $\tau$  with a corresponding friction velocity  $u_* = (|\tau|/\rho_w)^{1/2}$  where  $\rho_w$  is the density of water, and by a surface buoyancy flux  $B_0$  per unit area, given by

$$B_0 = -C_p^{-1}g\alpha\rho_0^{-1}Q + g\beta s\rho_0^{-1}(E - P)$$
<sup>(1)</sup>

where  $C_p$  is the specific heat of water,  $\alpha = -\rho^{-1}(\partial \rho / \partial T)_{p,s}$  is the coefficient of expansion of water at fixed pressure p and salinity s, Q is the net heat flux into the water,  $\beta = \rho^{-1}(\partial \rho / \partial s)_{p,T}$ , E is the evaporation rate and P is the rainfall rate (Gill, 1982). Here positive  $B_0$  corresponds to loss of buoyancy from the sea. The net heat flux into the sea is made up of insolation, which is distributed over an attenuation depth, minus net longwave back-radiation and latent and sensible heat loss rates which act at the surface.

The wind stress, at least in part, goes first into surface waves. In a wave field that is growing downwind, part of the momentum flux goes into the downwind increase of the wave momentum (associated with the Stokes drift of the irrotational waves) but for typical growth rates this fraction is only 3% or so (e.g. Richman and Garrett, 1977). Thus it is reasonable to assume that all of  $\tau$  is available to drive the mean rotational flow of the mixed layer.

The energy budget is more complicated. Some energy can be advected away in a growing wave field, but the more important question concerns the energy input. Is it  $\tau$  times the surface drift of the water, or  $\tau$  times the phase velocity of the longest and fastest travelling waves in the surface wave spectrum? Gemmrich et al. (1994) have examined the energy budget of the mixed layer and concluded that the momentum goes mainly into rather short waves, with a wavelength of about 0.25 m and a phase speed of about 0.6 m s<sup>-1</sup> so that the energy input is  $|\tau|$  times this. This has important consequences for turbulence levels near the surface. In fact, Gemmrich et al. (1994) based their conclusion largely on recent measurements reported by Anis and Moum (1992), Agrawal et al. (1992) and Osborn et al. (1992) which show an energy dissipation rate much more than the value  $u_*^3/\kappa z$ , for von Kármán's constant  $\kappa$  and distance z from the surface, which would apply for a conventional wall layer, but with a depth integral that is still much less than  $|\tau|$  times the wind speed (corresponding to the phase speed of the fastest wave for a fully developed sea). The result of Gemmrich et al. (1994) on the shortness of the

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generated waves is distinct from Thorpe's (1993) finding, from the observed frequency of wave breaking and the energy dissipation in a breaking event as a function of wave speed, that the breaking waves must be much shorter than the dominant waves.

The observed turbulent dissipation rate  $\epsilon$  greater than  $u_*^3/\kappa z$  means that in a convective situation the buoyancy flux does not dominate in the turbulent kinetic energy equation until it is rather deeper than the modulus  $u^3/\kappa B_0$  of the Monin-Obukhov length defined as minus this. Nonetheless, there are clearly many times, particularly at night and at the beginning of winter, when the deepening of the mixed layer is largely determined by surface buoyancy loss (e.g. Lombardo and Gregg, 1989). The main question is then whether there is a significant buoyancy jump at the base of the mixed layer and, if so, whether it is a consequence of penetrative convection or is caused by something other than the surface buoyancy flux.

# 2.2. Bulk models

Simple models of the surface mixed layer envisage a sharp jump  $\Delta b$  in buoyancy b (given by  $b = -g(\rho - \rho_0)/\rho_0$ , with  $\rho_0$  a reference density) at the base of the layer of depth h, and prescribe the entrainment velocity across this. As reviewed by Phillips (1977), one class of models parameterizes this entrainment rate as

$$w_{\star} = u_{\star} F(h\Delta b/u_{\star}^2) \tag{2}$$

 $(\mathbf{n})$ 

where  $Rb = h\Delta b/u_*^2$  is a bulk Richardson number based on  $u_*$  and  $\Delta b$ , and is the only dimensionless parameter if  $u_*$  is the only relevant velocity scale. The function F in (2) is presumably a decreasing function of Rb; if the rate of energy input to the mixed layer is assumed proportional to  $u_*^3$  and a fixed fraction of this input is assumed to go into the increasing potential energy of the layer as dense fluid is entrained upwards, then  $F \propto Rb^{-1}$  (Turner, 1973). Models based on this formulation include those of Denman and Miyake (1973), Niiler and Kraus (1977) and Garwood (1977).

