Equilibrium Wave Spectrum and Sea State Bias in Satellite Altimetry

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ABSTRACT

For a well-developed sea at equilibrium with a constant wind, the energy-containing range of the wavenumber spectrum for wind-generated gravity waves is approximated by a generalized power law $\beta(U^2/g)^{2\mu}k^{-4+2\mu}Y(k, \theta)$, where Y is the angular spread function and μ can be interpreted as a fractal codimension of a small surface patch. Dependence of μ on the wave age, $\xi = C_0/U$, is estimated, and the "Phillips constant," β, along with the low-wavenumber boundary, k_0, of the inertial subrange are studied analytically based on the wave action and energy conservation principles. The resulting expressions are employed to evaluate various non-Gaussian statistics of a weakly nonlinear sea surface, which determine the sea state bias in satellite altimetry. The locally accelerated decay of the spectral density function in a high-wavenumber dissipation subrange is pointed out as an important factor of wave dynamics and is shown to be also highly important in the geometrical optics treatment of the sea state bias. The analysis is carried out in the approximation of a unidirectional wave field and confined to the case of a well-developed sea characterized by $\xi > 1$.

1. Introduction

Sea surface roughness on scales of wind-generated gravity waves provides a peculiar example of turbulence developing through nonlinear wave–wave interactions within resonant wave tetrads. Due to the remarkable weakness of these interactions, the characteristic scales of wave field evolution are very large—hundreds of kilometers (e.g., Walsh et al. 1989). Conventional wave observations cover a relatively limited range of wind–wave interaction regimes, as characterized by rather short wind fetches and a relatively small local depth. More advanced stages of wave development, encountered in the open ocean, can be observed by satellite-borne instruments. However, oceanographic interpretation of satellite measurements requires adequate understanding of the sea surface's statistical geometry in relation to wave dynamics. Much work on the surface geometry is due to Longuet-Higgins (1957, 1962, 1984), while problems arising in connection with the multiple-scale variability of the surface elevation field have been addressed by Glazman (1986) and Glazman and Weichman (1989). Rapid development of ocean remote sensing techniques raises a number of new issues; some of them are addressed in this paper.

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Our attention is focused on dynamics and statistics of the wave field for a case of sufficiently well-developed seas, which means that there exists an extended range of wavenumbers dominated by inertial (Kolmogorov-type) cascades of energy and wave action. Dynamical problems are investigated in sections 2, 3, and 6 based on the weak-turbulence theory (WTT) of surface gravity waves (Zakharov and Filonenko 1966; Zakharov 1984; Zakharov and Zaslavskii 1981–1983). We discuss an effective exponent for the inertial range of wave spectra and estimate its dependence on the degree of the wave development. Furthermore, based on the integral balance of the wave energy and action, we derive the fetch laws, which during the past three decades have been known largely as an empirical fact. Section 6 focuses on the high-wavenumber rolloff of the wave spectrum associated with the energy dissipation due to wave breaking. Sections 4, 5, 7, and 8 are devoted to statistical geometry of the sea surface: various non-Gaussian statistics are estimated as functions of wind speed and the degree of the wave development. Such statistics are especially important for the correct interpretation of altimeter measurements. Therefore, we present many of our results in the form in which they can be readily applied to analysis of altimeter data. Comparison with experimental data of Fu and Glazman (1991) is also provided.

Theoretical studies (Glazman 1986; Glazman and Weichman 1989; Glazman 1990, 1991) and analyses of satellite data [satellite scatterometer (Glazman 1987; Glazman et al. 1988), altimeter (Glazman and Pilorz 1990), and microwave radiometer (Glazman 1991)] indicated that the degree of the wave development is an important factor of air–sea interactions that has a profound effect on statistical properties of a rough sea surface. This factor, expressed, for instance, via the wave age

$$\xi = \frac{C_0}{U}, \tag{1.1}$$

measures the overall effect of the conventional geometric fetch X and of the boundary-layer stratification (as discussed in section 2). Here U is the mean wind speed well above the surface, and C_0 is the phase velocity of dominant (spectral peak) waves. Typical values of ξ for open ocean waves lie in the range 1 to 3, with the mean about 2.3 (Glazman and Pilorz 1990; Fu and Glazman 1991). One of the most important properties of the sea surface controlled by the wave age is its effective fractal dimension D_H, which characterizes the cascade pattern in surface geometry (Glazman and Weichman 1989). In section 2, the term "effective" is defined quantitatively. Analysis of wave dynamics in sections 2 and 3 is confined to the case of open ocean waves, for which $D_H \geqslant 2.2$. When D_H is greater than 2 (2 is the topological dimension of a narrow-banded random surface), the surface experiences variations in a broad range of scales, and between each pair of dominant wave crests there appear many secondary (and higher-order) wave crests.

Surface curvature at the specular facets, surface slopes, and other parameters responsible for the observed radar cross section are strongly influenced by the fractal dimension of the sea surface (Glazman 1990, 1991a). Recently, Rodriguez and Chapman (1990) analyzed deconvoluted wave forms from the Geosat altimeter and concluded that the surface skewness, whose effect on the wave forms was parameterized by Jackson (1979), Barrick and Lipa (1985), and Srokosz (1986), exhibits a negative correlation with the wave age. Fu and Glazman (1991) analyzed errors in sea level measurements by the Geosat altimeter and found that the sea state bias decreases with an increasing wave age. One of the goals of the present work is to quantify these trends based on the theory for the sea state bias developed by Jackson, Barrick and Lipa, and Srokosz.

2. Equilibrium state of a well-developed sea and μ as a function of ξ

In the above-the-spectrum-peak range, the spectral density of the wave potential energy density per unit area (and divided by ρg) can be approximated as

$$k/k_0 \gg 1: \quad F(k, \theta) = \beta(U^2/g)^{2\mu} k^{-4+2\mu} Y(k, \theta), \tag{2.1}$$

where the nondimensional "Phillips constant" β can depend on the nondimensional fetch x, and μ tends to zero at short fetches (the Phillips law). This μ is a slowly growing function of x, attaining about ¼ at x of order 10^4 and going to its asymptotic limit of ⅓ (Zakharov and Zaslavskii 1982a, 1983a; Zakharov 1984; Kitaigorodskii 1987) when x tends to infinity (practically, at $x > 10^5$). The physical principles underlying Eq. (2.1) and its implications for the sea surface's geometry are discussed in Glazman and Weichman (1989), where μ is identified as the Hausdorff (fractal) codimension of a small surface patch corresponding to the equilibrium range: $\mu = D_H - 2$. A simple heuristic model relating μ to the wave age is offered in the end of this section.

