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A theory of the Earth's background free oscillations

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[1] It has recently been established that the Earth's free oscillations are continuously excited by phenomena other than earthquakes and that these oscillations constitute the background noise in the normal mode band at quiet sites. On the basis of evidence that the excitation source is at or just above the Earth's surface, a normal mode theory of the Earth's free oscillations excited by random atmospheric loading is developed. The displacement field is expressed in the frequency domain in general terms of the cross-spectral density of air pressure disturbance. For spatially homogeneous and isotropic disturbance the cross-spectral density is approximated by the power spectral density and the coefficient of coherence with a coherence length much shorter than the wavelengths of normal modes. With this approximation the spectrum of ground acceleration is represented as the product of the pressure force term and the Earth response term. The final expression of the acceleration spectrum includes the effect of the gravity attraction of a disturbed air mass. A synthetic spectrum is calculated, using a power law decaying air pressure spectrum consistent with observations, assuming a frequency-dependent coherence length of air pressure fluctuation, taking into account the effect of the gravity attraction of a disturbed air mass. This synthetic spectrum exhibits distinct peaks of fundamental modes and complex troughs consisting of overtone modes, in quantitative agreement with the peaks and troughs of the observed spectrum. *INDEX TERMS:* 7255 Seismology: Surface waves and free oscillations; 7260 Seismology: Theory and modeling; 7299 Seismology: General or miscellaneous; *KEYWORDS:* seismic excitation, atmospheric turbulence, normal mode theory, Earth ground noise, long-period seismology

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1. Introduction

[2] It is now firmly established that the Earth's free oscillations are incessantly excited by phenomena other than earthquakes [*Nawa et al.*, 1998; *Suda et al.*, 1998b; *Tanimoto et al.*, 1998; *Kobayashi and Nishida*, 1998a, 1998b; *Nishida and Kobayashi*, 1999]. The excited modes are almost exclusively fundamental spheroidal modes, the intensities of which clearly show annual variations [*Nishida et al.*, 2000] as well as diurnal variations [*Suda et al.*, 1998a]. For these background free oscillations there is little correlation between the excitations of neighboring modes and between the excitations of a mode on neighboring days [*Nishida and Kobayashi*, 1999]. The oscillations are found to be resonant with the acoustic free oscillations of the atmosphere through two frequency windows [*Nishida et al.*, 2000]. All of these observations indicate that globally distributed, random atmospheric pressure disturbance is the most likely source for background free oscillations, although other possibilities, including oceanic disturbance, may not be ruled out.

[3] Free oscillations of the solid Earth excited by atmospheric disturbance have been discussed theoretically by *Watada* [1995] and *Lognonné et al.* [1998] in order to explain the Rayleigh waves excited by volcanic explosions [*Kanamori and Mori*, 1992; *Widmer and Zürn*, 1992; *Zürn and Widmer*, 1996]. They viewed the media as a coupled system consisting of the atmosphere and the solid Earth, and the excitation source as an indigenous source of the system. *Kobayashi and Nishida* [1998a, 1998b], *Tanimoto* [1999], and *Tanimoto and Um* [1999], on the other hand, regarded atmospheric disturbance as an external source that generates a pressure force onto the surface of the solid Earth. The latter approach is justified because the coupling between the atmosphere and the solid Earth is, in general, weak [*Watada*, 1995; *Lognonné et al.*, 1998] and because the excitation source has been suggested to lie at or just above the Earth's surface [*Nishida et al.*, 2000]. *Kobayashi and Nishida* [1998a, 1998b] developed their theory through a dimensional analysis of the energy balance expected between the incoming solar flux, stirred atmospheric turbulence and the excited free oscillations of the Earth. *Tanimoto* [1999] and *Tanimoto and Um* [1999] modified the stochastic approach of solar seismology [*Goldreich and Keeley*, 1977] to derive their expressions for excitation of the

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Earth's free oscillations by atmospheric turbulence. Following *Kobayashi and Nishida* [1998a, 1998b], *Tanimoto* [1999], and *Tanimoto and Um* [1999], we also regard atmospheric turbulence as an external source, but our approach is more phenomenological than theirs in that we do not use any physical model of turbulence, but instead formulate the problem using atmospheric parameters that can in principle be observed. Our approach follows more strictly the normal mode theory of the Earth in deriving the relation between ground acceleration and atmospheric pressure disturbance applied to the Earth's surface. We develop our theory to obtain the whole spectrum rather than to describe modal peak amplitudes so that our expression can be compared more directly to the results of spectral analyses of background free oscillations.

2. Theory

[4] We develop our theory following the textbook of *Dahlen and Tromp* [1998] (hereinafter referred to as DT). We consider a surface force density $\mathbf{t}(\mathbf{x}, t)$ acting upon a surface element at point $\mathbf{x}$ of the Earth's surface Σ. The displacement $\mathbf{s}(\mathbf{x}, t)$ produced by such force density can be written as a sum of normal modes

$$\mathbf{s}(\mathbf{x},t) = \sum_k \frac{\mathbf{s}_k(\mathbf{x})}{\omega_k} \int_{-\infty}^{t} A_k(t') e^{-\frac{\omega_k(t-t')}{2Q_k}} \sin\omega_k(t-t')dt' \qquad (1)$$

where ω_k is the eigenfrequency with an index k, Q_k and $\mathbf{s}_k(\mathbf{x})$ are the associated quality factor and eigenfunction, respectively, and

$$A_k(t) = \int_\Sigma \mathbf{t}(\mathbf{x},t)\cdot\mathbf{s}_k(\mathbf{x})d\Sigma \qquad (2)$$

(DT, p. 121). Our interest is the surface force density acting vertically downward:

$$\mathbf{t}(\mathbf{x},t) = -\hat{\mathbf{r}}P(\mathbf{x},t) \qquad (3)$$

where $\mathbf{x} = (R, \theta, \varphi)$ and $\hat{\mathbf{r}}$ is the unit radial vector in this polar coordinate system. We neglect the effect of wind shear, which would apply traction tangential to the surface. The radial component of displacement at the surface ($r = R$) of the SNREI (Spherically symmetric, non-rotating, elastic and isotropic Earth model) Earth (DT, p.259) is written as

$$\begin{aligned} s_r(\mathbf{x},t) &= \mathbf{s}(\mathbf{x},t)\cdot\hat{\mathbf{r}} \\ &= \sum_n\sum_l \frac{U_l^2(R)}{\omega_l}\sum_m \mathcal{Y}_{lm}(\theta,\varphi) \\ &\int_{-\infty}^{t} B_{lm}(t')e^{-\frac{\omega_l(t-t')}{2Q_l}} \sin\omega_l(t-t')dt' \end{aligned} \qquad (4)$$

where U_l is the radial eigenfunction of the spheroidal mode ${}_nS_l^m$ with radial number n, $\mathcal{Y}_{lm}$ is the real scalar spherical harmonics of degree l and order m (DT, p. 269), and

$$B_{lm}(t) = -\int_\Sigma P(\mathbf{x},t)\mathcal{Y}_{lm}(\theta,\varphi)d\Sigma \qquad (5)$$

In equation (5), ω_l, Q_l, and U_l should be understood as ${}_n\omega_l$, ${}_nQ_l$, and ${}_nU_l$, respectively. Assuming that the processes involved are random and stationary, we introduce the autocorrelation function ϕ of radial displacement:

$$\phi(\mathbf{x},\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s_r(\mathbf{x},t)s_r(\mathbf{x},t+\tau)dt \qquad (6)$$

and the cross-correlation function ψ of pressure fluctuations at points $\mathbf{x}'$ and $\mathbf{x}''$:

$$\psi(\mathbf{x}',\mathbf{x}'';\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} P(\mathbf{x}',t)P(\mathbf{x}'',t+\tau)dt \qquad (7)$$