However, another possible velocity scale is the magnitude  $\Delta u$  of the difference between the average velocity of the surface mixed layer and the velocity of the underlying fluid. Various models for F(Rb), with  $\Delta u$  replacing u in (2), have been discussed by Phillips (1977) and generally have a rapid decrease with increasing Rb. A particularly simple form, suggested by Pollard et al. (1973), has  $F = \infty$  for Rb < 1 and F = 0 for Rb > 1, so that the mixed layer depth adjusts to have Rb always equal to unity during a deepening phase, though it is assumed not to 'unmix' if  $\Delta u$  decreases.

Of course  $\Delta u$  is a function of the history of  $u_{-}$  and the mixed layer depth, so that the two types of bulk model may predict somewhat similar behaviour, but there can also be major differences. For example, in the case of a mixed layer developing in response to an abrupt onset of wind above a stratified ocean, the first model will show a continued deepening of the layer whereas the second will give a mixed layer depth of no more than  $2^{3/4}u_*/(Nf)^{1/2}$  because of the rotation

of the mixed layer velocity by the Coriolis force (Pollard et al., 1973), with N the buoyancy frequency of the underlying fluid, and f the Coriolis frequency.

Price (1979) showed that an entrainment formula based on  $\Delta u$  was much more effective than one based on  $u_*$  in collapsing data from different laboratory studies. Thus the second type of bulk model, based on  $\Delta u$ , has become more common than the first in recent years, particularly with the popular model of Price et al. (1986) that stops the entrainment at Rb = 0.65 instead of unity and also smooths the interface at the base of the layer to give it a gradient Richardson number of 0.25.

A reasonable reaction to any remaining uncertainty is to argue that in reality the entrainment depends on the behaviour of turbulence in a stratified fluid and cannot be simply represented in terms of either  $u_*$  or  $\Delta u$ , though possibly some simple parameterization in terms of both of these can be extracted from more thorough studies. In fact, Deardorff (1983) reviewed studies that cast doubt on the relevance of some laboratory studies because of the influence of side walls, and presented entrainment formulae involving bulk Richardson numbers based on  $u_*$ ,  $\Delta u$  and a free convection velocity scale given by  $(B_0h)^{1/3}$ .

These bulk models essentially assume that the buoyancy is reasonably homogenized in the surface layer. They also appear to assume that the horizontal velocity is vertically uniform in the layer, but in fact do not do so, merely requiring that only its vertical average is relevant to the entrainment rate. Predicting the shear, and any vertical gradient of the buoyancy or other scalars, requires more detail than provided by the 'slab' approach.

## 2.3. Higher-order models

The success of turbulence closure models in other boundary layer situations has led to the development of a variety of models for application to the surface mixed layer of the ocean. The simplest of these assume simple profiles of eddy viscosity and diffusivity, as in the under ice model of McPhee (1991) and the recent model of Large et al. (1994). The latter has been used for successful simulation of various oceanic data sets, though success on seasonal time scales, rather than daily or for individual storms, depends on allowing for advection of water with different properties. The model also maintains an eddy viscosity of  $10^{-4}$  m<sup>2</sup> s<sup>-1</sup> and an eddy diffusivity of  $10^{-5}$  m<sup>2</sup> s<sup>-1</sup> below the base of the mixed layer. Moreover, for this as for all other models, comparison with data is made difficult by uncertainty in the surface heat and water fluxes.

Many other mixed layer models are based on various closure schemes proposed by Mellor and Yamada (1974, 1982). However, as reviewed by Gaspar et al. (1990), the commonly used versions suffer from uncertainty in the prescription, or computation, of the 'master length'. Gaspar et al. (1990) adopted a scheme with eddy viscosity and diffusivity determined by the turbulent kinetic energy (TKE) and a length scale. The TKE satisfies a prognostic equation involving production, dissipation and diffusion, but the length scale is given by simple consideration of the vertical distance a particle could travel up or down in converting its TKE to potential energy (Bougeault and André, 1986). Blanke and Delecluse (1993) found that this scheme produces better simulations for the tropical Atlantic Ocean than that of Philander and Pacanowski (1986), in which mixing coefficients are given prescribed values that are reduced by some function of the local Richardson number, but some discrepancies still occur. More recently, Kantha and Clayson (1994) have extended the modified Mellor-Yamada second moment closure scheme of Galperin et al. (1988), but at the base of the mixed layer they used the same ad hoc Richardson number dependent mixing formulae as employed by Large et al. (1994) in the transition zone.