Because of the important role which ξ, β, and μ play in our theory, we need some scaling laws relating these parameters to external factors of air–sea interactions. Such laws can be derived from the conservation principles, but this would require extending the model (2.1) to lower wavenumbers. A notorious feature of equilibrium sea is the existence of a rather steep cutoff of the energy spectral density at wavenumbers near k_0 which is associated with the dominant wavelength (the "outer scale"). Hence, the extension of (2.1) into the low-wavenumber range is obtained by multiplying it by a slightly smeared Heaviside function $H(k_0/k - 1)$ which provides a smooth cutoff near k_0. A simplest form of this function is given by $\exp[-(k_0/k)^2]$ [which also follows from the Pierson-Moskowitz (1964) spectrum]. Furthermore, since there is no commonly accepted form for the angular spread function Y, we shall conduct our analysis in the approximation of a unidirectional field of waves propagating along the (constant) wind direction (Y reduces to the Dirac delta function). It can be shown (e.g., see Zakharov and Zaslavskii 1983b) that if the spectrum is characterized by a narrow directional distribution, as is the case for wind-generated gravity waves, approximating $Y(\theta)$ by a delta-function gives a satisfactory spectrum model for wave energy and action balance analyses. Thus, for the energy-containing range, the one-dimensional spectral density of wave energy is

$$F(k) = \beta(U^2/g)^{2\mu} k^{-3+2\mu} \exp[-(k_0/k)^2] \quad (\equiv E(k)). \tag{2.2}$$

The dominant wavenumber can be presented in the form:

$$k_0 = K_U \xi^{-2} \tag{2.3}$$

where

$$K_U = g/U^2 \tag{2.4}$$

is interpreted as the characteristic wavenumber roughly corresponding to the spectral peak of the energy flux from wind to waves (implying the Miles-Phillips generation mechanism), and ξ is given by (1.1) with

$C_0 = \omega_0/k_0$. Furthermore, ξ is a function of the nondimensional fetch $x = gX/U^2$ (we consider wave duration to be infinite) which has the form

$$\xi = Ax^{\nu} \tag{2.5}$$

confirmed by numerous field observations. The values of ν found by different authors vary in a rather narrow range—from 0.22 to about ⅓, while A is somewhere between 6×10^{-2} and 8.6×10^{-2} (Phillips 1977; Donelan et al. 1985). As shown below, parameter ν is very important, and therefore we shall examine it in the following section.

At the present time, the generalized Phillips constant β is understood rather poorly. Field observations (e.g., those reviewed by Kitaigorodskii 1983) in moderately developed seas (i.e., when $\mu \approx$ ¼) estimate β as $\approx 2.3 \times 10^{-3}$. Earlier observations, dating back to JONSWAP, showed β as a homogeneous function of the nondimensional fetch:

$$\beta = Bx^{v}. \tag{2.6}$$

The reported values of v are typically small: $v \approx -0.2$, while $B \approx 0.0331$ (e.g., Hasselmann et al. 1976; Donelan et al. 1985). We believe that much of the uncertainty in the observed values of β or, alternatively, of B and v is due to the fact that an experimental estimation of β critically depends on the value of μ chosen to represent the slope of the spectrum. Most observations have been conducted at relatively short fetches (well under 100 km) and assuming μ to be a "universal constant." In the 1960s and much of the 1970s this constant was thought to be zero (the Phillips law), while in the past decade it was upgraded to ¼ on various grounds (Zakharov and Filonenko 1966; Kitaigorodskii 1983; Phillips 1985). Phillips (1985) presented arguments indicating that μ = ¼ can be obtained not only as a result of a purely inertial energy cascade but also as a result of a more complex energy balance when the source functions (energy dissipation and wind input) are included.

In order to relate μ to the nondimensional wind fetch, we recall that there are three special values, μ = 0, ¼, and ⅓, corresponding to three asymptotic regimes of wind–wave interaction. Two of them, relevant to open ocean waves, are the direct energy cascade (μ = ¼) and the inverse energy cascade (μ = ⅓) (Zakharov and Filonenko 1966; Zakharov and L'vov 1975; Zakharov and Zaslavskii 1982a). The corresponding values of μ were found by Zakharov and his collaborators as exact analytical solutions of the kinetic equation in the approximation of the weak-turbulence theory and assuming that the energy source is concentrated either at the low-frequency end of the spectrum (μ = ¼) or at the high-frequency end (μ = ⅓). A comprehensive review of the theory is given by Zakharov (1984). Although the actual spectral density of the energy flux from wind to waves is by no means a delta function of either ω or k (Phillips 1985), it is known to have a pronounced maximum due to a resonant mechanism of wind wave generation. In this respect, Eq. (2.4) represents a scaling relationship for the resonant wavenumber corresponding to this peak. The weak-turbulence theory offers a simplified approach to the parameterization of the overall spectrum width: at ξ near unity, the equilibrium wave spectrum is dominated by the direct inertial cascade (characterized by $\mu \approx$ ¼). As ξ increases well above one, the relative extent of the inverse cascade becomes appreciable leading to an increase in the effective value of μ.

Based on this idealization, one can roughly quantify the rate at which the effective value of μ should change as a function of the wave age. A crude model can be constructed starting with a composite spectrum as a prototype:

$$F_c(k) = \begin{cases} b(U^2/g)^{2/3}k^{-10/3}, & \text{if} \quad k_0 \leqslant k \leqslant K_U \\ b(U^2/g)^{1/2}k^{-7/2}, & \text{if} \quad K_U \leqslant k < \infty. \end{cases} \tag{2.7}$$

It is easy to check that at $k = K_U$ both branches meet at an arbitrary value of b. Let us find μ and β based on the requirement that the spectrum (2.1) must yield the same integral wave energy and wave action as would follow from (2.7). The resulting μ and β are said to be the effective fractal codimension and the generalized Phillips constant, respectively. Thus, we have two equations:

$$\left.\begin{aligned} \int_{k_0}^{\infty} E(k)dk &= \int_{k_0}^{\infty} E_c(k)dk \\ \int_{k_0}^{\infty} E(k)/\omega(k)dk &= \int_{k_0}^{\infty} E_c(k)/\omega(k)dk \end{aligned}\right\} \tag{2.8}$$

where

$$E(k) = \int_0^{\pi} F(k, \theta)kd\theta \approx \beta(U^2/g)^{2\mu}k^{-3+2\mu}. \tag{2.9}$$

Similarly,

$$E_c(k) = F_c(k)k \tag{2.10}$$

This system has an exact solution which, after some algebra, is

$$\left.\begin{aligned} \mu &= 1 - \frac{1}{4}\frac{M}{N\xi - M} \\ \beta &= b\frac{2(1-\mu)N}{\xi^{4(1-\mu)}} \end{aligned}\right\} \tag{2.11}$$

where

$$M = \frac{6}{11}\xi^{11/13} - \frac{1}{22} \quad \text{and} \quad N = \frac{3}{4}\xi^{8/3} - \frac{1}{12}.$$

Functions $\mu(\xi)$ and $1 - \beta(\xi)/b$ are plotted in Fig. 1.