By inserting equation (4) into equation (6) and using the definition (7), we obtain the expression for ϕ:

$$\begin{aligned} \phi(\mathbf{x},\tau) = &\sum_n\sum_{n'}\sum_l\sum_{l'} \omega_l^{-1}\omega_{l'}^{-1}U_l^2(R)U_{l'}^2(R) \\ &\sum_m\sum_{m'}\mathcal{Y}_{lm}(\theta,\varphi)\mathcal{Y}_{l'm'}(\theta,\varphi) \\ &\int_0^\infty dt'\int_0^\infty dt'' e^{-\omega_l t'/2Q_l} e^{-\frac{\omega_{l'}t''}{2Q_{l'}}} \sin\omega_l t' \sin\omega_{l'}t'' \\ &\int_\Sigma d\Sigma'\int_\Sigma d\Sigma''\mathcal{Y}_{lm}(\theta',\varphi')\mathcal{Y}_{l'm'}(\theta'',\varphi'') \\ &\psi(\mathbf{x}',\mathbf{x}'';t'-t''+\tau) \end{aligned} \qquad (8)$$

The Fourier transform of ϕ is the power spectral density (PSD) of radial displacement at $\mathbf{x}$, and the Fourier transform of ψ is the cross-spectral density of pressure fluctuations at $\mathbf{x}'$ and $\mathbf{x}''$:

$$\Phi(\mathbf{x},\omega) = \int_{-\infty}^{\infty}\phi(\mathbf{x},\tau)e^{-i\omega\tau}d\tau \qquad (9)$$

$$\Psi(\mathbf{x}',\mathbf{x}'';\omega) = \int_{-\infty}^{\infty}\psi(\mathbf{x}',\mathbf{x}'';\tau)e^{-i\omega\tau}d\tau \qquad (10)$$

By Fourier transforming equation (8) and using the real spherical-harmonic addition theorem (DT, p. 852), we obtain

$$\begin{aligned} \Phi(\mathbf{x},\omega) = &\sum_n\sum_{n'}\sum_l\sum_{l'}\frac{2l+1}{4\pi}\frac{2l'+1}{4\pi}U_l^2(R)U_{l'}^2(R) \\ &\left[-\frac{\omega_l}{2Q_l}+i(\omega_l+\omega)\right]^{-1}\left[-\frac{\omega_l}{2Q_l}+i(-\omega_l+\omega)\right]^{-1} \\ &\left[-\frac{\omega_{l'}}{2Q_{l'}}+i(\omega_{l'}-\omega)\right]^{-1}\left[-\frac{\omega_{l'}}{2Q_{l'}}-i(\omega_{l'}+\omega)\right]^{-1} \\ &\int_\Sigma d\Sigma'\int_\Sigma d\Sigma'' P_l(\cos\Theta')P_{l'}(\cos\Theta'')\Psi(\mathbf{x}',\mathbf{x}'';\omega) \end{aligned} \qquad (11)$$

where P_l is the Legendre polynomials of degree l and

$$\cos\Theta' = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi'-\varphi)$$

$$\cos\Theta'' = \cos\theta\cos\theta'' + \sin\theta\sin\theta''\cos(\varphi''-\varphi)$$

Polar angles Θ' and Θ'' are those between $\mathbf{x}'$ and $\mathbf{x}$ and between $\mathbf{x}''$ and $\mathbf{x}$, respectively, with the common apex at observation point $\mathbf{x}$. Equation (11) is the rigorous expression for the PSD of radial displacement due to pressure disturbance, the statistical property of which is given by cross-spectral density. Unfortunately, no information is available for the global distribution of the cross-spectral density of atmospheric pressure disturbance in the free oscillation band. We therefore must make several assumptions.

[5] We express the cross-spectral density of pressure disturbance as

$$\Psi(\mathbf{x}', \mathbf{x}''; \omega) = \sqrt{\hat{\Psi}(\mathbf{x}', \omega)\hat{\Psi}(\mathbf{x}'', \omega)}\rho(\mathbf{x}', \mathbf{x}''; \omega) \quad (12)$$

where ρ is the coefficient of coherence and $\hat{\Psi}(\mathbf{x}', \omega)$ and $\hat{\Psi}(\mathbf{x}''; \omega)$ are the PSDs of pressure disturbance at $\mathbf{x}'$ and $\mathbf{x}''$, respectively [*Hinich and Clay*, 1968; *Gossard and Hooke*, 1975, p. 333]. Many large-scale weather systems in the middle latitudes migrate eastward as a propagating wave. These "synoptic" weather systems have lower frequency than the seismic free oscillations. The atmospheric PSD is also likely to have spatial dependence, probably with larger amplitude associated with storm tracks in the mid latitudes. We, however, assume that the disturbance is spatially isotropic and homogeneous, and therefore allow no disturbance of propagating wave type. With this assumption, the cross-correlation function $\psi(\mathbf{x}', \mathbf{x}''; \tau)$ is symmetric with respect to time lag τ and hence the coefficient of coherence $\rho(\mathbf{x}', \mathbf{x}''; \omega)$ is a real function depending only on the ratio of distance $|\mathbf{x}' - \mathbf{x}''|$ to frequency-dependent coherence length $L(\omega)$:

$$\rho(\mathbf{x}', \mathbf{x}''; \omega) = h\left(\frac{|\mathbf{x}' - \mathbf{x}''|}{L(\omega)}\right) \quad (13)$$

where function $h(x)$ has the following property:

$$h(x) \sim \begin{cases} 1 & \text{if } x \ll 1 \\ 0 & \text{if } x \gg 1 \end{cases} \quad (14)$$

The PSD of atmospheric disturbance becomes position-independent:

$$\hat{\Psi}(\mathbf{x}', \omega) = \hat{\Psi}(\mathrm{x}'', \omega) = \hat{\Psi}(\omega) \quad (15)$$

and hence the PSD of ground displacement becomes also position-independent:

$$\Phi(\mathbf{x}, \omega) = \Phi(\omega) \quad (16)$$

The reported coherence length of atmospheric disturbance is, in general, less than 10 km at frequencies above 0.4 mHz [*Herron et al.*, 1969; *McDonald et al.*, 1971] and is much smaller than the wavelengths of normal modes in the 1–10 mHz band, which are on the order of 1000 km. In view of such a small coherence length it would be reasonable to adopt the simplest functional form for the coefficient of coherence:

$$h(x) = \begin{cases} 1 & \text{if } \mathrm{x} \le 1 \\ 0 & \text{if } \mathrm{x} \ge 1 \end{cases} \quad (17)$$