Further development and use of these models, whether bulk or higher order, is justified by the need for adequate simulation and prediction in a variety of situations. One might, however, question the value of some of the more elaborate schemes described above, and of the third moment closure scheme of André and Lacarrère (1985), in view of the fact that they do not explicitly incorporate the unique processes occurring in the oceanic surface layer. In this regard, Eric Kunze (personal communication, 1994) has compared higher-order closure models of the surface layer to geocentric pre-Copernican astronomy, in which epicycles were added to epicycles to account for the apparent motions of the planets. This was effective, but a real understanding and reliable prediction required Kepler's and Newton's laws. In the present problem, we must start to allow in the models for important upper ocean processes such as Langmuir circulation. We turn to this next.

#### 3. Langmuir circulation

Lines of surface convergence, roughly parallel to the wind, are frequently marked by 'windrows' of foam and other flotsam. The Nobel prize winning chemist Irving Langmuir was the first to describe the associated subsurface circulation pattern, of vortices of alternating sign, that now bears his name, and suggested that it is the key mechanism in producing the mixed layer. Pollard (1977) presented a schematic diagram of Langmuir circulation (Fig. 1) based on observations up till then. More detailed observations by Weller and Price (1988) have shown larger downwelling speeds, up to  $0.2 \text{ m s}^{-1}$ , beneath the surface convergences, and associated downwind jet speeds of comparable magnitude.

Pollard's (1977) review of theories of Langmuir circulation up until 1976 provoked Craik and Leibovich (1976) to revise an unsatisfactory earlier theory and present an elegant dynamical model, reviewed by Leibovich (1983), which has been widely accepted since then. The basic physics is an instability mechanism in which an infinitesimal downwind jet has its vertical vorticity, with opposite sign on the two sides of the jet, tilted by the Stokes drift of the surface waves to produce longitudinal rolls. These produce the surface convergence at the jet, and this is in turn reinforced because of the acceleration, by the wind stress, of the water moving towards the surface convergence.



Fig. 1. A schematic representation of Langmuir circulation. (Redrawn from Pollard (1977).)

Li and Garrett (1993) have examined the Craik–Leibovich (CL) model further and found that, for plausible values of the eddy viscosity in the model, the predicted downwelling speed is comparable with that measured, but the jet is weaker than observed. This suggests the need for more refined parameterization of the turbulence or, possibly, the importance of other physical processes such as the interaction of the surface waves with the circulation pattern.

Nonetheless, the vortex force associated with the Stokes drift does seem to be powerful, making Langmuir circulation different from longitudinal rolls in other boundary layer situations. For example, Li and Garrett (1995a, henceforth LG) recently investigated whether Langmuir circulation could be just convective rolls in a shear flow, given that, even under conditions of net heat input to the sea, insolation is penetrative and the sea surface is likely to be losing heat.

In their extension of the two-dimensional CL model, LG numerically solved the equations

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = v \nabla^2 u$$
(3)

$$\frac{\partial\Omega}{\partial t} + v \frac{\partial\Omega}{\partial y} + w \frac{\partial\Omega}{\partial z} = v \nabla^2 \Omega - \frac{\mathrm{d}u_s}{\mathrm{d}z} \frac{\partial u}{\partial y} + \alpha g \frac{\partial\theta}{\partial y}$$
(4)

$$\frac{\partial\theta}{\partial t} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = \kappa \nabla^2 \theta \tag{5}$$

where u is the downwind current,  $\Omega = (\partial w/\partial y) - (\partial u/\partial z)$  the downwind component of the vorticity and  $\theta$  the temperature. Turbulence is represented simply by eddy viscosity  $\nu$  and eddy diffusivity  $\kappa$ . The CL vortex force, associated with the

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Stokes drift  $u_s(z)$ , is given by the second term on the right-hand side of (4). If there is surface cooling, streamwise vorticity can also be generated by the buoyancy torque given by the last term in (4), with  $\alpha$  the expansion coefficient and g gravity. Internal heating is neglected here but can be added to (5).