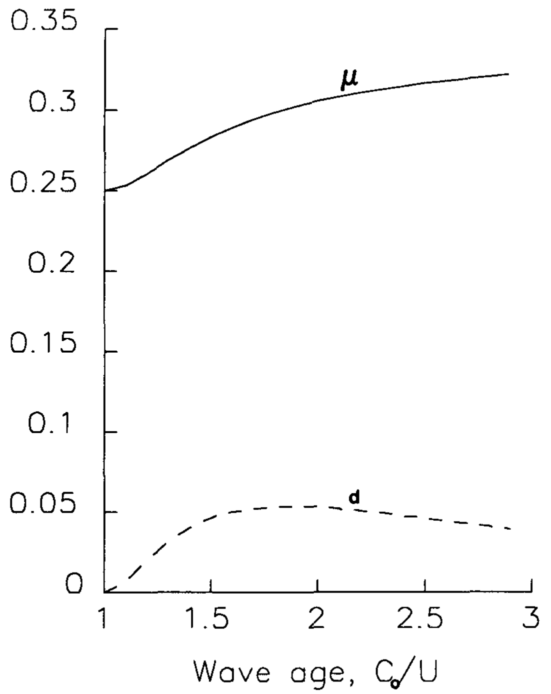

FIG. 1. The wave-age dependence of the fractal codimension, $\mu(\xi)$, and of the β-defect, $d(\xi) = 1 - \beta(\xi)/b$. Equations (2.11).

The latter function describes a (small) β-defect caused by our ad hoc model.

An alternative form of (2.7), as was originally presented by Zakharov (see also Kitaigorodskii 1987), is

$$F(k) = A_1 p^{1/3} k^{-10/3} \quad (2.12)$$

corresponding to the inverse inertial cascade of wave energy ($\mu = 1/3$), and

$$F(k) = A_2 q^{1/3} g^{-1/2} k^{-7/2}, \quad (2.13)$$

corresponding to the direct energy cascade ($\mu = 1/4$). Constant A_2 [estimated as $O(10^{-2})$] by Glazman and Weichman (1989) has a meaning similar to the Kolmogorov constant for the inertial range in isotropic three-dimensional turbulence, and q represents the surface density of wave energy flux (the rate of energy transfer from wind). Constant A_1 was estimated as O(1) by Zakharov and Zaslavskii (1983a), and p represents the wave action flux density.

However idealized, the spectrum forms (2.1), (2.2) proved to be highly useful for applications. This simple model (i.e., the use of a single μ for all wavenumbers) is motivated by the relatively short overall extent of the equilibrium range (roughly two decades in wavenumbers), as well as by the fact that the range of possible variations of μ is relatively small. A great practical advantage of (2.1) over more complicated forms (including composite spectra) is due to its simplicity. Spectra (2.1) and (2.2) allow one to readily relate various statistical properties of the wave field to external parameters and on that basis analyze and predict an impact of wind speed and fetch on air–sea fluxes, breaking wave statistics, microwave remote sensing signatures, etc. (Glazman 1990, 1991a; Glazman and Pilorz 1990). However, all such results depend ultimately on the scaling relationships (2.5) and (2.6) in which parameters ν and v play a crucial role. Therefore, it is important to better understand their physical meaning and constrain their values based on appropriate conservation principles.

3. Scaling law for ξ and β

Zakharov and Zaslavskii (1983b) derived the fetch dependence of the wave age based on the balance of the wave action and assuming that the wave spectrum is dominated by the inverse energy cascade ($\mu = 1/3$). We use a more general form for the wave spectrum and, in order to include the case of moderately developed seas ($\mu \approx 1/4$), consider the energy balance equation. However, the following procedure can be applied to the wave action as well.

Consider the energy conservation principle for deep-water gravity waves in the absence of mean currents. In an equilibrium sea, the total wave energy does not change with time. Due to the conservative nature of weakly nonlinear wave–wave interactions, the divergence of the spectral flux of wave energy in the wavenumber space, integrated over all wavenumbers, tends to zero, provided the direct energy cascade dominates the energy balance. Therefore, the energy conservation law is

$$\rho_w g \nabla \cdot \int_0^\infty C_g(k) E(k) dk = \rho_a Q. \quad (3.1)$$

The group velocity C_g is estimated based on the dispersion relationship for deep-water gravity waves. The left-hand side of (3.1) represents the divergence of the wave energy flow (Whitham 1974; Phillips 1977). The weakly nonlinear wave–wave interactions are included, in a parametric form, by allowing the spectrum parameters ξ and β to depend on the nondimensional fetch x. Finally, Q is the net flux density of energy supplied to waves, per unit mass of air (i.e., wind input minus dissipation). In what follows we assume that the energy dissipation due to wave breaking is negligible compared to the wind input (integrated over all wavenumbers). Equation (3.1) does not presume any particular spectral distribution of the source functions. Assuming constant wind direction and studying wave field variations only along the wind vector, we identify the distance along this vector as the conventional wind fetch, X. It is important to remember that the conservation equation (3.1) pertains to the field averaged over a horizontal area, which includes at least one dominant wavelength but is small by comparison to the characteristic variations of the wind fetch causing variations in the wave spectrum parameters.

At shorter wind fetches the waves experience the action of local wind only for a brief period $\tau \propto X/C_0$. Hence, we select the homogeneous boundary condition. Being interested in an equilibrium sea state, we further assume that the wind input can depend only on U and X. Ultimately,

$$Q = c_1(x)U^3. \qquad (3.2)$$

In general, c_1 may also depend on the stratification of the marine boundary layer above the surface being a decreasing function of atmospheric stability. However, such effects are beyond the scope of the present work. The role of c_1 is to account for gradual readjustment of the atmospheric boundary layer as the waves develop with fetch. However, since this readjustment occurs in response to the changing wave field, it is appropriate to view this interaction coefficient as a functional of the wave field itself, ultimately of the degree of wave development: $c_1 = f(\xi)$. The question then arises as to the characteristic rate at which c_1 changes as a function of ξ. The forthcoming proposition, like most other theoretical efforts in this difficult area, should be viewed as a hypothesis. We introduce into our discussion the term "generation range" (as used, for example, by Zakharov and Zaslavskii 1982b) to designate a fairly narrow band of wavenumbers near K_U where the wind is most strongly coupled with wave components. Considering sufficiently developed seas (i.e., $\xi \geqslant 1$) we notice that, apart from the wind speed, the most important parameter governing the surface's geometrical features in this range of scales is the fractal dimension of the surface patch (Glazman and Weichman 1989): its influence on the wave slope variance and on other major statistics of the surface geometry is more important than that of the dominant wavelength $2\pi/k_0$. According to the Miles theory, the wavebound component of the energy flux, which is coupled with the waves, is linearly proportional to the wave slope. Therefore, the interaction coefficient $c_1(\xi)$ can change only as fast as $\mu(\xi)$. This allows us to rewrite (3.1) as

$$g\int_0^\infty \frac{d\omega}{dk}E(k)dk = \int_0^X c(\mu)U^3 dX \approx \bar{c}U^3X \qquad (3.3)$$

where

$$c(\mu) = \frac{\rho_a}{\rho_w}c_1$$

and $\bar{c}$ is given by (3.7). Due to the assumed weak dependence of c_1 on X, Eq. (3.3) can be integrated over relatively short segments of X by keeping the interaction coefficient constant. In the end of this section the notion of the "relatively short segments of X" is quantified. Employing the deep-water dispersion relationship and substituting (1.1) and (2.2)–(2.4) into the left-hand side of (3.3), we ultimately arrive at the following scaling law:

$$\beta(x)\xi^{5-4\mu} \propto x. \qquad (3.4)$$

Using (2.5), this can be written in terms of the fetch alone:

$$\beta(x) \propto x^{1-5\nu+4\mu\nu}. \qquad (3.5)$$

It is interesting to notice that the most typical values of μ and ν reported by the field observations ($\mu = 1/4$ and $\nu = 1/4$) yield $\beta = \text{const}$.