The integration in equation (11) can now be performed analytically by rotating the original coordinates to those with the pole at $\mathbf{x}'$ and then to those with the pole at $\mathbf{x}$ and by using the relations for Legendre polynomials:

$$\int_0^{2\pi} P_l(\cos\alpha\cos\beta + \sin\alpha\sin\beta\cos\theta)d\theta = 2\pi P_l(\cos\alpha)P_l(\cos\beta)$$

$$\int_{-1}^{1} P_l(z)P_{l'}(z)dz = \frac{2}{2l+1}\delta_{ll'}$$

$$P'_{l+1}(z) - P'_{l-1}(z) = (2l+1)P_l(z)$$

The result of integration yields a simple expression for Φ:

$$\Phi(\omega) = \hat{\Psi}(\omega)\frac{R^4}{2}\sum_n\sum_l U_l^4(R)[P_{l-1}(\cos\theta_c) - P_{l+1}(\cos\theta_c)] \cdot \left[\left(\frac{\omega_l}{2Q_l}\right)^2 + (\omega_l + \omega)^2\right]^{-1}\left[\left(\frac{\omega_l}{2Q_l}\right)^2 + (\omega_l - \omega)^2\right]^{-1} \quad (18)$$

where θ_c is the arc length corresponding to the coherence length. Since

$$\theta_c(\omega) = L(\omega)/R \ll \pi/l \quad (19)$$

then

$$P_{l-1}(\cos\theta_c) - P_{l+1}(\cos\theta_c) \sim \frac{2l+1}{2}\theta_c^2(\omega) \quad (20)$$

Inserting this approximation into equation (18), we obtain our final expression for $\Phi(\omega)$ or for $\Upsilon(\omega) = \omega^4\Phi(\omega)$, which is the PSD of ground acceleration:

$$\Upsilon(\omega) = \left[\hat{\Psi}(\omega)L^4(\omega) \cdot \frac{4\pi R^2}{L^2(\omega)}\right]E(\omega) \quad (21)$$

where

$$E(\omega) = \sum_n\sum_l \frac{2l+1}{4\pi}\frac{U_l^4(R)}{4}\left[\left(\frac{\omega_l/\omega}{2Q_l}\right)^2 + \left(\frac{\omega_l}{\omega} + 1\right)^2\right]^{-1} \cdot \left[\left(\frac{\omega_l/\omega}{2Q_l}\right)^2 + \left(\frac{\omega_l}{\omega} - 1\right)^2\right]^{-1} \quad (22)$$

The bracket term in equation (21) is the excitation term, where $\hat{\Psi}L^4$ and $4\pi R^2/L^2$ represent the PSD of the effective force acting on a coherent area of L^2 and the number of mutually independent coherent areas over the whole globe, respectively. Hence, the excitation term stands for the total PSD of the effective force acting on the Earth's surface. $E(\omega)$ represents the response of the Earth to the unit PSD of such effective force. Although a vertical instrument senses not only the acceleration of inertial motion but also accelerations due to displacement relative to the Earth's gravity center and due to redistribution of mass in the Earth, the latter two effects can be neglected in the 1–10 mHz band [*Gilbert*, 1980]. Figure 1 shows response spectrum $E(\omega)$ in a frequency range from 1 to 10 mHz calculated for the Earth model PREM [*Dziewonski and Anderson*, 1981]. Since we have truncated the normal mode summation at 10 mHz, a sharp spectral fall appears near 10 mHz, which should be regarded as an artifact. Resonant peaks are mostly of fundamental spheroidal modes, but the first overtone peaks

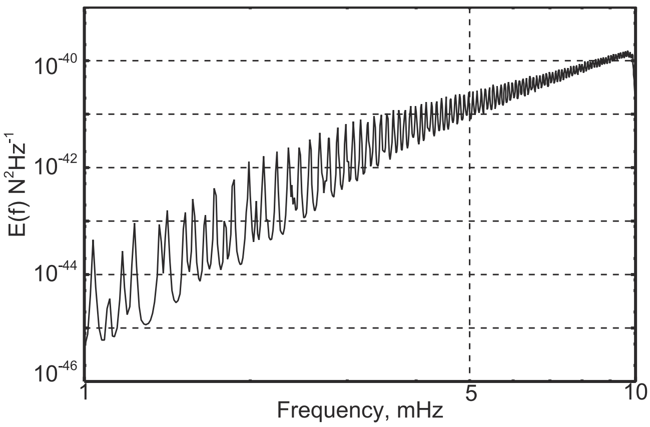

Figure 1. Earth response function $E(f)$ in kg^{-2} to the unit PSD of effective force (= 1 N^2 Hz^{-1}) per a unit area of the Earth's surface (see equation (22) in the text).

are also significant at the lowest frequencies. On the other hand, the shape of spectral troughs is largely controlled by the excited overtone modes. In what follows, we regard $\hat{\Psi}(\omega)$ and $\Upsilon(\omega)$ as one-sided spectra by multiplying the original PSD values by a factor of 2 for inclusion at negative frequencies.

3. Comparison With Observations

[6] We compare the above theory with the background free oscillation spectrum obtained by *Nishida et al.* [2000]. They analyzed 10 s sampling records of STS-1 vertical seismometers at 17 quiet stations of the IRIS (Incorporated Research Institutions for Seismology) network and 8 quiet stations of the GEOSCOPE network between 1989 and 1998. For each station, *Nishida et al.* [2000] divided the whole record into 1 day segments with an overlap of 3 hours, discarding all segments affected by earthquakes with moments greater than 10^{18} N m and affected by nonstationary local ground or instrumental noise using the method described by *Nishida and Kobayashi* [1999], with the prescribed threshold powers of 1×10^{-18} m^2s^{-3} and 3×10^{-18} m^2s^{-3}, respectively. *Nishida et al.* [2000] stacked the power spectra of the remaining 1 day segments over all the stations and over the whole period. Figure 2 shows the acceleration power spectrum so obtained in a frequency range from 1 to 10 mHz, which is considered to be free from the effect of earthquakes and the effect of local or instrumental nonstationary noise. The observed spectrum exhibits clearly the fundamental spheroidal peaks at levels on the order of 0.8×10^{-18} m^2s^{-3} in a frequency band from less than 2 mHz to more than 8 mHz, which are explained by the theory developed in the previous section.

[7] Since in equation (21) the theoretical acceleration spectrum is expressed as the product of the excitation term and response term and since the response term has been obtained as in Figure 1, the synthetic spectrum can in principle be calculated if the excitation source is specified. We, however, note that the observed acceleration spectrum includes the effect of gravitational attraction of atmospheric disturbance, which is not taken into account in the expression of equation (21). We also note that the observed spectrum is the one for tapered records, while the effect of tapering is not taken into account in equation (21). The expression including these effects is given by

$$\Upsilon(\omega) = \left[\hat{\Psi}(\omega)L^4(\omega)\cdot\frac{4\pi R^2}{L^2(\omega)}\right]\left[\frac{\Gamma(\omega) * E(\omega)}{2\pi}\right] + (2\pi G)^2 M(\omega) + N(\omega) \tag{23}$$

where $N(\omega)$ represents noise. The first term of equation (23) corresponds to equation (21) but $E(\omega)$ is convolved with the PSD of the taper function $\Gamma(\omega)$. This convolution is required because if we sample many records with a finite length T from a stationary random process with its PSD $S(\omega)$ and apply a taper function to each, then the average PSD of the tapered records gives an unbiased estimate of $S(\omega)$ convolved with $\Gamma(\omega)$ [*Hinich and Clay*, 1968]. Thus the theoretical spectrum (21) can be corrected for the effect of tapering by convolving the right-hand side of equation (21) with the PSD of the window. Since in equation (21) the atmosphere-related quantities $\hat{\Psi}(\omega)$ and $L(\omega)$ are fairly flat in a narrow band at any particular frequency, as implied in equation (25), the correction effectively replaces the response spectrum $E(\omega)$ by its convolution with the window spectrum $\Gamma(\omega)$ as in equation (23). We used the Welch window for tapering, the PSD of which is

$$\Gamma(\omega) = \frac{15}{2}T\left[\frac{1}{X^2}\left(\frac{\sin X}{X} - \cos X\right)\right]^2, \quad X = \omega T/2$$

