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# Space and time variability of uncertainty in morphodynamic simulations

A.B. Fortunato *, X. Bertin, A. Oliveira

*National Laboratory of Civil Engineering, Estuaries and Coastal Zones Division, Av. do Brasil, 101, 1700-066 Lisbon, Portugal*

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ABSTRACT

The uncertainty associated with simulations of process-based coastal area morphodynamic models is assessed through numerical experimentation. Appropriate metrics of uncertainty are defined based on the standard deviation of the model results at each location and each time step. Uncertainty is examined using a set of realistic one year morphodynamic simulations of the evolution of a highly dynamic tidal inlet. Results indicate that uncertainty increases linearly with time, and suggest that its rate grows with increasing sediment fluxes. Hence, the limits of predictability of morphodynamic model applications are higher for slowly varying systems. Attempts to reduce uncertainty by aggregating model results at larger spatial scales met with limited success. Ensemble simulations are suggested as a possible avenue to investigate the long-term evolution of tidal inlets using process-based models.



## 1. Introduction

Coastal engineers and scientists are often faced with the need to predict the morphological evolution of estuaries and tidal inlets. Several types of models, including empirical, physical and data-driven, have extensively been used for this purpose over the years. However, as in other scientific areas, developments in process-based numerical models make them increasingly attractive. Coastal area morphodynamic models couple modules for hydrodynamics, wave propagation, sediment transport and bottom updates, and can provide detailed predictions of morphological evolution of coastal systems.

Yet, despite the extensive developments in these models, and some clear successes (e.g., Cayocca, 2001), coastal area models have seen comparatively few practical engineering applications. Together with the numerical problems associated with these models (high computational costs, numerical instabilities, excessive numerical diffusion), the perception of the large errors associated with the evaluation of sediment fluxes may explain their limited use in engineering applications.

These errors can have two distinct sources. The first one is associated to limitations of the models themselves. Empirical sediment transport formulae, often used in morphodynamic models, are seldom correct by more than a factor of two, even in controlled laboratory experiments (Van Rijn, 1990). Using more sophisticated models that compute sediment fluxes by multiplying the velocity by the concentration does not necessarily produce better estimates (Davies and Villaret, 2002). The second source of errors in the evaluation of sediment fluxes is related to inaccuracies in the model inputs. Errors associated with the sediment characteristics and the input flow velocities deteriorate the prediction of sediment fluxes (Pinto et al., 2006), because sediment fluxes are extremely sensitive to these parameters (Van Rijn, 2007). In tidally-driven flows, errors in the evaluation of the high frequency tidal constituents can also contribute significantly to the errors in the evaluation of sand fluxes (Fortunato, 2007). Finally, because velocities are very sensitive to bathymetric errors (Blumberg and Georgas, 2008), it is possible that feed-back loops between errors in the bathymetry changes and in the velocity fields lead to a very rapid growth of the uncertainty in morphological predictions. The present work addresses this second source of errors.

As in other domains, model calibration can reduce uncertainty. However, the calibration of morphodynamic models is usually limited by the unavailability of good quality bathymetric data at decadal to century time scales, and by the large number of parameters and processes involved. In rapidly evolving systems, the limitations of the data may also be associated with the insufficient frequency of the measurements. For instance, meanders in the Óbidos lagoon form and disappear on monthly time scales (Oliveira et al., 2006), while bathymetry is only measured twice a year, at best.

Although the poor accuracy associated with the evaluation of sediment fluxes is well established, it is still unclear how this accuracy affects the uncertainty of the morphodynamic model predictions. Understanding this uncertainty is becoming increasingly important, as computational resources now allow predictions at century and millennia time scales using process-based models (e.g., Hibma et al., 2003; Van der Wegen and Roelvink, 2008), potentially enabling us, for instance, to examine the effects of climate change on the morphodynamics. However, the ability of process-based models to hindcast or predict the evolution of tidal inlets at decadal to century time scales

* Corresponding author.
*E-mail address:* afortunato@lnec.pt (A.B. Fortunato).

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remains questionable (Stive, 2006). As computational resources grow, uncertainty becomes increasingly important in determining the limits of predictability of morphodynamic models.

Hence, the first goal of this paper is to determine how the uncertainty in morphological predictions evolves over time, and whether errors tend to add up during the simulation, or to compensate one another.

In addition to understanding uncertainty in these models, approaches to reduce the effect of this uncertainty should be sought. A possible avenue is to tap on the relative success of some box (or aggregated) models (e.g., Cheung et al., 2007). These models describe the evolution of the major morphological features of tidal inlets (e.g., channels, sand banks) by solving mass balance equations together with empirical relations between aggregate quantities (e.g., tidal prisms, channel cross-sections, sand bank volumes). Unlike process-based models, which target a detailed spatial description of the model domain, box models only provide coarse representations of the system features. Yet, in many cases, these representations may provide adequate answers for management questions.

This approach suggests the possibility that by increasing the spatial scales of interest one could reduce uncertainty, thereby extending the limits of predictability of the models. The second goal of this paper is therefore to understand how this uncertainty depends on the spatial scales of interest, and if there is a spatial scale which minimizes uncertainty.

The answers to these questions are vital to determine the limits of predictability of morphodynamics modeling systems, and to help the modeler in extracting the most meaningful information from the model results.

The answers to these questions are sought through numerical experimentation using the morphodynamic modeling system MORSYS2D (Fortunato and Oliveira, 2004, 2007a; Bertin et al., 2009a). MORSYS2D is applied to the Óbidos lagoon, a rapidly evolving system on the Portuguese coast (Oliveira et al., 2006; Fortunato and Oliveira, 2007b). A set of possible bed evolution predictions is obtained by varying several input parameters (the sediment characteristics, the tidal forcing and the sediment transport formula) within ranges of realistic values. The variability of the predictions across simulations quantifies the model uncertainty and its spatial variability. Appropriate metrics for uncertainty are defined, based on the standard deviation of depth across a large number of simulations, integrated at different spatial scales.

