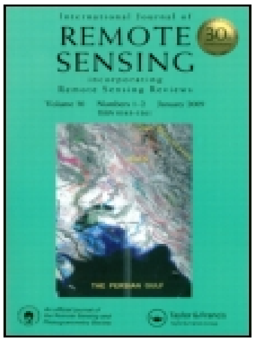

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The probability density of the total curvature of a uniform random Gaussian sea surface in the specular points

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Abstract. Assuming that sea surface is a random uniform Gaussian function, an analytical expression is developed for the probability distribution density of the total curvature of the surface in specular reflection points. The density can be expressed in terms of an integral, and its asymptotics acquire a simple form for large values of the total curvature. A numerical experiment to verify the probability distribution density is also given.

1. Introduction

The study of sea wave variables have been a subject of considerable interest in oceanography. For this purpose, optical methods have been used.

Let S be the projection of the part of ocean's surface being illuminated by parallel light rays travelling in the direction of the unit vector $\vec{s}_0$, and the surface is being viewed in the direction of the unit vector $\vec{s}$. It is well known (Keller and Keller 1950, Bass and Fuks 1972, Shifrin and Gardachov 1985) that the mean value of intensity $\langle I \rangle$ of light reflected by a rough sea surface is given by

$$\langle I \rangle = k \langle N \rangle \left\langle \frac{1}{|\Omega|} \right\rangle S \tag{1}$$

where, $k = k(\vec{s}_0, \vec{s})$ is the coefficient not depending on surface geometry; $\langle N \rangle$ is the average number of specular points per unit horizontal area; and Ω is the total curvature at the specular point and $\langle 1/|\Omega| \rangle$ the mean value of $1/|\Omega|$.

To define the fluctuation of the intensity (I) and its probability density ($w(I)$), a knowledge of the total curvature probability density at the specular points ($W(\Omega)$) is required. The statistical distribution of the total curvature (Ω) for a Gaussian uniform surface $z = \zeta(x, y)$ (of which elevation ζ and derivatives $\partial^{n+m}\zeta/\partial x^n \partial y^m$ are distributed normally and $\langle \zeta(x, y) \rangle = 0$), has been derived by Longuet-Higgens (1958, 1969). The function $W(\Omega)$ has been expressed in terms of contour integrals and its asymptotics by special functions.

The main object of the present study was to obtain a comparatively simple expression for $W(\Omega)$, convenient for use in practical computations. It is shown here that the distribution $W(\Omega)$ has an integral representation in terms of error function. Verification of the given expression by numerical experiments was also performed. A computation using this expression gave exactly the same result as that obtained by the Longuet-Higgins (1969) formula.

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Let the surface of the ocean (which is assumed to be Gaussian uniform) be defined by

$$z = \zeta(x, y) \tag{2}$$

where x and y are horizontal coordinates and z is the elevation. Then, the total curvature of the surface (Ω) is given by

$$\Omega = \frac{\zeta_{xx}\zeta_{xy} - \zeta_{xy}^2}{(1 + \zeta_x^2 + \zeta_y^2)^2} \tag{3}$$

where ζ_x, ζ_y, ζ_{xx}, ζ_{xy}, ζ_{yy} denotes derivatives $\partial\zeta(x,y)/\partial x$, $\partial\zeta(x, y)/\partial y$, $\partial^2\zeta(x, y)/\partial x^2$, $\partial^2\zeta(x, y)/\partial y\partial x$, $\partial^2(x, y)/\partial y^2$, respectively.