The second term of equation (23) represents the gravitational effect of air mass disturbance in Bouguer plate approximation, where G is the gravitational constant and $M(\omega)$ is the PSD of air mass fluctuation in a vertical column with a unit cross-sectional area. If the air mass fluctuation is the only cause for pressure fluctuation, $M(\omega)$ is proportional to $\hat{\Psi}(\omega)$:

$$(2\pi G)^2 M(\omega) = c^2\hat{\Psi}(\omega) \tag{24}$$

where $c = 2\pi G/g$ (= 4.3 nm s^{-2}/hPa) and g is gravitational acceleration. In equation (24), c may be understood as the

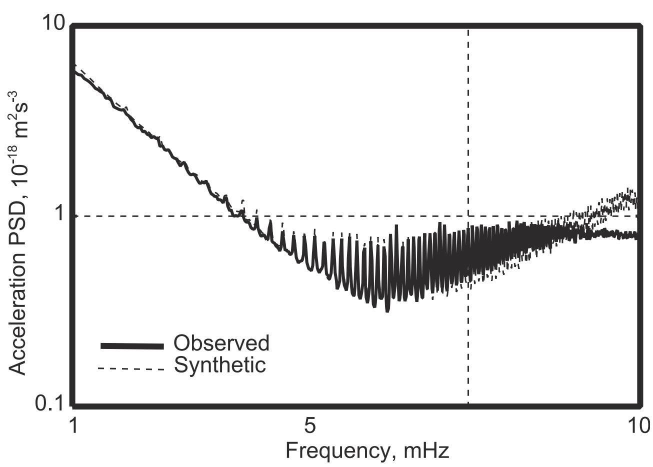

Figure 2. Averaged power spectrum of acceleration seismograms in seismically quiet days [*Nishida et al.*, 2000] in comparison with the theoretical spectrum $\Upsilon(f)$ (see equation (23)).

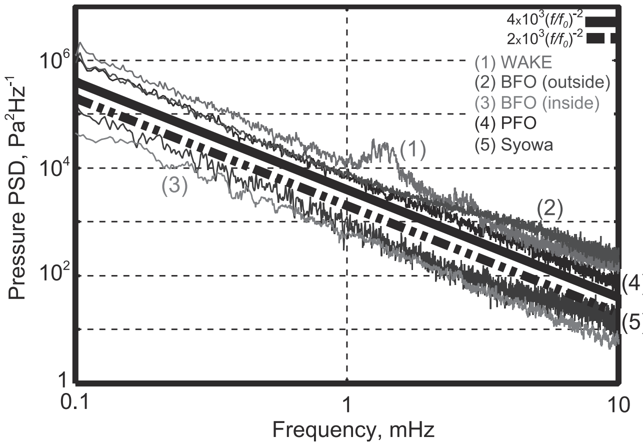

Figure 3. Averaged power spectra of atmospheric pressure at several IRIS stations and at Syowa station, Antarctica, in comparison with the theoretical spectra $\hat{\Psi}(f)$ (see Table 1).

conversion factor (admittance) from atmospheric pressure change to gravity change in Bouguer plate approximation. Equation (24) is a good approximation in a period range from a few hours to one year [*Torge*, 1989, p.382]. In the period range of our interest, however, $\hat{\Psi}(\omega)$ would involve not only the pressure loading effect of air mass redistribution but would also involve the effect of dynamic pressure of winds hitting the Earth's surface. This would effectively introduce frequency dependence into the conversion factor c in equation (24). Hence we replace the second term of equation (23) with equation (24) where c is understood as $c(\omega)$. The third term of equation (23), the noise term, should be smaller than the first term in a frequency range where free oscillation peaks are clearly visible.

[8] In equation (23) the unknown quantities are $\hat{\Psi}(\omega)$, $L(\omega)$, $M(\omega)$ (or $c(\omega)$) and $N(\omega)$. Within these the behavior of $\hat{\Psi}(\omega)$ is relatively well known. Figure 3 shows the power spectra of atmospheric pressure variations at several IRIS stations (WAKE, BFO (inside the vault), BFO (outside the vault) and PFO) and at Syowa station, Antarctica, in a frequency range from 0.1 to 10 mHz. The spectrum of Syowa station is based on the barographic records on the quietest 18 days of 1994, while the spectra of the IRIS stations are obtained from the barographic records on relatively quiet days (roughly 100 days) in each 1-year period between 1992 and 1998. It will be shown later that the PSD level of barographic records for only quiet days are consistent with the ground acceleration spectrum. Although the PSD values differ considerably among stations, their spectral trends change little and they all decay approximately as f^{-2} where f is frequency. Since this f^{-2} decay is a well-established observation in atmospheric science [*Gossard and Hooke*, 1975, p. 7], it will be fixed in the following analysis, with the additional assumption that L and c also follow power law decays:

$$\hat{\Psi}(f) = \hat{\Psi}_0\ (f_0/f)^2$$

$$L(f) = L_0\ (f_0/f)^\beta$$

$$c(f) = c_0\ (f_0/f)^\gamma \quad (25)$$

Table 1. Frequency Dependence of Atmospheric Quantities[a]

PSD of Pressure $\hat{\Phi}$, Pa^2 Hz^{-1}	Correlation Length L, km	Conversion Factor c, nm s^{-2} hPa^{-1}
$4 \times 10^3 \times \left(\frac{f}{f_0}\right)^{-2}$	$0.6 \times \left(\frac{f}{f_0}\right)^{-0.12}$	$4 \times \left(\frac{f}{f_0}\right)^{-0.4}$
$2 \times 10^3 \times \left(\frac{f}{f_0}\right)^{-2}$	$1 \times \left(\frac{f}{f_0}\right)^{-0.15}$	4

[a] Here f, frequency; f_0, reference frequency (=1 mHz).

where f_0 is the reference frequency, which is taken to be 1 mHz. The quantities with subscript 0 are those at this reference frequency. Since conversion factor c has to take a Bouguer correction value of 4.3 at periods of a few hours [*Torge*, 1989, p. 382], $c(f)$ is assumed to be 4 at the reference frequency of 1 mHz (c_0 = 4 nm s^{-2} hPa^{-1}). At higher frequencies, $c(f)$ is expected to decrease where the air mass effect becomes less important [*Banka and Crossley*, 1999]. Among other parameters, the power indices β and γ control the trend of the spectral curve, while the constants $\hat{\Psi}_0$ and L_0 control the amplitude level of the spectrum. The free oscillations amplitudes are sensitive to β but insensitive to γ. The low frequency trend of the acceleration spectrum is sensitive to γ but insensitive to β. The residual of the observed PSD from the first two terms in the right-hand side of equation (23) is regarded as noise and the noise level is estimated by assuming a linear trend, $N(f) = N_0 f/f_0$. We determine these parameters by a trial and error method to obtain a good match in both the spectral trend and amplitude level between the observation and calculation. There is little trade-off in this parameter search if the searched values are limited to one or two digits and if the fitting is attempted in a frequency range below ~7 mHz. Note that the observed spectral trend changes significantly across ~7 mHz and that this change cannot be accounted for by our parameterization as in equation (25). Table 1 summarizes the result of this simple parameter search. In this table we have also added the result for a case of frequency-independent conversion factor (c = 4), which was obtained by limiting our interest at frequencies above 2