This paper is organized in three sections besides this introduction. Section 2 describes the methodology, including the modeling system, the study area and the metrics used to assess uncertainty. Section 3 presents and discusses the results. The paper closes with a summary of the major conclusions.

## 2. Methodology

### 2.1. Model description

MORSYS2D is a 2D coastal area morphodynamics modeling system which offers a choice between different modules for waves and currents. Only the modules used herein are described. Previous versions of the model are described in detail in Fortunato and Oliveira (2004, 2007a) and Oliveira et al. (2005). A detailed description and validation of the present version, including short wave forcing and interaction with currents, is given in Bertin et al. (2009a).

The hydrodynamics are computed with ELCIRC (Zhang et al., 2004), a 3D baroclinic shallow water model run in 2D mode herein. ELCIRC solves the shallow water equations using Eulerian–Lagrangian methods with a semi-implicit time discretization. Space is discretized using a combination of finite volumes and finite differences on an unstructured grid. This combination of space and time discretization allows for detailed spatial resolution in the areas of interest at low computational costs.

Sand transport and bottom updates are computed with SAND2D (Fortunato and Oliveira, 2004, 2007a). After sand fluxes are computed with one of the various empirical formulae implemented, the Exner equation is solved to compute the bathymetry at the next time step:

$$\Delta h^i = \frac{1}{1-\lambda}\nabla Q_*^i \tag{1}$$

where $\Delta h_i$ is the bottom variation over a time step, $\lambda$ is the sediment porosity and $Q_*$ is the sediment flux integrated over the time step. The integrated flux includes both the advective flux $Q$, computed through an empirical formula, and a diffusive flux:

$$Q_* = Q + \varepsilon(1-\lambda)\left(|Q_x|\frac{\partial h}{\partial x}, |Q_y|\frac{\partial h}{\partial y}\right) \tag{2}$$

where $\varepsilon$ is a dimensionless diffusion coefficient.

The solution of the Exner equation involves several difficulties. First, shock waves are naturally produced, because the crest of the wave travels faster than the trough (Kubatko and Westerink, 2007). Secondly, the combination of the shallow water equations with the Exner equation forms a strongly non-linear system, because sediment fluxes depend on depth both directly and indirectly (through the velocities). Dealing with both difficulties poses a significant challenge from a numerical viewpoint. The most successful models require filtering or some numerical diffusion to avoid spurious oscillations and some degree of implicitness in the solution of the Exner equation (Johnson and Zyserman, 2002; Callaghan et al., 2006; Kubatko et al., 2006). SAND2D solves the Exner equation using a node-centered finite volume technique on an unstructured triangular grid. Stability is sought through the use of the non-linear filter of Oliveira and Fortunato (2002) for unstructured grids, and a predictor–corrector scheme (Fortunato and Oliveira, 2007a).

To improve stability and reduce the computational cost, the time step is automatically adjusted to keep the maximum Courant number close to a target value (Bertin et al., 2009a), taken here as unity. At the end of each morphodynamic time step, the Courant number is estimated as (Roelvink, 2006):

$$Cu \approx \frac{bQ_*}{H\Delta x} \tag{3}$$

where $b$ is the velocity power in the transport formulae (typically between 3 and 5, depending on the specific formulation) and $H$ is the total depth. The determination of the next morphological time step is based on two assumptions: 1) the logarithm of $\alpha_i$, the maximum Courant number at time $i$ divided by the time step, varies linearly in time; and 2) the morphological time step varies slowly in time. The first assumption, which was based on observations from runs with a constant time step, can be expressed as:

$$\alpha_{i+1} = \alpha_i^2 / \alpha_{i-1}. \tag{4}$$

Eq. (4) provides a first estimate of the time step $\Delta t_{i+1}^*$ that targets a Courant number of 1:

$$\Delta t_{i+1}^* = \alpha_{i-1} / \alpha_i^2 = \frac{Cu_{i-1}}{Cu_{i-1}^2}\frac{\Delta t_i^2}{\Delta t_{i-1}} \tag{5}$$

where $Cu$ represents an estimate of the maximum Courant number. The second approximation is enforced in the model by using a relaxation factor that prevents rapid variations in the time step:

$$\Delta t_{i+1} = \theta\Delta t_{i+1}^* + (1-\theta)\Delta t_i \tag{6}$$

where $\theta$ is a relaxation factor taken as 0.7.

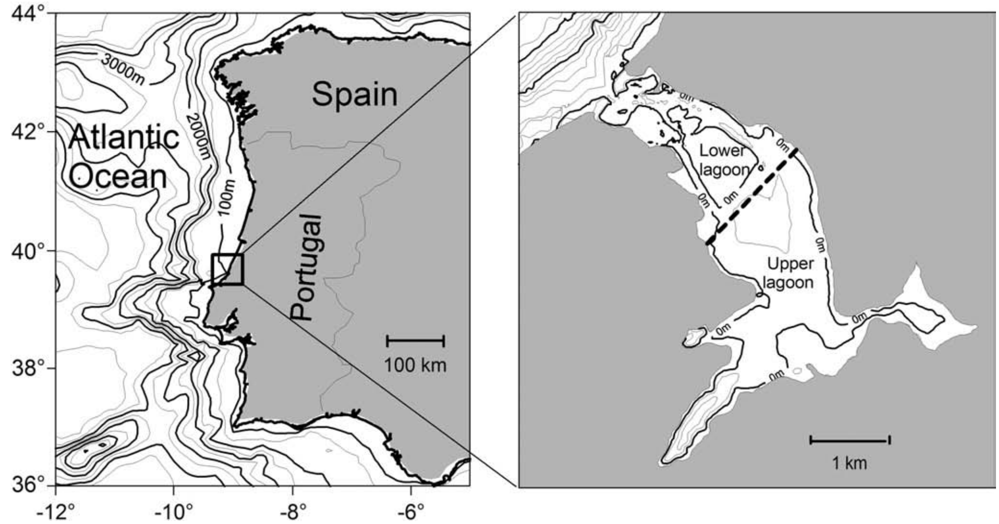



**Fig. 1.** The Óbidos lagoon (Portugal): location and place names.

This adaptive procedure leads to time steps that vary from about 2 min when sand fluxes are maximum (typically around mid-tide or for very energetic wave conditions) to 45 min when sand fluxes are minimum (around high and low tide or when waves are small). To prevent excessively large time steps, in particular during the warm-up period at the beginning of the simulation, the user specifies a maximum time step, which we typically set to 1 h. The minimum morphodynamic time step is determined by the hydrodynamic model time step. The performance of this adaptive procedure in the Óbidos lagoon is illustrated in Bertin et al. (2009a).