The last and most crucial step in our development is the assumption that β is indeed a universal constant, as was anticipated by Kitaigorodskii (1983) for a special case of $\mu = 1/4$. This assumption yields:

$$\xi^{5-4\mu} \propto x$$

or

$$\nu = \frac{1}{5-4\mu} \qquad (3.6)$$

where the effective codimension μ can be roughly estimated from (2.11). The function $\xi(x)$ is plotted in Fig. 2. [In order to quantify the actual range of the nondimensional fetch, we employed in this calculation a concrete value of the proportionality coefficient A introduced in (2.5): $A = 8.62 \times 10^{-2}$, as suggested by Donelan et al. 1985.] Our theory thus resulted in an unlimited monotonic growth of the wave age with fetch. Apparently, by imposing some mechanism that would

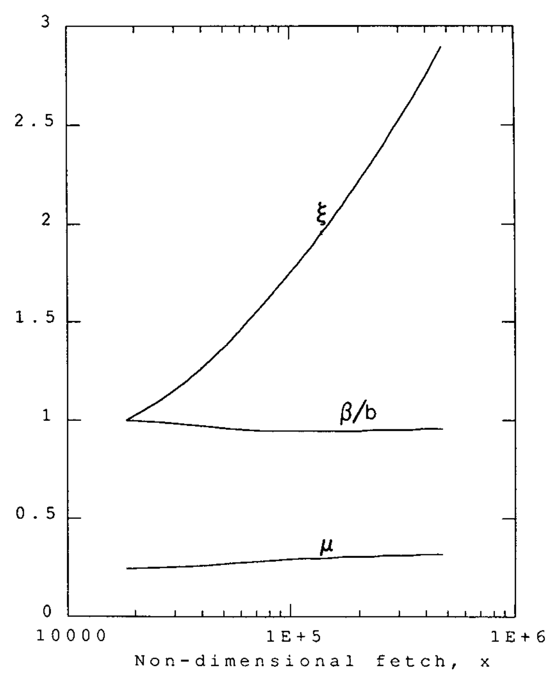

FIG. 2. The wave age ξ (3.6), the codimension μ and the generalized Phillips constant (in relative units) β/b (2.11), as functions of the nondimensional wind fetch.

make Q in (3.1) a decreasing function of x, we could obtain a regime in which the wave age as a function of x eventually reaches a limit. However, the search for such a mechanism is outside the scope of the present work. Besides, the previous experimental work (Glazman and Pilorz 1990) yields results that strongly disagree with the notion of the fully developed sea as presented by Pierson and Moskowitz (1964) and characterized by $\xi = 1.4$ as the limit. The limiting value of ξ remains unknown, and we find that values as large as 3 are quite feasible (Glazman 1991b).

The physical significance of the constant β assumption can be clarified in a special case of $\mu = 1/4$ for which equation (2.1) can be interpreted in terms of more fundamental quantities introduced by (2.13). Namely, $q = c(\mu)U^3$ and A_2 plays the role of the Kolmogorov constant—it controls the energy dissipation rate. Hence, $\beta = A_2 c^{1/3}(\mu)$. The fetch-independent β means that, once the sea has attained an essentially fractal regime (as defined in Glazman and Weichman 1989), the rate of energy transfer does not change appreciably with further increase of the fetch. Let us examine the probability of wave-breaking events observed instantaneously (Glazman and Weichman 1989) in a given surface area (defined as the expectation of the number of breaking wavelets per total number of gravity wavelets observed): $P_{BW} \propto (\Gamma/\gamma)^2 \exp(-\Gamma^2/2\gamma^2)$, where γ^2 is the wave slope variance and Γ is the characteristic slope attained by gravity-range wavelets prior to breaking. It is easy to show that in an essentially fractal regime [see also (6.9)–(6.10) of the present paper], γ^2 is proportional to β but is virtually independent of k_0. Therefore, assuming β to be independent of x is equivalent to assuming P_{BW} (hence, the Kolmogorov constant A_2) to be independent of x. Thus, in the case of a direct energy cascade, our description of the energy flow is, at least, self-consistent.

The interaction coefficient $c(\mu)$ is difficult to estimate theoretically. Let us express it in terms of the empirical parameters A and β introduced in section 2. To this end, we substitute (2.2)–(2.5) and (3.6) into (3.3) and arrive at

$$\bar{c}(\mu) = \frac{\Gamma(5/4 - \mu)}{4} \beta A^{5-4\mu} \approx \frac{\beta}{4} A^4 \ (\approx \text{const}), \quad (3.7)$$

which lends further credibility to our model and permits a semiempirical estimation of $\bar{c}$.

Finally, in Fig. 2 we also plotted μ and β to illustrate a considerable difference in the characteristic scales of X—variations needed to cause comparable relative changes in ξ, μ, and β. An order of magnitude increase of x causes 100% increase of ξ but only 20% growth of μ and about 3% change of β. It is these differences that justify our simplified theory.

Employing the wave action balance equation instead of (3.1), the fetch dependence of the wave age is found to be

$$\xi^{6-4\mu} \propto x$$

or

$$\nu = \frac{1}{6 - 4\mu}. \quad (3.8)$$

This result should be appropriate mainly for very high degrees of wave development.

4. Sea state bias in satellite altimetry

The contemporary view of the sea state bias is based on the geometrical optics treatment of the reflection of a finite-length pulse from a weakly nonlinear wave surface at nadir incidence. If we consider all specular points as reflectors of equal strength, the distribution of their heights about the mean sea level is estimated, and the sea state bias is then determined as a deviation in the reported sea level height due to a shift in the position of the half-power point of the altimeter waveform caused by departures of the surface wave field statistics (i.e., of the joint probability density function for surface elevations and slopes) from the Gaussian distribution (Jackson 1979; Barrick and Lipa 1985; Srokosz 1986, 1987). The bias is given by

$$\text{bias} = -\frac{1}{8}\left(\frac{1}{3}\lambda_0 + \lambda_1\right)H_{1/3}, \quad (4.1)$$

where $H_{1/3} = 4\langle\zeta^2\rangle^{1/2}$ is the significant wave height, $\lambda_0 = \langle\zeta^3\rangle/\langle\zeta^2\rangle^{3/2}$ is the surface skewness, and λ_1 (usually denoted by γ but re-denoted here to avoid conflict with the rms wave slope) was called "cross-skewness" by Srokosz (1986) (relating elevation to slope squared: $\langle\zeta(\nabla\zeta)^2\rangle$). This latter quantity obtains simple geometrical interpretation by considering the mean height of specular points which was estimated by Srokosz (1986):

$$\int_{-\infty}^{\infty} \zeta f(\zeta) d\zeta = -\frac{\lambda_1}{8} H_{1/3}, \quad (4.2)$$

where $f(\zeta)$ is the (conditional) probability density function for the heights of the surface points with zero slope ($\nabla\zeta = 0$). Therefore, we shall call λ_1 the "specular relative height" (or just "specular height"), which better describes its meaning. It can be shown (Longuet-Higgins 1963; Barrick and Lipa 1985; Srokosz 1986) that the two parameters λ_0 and λ_1 may be expressed in terms of integrals of the two-dimensional wavenumber spectrum of the waves. In particular,

$$\lambda_0 = \frac{3\int dk F(k) \int C(k, k')F(k')dk'}{\sigma^3}. \quad (4.3)$$

Here we introduced the total variance of the surface elevation:

$$\sigma^2 = \int F(k)dk \quad (= \langle \zeta^2 \rangle). \tag{4.4}$$

The expression for λ_1 is more complicated and is given in full by Srokosz (1986), along with the form of the coefficient $C(k, k')$.