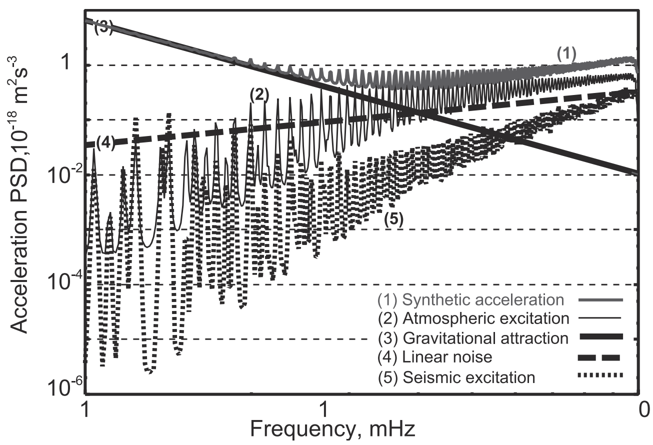

Figure 4. Synthetic acceleration power spectrum $\Upsilon(f)$ that consists of the air mass gravity attraction term, atmospheric excitation term and linear noise term (see equation (23) in the text). The power spectrum of the synthetic seismogram for seismically quiet 437 days in the 10-year period is also shown [*Suda et al.*, 1998b].

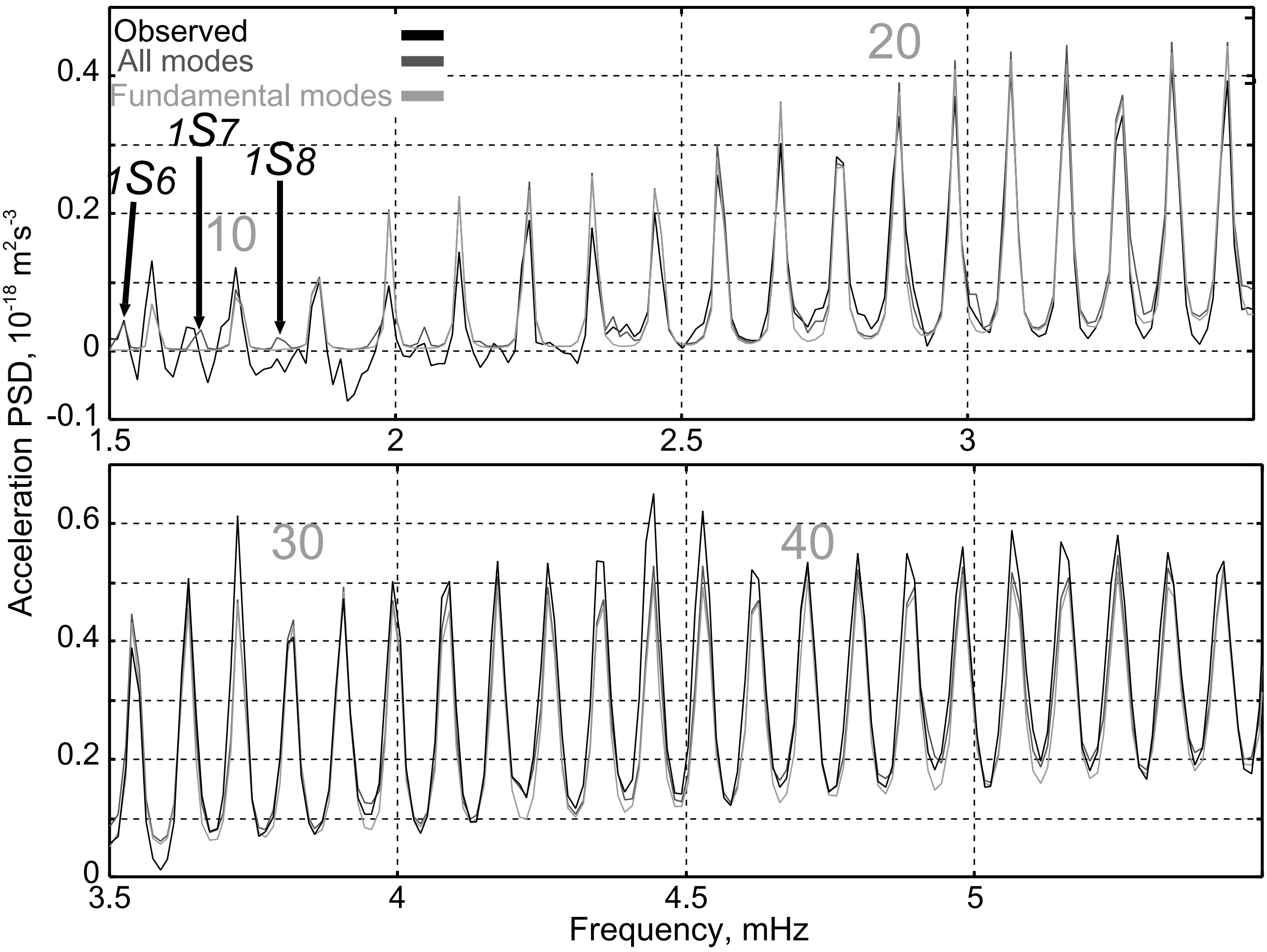

Figure 5. Averaged power spectrum of acceleration seismograms in seismically quiet days [*Nishida et al.*, 2000]. The spectrum is detrended in the frequency range 1.5–5.5 mHz so that it can be compared directly to the free oscillation term of the synthetic spectrum $\Upsilon(f)$ (the first term of equation (23)). Substraction of the trend component makes the spectrum oscillatory around the zero level at frequencies below 2 mHz. The synthetic spectrum excluding the overtone modes is also shown. Several fundamental modes are indicated by their angular order numbers. The subtle undulation of the observed spectrum especially at the trough part can only be explained by the synthetic spectrum including the overtone modes. See color version of figure at back of this issue.

mHz. A good match cannot be obtained below 2 mHz in this case, but at higher frequencies the fit is as good as in the first case.

[9] In equation (23) the acceleration PSD in the 1–10 mHz band consists of the gravity attraction term, the free oscillation term and the remaining noise term which measures the noise level in the 1–10 mHz band. Figure 4 indicates, for the frequency-dependent admittance model, how each of these terms contributes to the resultant acceleration PSD. The greatest contributor at frequencies below 3 mHz is the gravity attraction of air mass and the greatest contributor above 3 mHz is the Earth's free oscillations due to random atmospheric loading. At frequencies above 4 mHz the linear noise term exceeds the gravity attraction term, yet remains well below the free oscillation term. One might suspect that this noise term approximates the cumulative effect of free oscillations long after large earthquakes and immediately after small earthquakes. We evaluate the cumulative seismic effect using the 10-yearlong synthetic seismogram at station SUR of the IDA (Incorporated Research Institute for Seismology) network calculated by *Suda et al.* [1998b]. They took into account all earthquakes with magnitudes greater than 4.6 occurring in the 1986–1995 period and selected 437 seismically quiet days from these 10 years to obtain the average power spectrum for the quiet days. In Figure 4 we reproduce the seismic spectrum so calculated, which is well below the noise term except near the high-frequency end, indicating that the noise term approximates disturbances other than the cumulative effect of earthquakes. We note at the same time that at frequencies below 1.5 mHz the peak heights of the modes excited by atmospheric surface disturbance and those by earthquakes are almost coincident. Obviously, a more stringent criterion is required to define seismically quiet days than the one adopted by *Suda et al.* [1998b] to isolate the lowest frequency modes of background free oscillations from those excited by earthquakes.