*2.2. Model application*

The numerical tests are performed in the Óbidos lagoon, a small coastal system in the Western Portuguese coast (Fig. 1). The Óbidos lagoon was chosen for its very rapid evolution. This lagoon is known to close occasionally since at least the XV century (Henriques, 1992), and several aerial photographs from the past 60 years show that the number, extension and position of its tidal channels can vary drastically over the years. Tidal elevations inside the lagoon indicate the existence of a yearly cycle in the bathymetry of the tidal inlet: the amplitudes of the M2 roughly double during the maritime summer, when waves abate, then decrease in the maritime winter (Oliveira et al., 2006). At monthly time scales, meanders can form and disappear (Oliveira et al., 2006; Fortunato and Oliveira, 2007b). Bathymetric comparisons showed that the tidal inlet had a strong tendency for accretion the past few years, which has been partly compensated by frequent dredging operations (Fortunato and Oliveira, 2007b).

The lagoon bathymetry was taken from two different surveys. The upper lagoon data were measured in 2000, while the lower lagoon data

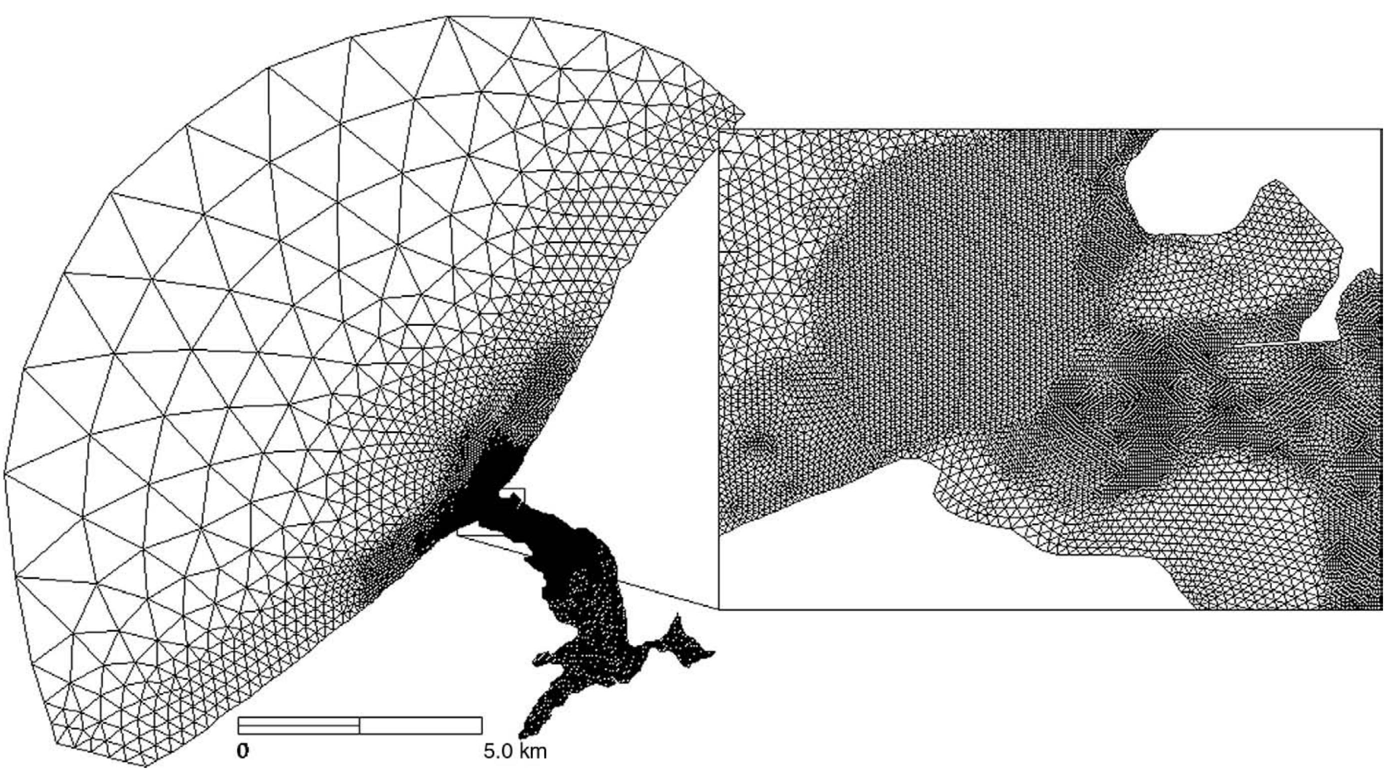



**Fig. 2.** Grid used for the morphodynamic modeling system, with 26,000 nodes. The inset is amplified tenfold.

**Table 1**
Sources of uncertainty in the simulations.

| Tidal forcings | d50 (mm) | Mean sea level (m, above chart datum) | Sediment transport formula |
|---|---|---|---|
| M2, S2 | 0.3 | 2.0 | Ackers and White, with $\varepsilon=3$ |
| M2, S2, N2, O1, K1 | 0.5 | 2.2 | Bhattacharya et al., with $\varepsilon=1$ |

are from July 2001. The 2001 bathymetry was measured one month after a major dredging operation in which the northern channel was dredged and the inlet mouth was relocated in a central position. Hence, rapid bathymetric changes are expected to occur. Outside the lagoon, bathymetric data are scarce. Some data were measured in June 2000 seaward of the surf zone, but data within the surf zone are unavailable.