The subject is to obtain a statistical distribution of Ω at the specular points, where the following system of equations are satisfied:

$$\left.\begin{aligned} \zeta_x(x, y) &= \gamma_x = \text{const.} \\ \zeta_y(x, y) &= \gamma_y = \text{const.} \end{aligned}\right\} \tag{4}$$

Let, $w(\zeta_x, \zeta_y, \zeta_{xx}, \zeta_{xy}, \zeta_{yy})$ denote a probability density of $\zeta_x, \zeta_y, \zeta_{xx}, \zeta_{xy}, \zeta_{yy}$. Since the first derivatives, ζ_x and ζ_y, are statistically independent of the second derivatives, ζ_{xx}, ζ_{xy}, and ζ_{yy}, we obtain

$$w(\zeta_x, \zeta_y, \zeta_{xx}, \zeta_{xy}, \zeta_{yy}) = w(\zeta_x, \zeta_y) \times w(\zeta_{xx}, \zeta_{xy}, \zeta_{yy}) \tag{5}$$

The probability density functions $w(\zeta_x, \zeta_y)$ and $w(\zeta_{xx}, \zeta_{xy}, \zeta_{yy})$ are normal distributions and are given by

$$w(\zeta_x, \zeta_y) = \frac{1}{2\pi\sqrt{\Delta_2}} \exp\left(-\frac{m_{02}\zeta_x^2 - 2m_{11}\zeta_x\zeta_y + m_{20}\zeta_y^2}{2\Delta_2}\right) \tag{6}$$

and

$$\begin{aligned} w(\zeta_{xx}, \zeta_{xy}, \zeta_{yy}) = &\frac{1}{(2\pi)^{3/2}\sqrt{\Delta_3}} \\ &\times \exp\Big[-\frac{1}{2}(M_{11}\zeta_{xx}^2 + M_{22}\zeta_{xy}^2 + M_{33}\zeta_{yy}^2 \\ &+ 2M_{12}\zeta_{xx}\zeta_{yy} + 2M_{13}\zeta_{xx}\zeta_{yy} + 2M_{23}\zeta_{xy}\zeta_{yy})\Big] \end{aligned} \tag{7}$$

where

$$\Delta_2 = \det E_2, E_2 = \begin{pmatrix} m_{20} & m_{11} \\ m_{11} & m_{02} \end{pmatrix} \equiv \begin{pmatrix} \langle \zeta_x^2 \rangle & \langle \zeta_x\zeta_y \rangle \\ \langle \zeta_x\zeta_y \rangle & \langle \zeta_y^2 \rangle \end{pmatrix} \tag{8}$$

$$\Delta_3 = \det E_3, E_3 = \begin{pmatrix} m_{40} & m_{31} & m_{22} \\ m_{31} & m_{22} & m_{13} \\ m_{22} & m_{13} & m_{04} \end{pmatrix} \equiv \begin{pmatrix} \langle \zeta_{xx}^2 \rangle & \langle \zeta_{xx}\zeta_{yy} \rangle & \langle \zeta_{xx}\zeta_{yy} \rangle \\ \langle \zeta_{xx}\zeta_{xy} \rangle & \langle \zeta_{xx}\zeta_{yy} \rangle & \langle \zeta_{xy}\zeta_{yy} \rangle \\ \langle \zeta_{xx}\zeta_{yy} \rangle & \langle \zeta_{xy}\zeta_{yy} \rangle & \langle \zeta_{yy}^2 \rangle \end{pmatrix} \tag{9}$$

and $\{M_{ij}\}$ is the matrix, inverse to E_3.

The conditional probability density $w(\zeta_{xx}, \zeta_{xy}, \zeta_{yy}|\gamma_x, \gamma_y)$ of the random variables $\zeta_{xx}, \zeta_{xy}, \zeta_{yy}$ at the specular points with a gradient of (ζ_x, ζ_y), which satisfies equation (4), is defined by

$$w(\zeta_{xx}, \zeta_{xy}, \zeta_{yy}|\gamma_x, \gamma_y) = \frac{1}{\langle N \rangle} w(\gamma_x, \gamma_y) \times w(\zeta_{xx}, \zeta_{xy}, \zeta_{yy})|\zeta_{xx}\zeta_{yy} - \zeta_{xy}^2| \tag{10}$$