LG showed that this problem is characterized by three non-dimensional numbers. The first is the Langmuir number introduced by Leibovich (1977) and given by

$$La = \left(\frac{\nu\beta}{u_*}\right)^{3/2} \left(\frac{u_*}{S_0}\right)^{1/2}$$
(6)

where the Stokes drift is  $u_s(z) = 2S_0 \exp(2\beta z)$ . The second is the eddy Prandtl number  $\Pr = \nu/\kappa$  and the third is the Hoenikker number

$$Ho = \frac{-\alpha Q}{C_p \rho_w S_0 \beta u_*^2}$$
(7)

This parameter can be expressed as the ratio of a length scale for the wave field to the Monin–Obukhov length, but it is more useful to recognize that HoPr represents the ratio, in the downwind vorticity equation, of convective forcing by surface heat loss -Q, with  $C_p$  the specific heat of water, to wave forcing. From numerical solutions and scale analysis, LG showed that, for Pr = 1, Ho must be as big as about 3 for convective forcing to compare with wave forcing at appropriately small values of La. Realistic values for the surface heat flux, Stokes drift and wind stress, however, give Ho significantly less than 0.1. It thus appears that the surface heat flux is unimportant to the dynamics of the cells, a conclusion that also holds for



Fig. 2. Quasi-steady Langmuir cells at La = 0.02, Ho = 0.05 and Pr = 1. (a) Downwind current. (b) Temperature. (c) Streamwise vorticity. (d) Streamfunction. The depth is scaled with the surface wave scale  $\beta^{-1}$ . (From Li and Garrett (1995a). Reprinted with permission from J. Phys. Oceanogr.)



Fig. 3. Vertical profiles (with depth scaled by  $\beta^{-1}$ ) at La = 0.02, Ho = 0.05 and Pr = 1 for (a) non-dimensionalized temperature and (b) heat fluxes. (From Li and Garrett (1995a). Reprinted with permission from J. Phys. Oceanogr.)

other values of Pr, for net heating (Ho < 0) and for depth-distributed heating. The temperature then behaves as a passive scalar in these numerical experiments in which a heat flux is applied to previously homogeneous water.

Fig. 2 shows the non-dimensional fields obtained in a steady-state solution for La = 0.02, Ho = 0.05 and Pr = 1. In fact, with surface heat flux only, the *u* and  $\theta$  fields are proportional for Pr = 1. For plausible values of the heat loss and other parameters, the predicted surface temperature difference  $\delta\theta$  from divergence to convergence is  $O(10^{-2})$  K, comparable with values reported by Thorpe and Hall (1982) and Weller and Price (1988). Scale analysis, supported by numerical solutions, shows that  $\delta\theta \propto \text{HoPr}^{1/2}\text{La}^{-1/6}$ . This implies weak dependence ( $\nu^{-1/4}$ ) on the eddy viscosity  $\nu$ , but more dependence on Pr. Doubts also remain, of course, about the validity of the simple parameterizations of turbulence.

Fig. 3(a) shows, for the same parameters as Fig. 2, the non-dimensionalized vertical profiles of the cross-stream average temperature  $\langle \theta \rangle$  made up of the conductive solution T and a change  $\langle \theta' \rangle$  due to the Langmuir cells. The cells homogenize the temperature below a thin conductive surface layer and Fig. 3(b) shows that the total vertical heat flux  $Q_t$  is largely associated with the advective heat flux  $Q_a$  due to the cells, rather than the conductive heat flux  $Q_c$ . We conclude that Langmuir circulation is a powerful stirring agent of passive scalars in the surface mixed layer, and that temperature is indeed passive if realistic cooling or heating is applied to homogeneous water.

A more important question, however, concerns the interaction between the circulation and pre-existing stratification. Li and Garrett (1995b) have used the same model to investigate this. Their results show the expected result that, if wind stress is applied to a density-stratified ocean, Langmuir circulation quickly develops near the surface and, while the cells are small with a shallow penetration depth, tends to 'engulf' and homogenize the temperature as if it were a passive scalar. As the cells merge and grow in scale, however, they do not penetrate as deeply as in homogeneous water and give instead a well-mixed layer above a jump in buoyancy to the stratified ocean beneath. Results suggest that, for this and other initial stratification profiles, the mixed layer stops its rapid growth into a stratified fluid when the buoyancy jump across its base is  $cu_*^2/h$ , where c is a constant of

proportionality that depends on the sea state and the uncertain eddy viscosity in this model, but takes a value of about 50 for fully developed seas. As discussed by Li and Garrett (1995b), this result may be understood in terms of a constant Froude number based on the downwelling speed in the cells.