Sediments are mostly muddy in the upper lagoon (Henriques, 1992). In the lower lagoon, sediment diameters are known at various locations of the tidal inlet from a 1997 field survey. The mean sediment diameter increases seaward, from about 0.3 mm near the lower lagoon to about 0.6 mm close to the inlet mouth. The sediment characteristics also differ from the channels to the sand banks, as well as over depth. However, it is difficult to define an adequate parameterization of the spatial variation of the sediment characteristics due to the scarcity of the data. Hence, a fixed sediment diameter is often used in sediment transport calculations (e.g., Oliveira et al., 2005; Fortunato and Oliveira, 2007b; Bertin et al., 2009a).

The finite element grid used to model both hydrodynamics and sediment transport has a maximum resolution of about 10 m in the inlet mouth and channels (Fig. 2). The flow is forced by tides taken from the regional model of Fortunato et al. (2002), and river flow is neglected. Root mean square errors of tidal elevations inside the lagoon, obtained in simulations with a fixed bathymetry, are about 10 cm (Oliveira et al., 2006).

### 2.3. Uncertainty metrics

Two different parameters are used to measure uncertainty. The first provides information on the spatial distribution of uncertainty, thus allowing the identification of the areas where the model is more or less reliable. The second parameter is an aggregate index to assess the overall uncertainty of the model.

The spatial distribution of uncertainty is measured through the standard deviation of the predictions for a set of simulations. Because the uncertainty can potentially depend on the spatial scale at which it is evaluated, predicted depths at each particular time $t$ are first averaged over square cells with a side length $\Delta$. Hence, the uncertainty is measured by the standard deviation $\sigma$ of the predictions, and varies with the horizontal position, time and spatial scale: $\sigma(x,y,t,\Delta)$.

The adequacy of $\sigma$ as a measure of uncertainty depends, to a large extent, on the information used to compute its value. The simulations used to compute $\sigma$ must be realistic, and cover the adequate range of uncertainty of the major sources of errors in the simulations.

Borrowing from the concepts used in the Brier Skill Score (Sutherland et al., 2004), an index used to measure the skill of morphodynamic models, a more compact measure of uncertainty is obtained by aggregating the standard deviation $\sigma$ in space, and by making it dimensionless. The Confidence Index $C_{t\Delta}$ is defined as:

$$C_{t\Delta} = 1 - \frac{\sum_{\Omega} \sigma^2(x,y,t,\Delta)}{\sum_{\Omega} \left(\mu(x,y,t,\Delta) - h_i(x,y,\Delta)\right)^2} \quad (7)$$

where $\Omega$ is the domain, $\mu$ is the average of the depth predictions at time $t$ for the set of simulations, and $h_i$ is the initial depth. For consistency, both $\mu$ and $h_i$ are averages within cells with side length $\Delta$.

The Confidence Index is unity when there is no uncertainty, and decreases with increasing uncertainty. When $C_{t\Delta}$ is zero, the uncertainty is of the same order as the depth changes predicted by the model. At this stage, the predictions are useless. An important characteristic of the Confidence Index is that it is unaffected by the inclusion of an additional inactive area to the domain. Indeed, in an area where the bathymetry does not change, $\sigma$ is zero and $\mu$ equals $h_i$. Hence, both summations in Eq. (7) do not change.