It can be shown that the matrix E_3 is a positive-definite; hence, by a real linear transformation of variables, the exponent in equation (7) can be reduced to a unit form and, at the same time, the expression

$$\omega = \zeta_{xx}\zeta_{yy} - \zeta_{xy}^2 \tag{11}$$

can be reduced to a diagonal form (Longuet-Higgins 1958). Thus, for transformation from variables ζ_{xx}, ζ_{xy}, ζ_{yy} to new variables η_1, η_2, η_3 is obtained

$$\left.\begin{array}{c} M_{11}\zeta_{xx}^2 + M_{22}\zeta_{xy}^2 + M_{33}\zeta_{yy}^2 + 2M_{12}\zeta_{xx}\zeta_{yy} + 2M_{13}\zeta_{xx}\zeta_{yy} + 2M_{23}\zeta_{xy}\zeta_{yy} \\ = \eta_1^2 + \eta_2^2 + \eta_3^2 \\ \omega = \zeta_{xx}\zeta_{yy} - \zeta_{xy}^2 = l_1\eta_1^2 + l_2\eta_2^2 + l_3\eta_3^2 \end{array}\right\} \tag{12}$$

where l_1, l_2, l_3 are the roots of the cubic equation

$$4l^3 - 4Hl - \Delta_3 = 0 \tag{13}$$

and H is determined from $H = ⅓(m_{40}m_{04} - 4m_{31}m_{13} + 3m_{22}^2)$. Since the roots l_1, l_2, l_3 are all real, we must obtain

$$0 \le \frac{\Delta_3^2}{H^3} \le 1 \tag{14}$$

and

$$\left.\begin{array}{c} l_1 + l_2 + l_3 = 0 \\ l_2 l_3 + l_3 l_1 + l_1 l_2 = -\tfrac{3}{4}H \le 0 \\ l_1 l_2 l_3 = \tfrac{1}{4}\Delta_3 \ge 0 \end{array}\right\} \tag{15}$$

$$l_3 \le l_2 \le 0 \le l_1 \tag{16}$$

It has been shown (Longuet-Higgins 1969) that average density of specular points with gradient $\zeta_x(x, y) = \gamma_x$, $\zeta_y(x, y) = \gamma_y$ can be defined by

$$\langle N \rangle = \frac{4}{\pi} l_1 \Phi\left(-\frac{l_2}{l_1}\right) w(\gamma_x, \gamma_y) \tag{17}$$

where $\Phi(x)$ is a very slowly varying monotonic function with maximum and minimum values of $\Phi(0) = 1$ and $\Phi(½) = \pi/2\sqrt{3} \approx 0.907$, respectively.

2. Expression for probability density of total curvature

Following this introduction, we now turn to obtain an expression for the probability density $W(\Omega)$ of the total curvature Ω at the specular points, determined by equation (4). From equations (10), (12) and (17), for a distribution of η_1, η_2, η_3 at the specular points, we have

$$w(\eta_1, \eta_2, \eta_3) = \frac{1}{8\sqrt{2\pi}} |l_1\eta_1^2 + l_2\eta_2^2 + l_3\eta_3^2| \exp[-(\eta_1^2 + \eta_2^2 + \eta_3^2)/2] \tag{18}$$

First, we shall find the statistical distribution $w(\omega)$, of the random variable

$$\omega = l_1\eta_1^2 + l_2\eta_2^2 + l_3\eta_3^2 \tag{19}$$

at the specular points. Now, from equation (19), we have

$$\left.\begin{aligned} \eta_1 = \eta_1(\eta_2, v_3, \omega) &= \frac{1}{\sqrt{l_1}} \sqrt{\omega - l_2\eta_2^2 - l_3\eta_3^2} \\ \frac{\partial \eta_1}{\partial \omega} &= \frac{1}{2\sqrt{l_1}} \frac{1}{\sqrt{\omega - l_2\eta_2^2 - l_3\eta_3^2}} \end{aligned}\right\} \tag{20}$$