[10] In Figure 2 we superpose the resultant synthetic power spectrum onto the observed spectrum. The agreement is excellent, although the linearly trending noise term cannot account for the change in the observed spectral trend at ~7 mHz. The agreement extends not only to the overall spectral shape but also to some detailed features. In order to highlight the latter point, we detrend the observed spectrum between 1.5 and 5.5 mHz. Figure 5 shows a comparison of such a detrended spectrum with the first term of the synthetic spectrum (23). Also added is the corresponding synthetic spectrum that excludes the overtone modes and takes into account only the fundamental modes. The major peaks are all those of the fundamental modes. The peak heights of the observed modes vary in a complex way with frequency. For example, we observe lowering of peak heights for the modes ${}_0S_{19}$, ${}_0S_{24}$, ${}_0S_{27}$, and ${}_0S_{30}$. Such complex variation is reasonably well reproduced by the synthetic spectrum. Even more remarkably, the observed spectrum shows subtle undulation at the trough part, which is again well simulated by the synthetic spectrum. On the other hand, the synthetic spectrum, including only the fundamental modes, does not show such undulation and hence the agreement with the observed spectrum is much worse. For example, the observed spectrum at frequencies below 2 mHz shows relative peaks in the trough parts of the fundamental modes, which may correspond to the first higher modes ${}_1S_6$, ${}_1S_7$, and ${}_1S_8$. In the 2–3 mHz frequency range, higher mode contaminations are observed between two adjacent peaks of fundamental modes with degrees 12–13, 15–16, and 18–19, respectively. At frequencies above 3 mHz, the bottom parts are lifted by excitation of the higher modes between two adjacent peaks of fundamental modes with degrees 26–27, 31–32, 34–35, 36–37, 39–40, 42–43, 44–45, and 47–48, respectively. Thus, Figure 5 demonstrates that the troughs of the observed spectrum consist of the overtone modes of the background free oscillations rather than some unknown noise, and hence that the relative excitation of overtone modes against fundamental modes is in good agreement between the observation and calculation. This agreement strongly supports our basic assumptions for the excitation source. We also note that both the observed and synthetic peak heights gradually decrease with decreasing frequency below 3 mHz, in an opposite tendency to the peak heights of the synthetic spectrum for earthquakes (Figure 4).

4. Discussion

4.1. Atmospheric Disturbance Responsible for Free Oscillations

[11] We have determined the PSD level of air pressure spectrum $\hat{\Psi}$ so as to obtain a good match between the observed and synthetic acceleration spectra Υ (Table 1). The PSD level calculated for each of the two cases in Table 1 is shown in Figure 3, which is well among the observed pressure spectra of atmospherically quiet days. It is perhaps meaningless to discuss why the agreement is better for quiet days where the PSD level is an order of magnitude smaller than the PSD level at the same station on noisy days, in part because we have grossly approximated air pressure disturbance to be isotropic and homogeneous. The PSD in this approximation may correspond to one after a spatial average of pressure fluctuation over a coherent area, which should be different from the PSD at each observational site. Similarly, the coherence length in this approximation may correspond to the one after an azimuthal average, which should be different from the coherence length measured at each observational site.

[12] We can calculate the coherence length of atmospheric disturbance and the conversion factor of air mass effect, using the formulae given in Table 1. The frequency-dependent conversion factor $c(f)$ decreases as frequency increases (from 4 at 1 mHz to 2 at 6 mHz), a tendency expected if wind pressure becomes more important and hence air mass loading becomes less significant as frequency increases. The coherence length decreases slowly as frequency increases from 0.6 (1.0) km at 1 mHz to 0.5 (0.8) km at 6 mHz for the frequency-dependent (frequency-independent) admittance model. Such a tendency has been reported through observations in a lower frequency range where the coherence length decreases from about 10 km at 0.4 mHz to about 2 km at 2 mHz [*Herron et al*, 1969], showing an f^{-1} decay of coherent length. This trend suggests a coherence length of 0.7 km at 6 mHz in crude agreement with the value obtained in our analysis, although the frequency dependence is different. We have to be careful when directly comparing the frequency dependence of coherence length, however, because we assumed isotropic and homogeneous disturbance while observed pressure fluctuations usually move with wind [*Herron et al*, 1969].

4.2. Evidence for Coupling With Atmospheric Free Oscillations

[13] In Figure 5 the observed amplitude of the mode ${}_0S_{29}$ is significantly greater than the calculated amplitude, while the amplitudes of the adjacent modes are all in good agreement between the observation and calculation. Similarly the observed amplitudes of ${}_0S_{37}$ and its neighboring modes are greater than not only those of the adjacent modes, but also of the calculated amplitudes of ${}_0S_{37}$ and its neighboring modes. The excess amplitudes of these modes in the background free oscillations have been found for the first time by *Nishida et al.* [2000] and have been interpreted as evidence for the acoustic coupling of free oscillations between the solid Earth and the atmosphere [*Watada*, 1995; *Lognonné et al.*, 1998]. According to *Lognonné et al.* [1998] the fundamental modes with degrees 28–29, 34–37, and 42–44 are coupled with the trapped or partially trapped atmospheric acoustic modes of the fundamental and first- and second- higher mode branches in the frequency windows of 3.68, 4.40–4.65, and 5.07 mHz, respectively. Clearly, the excess amplitudes *Nishida et al.* [2000] detected in the background free oscillations are those in the first two predicted windows. In addition, Figure 5 indicates an excess of the observed amplitudes of ${}_0S_{44}$ and its neighboring modes with respect to the calculated modes in the predicted third window near 5.07 mHz. Thus, the observed background free oscillations (Figure 5) show the excess amplitudes in all the first three frequency windows in agreement with the coupling theory between the solid Earth and the atmosphere. The calculated amplitudes based on our theory indicate that these excess amplitudes are not due to the interference of other seismic modes.

4.3. Comparison With Other Theories

[14] If the interference with other modes is ignored, the PSD of modal acceleration at frequency ω_l is approximated, using equations (21) and (22) and taking into account that $Q_l \gg 1$, as

$$\Upsilon(\omega_l) = \left[\hat{\Psi}(\omega_l) L^4(\omega_l) \cdot \frac{4\pi R^2}{L^2(\omega_l)}\right] \left[\frac{2l+1}{4\pi}\left(Q_l \frac{U_l^2(R)}{2}\right)^2\right] \quad (26)$$

We multiply (26) with an effective frequency width ω_l/Q_l [*Aki and Richards*, 1980, p. 376] and take the square root to obtain the modal RMS acceleration amplitude:

$$A_l \sim \left[\sqrt{\hat{\Psi}(\omega_l)\omega_l} L^2(\omega_l) \cdot \frac{R}{L(\omega_l)}\right] \times \left[\sqrt{2l+1}\sqrt{Q_l} \frac{U_l^2(R)}{2}\right] \quad (27)$$