Although this vigorous engulfment process is one consequence of the Langmuir circulation, the model also shows that the downwind velocity field is also fairly well homogenized in the mixed layer by the cells. This leads to small values of the Richardson number at the base of the layer, and would presumably lead to shear instability (which is not possible in the present model owing to its independence of the downwind coordinate x). The fluid mixed in this shear instability could then be picked up by the cells and stirred throughout the upper layer, though this is probably a slower process than the initial rapid engulfment.

It is hoped that this work, and extensions to include three-dimensional effects and more realistic treatment of subgrid-scale turbulence (as in the large eddy simulation of Skyllingstad and Denbo (1995)), will lead to a plausible blend of bulk parameterizations of entrainment in terms of  $u_{\star}$  (as for the initial engulfment) and  $\Delta u$  across the base of the layer (as for the later shear instability and stirring). Our expectations are that allowance for Langmuir circulation will mainly reduce the occurrence of some of the very shallow mixed layers which can occur with other schemes, though it may also delay springtime restratification.

# 4. The role of horizontal buoyancy gradients

Most of the emphasis in mixed layer modelling has been on the parameterization of mixing in one-dimensional models. This is clearly still the key problem, but there are increasing attempts to assess the importance of the frequently observed horizontal variations in mixed layer parameters. De Szoeke (1980) and De Ruijter (1983) have shown that the depth-varying advection of horizontal temperature variations by wind-driven shear currents can be important, and this effect has been included in the model of Lascaratos et al. (1993). Recent observations (Brainerd and Gregg, 1993a,b) have suggested that restratifying currents may be driven by the horizontal buoyancy gradients themselves rather than just by the wind, and this is providing impetus for a number of theoretical studies.

The simplest possible problem, reviewed by Tandon and Garrett (1994), is of the adjustment of a constant horizontal buoyancy gradient  $b_x = -(g/\rho)(\partial \rho / \partial x)$  in a vertically well-mixed layer of depth h initially at rest. The geostrophically adjusted state has a vertical buoyancy gradient with  $N^2 = -(g/\rho)(\partial \rho / \partial / z)$  given by the simple formula

$$N^2 = M^4 / f^2 \tag{0}$$

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where  $M^2 = |b_x|$ . This restratification can be significant in some frontal situations in the deep ocean, at low latitudes and on the continental shelf. The adjustment occurs on a time scale comparable with the inertial period and is accompanied by



Fig. 4. Horizontal buoyancy gradient in the surface mixed layer owing to variable mixing. (From Tandon and Garrett (1995). Reprinted with permission from J. Phys. Oceanogr.)

inertial oscillations which cause the stratification to oscillate between zero and twice the value given by (8).

An interesting question concerns the shear stability of the solution to this problem; the development of stratification is accompanied by sheared currents in the direction of the buoyancy gradient and normal to it due to the action of the Coriolis force. In fact, the steady geostrophically adjusted state would have Ri = 1 whereas the full time-dependent solution has  $Ri = \frac{1}{2}$  for all z and t, implying stability. Ou (1984) has examined the adjustment when  $b_x$  is not constant; Tandon and Garrett (1994) found that  $Ri < \frac{1}{4}$  is possible if the frontal region is sharp enough.

These models are, however, limited by the assumption of a horizontally constant mixed layer depth that does not vary with time. A more plausible starting point (Fig. 4) has horizontal variation in both buoyancy and mixed layer depth, as might be caused by spatially varying mixing of an ocean that was initially uniformly stratified. Tandon and Garrett (1995) showed that in this and related problems, with a plausible 'wide front' approximation, there is a slight flattening of the interface slope but restratification given by (8) is still achieved locally. This holds promise for extensions of a general subinertial mixed layer model developed by Young (1994) but hitherto limited by the assumption of a large density jump across the base of the mixed layer, an assumption which rules out changes in the interface.

## 5. Is the nonlinearity of the equation of state important?

The density of seawater depends in a nonlinear way on the pressure and on the temperature and salinity of the water. For the surface mixed layer the temperature dependence is the most important, and can give rise to the interesting and potentially misleading existence of a net annual buoyancy flux into the sea even in a situation where the net annual heat flux is zero!