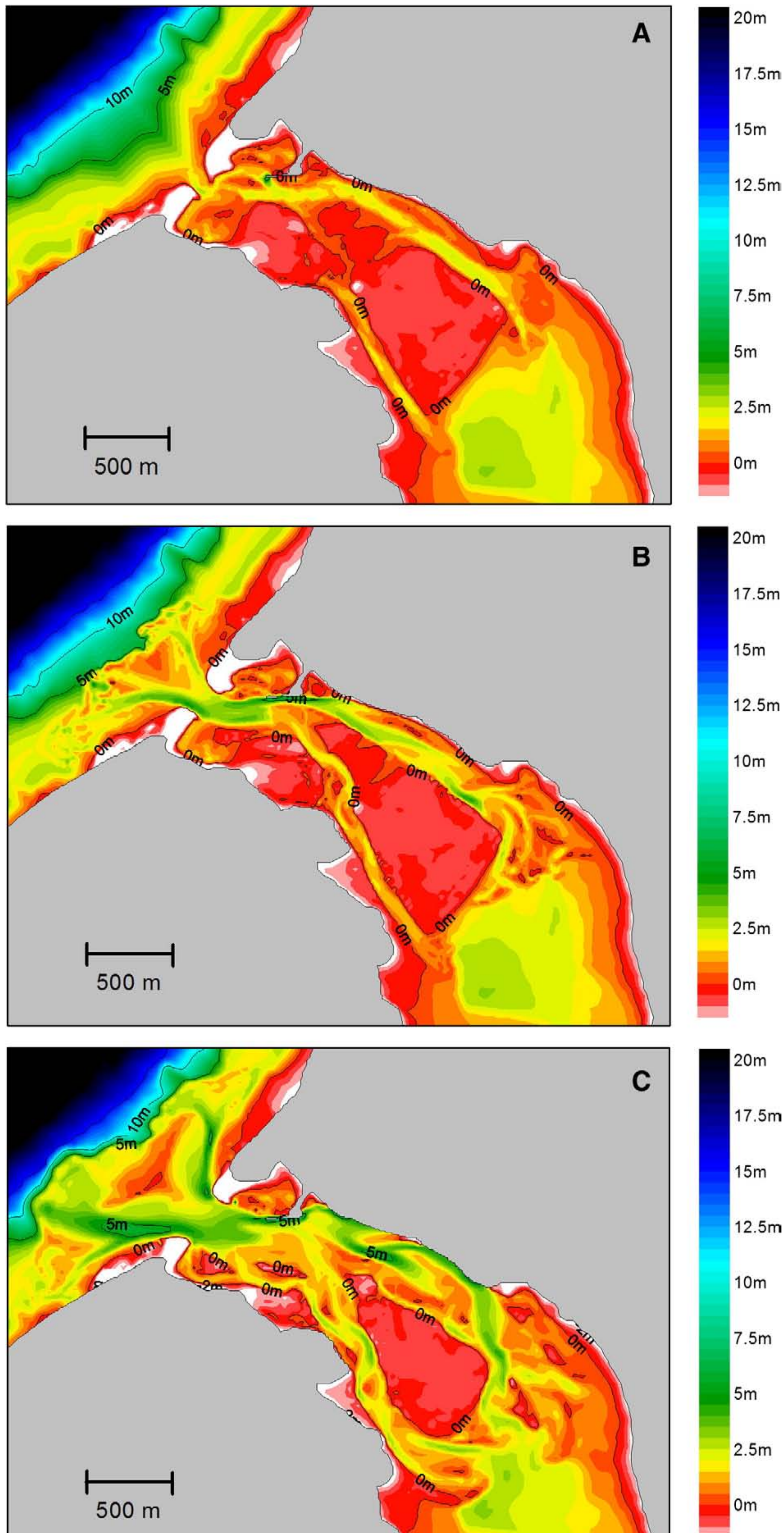
### 2.4. Numerical tests

Many factors affect the uncertainty of model predictions. We focus here on the most important ones.

Pinto et al. (2006) analyzed the sensitivity of various empirical sediment transport formulae for steady flow to several physical quantities (velocity, depth, d50 and sediment distribution), and concluded that sediment fluxes were mostly sensitive to velocity and d50. These two quantities are therefore selected for the present analysis. Fortunato (2007) showed that, in tidally-driven flows, errors in the evaluation of even harmonics (in particular fourth-diurnal constituents) can also contribute significantly to the errors in the tidally-averaged sediment transport fluxes. Our analysis includes therefore variations in overtides and compound tides. Finally, the predictions provided by different transport formulae typically differ by a factor of at least 2. Hence, simulations are performed with two alternative formulae.

Simulations involved therefore varying four different sources of uncertainty:

- Small, yet realistic, changes in the velocity fields are obtained by varying the tidal constituents used to force the model as boundary conditions. These variations in forcings aim at mimicking errors in the velocity fields. Simulations are either forced with the two major tidal constituents alone (M2 and S2), or include the third major semi-diurnal constituent (N2) and the two major diurnal constituents (O1 and K1) as well. The addition of the three astronomic constituents increases the maximum velocities in the channels by roughly 10%, for the initial bathymetry.
- Two alternative values of d50 are used: 0.5 and 0.6 mm. Available data for this coastal system indicate that the mean diameter of the superficial sediments varies from about 0.3 mm in the upstream part of the channel to about 0.6 mm at the inlet mouth. A few centimeters below the bottom surface, sediment diameters can change differently depending on the location: they decrease in the flood sand bank and increase on the beach. Although considering a realistic spatial variability of d50 can improve sediment transport calculations (Bertin et al., 2009b), a constant value of d50 is often used (Fortunato and Oliveira, 2007b; Bertin et al., 2009a) because available data are too scarce to allow for an accurate specification of the spatial variation of sediment diameters. The values of d50 are selected as representative of the lower lagoon because the comparison of historical bathymetries and aerial photographs indicates that the upstream part of the channel is fairly stable compared to the inlet mouth (Fortunato and Oliveira, 2007b).
- Variations in the generation of overtides are achieved by varying both the tidal forcing, as explained above, and the mean sea level. By neglecting N2 in some of the simulations, we also remove some quarter-diurnal constituents (MN4, MS4 and N4). The uncertainty in the mean sea level stems from both the sea level rise and its dependence on waves and meteorological factors. Varying the mean sea level affects the finite amplitude term in the momentum equations, which, in a very shallow lagoon, is expected to be the main term responsible for the generation of even harmonics.
- Finally, two alternative sediment transport formulae are applied (see Appendix): a traditional formula (Ackers and White, 1973) and a more recent one (Bhattacharya et al., 2007). Besides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12.5m
10m
7.5m
5m
2.5m
0m

**Fig. 4.** Aerial photograph of the Óbidos lagoon tidal inlet from 1991. The surf zone suggests the presence of an ebb sand bank flanked by two channels in front of the inlet.

providing different sediment flux predictions, these two formulae also lead to different numerical behaviors of the model. The model is more stable with the latter, hence a diffusion coefficient of $\varepsilon = 1$ is used. For the former formula, the diffusion coefficient is set to 3 to prevent numerical oscillations. Hence, an additional source of uncertainty is also indirectly included in the analysis. This source combines the numerical properties of the model, which may force the use of additional diffusion, and gravitational transport, represented by the diffusion term. A comparison between results for the Bhattacharya et al. formulation with different values of $\varepsilon$ (1 and 3) indicates that gravitational transport introduces significant differences in the results (not shown).

All 16 combinations of these parameters (Table 1) were simulated. Simulations are performed for one year (370 days). During a spin-up period, taken as 30 days, fluxes are multiplied by a sinusoidal ramp function. This large spin-up period is necessary due to the unavailability of bathymetric data within the surf zone. Because the model bathymetry in that area is interpolated, the initial conditions do not include the channels and ebb sand banks that are known to exist from aerial photographs. As a result, these morphological features are rapidly generated by the model in the first days of simulation, and the model bathymetry tends to evolve very fast. The ramp function prevents numerical problems associated with the generation of a realistic initial bathymetry by the model.