Then, the distribution of ω is given by

$$w(\omega) = \begin{cases} 2\iint_{-\infty}^{\infty} w(\eta_1(\eta_2, \eta_3, \omega), \eta_2, \eta_3) \dfrac{\partial \eta_1}{\partial \omega} \mathrm{d}\eta_1 \, \mathrm{d}\eta_2 & \text{if} \quad \omega > 0 \\ 2\iint_{(G)} w(\eta_1(\eta_2, \eta_3, \omega), \eta_2, \eta_3) \dfrac{\partial \eta_1}{\partial \omega} \mathrm{d}\eta_1 \, \mathrm{d}\eta_2 & \text{if} \quad \omega < 0 \end{cases} \tag{21}$$

where, (G) is a region on the plane (η_2, η_3), points of which satisfy the inequality $\omega \geq l_2\eta_2^2 + l_3\eta_3^2$.

If $\omega > 0$ from (21) we obtain

$$w(\omega) = \frac{1}{8\sqrt{2\pi} l_1^{3/2} \Phi(-l_1/l_2)} \omega \exp\left(-\frac{\omega}{2l_1}\right) J_1 \tag{22}$$

where,

$$J_1 = \iint_{-\infty}^{\infty} \frac{\exp[-½(\alpha_2\eta_2^2 + \alpha_3\eta_3^2)]}{\sqrt{\omega - l_2\eta_2^2 - l_3\eta_3^2}} \mathrm{d}\eta_2 \mathrm{d}\eta_3 \tag{23}$$

and $\alpha_2 = 1 - l_2/l_1 > 0$, $\alpha_3 = 1 - l_3/l_1 > 0$. To evaluate the integral J_1, we use the substitutions

$$\left.\begin{aligned} \eta_2 &= r\cos\alpha/\sqrt{-l_2} \\ \eta_3 &= r\cos\alpha/\sqrt{-l_3} \end{aligned}\right\} \tag{24}$$

and find that

$$\begin{aligned} J_1 &= \frac{1}{\sqrt{l_2 l_3}} \int_0^{2\pi} \mathrm{d}\alpha \int_0^{\infty} \frac{r\exp[-r^2\mu(\alpha)]}{\sqrt{\omega^2 + r^2}} \mathrm{d}r \\ &= \frac{\sqrt{\pi}}{2\sqrt{l_2 l_3}} \int_0^{2\pi} \frac{\exp[\omega\mu(\alpha)]}{\sqrt{\mu(\alpha)}} [1 - F(\sqrt{\omega\mu(\alpha)})] \, \mathrm{d}\alpha \end{aligned} \tag{25}$$

where $\mu(\alpha) = 1/2l_1(1 - l_1/l_2\cos^2\alpha - l_1/l_3\sin^2\alpha)$ and $F(x) = 2/\sqrt{\pi}\int_0^x \exp(-t^2)\mathrm{d}t$ is the error function.

Hence, if $\omega > 0$, for the probability density $w(\omega)$, we have an integral representation in the form

$$w(\omega) = \frac{1}{16\sqrt{2}\Phi(-l_1/l_2)\sqrt{l_1 l_2 l_3}}\,\omega \exp\left(-\frac{\omega}{2l_1}\right)$$
$$\int_0^{2\pi} \frac{\exp[\omega\mu(\alpha)]}{\sqrt{\mu(\alpha)}}[1 - F(\sqrt{\omega\mu(\alpha)})]\,\mathrm{d}\alpha \tag{26}$$

If $\omega < 0$. from equation (21), we have

$$w(\omega) = \frac{1}{8\sqrt{2\pi}\,l_1^{3/2}\,\Phi(-l_1/l_2)}(-\omega)\exp\left(-\frac{\omega}{2l_1}\right)J_2 \tag{27}$$

where,

$$J_2 = \iint_{(G)} \frac{\exp[-½(\alpha_2\eta_2^2 + \alpha_3\eta_3^2)]}{\sqrt{\omega - l_2\eta_2^2 - l_3\eta_3^2}}\,\mathrm{d}\eta_2\,\mathrm{d}\eta_3$$
$$= \frac{\sqrt{\pi}}{2\sqrt{l_2 l_3}}\int_0^{2\pi} \frac{\exp[\omega\mu(\alpha)]}{\sqrt{\mu(\alpha)}}\,\mathrm{d}\alpha \tag{28}$$