A suggestion was made to estimate the components of the sea state bias by postprocessing the altimeter waveform data (Srokosz 1986; Rodriguez 1988; Rodriguez and Chapman 1989, 1990), but this is a time-consuming task and it may not be possible to get accurate results (Rodriguez and Chapman 1990). Here, based on the knowledge of the wave spectrum, we examine the two components of the bias analytically and numerically. Our practical goal is to relate the sea state bias to wind and wave data available from satellite measurements. In particular, we will present the sea state bias as a function of wave age and significant wave height, which can be estimated from altimeter measurements (as, for instance, done by Fu and Glazman 1991).

First we notice that λ_0 contains no information about surface gradients (i.e., wave slopes), which are known to be strongly influenced by the high-wavenumber tail of the wave spectrum. All integrals in (4.3) converge very rapidly. Therefore, λ_0 can be estimated with sufficient accuracy even if the tail of the spectrum is specified incorrectly. In contrast, parameter λ_1, related to wave slope statistics, critically depends on the scales of surface variations taken into account. Unless irrelevant small-scale variations of the surface elevation field, pertaining to the tail of the wave spectrum, are filtered out, the corresponding integrals would diverge (Glazman 1986; Glazman and Weichman 1989). In section 6 we provide a brief review of basic notions related to this issue, which will allow us to evaluate λ_1 in section 7, while λ_0 is examined in the following section.

5. Sea surface skewness in a unidirectional sea

Let us consider a simple case of a unidirectional wave field. This case allows one to obviate a difficult problem of selecting an appropriate angular spread function $Y(k, \theta)$ in (2.1) and to drastically reduce the amount of computations.

The skewness takes the form (Jackson 1979) (after introducing minor corrections pointed out by Srokosz and Longuet-Higgins 1986):

$$\lambda_0 = \frac{6\int_0^\infty E(k)G_0(k)dk}{\sigma^3} \tag{5.1}$$

where

$$G_0(k) = \int_0^k k'E(k')dk'. \tag{5.2}$$

Substituting (2.2), the inside integral (5.2) is found as

$$G_0(k) = \beta(U^2/g)^{2\mu}\int_0^k (k')^{-2+2\mu}\exp[-(k_0/k')^2]dk'. \tag{5.3}$$

We reduce this to a table form by replacing $(k_0/k')^2 = y$, which yields

$$G_0(k) = \frac{\beta}{2}(U^2/g)^{2\mu}k_0^{-1+2\mu}\Gamma\left(\frac{1}{2} - \mu, \left(\frac{k_0}{k}\right)^2\right). \tag{5.4}$$

In Fig. 3 this incomplete gamma function is plotted to show that its growth with an increasing k at $k > k_0$ is very weak. Due to a rapid decay of $E(k)$ at high k [which is not offset by the slow growth of $G_0(k)$], the outside integral in the numerator of (5.1) is dominated by the spectral peak range. This integral, as well as the denominator in (5.1), can be calculated exactly, using (6.455) and (3.381) of Gradshteyn and Ryzhik (1980). The final results are

$$\sigma^2 = \frac{\beta}{2}(U^2/g)^{2\mu}k_0^{-2(1-\mu)}\Gamma(1-\mu) \tag{5.5}$$

$$\lambda_0 = 3(\beta\xi^{-4\mu})^{1/2}R_0(\mu) \tag{5.6}$$

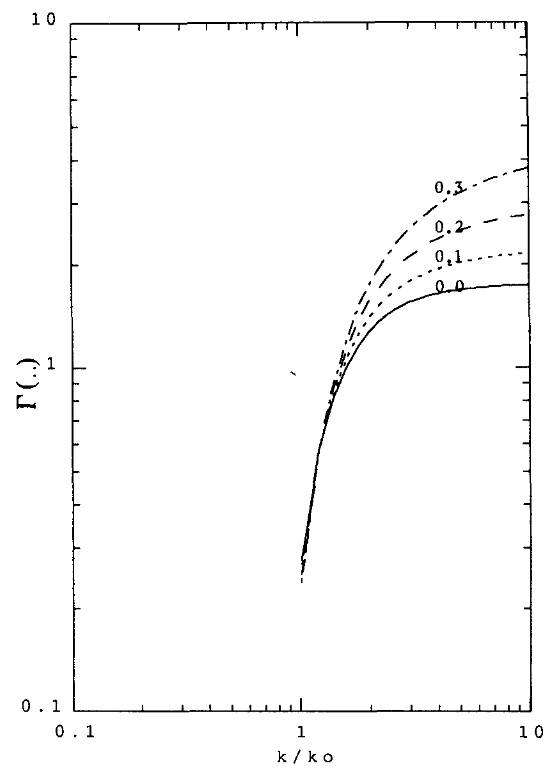

FIG. 3. The incomplete gamma function (5.4) for several values of μ as indicated at the curves.

where

$$R_0(\mu) = \frac{2^{2\mu}\Gamma(3/2 - 2\mu)_2F_1(1, 3/2 - 2\mu; 2 - \mu; 1/2)}{2(1 - \mu)[\Gamma(1 - \mu)]^{3/2}} \quad (5.7)$$

and ${}_2F_1$ is the Gauss hypergeometric function, readily evaluated using a series expansion [e.g., (9.100) of Gradshteyn and Ryzhik 1980]. The factor 3 is introduced in (5.6) in order to avoid notational inconvenience when comparing two terms of (4.1) (section 8).

As shown in Fig. 4, the factor R_0, for the range of μ of interest to the given problem, is a weak function of μ; hence, the behavior of λ_0 is controlled primarily by the nondimensional fetch (or the wave age) through β and ξ. Employing (3.4) we derive this dependence:

$$\lambda_0 \propto (x\xi^{-5})^{1/2}. \quad (5.8)$$

Two alternative final forms for λ_0 follow from (3.6):

$$\lambda_0 \propto x^{-2\mu/(5-4\mu)}$$
$$\propto \xi^{-2\mu}. \quad (5.9)$$

In the case of $\mu = 1/4$, these yield $\lambda_0 \propto x^{-1/8}$ and $\lambda_0 \propto \xi^{-1/2}$. In other words, as the sea becomes more mature, the overall surface skewness λ_0 decreases.