Waves play a major role in the dynamics of most tidal inlets, and the Óbidos lagoon is not an exception. However, waves are not considered here for practical reasons. First, they increase significantly the computational cost, not only due to the need to run a wave model, but also because the grid spacing and the time step have to be reduced substantially to resolve the breaking zone and to deal with larger sand fluxes. Because computational costs are a major concern in this analysis due to the large number of simulations required, including the wave forcing would have to be compensated by shorter simulations. Secondly, including the wave forcing would require the consideration of additional sources of uncertainty, such as the wave characteristics themselves or the wave model parameters (bottom friction or the parameterization of wave breaking). Again, the duration of the simulations would have to be reduced to accommodate a larger number of simulations.

## 3. Results and discussion

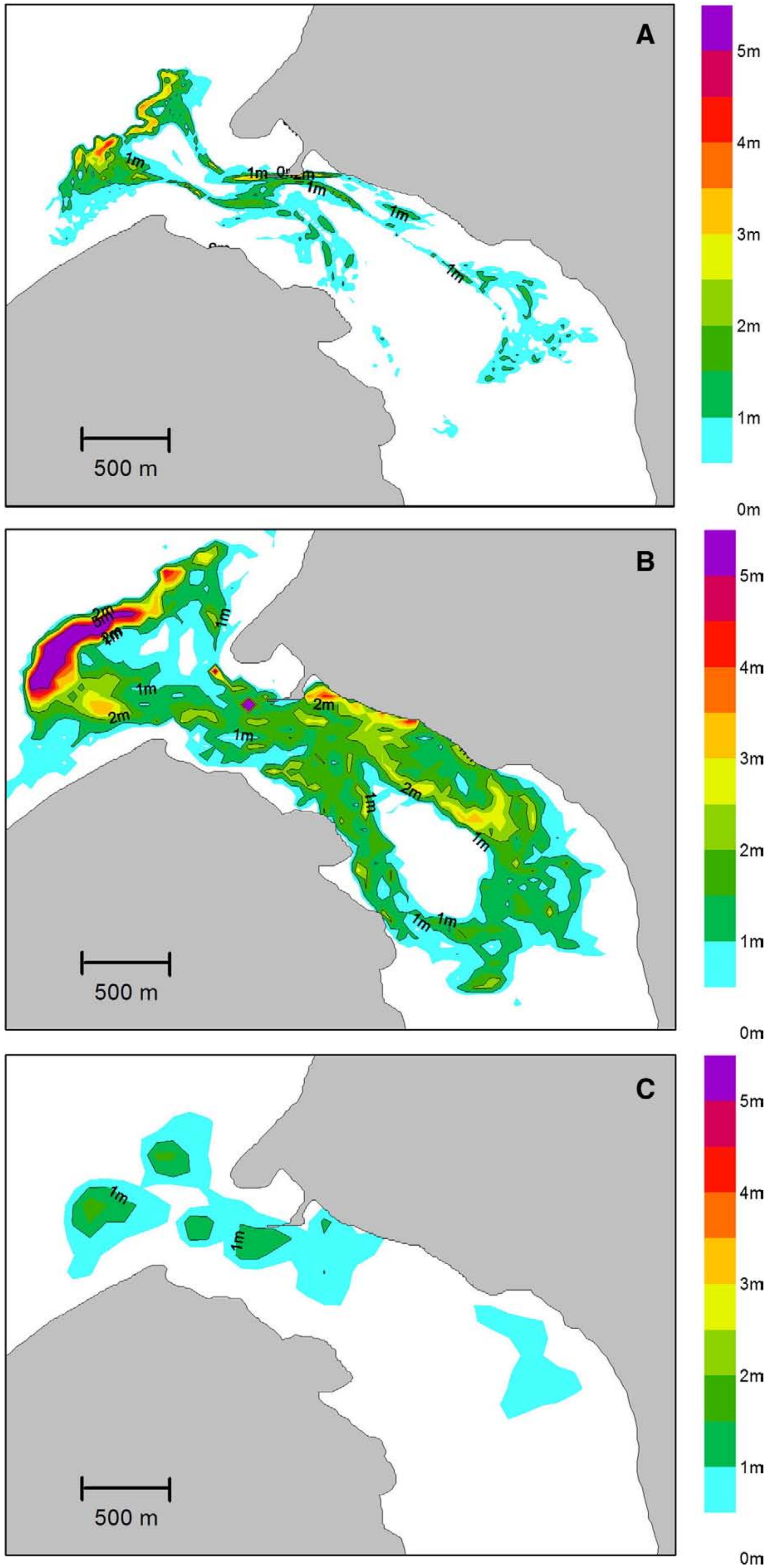
The one year simulations exhibit several realistic features (Fig. 3). Changes occur mostly near the lagoon mouth, while the upper lagoon and the flood sand banks remain largely unchanged. A realistic ebb sand bank is generated seaward of the mouth, flanked by two channels. Some of the existing aerial photographs suggest a similar configuration of the bank (Fig. 4). Also, the connection between the southern channel and the northern channel deepens. This evolution is consistent with the known behavior of the downstream part of the southern channel, which formed naturally after the upstream part of the channel was artificially created in 1995.

The mouth and the channels widen during the simulation. This behavior, attributed to the absence of waves and the lack of corresponding littoral drift in the model simulations, is consistent with the known increase of tidal amplitude inside the lagoon, which can double during the maritime summer, when waves abate (Oliveira et al., 2006; Bertin et al., 2009b). As the mouth and channels grow wider, the velocities in the lagoon increase.