Hence, if $\omega < 0$, for the probability density $w(\omega)$, we have an integral representation in the form

$$w(\omega) = \frac{1}{16\sqrt{2}\Phi(-l_1/l_2)\sqrt{l_1 l_2 l_3}}(-\omega)\exp\left(-\frac{\omega}{2l_1}\right)$$
$$\int_0^{2\pi} \frac{\exp[\omega\mu(\alpha)]}{\sqrt{\mu(\alpha)}}\,\mathrm{d}\alpha \tag{29}$$

From equations (26) and (29), it can be shown that

$$\int_{-\infty}^{0} w(\omega)\,\mathrm{d}\omega = \int_0^{\infty} w(\omega)\,\mathrm{d}\omega = \frac{1}{2} \tag{30}$$

which indicates that the number of elliptical ($\omega > 0$) type of specular points are equal to the number of saddle ($\omega < 0$) type of specular points.

Thus, equations (26) and (29) give statistical distribution of the random variables ω at the specular points.

3. The asymptotics of probability density

The asymptotic expression for distribution $w(\omega)$ can also be obtained. From equation (26) for large values of ω, we obtain:

$$w(\omega) \approx \frac{1}{8\Phi(-l_2/l_1)}\sqrt{\frac{2\pi}{l_1(l_1 - l_2)(l_1 - l_3)}}$$
$$\sqrt{\omega}\exp\left(-\frac{\omega}{2l_1}\right) \quad \omega \to +\infty \tag{31}$$

Evaluating the asymptotic of the integral in equation (29), as $\omega \to -\infty$, we find

$$w(\omega) \approx \frac{1}{8\Phi(-l_2/l_1)} \sqrt{\frac{2\pi(-l_3)}{(l_1-l_3)(l_2-l_3)}} \sqrt{-\omega} \exp\left(-\frac{\omega}{2l_3}\right) \quad \omega \to -\infty \tag{32}$$

In the following two special cases, $w(\omega)$ can be expressed in terms of known functions.

Case 1^0: Let $l_2 = 0$. Then $l_1 = -l_3$. Which is the case when the surface $\zeta(x, y)$ really consists of two distinct system of long-crested waves intersecting each other at a small angle. It can thus be shown that distribution $w(\omega)$ is defined by

$$w(\omega) = \frac{1}{8l_1^2} |\omega| K_0\left(\frac{|\omega|}{2l_1}\right) \tag{33}$$

where, $K_0(x) = \int_1^{+\infty} e^{-xt}/\sqrt{t^2-1}\, dt$ $(x > 0)$ is the modified Bessel function with imaginary argument. The distribution is symmetrical about the origin.

Case 2^0: Let $l_2 = l_3$. Then $l_1 = -2l_2$. This might occur in a variety of circumstances—for instance, when the surface is isotropic or when the angular spread of energy is small and has a certain 'peakedness'. Since

$$\mu(\alpha) = \frac{1}{2l_1}\left(1 - \frac{l_1}{l_2}\cos^2\alpha - \frac{l_1}{l_3}\sin^2\alpha\right) = \frac{1}{2l_1}\left(1 - \frac{l_1}{l_2}\right) = \frac{3}{2l_1} = -\frac{3}{4l_2}$$

from equations (26) and (29), we find

$$w(\omega) = \frac{1}{8l_2^2}\omega \exp\left(-\frac{\omega}{2l_1}\right)\left[1 - F\left(\sqrt{-\frac{3\omega}{4l_2}}\right)\right] \quad (\omega > 0) \tag{34}$$

and

$$w(\omega) = \frac{1}{8l_2^2}(-\omega)\exp\left(-\frac{\omega}{2l_1}\right) \quad (\omega < 0) \tag{35}$$

respectively.