This expression has the form [excitation term] × [Earth response term]. In the excitation term $\sqrt{\hat{\Psi}(\omega_l)\omega_l}$ is the RMS amplitude of atmospheric pressure variation in a frequency band ω_l around the central frequency ω_l, $\sqrt{\hat{\Psi}(\omega_l)\omega_l} L^2(\omega_l)$ represents the RMS amplitude of the effective force acting on the coherent area, and R/L gives a one-dimensional measure for the number of coherent areas over the Earth's surface. Expression (27) may be compared to that obtained by *Kobayashi and Nishida* [1998a, 1998b]:

$$A_l \sim \left[\left(p_H \frac{\omega_H}{\omega_l}\right) H^2 \cdot \frac{R}{H}\right] \times \left[Q_l \frac{4}{M_l}\right] \quad (28)$$

where $\omega_H = 2\pi/\tau_H$ and τ_H is the characteristic time of atmospheric convective circulation, p_H the air pressure at frequency ω_H and H is the pressure scale height of atmosphere. These quantities are related to each other as $p_H \sim \rho_a (H/\tau_H)^2$ through air density ρ_a. In the excitation term the RMS amplitude of the effective force is given by $p_H (\omega_H/\omega_l) H^2$. In the response term M_l is the effective mass involved in the lth mode oscillation. *Kobayashi and Nishida* [1998a, 1998b] evaluated M_l using the approximate relation $M_l \sim 4\pi R^2 \rho_s \lambda_l$, where ρ_s is the density of the solid Earth and λ_l is the modal wavelength. Using the orthonormality relation for the radial eigenfunctions (DT, p. 279), the effective mass may also be defined as

$$M_l \sim \frac{\int_0^R \rho_s (U_l^2 + V_l^2) 4\pi r^2 dr}{U_l^2(R) + V_l^2(R)} = \frac{4\pi}{U_l^2(R) + V_l^2(R)} \quad (29)$$

where V_l is the radial eigenfunction for horizontal displacement of the spheroidal oscillations as defined in DT (p.268). Equation (29) implies an approximate relation:

$$\frac{4}{M_l} \sim \frac{U_l^2(R)}{2} \quad (30)$$

Thus the Earth response terms in equations (27) and (28) are almost identical except for the factors $\sqrt{2l+1}$ and $\sqrt{Q}$. The difference of $\sqrt{2l+1}$ comes from the fact that equation (28) is valid for a singlet while equation (27) has been derived for a multiplet (the $2l + 1$ oscillations associated with a given eigenfrequency ω_l). Since what we observe are multiplet peaks, the right-hand side of equation (28) has to be multiplied by $\sqrt{2l+1}$ when it is compared to observations. The difference in Q arises from the fact that equations (27) and (28) deal with the response of the Earth to temporarily random excitation and to coherent excitation, respectively. For stationary random disturbance, an oscillator accumulates power in proportion to its life time cycle Q so that the RMS amplitude is proportional to $\sqrt{Q}$. In equation (28), therefore, Q must be replaced with $\sqrt{Q}$.

Table 2. Frequency Dependence of Air Pressure Disturbance[a]

	Local Pressure $\sqrt{\hat{\Psi}(f)f}$		Effective Force $\sqrt{\hat{\Psi}(f)fL^2(f)}$		Excitation Force $\sqrt{\hat{\Psi}(f)fL^2\frac{R}{L(f)}}$	
	PSD	RMS Amplitude	PSD	RMS Amplitude	PSD	RMS Amplitude
KN	–	–	f^{-3}	f^{-1}	f^{-3}	f^{-1}
T	f^{-3}	f^{-1}	f^{-9}	f^{-4}	f^{-6}	$f^{-\frac{5}{2}}$ [b]
Present study	f^{-2}	$f^{-\frac{1}{2}}$	$f^{-2.6}$	$f^{-0.8}$	$f^{-2.3}$	$f^{-0.65}$ [c]

[a] KN, *Kobayashi and Nishida* [1998a, 1998b]; T, *Tanimoto* [1999]
[b] Based on our arrangement (31) with constant κ.
[c] In case of constant conversion factor.

[15] Our expression (27) may also be compared to that obtained by *Tanimoto* [1999]:

$$A_l \sim \left[\left(\kappa p_H \frac{\omega_H}{\omega_l}\right) \lambda^2(\omega_l) \cdot \frac{R}{\lambda(\omega_l)}\right] \times \left[\sqrt{Q_l} \frac{U_l(R)\sqrt{U_l^2(R) + V_l^2(R)}}{2}\right] \quad (31)$$

where $\kappa = 46/(2\pi)^{\frac{5}{2}} \sim 0.46$ and $\lambda(\omega_l) = H(\omega_H/\omega_l)^{\frac{3}{2}}$. Note that we rearranged Tanimoto's expression as in equation (31) for ease of comparison with equations (27) and (28). In the excitation term $p_H(\omega_H/\omega_l)$ represents the RMS amplitude of local pressure fluctuation and the RMS amplitude of the effective force is given by $\kappa p_H(\omega_H/\omega_l)\ \lambda^2(\omega_l)$. *Tanimoto and Um* [1999] proposed κ to be frequency-dependent such that the above value of 0.46 is the one at the high frequency limit in a case where two adjustable parameters take particular values. The constant κ was used by *Tanimoto* [1999] to explain the observed low-frequency noise of seismograms below 3 mHz in terms of the Earth's free oscillations excited by atmospheric turbulence, although his view has now been rejected [*Tanimoto and Um*, 1999]. The frequency-dependent κ was used by *Tanimoto and Um* [1999] to explain the observed background free oscillation amplitudes in a frequency range from 3 to 7 mHz. The response term in equation (27) and that in equation (31) have the same Q dependence but different eigenfunction dependence. The factor $\sqrt{2l+1}$ is absent in the latter.

[16] Table 2 summarizes the frequency dependence of air pressure disturbance for the physical models of *Kobayashi and Nishida* [1998a, 1998b] and of *Tanimoto* [1999] and for the phenomenological model in the present study. In Table 2, air pressure disturbance is specified by the local pressure fluctuation at an observational site, the effective force acting on a coherent area of L^2 and the excitation force contributing directly to the excitation of a modal oscillation. The local pressure fluctuation in the present study is given on the observational ground such that its power spectrum follows the well-known f^{-2} decay (Figure 3). In the model of

Kobayashi and Nishida [1998a, 1998b] local pressure fluctuation is assumed to contribute little to background free oscillations, so this term is blanked in Table 1. *Tanimoto* [1999] and *Tanimoto and Um* [1999] referred to the Kolmogorov convective turbulence model [*Landau and Lifshitz*, 1987] to argue for f^{-1} decay of the RMS amplitude of pressure fluctuation. We note that this f^{-1} decay is an expression for an observer moving with the mean flow of fluid and is different from the expression at a fixed point of observation (see the difference between equations (33.7) and (33.8) of *Landau and Lifshitz* [1987]). Observations of pressure fluctuation are usually made at a fixed point, where the $f^{-2/3}$ decay of RMS amplitude, or equivalently the $f^{-7/3}$ decay of PSD, is the appropriate expression for the Kolmogorov model. In fact, the $-7/3$ slope of the PSD has often been referred to as one in the Kolmogorov model of atmospheric turbulence [e.g., *Hauf et al.*, 1996]. *Tanimoto and Um* [1999] analyzed air pressure records to obtain a result consistent with the well known f^{-2} decay of PSD [*Gossard*, 1960], which is equivalent to the $f^{-1/2}$ decay of RMS amplitude. This $-1/2$ slope of the observed RMS amplitude is less steep than the -1 slope of the model RMS amplitude in equation (31).