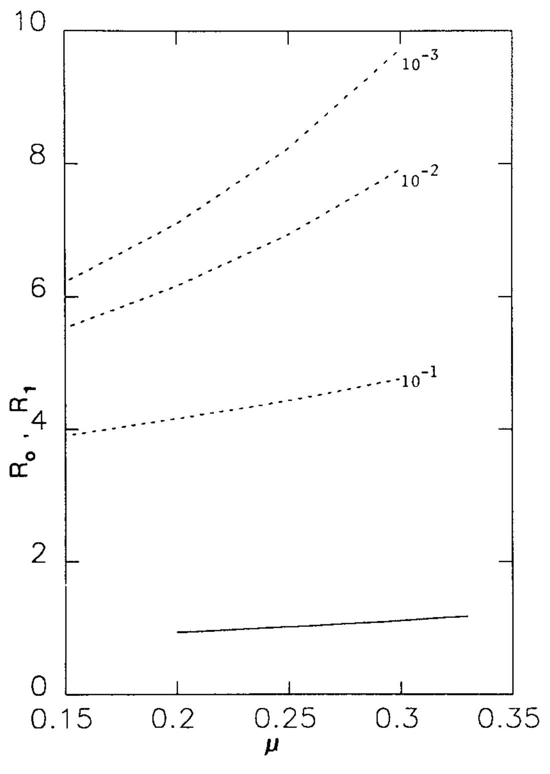

FIG. 4. Nondimensional factors R_0 (solid curve) and R_1 (dashed curves), (5.7) and (8.3), respectively. The values of δ are plotted at each curve.

However, as shown in section 7, the surface skewness, although of considerable intrinsic interest, has little influence on the sea state bias, as the typical values of $R_0(\mu)$ turn out to be rather small by comparison to the similar factor of the "specular height" λ_1.

6. The dissipation range and the intrinsic inner scale

In recent studies (Kitaigorodskii 1983; Phillips 1985; Glazman 1985, 1986), the existence of an intrinsic inner scale h associated with energy dissipation due to wave breaking was predicted. A simple argument (Glazman 1985, 1986; Glazman and Weichman 1989) that yields an estimate of this scale is that the wave slope variance

$$\gamma^2 = \int_0^\infty k^2 E(k) dk \quad (6.1)$$

would be infinite for all $\mu \geqslant 0$ unless the spectrum experiences a rapid rolloff within certain transitional (i.e., "dissipation") subrange. A physical mechanism limiting possible values of γ is due to hydrodynamic instability of steep wavelets leading to a vigorous breaking of exceedingly steep crests. An important feature of developed seas is the statistical self-affinity of surface profiles, which appears in the fractal regime (Glazman and Weichman 1989). That means, in particular, that shorter wavelets tend to be steeper than the longer ones:

$$\langle(\Delta\zeta/\Delta r)^2\rangle \propto (\Delta r)^{-2\mu}. \quad (6.2)$$

Therefore, shorter wavelets are more likely to break. With respect to the overall surface geometry, the breaking of the wavelets results in an effective low-pass filtering of the surface elevation field. In terms of the wavenumber spectrum (representing a large surface area including many breaking events), the "filtered" surface can be described by

$$\bar{E}(k) = E(k) \exp[-(k/k_h)^2] \quad (6.3)$$

where $E(k)$ is given by (2.2). According to (2.2), the spectrum (6.3) increases as a function of wind. On the contrary, the spectrum behavior at high wavenumbers, as reported by Banner et al. (1989), is virtually independent of the wind speed (and of the wind friction velocity) and is given by

$$E_h(k) = \beta' k^{-p}, \quad \text{for} \quad k \gg k_h \quad (6.4)$$

where $p \approx 4$ and $\beta' = 2 \times 10^{-3}$. As a result of these marked differences in the spectrum behavior above and below the "transitional" range associated with the intrinsic inner scale h, a profound mismatch between (2.2) and (6.4) appears. This mismatch would grow with an increasing wind. Apparently, the low-pass filter introduced by (6.3) closes the gap.

We interpret Banner's results as indirect experimental evidence confirming the existence of the "transitional" dissipation range predicted by the authors mentioned above. Indeed, after a certain (actually, rather small) amount of energy is removed from the inertial subrange due to the intermittent wave breaking, the spectral density at smaller scales can resume its fractal behavior (i.e., can continue its power-law decay at the rate k^{-4} or slower).

According to previous estimates (Glazman 1986; Glazman and Weichman 1989), the value of $h \equiv 1/k_h$ is about one-half meter.

Substituting the filtered spectrum into (6.1), one evaluates the wave slope variance corresponding to the gravity-range surface wavelets exactly, as

$$\gamma^2 = \frac{\beta}{2} \xi^{-4\mu} I_0(\mu, \delta) \tag{6.5}$$

where

$$I_0(\mu, \delta) = \int_0^\infty x^{-1+\mu} \exp(-x^{-1} - \delta^2 x) dx$$
$$= 2\delta^{-\mu} K_\mu(2\delta) \tag{6.6}$$

is obtained by replacing $(k/k_0)^2 = x$ and using (3.471.9) of Gradshteyn and Ryzhik (1980). Here,

$$\delta = \frac{k_0}{k_h} \tag{6.7}$$

and K_μ is the modified Bessel function of the third kind. A more traditional (approximate) form of (6.5) is based on

$$I_0(\mu, \delta) \approx \int_1^\infty x^{-1+\mu} e^{-\delta^2 x} dx = \delta^{-2\mu} \Gamma(\mu, \delta^2) \quad (6.6')$$

which corresponds to a sharp low-wavenumber spectral cutoff.

Using (2.3), δ can be written in a more informative form as

$$\delta = \delta_0 \xi^{-2} \tag{6.8}$$

where $\delta_0 = K_U/k_h = gh/U^2$. The so-called essentially fractal regime takes place when

$$\delta^{2\mu} \ll \Gamma(1 + \mu) \tag{6.9}$$

which reduces (6.5) and (6.6′) to

$$\gamma^2 \approx \frac{\beta}{2} \left(\frac{U^2}{gh}\right)^{2\mu} \Gamma(\mu) \tag{6.10}$$

(Glazman and Pilorz 1990; Glazman 1990, 1991a). Based on the analysis of dimensions, Kitaigorodskii (1983) suggested a gravity-wave version of the Kolmogorov inner scale: $h \approx q^{2/3}/g$, which translates into $h \propto U^2/g$. The latter would yield $\delta_0 =$ const. If δ_0 were indeed a constant, all the wind-speed dependence of γ would be due exclusively to the weak dependence of the fractal codimension on the wave age ξ. However, a direct transplantation of the Kolmogorov argument to the case of gravity waves is questionable because the possibility of the inverse cascade of wave energy diversifies feasible mechanisms of energy dissipation. In particular, the growth of the long-wave part of the wave spectrum decreases the role of small-scale wavelets as a factor of the dynamical equilibrium. Hence, the inner scale is unlikely to be entirely determined by U. In our forthcoming calculations, we shall use several characteristic values of δ_0 as based on earlier estimates (Glazman 1986; Glazman and Weichman 1989) according to which h is roughly half a meter; hence, δ_0 is between 0.01 and 0.1.