The increase of velocities is particularly marked for the simulations with the Bhattacharya et al. (2007) sediment transport formula (Fig. 3C), which predicts higher sediment fluxes. As a result of these large velocities, unrealistic meanders form in the channels. In contrast, simulations with the Ackers and White (1973) formula exhibit smaller changes and more stable channels (Fig. 3B). In general, the simulations with the latter formula appear more realistic. These results suggest that the Bhattacharya et al. (2007) formula overpredicts sediment fluxes, even though the absence of waves during a full year could conceivably lead to the behavior predicted by this formula. After this paper was submitted, a comparison between the Bhattacharya et al. formula and several other formulae showed that it consistently predicts significantly higher fluxes than the others (Silva et al., 2009).

The standard deviation of the predictions indicates that, in general, uncertainty is largest in areas where bathymetric changes are more pronounced: in the channels, inlet mouth and ebb sandbank (Fig. 5A and B). In particular, the largest standard deviations coincide with the edges of these features, an indication that the exact location of a morphological feature may be difficult to determine. For instance, the oceanward extent of the ebb sand bank tends to increase with finer sediments, stronger tidal forcing and higher mean sea level. In contrast, the existence and approximate location of these features appear more reliable, as they are fairly constant across simulations. For instance, there is little doubt about the generation of the ebb sand

**Fig. 3.** Evolution of the Óbidos lagoon, with d50 = 0.5 mm, 5 astronomic constituents and Z0 = 2 m: A) initial conditions; B) results after 1 year for the Ackers–White formula; C) results after 1 year for the Bhattacharya et al. formula.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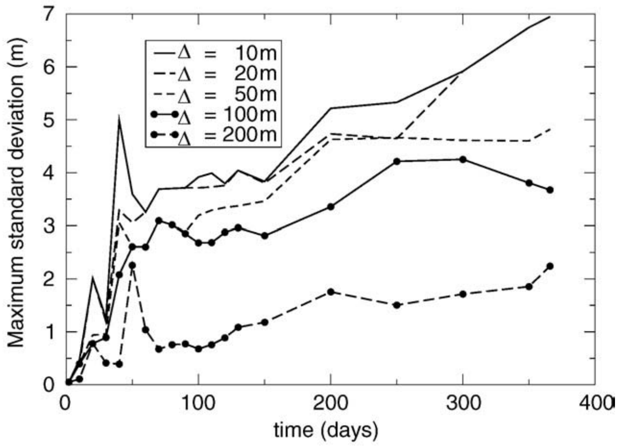


**Fig. 6.** Maximum standard deviation of the predictions for the Ackers and White formulation.

bank, since the standard deviation in its center after one year is smaller than 1 m.

Uncertainty is higher and more widespread for the Bhattacharya et al. than for the Ackers–White formula. This difference between the two formulations is probably associated with the larger sediment fluxes that occur with the former, and the ensuing meandering of the channels.

Increasing the scale of integration, Δ, clearly reduces the maximum uncertainty (Fig. 6). However, this reduction occurs to a large extent at the expense of the spreading of uncertainty (Fig. 5C). The Confidence Index, an integral measure of uncertainty, is only slightly affected by the scale of integration (Fig. 7).

The Confidence Index decreases with time (Fig. 7), as errors build up during the simulation. During the first month of simulation, $C_{t\Delta}$ decreases slowly because sediment fluxes are multiplied by a ramp function. However, $C_{t\Delta}$ decreases linearly with time from the second month on. This behavior is common to both formulations. Assuming that the trend remains constant, these results indicate that meaningful simulations (i.e., $C_{t\Delta}$ positive) could be pursued for 140 and 20 years for the Ackers–White and the Bhattacharya et al., respectively. These estimates were obtained by a linear extrapolation of the curves on Fig. 7, whose linear regression coefficients exceed 0.99 in all cases.

Results confirm the possibility of reducing uncertainty by averaging the predictions over large cells. The Confidence Index is roughly unchanged for values of Δ between 10 and 50 m, but increases for Δ = 100 m. However, gains are modest and difficult to guarantee. For Δ = 200 m, $C_{t\Delta}$ can decrease relative to smaller values of Δ. These results suggest that model outputs can be averaged at larger spatial scales to reduce uncertainty, but this procedure cannot be done blindly. Instead, the procedure should take into account the major morphological features. As an alternative, ensemble simulations such as the ones presented herein can be used to determine the degree of uncertainty associated with the different characteristics of the predictions.

## 4. Conclusions

Several sources of uncertainty can limit the quality of the predictions of the morphological evolution of coastal systems through the application of process-based models: the physical characteristics of the sediments, the forcings and the sediment transport formulae themselves. The numerical experiments presented above provide new

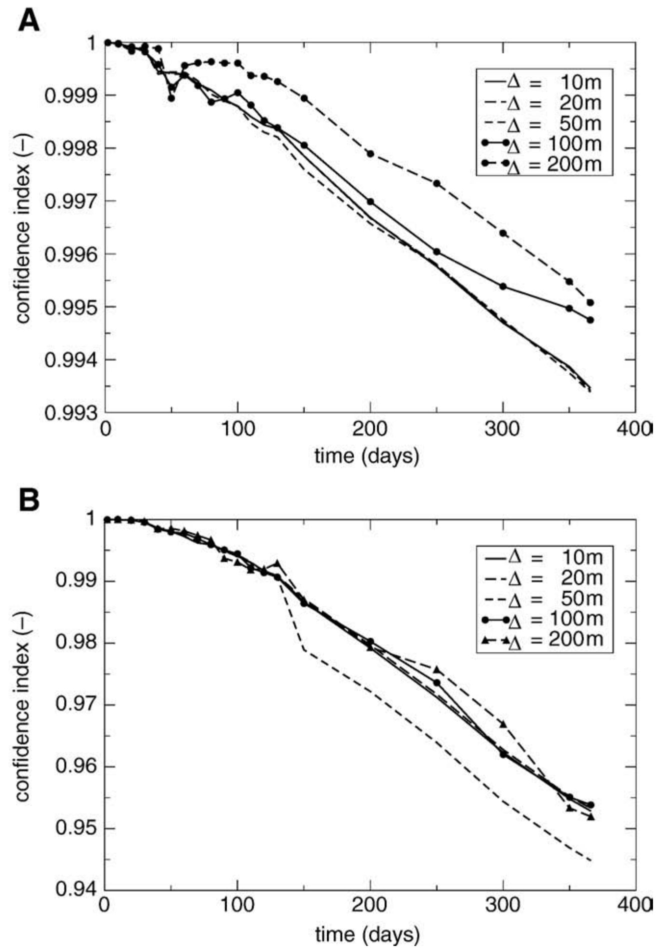


**Fig. 7.** Evolution of the Confidence Index over time for: A) the Ackers and White formulation; B) the Bhattacharya et al. formulation.

insight into the uncertainty associated with the predictions of the morphological behavior of a particular coastal lagoon. While the general behavior of the lagoon, such as the development of different morphological features, is fairly similar across simulations, the extension of these features is more difficult to predict, as uncertainty grows at their edges.