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This is easily seen by inspection of (1). The relevant nonlinearity in the equation of state corresponds to a temperature dependence of the expansion coefficient  $\alpha$ and the annual average buoyancy flux has a term  $C_p^{-1}p_0^{-1}g\overline{\alpha'}Q'$ , where  $\alpha', Q'$  are the departures of the expansion coefficient and the total heat flux away from their annual means. We shall ignore the depth dependence of the insolation, or assume that it is distributed over a depth in which the temperature does not change from its surface value. Hence we may write  $\alpha' = (\partial \alpha / \partial T)T'$  with T' the departure of the surface temperature from its annual average T, and so we are interested in evaluating the annual average  $\overline{T'Q'}$ . Now if  $\Pi$  represents the total heat content of the ocean (down to the greatest depth affected by the annual cycle) we have

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{Q}{\rho_0 C_p} \tag{9}$$

If the mixed layer were well mixed of constant depth, (9) would give  $(dT'/dt) \propto Q'$ and hence  $\overline{T'Q'} = 0$  as T', Q' would be in quadrature. In general, however, (9) gives (for  $\overline{Q} = 0$ )

$$\overline{T'Q'} = \overline{TQ} = \rho_0 C_p \overline{T(dH/dt)}$$
(10)

$$=\rho_0 C_p t_0^{-1} \oint T \mathrm{d}H \tag{11}$$

where  $t_0 = 1$  year and  $\phi T dH$  is the area enclosed by the curve of T vs. H, proceeding clockwise (Fig. 5).

Data and models (Gill and Turner, 1976; Zahariev and Garrett, 1995) show a tendency for this area to be positive (with increasing time corresponding to clockwise rather than anticlockwise circulation around the curve in Fig. 5). Equivalently, there is a tendency for the heating of warm water and cooling of cold water, with the former expanding more than the latter contracts. The effect can be significant; for the Mediterranean Sea the associated buoyancy flux is comparable with other terms and equivalent to a heat input of 6 W m<sup>-2</sup> (Garrett et al., 1993).

This possibility of a net surface buoyancy input for no net heat input does not mean that the ocean becomes steadily less dense. As discussed by Zahariev and Garrett (1995), there is a compensating loss of buoyancy owing to cabbeling, or densification on mixing, whenever the mixed layer entrains colder water from below. The effect does, however, emphasize the need for care in evaluating the thermodynamics as well as the dynamics of the surface mixed layer.

### 6. Water mass formation

The exchange of water between the surface mixed layer and the underlying ocean is a topic of great current interest, with particular attention being paid to the quantity and properties of water that is subducted into the main thermocline (e.g. Marshall et al., 1993). One approach (Walin, 1982; Speer and Tziperman, 1992) starts from consideration of the buoyancy flux, across the sea surface,



Fig. 5. Temperature vs. heat content of the surface mixed layer over an annual cycle which proceeds clockwise, with J marking 15 January.

between the surface outcroppings of neighbouring isopycnals. This buoyancy flux must be balanced by a diapycnal buoyancy flux, but with contributions from both diapycnal advection (related to water mass formation) and diapycnal mixing. Garrett et al. (1995) point out that it is hard to separate the relative contributions of advection and mixing, though the diapycnal buoyancy flux may all be advective in the special case of a well-mixed surface layer and an adiabatic interior. This result does, however, ignore the horizontal diapycnal eddy fluxes in the surface mixed layer owing to mesoscale eddies. It also leaves somewhat unresolved questions arising from the seasonal cycle in the properties of the surface layer.

## 7. Discussion

One-dimensional models remain the backbone of studies of the surface mixed layer, and improved observations and greater computer power now make it possible to conduct serious investigations of dominant physical processes such as Langmuir circulation which have hitherto been bypassed in bulk models or ignored in higher-order models. Most investigations of fully nonlinear Langmuir circulation are still two-dimensional, but are beginning to be supported and extended by

three-dimensional 'large eddy simulations' (Skyllingstad and Denbo, 1995). It is important, however, that the detailed results of these investigations be interpreted in ways that lead to simple parameterizations, suitable for operational use.

Internal waves are a process that I have not discussed in this brief review, although the oscillatory vertical displacements and vertical shears of horizontal currents that they produce may well affect the average behaviour of the surface mixed layer and need to be parameterized (Mellor, 1989).

Horizontal inhomogeneities of mixed layer properties also give rise to interesting physical effects that may need to be incorporated better into mixed layer models, especially if they are being used as part of a study of larger-scale issues of ocean circulation and climate. There are still many exciting observational, theoretical, computational and practical questions awaiting solution.

### Acknowledgements

This limited review was prepared for the Fourth International Symposium on Stratified Flow at Grenoble in July 1994. It has emphasized, perhaps excessively, my current interests and joint work with colleagues whose papers are cited and whom I thank for their contributions. The support of Canada's Natural Sciences and Engineering Research Council and Department of Fisheries and Oceans, and of the US Office of Naval Research, is also gratefully acknowledged.

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