Finally, a question arises whether k_h presents the appropriate upper boundary for the range of wavenumbers falling into the framework of geometrical optics for radar (X- and C-band) backscatter at nadir incidence. A detailed discussion of this issue is given by Glazman (1990), and here we note only that, although wavenumbers higher than k_h might also satisfy the criterion of geometrical optics, the use of the higher-wavenumber portion of the spectrum would not change the character of the wind-speed and wave-age dependences of the sea state bias. This is so because, as mentioned above, the "Banner range" of the spectrum is independent of the wind and other external parameters. In other words, by adding the contribution from (6.4) to our integrals, we would slightly increase the value of γ, but such an essentially constant component would add no new information of oceanographic significance.

7. "Specular height" of a unidirectional sea

The integral representation for λ_1 in a unidirectional sea, as originally obtained by Jackson (1979), is

$$\lambda_1 = 2 \frac{\int_0^\infty E(k) G_1(k) dk}{\sigma \gamma^2} \tag{7.1}$$

where σ was introduced by (4.4) and γ^2 is given by (6.1). Furthermore,

$$G_1(k) = \int_0^k (k'^3 + 2k^2 k') E(k') dk'. \tag{7.2}$$

High-order spectral moments appearing in (7.1) and (7.2) necessitate the use of the filtered wavenumber spectrum (6.3). Here again, we replace $(k_0/k)^2 = x$ and $(k_0/k')^2 = y$ and represent the numerator of (7.1) as

$$\int E(k) G_1(k) dk = a(I_1 + I_2) \tag{7.3}$$

where

$$I_1(\mu, \delta) = \int_0^\infty \int_x^\infty \exp[-(x + y)$$
$$- \delta^2 (x + y)/xy](xy)^{-\mu} y^{-3/2} dy dx, \tag{7.4}$$

$$I_2(\mu, \delta) = 2\int_0^\infty \int_x^\infty \exp[-(x+y) - \delta^2(x+y)/xy](xy)^{-(1+\mu)}y^{1/2}dydx, \quad (7.5)$$

$$a = \frac{(\beta\xi^{-4\mu})^2}{4k_0}. \quad (7.6)$$

On account of (5.5) and (6.5),

$$\sigma\gamma^2 = \frac{(\beta\xi^{-4\mu})^{3/2}}{2\sqrt{2}k_0} I_3 \quad (7.7)$$

where

$$I_3(\mu, \delta) = 2\delta^{-\mu}K_\mu(2\delta)[2\delta^{1-\mu}K_{1-\mu}(2\delta)]^{1/2} \approx 2\delta^{-\mu}K_\mu(2\delta)\sqrt{\Gamma(1-\mu)}. \quad (7.8)$$

In terms of these quantities, (7.1) becomes

$$\lambda_1 = (\beta\xi^{-4\mu})^{1/2}R_1(\mu, \delta) = (x\xi^{-5})^{1/2}R_1(\mu, \delta) \quad (7.9)$$

where the latter equality results from (3.4), and

$$R_1(\mu, \delta) = \frac{\sqrt{2}(I_1 + I_2)}{I_3}. \quad (7.10)$$

Functions I_1 and I_2 are evaluated numerically and plotted in Fig. 5 to show that I_1 is small by comparison to I_2.

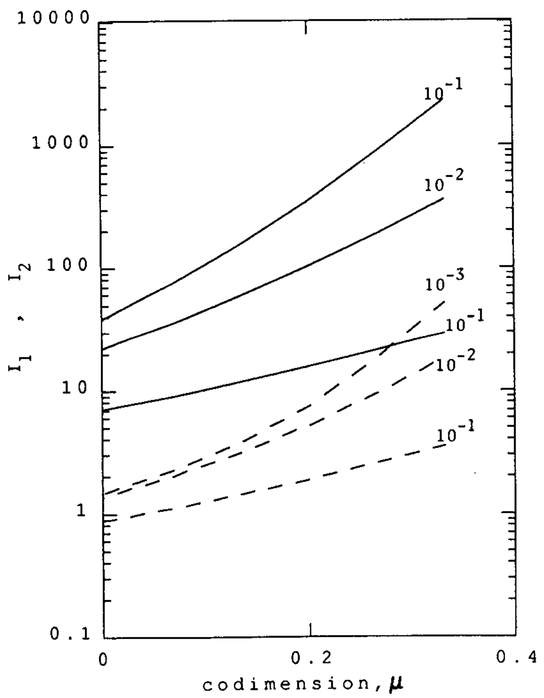

FIG. 5. Functions $I_1(\mu, \delta)$ (7.4) (dashed curves) and $I_2(\mu, \delta)$ (7.5) (solid curves) for selected values of δ as designated at each curve.

8. Sea state bias as a function of the wave age

Taking into account (3.4), (5.6), and (7.9), Eq. (4.1) can be written in a convenient form:

$$\text{bias} = -\frac{(x\xi^{-5})^{1/2}}{8}[R_0 + R_1]H_{1/3}. \quad (8.1)$$

In Fig. 4 functions R_0 and R_1 are compared to show that, for practical purposes, the component of the sea state bias due to the surface skewness, λ_0, can be neglected. Hence, the sea state bias in a developed sea becomes simply:

$$\text{bias} \approx -\frac{\lambda_1}{8}H_{1/3}, \quad (8.2)$$

where λ_1 is given by (7.9) with

$$R_1 \approx \frac{\sqrt{2}I_2}{I_3}. \quad (8.3)$$

That is, the bias is chiefly due to the difference between the zero-valued mean level and the mean level of the specular points. Taking β as 2.3×10^{-3} and employing (2.11) and (6.8) in the evaluation of R_1, we find $\lambda_1/8$ as a function of ξ for several values of δ_0; see Fig. 6. Evidently, the sea state bias is always negative. Its absolute value tends to decrease as the wave age increases (i.e., as the width of the wavenumber spectrum grows). This could be anticipated a priori based on the following simple argument. The wave spectrum width is controlled by μ and δ, both of which are functions of the nondimensional fetch. As the spectrum broadens with an increasing wave age, the mean number of specular facets per unit area of the sea surface also increases. In view of the central limit theorem, this increase should lead to a greater statistical normality in the distribution of specular point elevations.