The choice of the transport formulae proved to be the major source of uncertainty in the simulations performed herein: the behavior of the simulations differed markedly for the two formulae. This conclusion justifies continuing efforts to develop more accurate transport formulae (e.g., Bhattacharya et al., 2007; Van Rijn, 2007; Camenen and Larson, 2008). Also, this conclusion apparently contradicts the analysis of Pinto et al. (2006), which indicates that errors in sediment fluxes depend more on errors in the physical factors that affect sediment transport than on the choice of the particular sediment transport formula. A possible explanation for this contradiction lies on the relatively small input errors used in the present analysis. For instance, Pinto et al. (2006) consider up to 40% errors in d50, while only a 20% variation was admitted here.

An attempt to reduce uncertainty by increasing the spatial scale at which results are integrated met with limited success. While the maximum and the average uncertainty do decrease at appropriate scales, this reduction is not always guaranteed. Also, the reduction of the maximum uncertainty is obtained at the expense of the spreading of the local uncertainty.

**Fig. 5.** Standard deviation of the predictions after 1 year for: A) the Ackers and White formulation and Δ = 10 m; B) the Bhattacharya et al. formulation and Δ = 10 m; C) the Ackers and White formulation and Δ = 100 m.

An extrapolation of the results obtained for one year indicates that the simulations become useless after 20 or 140 years, depending on the transport formula. After this simulation time, the Confidence Index becomes negative, an indication that the results are totally unreliable. Clearly, this conclusion cannot be extended to other systems. The fact that uncertainty is larger in areas with larger changes and also for the formula that predicts the highest sand fluxes suggests that uncertainty will be smaller (hence meaningful predictions can be performed for longer periods of time) in systems that evolve slower. In contrast, the linearity of the decrease of the Confidence Index in time is probably valid for other systems, as it occurred for both the Ackers–White and the Bhattacharya et al. formulae.

The existence of limits of predictability associated with various error sources suggests that a possible avenue to investigate the long-term evolution of tidal inlets using process-based models is to perform ensemble simulations, such as the ones presented above. This approach provides not only a prediction, but a measure of its reliability as well. The Confidence Index introduced herein can be an easy way to measure this reliability, although more work is necessary to associate quantitative values of $C_{t\Delta}$ with a qualitative measure of uncertainty.

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## Appendix A. Sediment transport formulae

Ackers and White (1973) developed a general sediment transport function to calculate sediment flux based on experiments with established sediment movement:

$$q_s = \frac{s_d k d_{35}}{d}\left(\frac{\bar{u}}{u_*}\right)^n\left(\frac{F_{gr}-A}{A}\right)^m \tag{8}$$

$$F_{gr} = \frac{u_*}{\sqrt{g d_{35}(s_d - 1)}}\left[\frac{\bar{u}}{\sqrt{32}\log(10d/d_{35})}\right]^{1-n} \tag{9}$$

where $\bar{u}$ is the mean flow velocity, $s_d = \rho_s/\rho$ is the specific density, $g$ is the acceleration of gravity, $F_{gr}$ is the sediment mobility number, $u_*$ is the stress velocity, $A$ is the value of $F_{gr}$ at nominal initial motion; $k$, $m$ and $n$ are empirical coefficients and $d$ is the flow depth. The values of $A$, $k$, $m$ and $n$ were obtained from experimental studies that showed how these parameters vary with the dimensionless particle size, $d_{gr} = d_{35}[g(s_d-1)/\nu^2]^{1/3}$, where $\nu$ is the kinematic viscosity of the fluid. If $d_{gr} > 60$ then $n = 0.00$, $m = 1.50$, $A = 0.17$, and $k = 0.025$ and if $1 < d_{gr} \leq 60$ then $n = 1.00 - 0.56\log\, d_{gr}$, $m = (9.66/d_{gr}) + 1.34$, $A = \left(0.23/\sqrt{d_{gr}}\right)$, and $\log\, k = 2.86\log\, d_{gr} - (\log\, d_{gr})^2 - 3.53$. Eq. (8) is applicable to sediment mixtures with $d_{gr} > 1$, i.e., $d_{35}$ above about 0.04 mm.

Bhattacharya et al. (2007) proposed a formulation to compute the total transport directly:

$$q_s/\sqrt{(s_d - 1)g d_{50}^3} = \begin{cases} 0.072078\dfrac{T^{0.893}}{D_*^{0.353}}\left(\dfrac{H}{d_{50}}\right)^{0.486} & T > 2.22 \\ 0.0000782\dfrac{T^{0.54}}{D_*^{0.00407}}\left(\dfrac{H}{d_{50}}\right)^{1.16} & T \leq 2.22 \end{cases} \tag{10}$$

where $T = (\theta' - \theta_{cr})/\theta_{cr}$ is the transport stage parameter, $\theta'$ is the mobility parameter relative to grain roughness, $\theta_{cr}$ is the Shield critical shear stress, and $D^* = d_{50}((s_d - 1)g/\nu^2)^{1/3}$ is the dimensionless sediment grain size.

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