Since the statistical distribution of ω has already been found, the distributions of
$\Omega = \omega/q$, $[q = (1 + \gamma_x^2 + \gamma_y^2)^2]$ and $\rho = 1/|\Omega|$ can also be readily defined:

$$W(\Omega) = \frac{1}{q} w\left(\frac{\omega}{q}\right) \tag{36}$$

$$W(\rho) = \frac{1}{\rho^2}\left[W\left(\frac{1}{\rho}\right) + W\left(-\frac{1}{\rho}\right)\right] \tag{37}$$

Then, the average value of $\rho = 1/|\Omega|$ can be simply evaluated by

$$\langle \rho \rangle = \left\langle \frac{1}{|\Omega|} \right\rangle = \int_{-\infty}^{+\infty} \frac{1}{|\Omega|} W(\Omega)\, d\Omega = \frac{\pi q}{4l_1\Phi(-l_2/l_1)} \tag{38}$$

The second moment of $W(\rho)$ and its higher moments, are all infinite.

4. Numerical experiments and discussion

The validity of the formulas obtained for $W(\Omega)$ were tested by carrying out numerical experiments, using $E(k_x, k_y)$, the energy spectrum of the surface $z=\zeta(x, y)$, for wind waves. All harmonics were taken from the gravitational part of the spectrum (with $\Lambda_{\min}=2.5\,\mathrm{m}$, $\Lambda_{\max}=1020\,\mathrm{m}$) and, at the wind speed $v=10\,\mathrm{m\,s^{-1}}$ were found: $l_1=0.00390$, $l_2=-0.00194$, $l_3=-0.00196$. The average number of specular points per unit area and the average radius, determined by equations (17) and (38), were $\langle N\rangle=0.128$ (m^{-2}) and $\langle\rho\rangle=\langle 1/|\Omega|\rangle=221.7$ (m^2), respectively. The graph of $W(\Omega)$ for $\gamma_x=0$, $\gamma_y=0$ is shown in figure 1.

Further, the uniform Gaussian surface is generated by

$$z=\zeta(x, y)=\sum_{n=1}^{N} c_n \cos(k_{xn}x+k_{yn}y+\phi_n) \tag{39}$$

where the phases ϕ_n were taken as being randomly and uniformly distributed between 0 and 2π, and the amplitudes (c_n) were such random positive variables that in any small region $[k_x, k_x+\mathrm{d}k_x]\times[k_y, k_y+\mathrm{d}k_y]$ of the plane of the wave numbers (k_x, k_y) satisfy

$$\sum_n \frac{1}{2} c_n^2 = E(k_x, k_y)\,\mathrm{d}k_x\mathrm{d}k_y \tag{40}$$

which is the summation of all amplitudes with (k_x, k_y), belonging to the region $[k_x, k_x+\mathrm{d}k_x]\times[k_y, k_y+\mathrm{d}k_y]$.