Since the endpoint result depends, among other parameters, on $\delta_0 = gh/U^2$, any prediction of the sea state bias as a function of external factors requires an explicit specification of the intrinsic inner scale h. As was mentioned in section 6, there exists neither physical nor experimental justification for assuming h to be a function of the wind speed in general. Only at short fetches, when the dissipation due to breaking waves is important in the entire equilibrium range, can the value of h be expected to depend on external factors. A plausible dependence was proposed by appealing to the concept of the Taylor microscale (Glazman 1986). Considering the present case of mature seas, we shall continue treating h as a constant. So far, this assumption produced results that have been successfully confirmed by indirect observations. For instance, the character of the wind dependence for the nadir radar cross section at high degrees of wave development agrees rather well with altimeter measurements (Fig. 3 in Glazman and Pilorz 1990). Also using constant h, Glazman and Weichman (1989, 1990) arrived at a

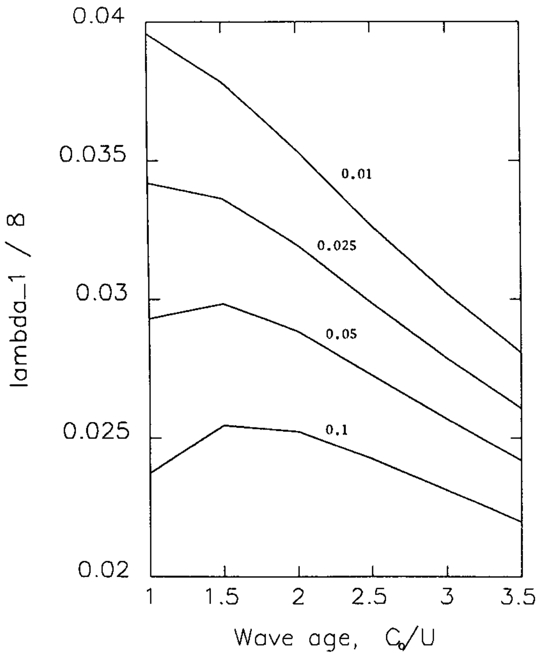

FIG. 6. The wave-age-dependent factor of the sea state bias, $\lambda_1/8$, based on (7.9), (8.3) and (6.8), and (2.11), for several values of δ_0 as designated at each curve.

rather realistic description of breaking wave statistics, including fluctuations of breaking wave rates observed over a small surface area.

In the following numerical example, we assume constant h and write (6.8) in the form $\delta = (h/X)x\xi^{-2}$ where the definition $x = gX/U^2$ was employed to exclude U from δ_0. The advantage of this particular form will be explained below. Since the nondimensional fetch x and wave age ξ are unambiguously related by (2.5) with ν given by (3.6), one ultimately arrives at

$$\delta = A^{-5+4\mu} \frac{h}{X} \xi^{3-4\mu}. \tag{8.4}$$

The choice between (8.4) and the alternative form $\delta = (gh/U^2)\xi^{-2}$ depends on the specific experimental conditions implied, that is, at a constant wind U, different degrees of wave development (quantified by either x or ξ) can be observed at different distances X along the wind vector. Such a situation is rather typical, for instance, of aircraft experiments in which the wave field is monitored along the flight path normal to the coast line as the wind remains unchanged. Therefore, this particular case should be modeled by keeping δ_0 constant, so that $\delta = \text{const} \times \xi^{-2}$. Figure 6 is most relevant to this case. On the contrary, sampling the ocean surface on the global scale (the case of satellite observations), one typically finds the wind speed in a global sample varying in a very wide range (say, from 0.5 to 25 m s^{-1}), while the fetch (as, for example, associated with the characteristic curvature of surface isobars) varies in a relatively narrower range (say, 70 to 300 km). Moreover, due to the linear dependence of x on X, as opposed to the inverse quadratic dependence of x on U, the wave age is much more sensitive to wind-speed variations than to fetch variations. Therefore, attributing most of the wave-age dependence to wind speed variations [i.e., employing (8.4) with a constant value of X], we have a better chance to correctly represent statistics of global satellite observations.

For the situation described by (8.4), we plot the sea state bias coefficient $\epsilon = -\lambda_1/8$ versus wave age for several values of h/X (in Fig. 7). The curves closely resemble hyperbolas of the form ξ^{-m} with m of order one. The dashed line represents an empirical fit found by Fu and Glazman (1991) based on analysis of 2.7 years worth of data gathered by the Geosat altimeter. Evidently, the theory produces best agreement with the experimental results if the values of h and X are taken as, e.g., 0.5 m and 125 km. These values are quite reasonable. Hence, taking into account both the simplicity of our idealized theory and the inevitable errors and biases of the experimental procedure employed in Fu and Glazman (1991), we conclude that the theory does bring out major features of the process.

9. Conclusions

Our main conclusion is that the weak-turbulence theory yields correct description of wave dynamics for equilibrium or near-equilibrium sea states. The theoretical fetch dependence of the wave energy, wave age,

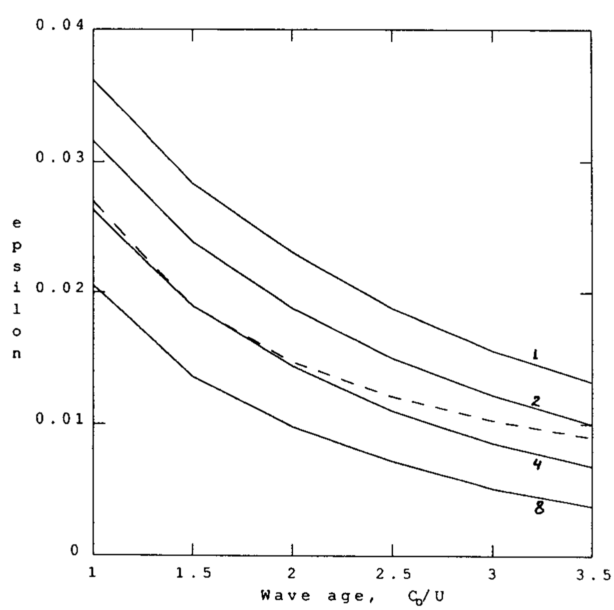

FIG. 7. The sea state bias coefficient $\epsilon \approx -\lambda_1/8$ for several values of h/X, where h is the inner scale and X is the dimensional fetch. The numbers at the curves give $(h/X) \times 10^6$.

surface skewness, "specular height," and other properties of a weakly nonlinear sea agrees reasonably well with the observed trends. For instance, substituting (2.3) and (2.4) into equation (5.5) and dividing it by $(U^2 g^{-1})^2$ produces a relationship between the wave age and the dimensionless wave energy, e:

$$\xi = \text{const}\, e^r, \quad \text{const} = [\beta\Gamma(1-\mu)/2]^{-r}, \quad r = 1/(4-4\mu). \tag{9.1}$$

For $\mu = 1/4$, we find that both r and const are in good agreement with the experimental data of Donelan et al. (1985), Dobson et al. (1989), and Glazman and Pilorz (1990). The "fetch laws" for the wave age, (3.6) and (3.8), also agree with the trends observed by different investigators, as summarized by Dobson et al. (1989).

Surface statistics responsible for the sea state bias are strong functions of D_H and wave age. Statistical description of surface geometry given in sections 4 through 8 allows one to relate sea surface properties accessible to remote sensing measurement (wave slope variance, sea surface skewness, etc.) with factors of air–sea interactions. Such relationships are particularly useful for studies of wave dynamics and air–sea exchanges in the open ocean where satellite techniques have obvious advantages over conventional observations.

Since a radar altimeter provides estimates of wind speed and significant wave height, it is reasonable to seek the sea state bias correction in the form

$$\text{bias} = B(\xi/\bar{\xi})^{-m} H_{1/3}. \tag{9.1}$$

Here, $\bar{\xi}$ is the mean value of the wave age over the World Ocean. The actual wave age can be estimated roughly by using $H_{1/3}$ and U as suggested in Glazman and Pilorz (1990), and coefficients B and m can be found empirically. In this respect, the attempt reported by Fu and Glazman (1991) appears to present an encouraging result.

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