[17] In Table 2 the frequency dependence of either the effective force or excitation force in our analysis was obtained so as to be consistent with the observed ground acceleration at frequencies below $\sim$7 mHz, above which the observed acceleration spectrum shows a different trend for reasons not clear at present. *Kobayashi and Nishida* [1998a, 1998b] derived frequency dependence from a dimensional analysis of fluctuation of the atmospheric convection cell, assuming that the cell size is comparable to the pressure scale height and that the effect of convective eddies with smaller sizes is insignificant. The frequency dependence of *Tanimoto* [1999] and *Tanimoto and Um* [1999] was obtained by summing all the contributions from various wavelength components of turbulent eddies following the formalism of *Goldreich and Keeley* [1977] and assuming a Kolmogorov type of wavelength dependence for pressure fluctuation. In our phenomenological approach, on the other hand, we do not use a physical model of turbulence and hence do not decompose the cross-correlation function (7) into each wavelength component. Among local pressure, effective force and excitation force (Table 2), the last contributes most directly to a modal oscillation. The slope of PSD of the excitation force is -3, -6 and -2.3 in the models of *Kobayashi and Nishida* [1998a, 1998b], *Tanimoto* [1999] and the present study, respectively. The excitation force in the model of *Tanimoto* [1999] decays very rapidly with increasing frequency as compared to those in the other two models. *Tanimoto and Um* [1999] proposed a frequency-dependent κ in equation (31), which makes the excitation force less strongly frequency-dependent.

5. Conclusion

[18] We have developed a normal mode theory of atmospheric excitation of the Earth's spheroidal oscillations to derive the expression that can be directly compared to the observed spectrum of background free oscillations. The calculated spectrum exhibits distinct peaks of fundamental modes and troughs complicated by overtone modes, which well simulate the peaks and troughs of the observed spectrum. The observed and calculated spectra are in marked contrast to the one calculated by assuming that free oscillations are excited only by earthquakes, not only in terms of the amplitude level but also in terms of the spectral behavior. These results clearly rule out the possibility of the excitation source as being earthquakes in origin and strongly supports the idea of atmospheric excitation of background free oscillations proposed by *Kobayashi and Nishida* [1998a, 1998b], although the possibility of oceanic origin remains to be explored. We have also shown that the observed modal amplitudes of ${}_0S_{29}$, ${}_0S_{37}$, and ${}_0S_{44}$ are greater than the synthetic amplitudes even if the interference of other modes is taken into account. The observed excess amplitudes have to be attributed to the effect of acoustic coupling between the free oscillations of the solid Earth and the atmosphere [*Nishida et al.*, 2000]. In order to discuss this phenomenon quantitatively, the excitation problem must be solved by viewing the solid Earth and the atmosphere as a coupled system and by viewing the force system as an indigenous source [*Watada*, 1995; *Lognonné et al.*, 1998]. Our expression of the background free oscillation spectrum involves three atmospheric parameters, the air pressure PSD, the coherence length of pressure fluctuation, and the pressure-gravity admittance. The latter two quantities are poorly known in the 1–10 mHz band and are determined in the present study in order to obtain a good fit to the observed acceleration spectrum. These quantities are expected to be determined independently from meteorological data in the future.

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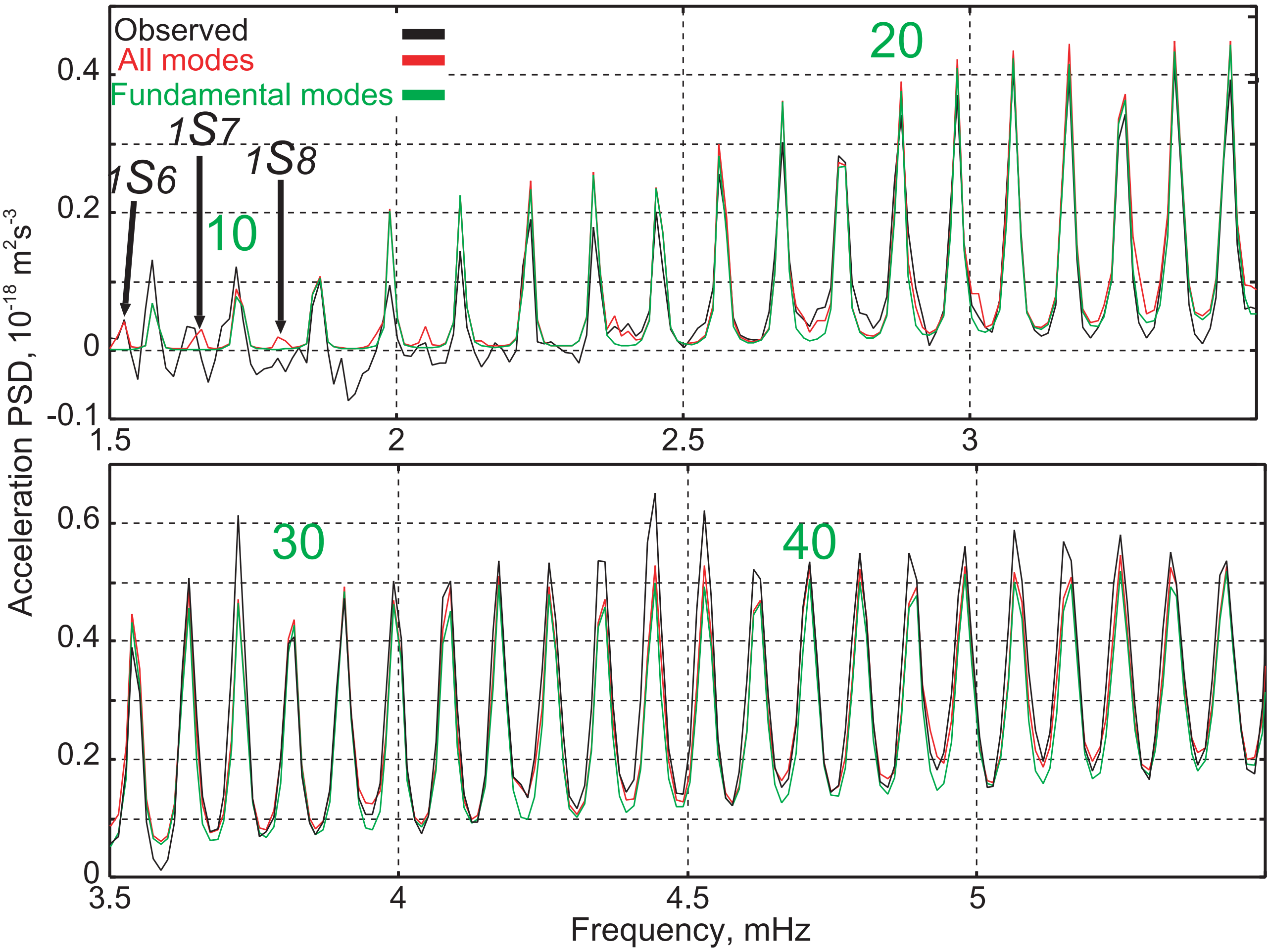

Figure 5. Averaged power spectrum of acceleration seismograms in seismically quiet days [*Nishida et al.*, 2000]. The spectrum is detrended in the frequency range 1.5–5.5 mHz so that it can be compared directly to the free oscillation term of the synthetic spectrum $\Upsilon(f)$ (the first term of equation (23)). Substraction of the trend component makes the spectrum oscillatory around the zero level at frequencies below 2 mHz. The synthetic spectrum excluding the overtone modes is also shown. Several fundamental modes are indicated by their angular order numbers. The subtle undulation of the observed spectrum especially at the trough part can only be explained by the synthetic spectrum including the overtone modes.