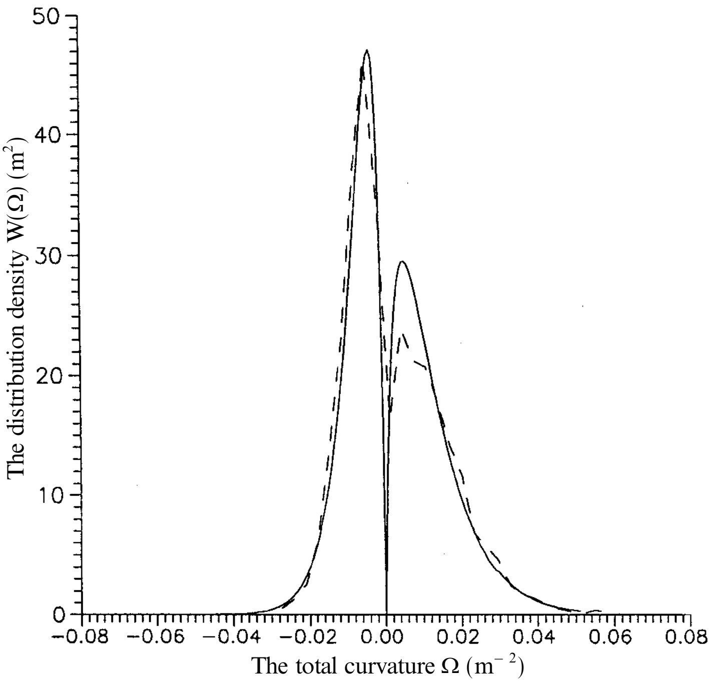

Figure 1. Probability distribution density of the total curvature at the specular points: (solid line) were obtained by using equations (26) and (29); (dashed line) obtained by using numerical experiments.

The number of harmonics (N) in equation (39) were taken as equal to 80, for the distribution of $\zeta(x, y)$ at this value of N becomes practically a normal one.

The system of equation (4), which takes the form

$$\left.\begin{aligned}\zeta_x(x,y) &= -\sum_{n=1}^{N} c_n k_{xn} \sin(k_{xn}x + k_{yn}y + \phi_n) = \gamma_x \\ \zeta_x(x,y) &= -\sum_{n=1}^{N} c_n k_{yn} \sin(k_{xn}x + k_{yn}y + \phi_n) = \gamma_y\end{aligned}\right\} \tag{41}$$

were solved numerically and $\{(x_i, y_i),\ i = 1, 2, 3 \dots M\}$ sets of specular points and their total curvatures $\{\Omega_i = \zeta_{xx}(x_i, y_i)\zeta_{xy}(x_i, y_i) - \zeta_{xy}(x_i, y_i)^2/(1 + \zeta_x^2 + \zeta_y^2)^2,\ \ i = 1, 2, 3 \dots M\}$ were found. Using the set $\{\Omega_i,\ i = 1, 2, 3 \dots M\}$, the statistical distribution (histogram) of Ω, which was denoted by $W_e(\Omega)$, were evaluated. The distribution $W_e(\Omega)$, obtained for 6500 specular points ($M = 6500$), is shown in figure 1 by a dashed line. In general, the curves of $W(\Omega)$ and $W_e(\Omega)$ differ non-essentially, showing, therefore, that the formula obtained for $W(\Omega)$ herein is correct.

A striking feature of the distribution $W(\Omega)$ is its nonsymmetricity, which is the general property of random gaussian uniform surfaces.

5. Application

As one of the possible applications, we give an idea of the method of indication of oil films on the sea surface by using the distribution $W(\rho)$ of the reciprocal of curvature $\rho = 1/|\Omega|$. It is well known that oil films extinguish the high-frequency region of the wave spectrum. According to the measurements of Cox and Munk (1954), for a clear sea surface the dispersion of surface slopers (gradients) $\zeta_x(x, y)$, $\zeta_y(x, y)$ are

$$\left.\begin{aligned}\alpha_x^2 &= 3.16 \times 10^{-3}\, v \\ \sigma_y^2 &= 0.003 + 1.92 \times 10^{-3}\, v\end{aligned}\right\} \tag{42}$$

and oil films covering the surface have a dispersion 2–3 times less than that of a clear sea surface. For oil covered surface, we take

$$\left.\begin{aligned}\bar{\sigma}_x^2 &= \sigma_x^2/3 \\ \bar{\sigma}_y^2 &= \sigma_y^2/3\end{aligned}\right\} \tag{43}$$

Therefore, to get the same values of dispersions σ_x^2, σ_y^2 and $\bar{\sigma}_x^2$, $\bar{\sigma}_y^2$, clear and oil-film-covered sea surfaces have been modelled by the wave spectrum that includes harmonics with wavelengths $\lambda \geq \lambda_{\min} = 4.5\,\mathrm{cm}$ and $\lambda \geq \bar{\lambda}_{\min} = 60.0\,\mathrm{cm}$, respectively. In this case the parameters l_1, l_2, l_3, which were evaluated from the energy spectrum $E(k_x, k_y)$ at $v = 10\,\mathrm{m\,s^{-1}}$, were found as $l_1 = 63.0\,\mathrm{m^{-2}}$, $l_2 = -30.8\,\mathrm{m^{-2}}$, $l_3 = -32.2\,\mathrm{m^{-2}}$ and $\bar{l}_1 = 0.090\,\mathrm{m^{-2}}$, $\bar{l}_2 = -0.044\,\mathrm{m^{-2}}$, $\bar{l}_3 = -0.046\,\mathrm{m^{-2}}$, for clear and oil-film-covered surfaces, respectively.

The graphs of distribution density $W(\rho)$, calculated by the theoretical formulae (26) and (29) for clear and oil-covered surfaces, are shown in figure 2. The mean value of ρ evaluated by equation (38) was $\langle \rho \rangle = 0.014\,\mathrm{m^2}$ for a clear surface and $\langle \bar{\rho} \rangle = 9.6\,\mathrm{m^2}$ for an oil-covered surface. The average number of specular points N calculated by equation (17) was $\langle N \rangle = 409.3\,\mathrm{m^{-2}}$ and $\langle \bar{N} \rangle = 1.8\,\mathrm{m^{-2}}$ for clear and oil films covered surfaces, respectively. As is seen, when an oil film exists on the sea

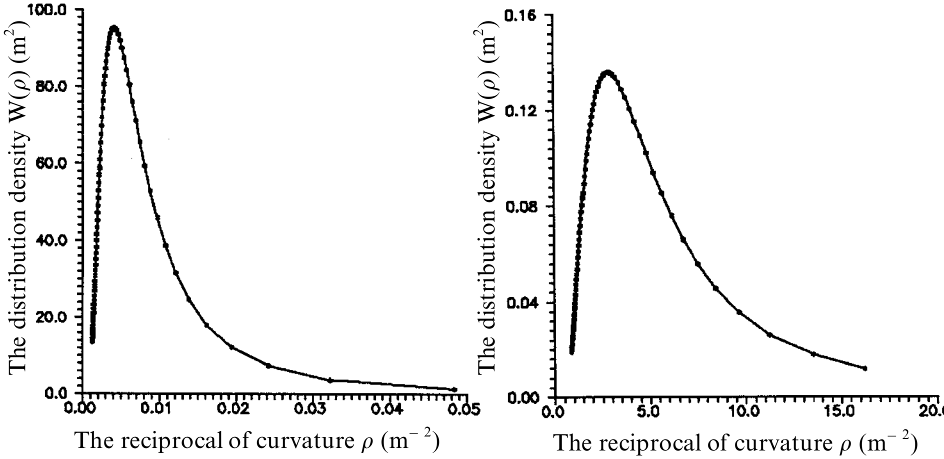

Figure 2. Probability distribution density of the reciprocal of curvature at the specular points: (*a*) clear sea surface; (*b*) oil-covered sea surface.

surface, the number of specular points decreases by about 200 times; the mean value of ρ (in other words, the mean value of the sizes of images of the specular point-glitters) increases by about 700 times, and the graph of $W(\rho)$ shifts essentially to the right. By designing an appropriate optical system to sense the changes occurring in these parameters, the problem of remote sensing of oil films existent on the sea surface can be solved.

6. Conclusion

It is seen that the distribution $W(\rho)$ of radius of curvature $\rho = 1/|\Omega|$ depends on two parameters (any two of l_1, l_2, l_3 or H, Δ_3), which can be determined by the energy spectrum of surface waves. A comparatively simple form of $W(\rho)$ is convenient for the calculation of statistical characteristics of a reflected light from the sea surface.

The number of specular points and radia of curvatures are quantities very sensitive to the surface geometry structures. Therefore, these variables can be used for solving various remote sensing problems, such as studies of currents and internal waves, near-by surface processes, etc., which influence the sea surface geometry structures.

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