On the Statistical Distribution of Wave Heights in a Storm

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There has been recent controversy over how well the Rayleigh distribution matches the observed distribution of wave heights. Most of this controversy stems from comparisons based on different definitions of the significant wave height. Once consistent definitions are used, all available data support the conclusion that the Rayleigh distribution overpredicts the heights of the higher waves in a record. Analysis of 116 hours of hurricane-generated waves in the Gulf of Mexico permitted the empirical fitting of the data to a Weibull distribution. Statistics developed from the empirical distribution include the prediction that the highest wave in 1000 is only 0.907 times the height predicted by the Rayleigh distribution.

INTRODUCTION

The surface of the sea is usually a complex and irregular function of space and time, best described by statistical measures. *Sverdrup and Munk* [1947] thus decided that the energy of the sea state was the proper quantity to be predicted by their significant wave method of hindcasting. *Pierson* [1952] greatly improved upon that idea by showing the practicality of hindcasting the directional spectrum, that is, the partition of the energy into various frequency and direction bands. The development of the directional spectral hindcasting method for hurricanes has been described by *Cardone et al.* [1976].

Useful as the spectral concept may be in hindcasting, it is rarely used directly by the design engineer. The designer is usually most interested in the expected values of maximum wave heights, although the details of the wave shape and the directional spreading of wave energy can also influence forces. A description of the long and complicated path from historical meteorological data to long-term wave statistics has been given by *Jahns and Wheeler* [1973]. One crucial step in this hindcasting process is the determination of the distribution of wave heights during a short time interval from the spectrum of the sea during that interval.

A number of years ago, *Longuet-Higgins* [1952] pointed out that if the spectrum is sufficiently narrow, the wave heights should be approximated by a Rayleigh distribution. A mathematical expression for this prediction is

$$P(H > H_0) = \exp(-H_0^2/8m_0) \quad (1)$$

where the left-hand side denotes the probability that the wave height H is greater than H_0 and m_0 is the total variance in the spectrum. This variance is, of course, one of the products of the wave hindcast program.

Given an actual wave record, the accuracy of (1) can be easily checked, and this has been done many times over the years. Surprisingly, there is still controversy over how well the observations match the theory. Part of this controversy stems from implicit disagreement over how good the fit should be to be called good and how much emphasis is given to the high wave tail of the distribution, but we will show here that much of the disagreement stems from confusion over definitions and terminology. Further, we will show that (1) consistently overpredicts the heights of the higher waves in a record.

High-quality data are required to resolve overpredictions of only a few percent. Thus when *Longuet-Higgins* [1952] compared his theory to the relatively scanty data available in 1952, he found 'quite close agreement.' However, much higher quality data are now available, and removal of a well-established bias in the hindcasts of 10% would be important.

In more recent data comparisons, *Wu* [1973] found data from weather station Papa 'slightly lower than the theory predicts' at moderately low probabilities but said that 'in general, the data shows no significant deviation from the Rayleigh distribution.' *Chakrabarti and Cooley* [1977] found 'reasonably good correlation' between the data and theory for measurements made by the O.W.S. *Weather Reporter* in a 1961 Atlantic storm. On the other hand, *Thompson* [1974] used Coastal Engineering Research Center (CERC) coastal stations to extend the measured distributions to very low probabilities and found that 'at the high wave end of the distribution, the data deviate significantly from the Rayleigh distribution.'

The clearest disagreement concerns data from a platform-mounted wave staff in Hurricane Camille. Using data from Ocean Data Gathering Program (ODGP) station 1, *Earle* [1975] found agreement with the Rayleigh distribution to within 2%. Using the same data, along with a number of other storm wave records, *Haring et al.* [1976] found that the Rayleigh distribution overpredicted the highest waves by about 10%.

DATA ANALYSIS

In hopes of resolving some of the discrepancies, we also analyzed the Camille data as well as a number of other records of hurricane waves in the Gulf of Mexico that were available to us. The measurements were all made by wave staffs fixed to oil production platforms. The staffs were calibrated in the field at least once a month by shorting them at measured distances. These calibrations indicated that the absolute water level was known to within a third of a meter and that the measurements were linear to within 1–2%. The locations and times of the measurements are summarized in Table 1, and the measurement locations and storm tracks are shown in Figure 1. The ODGP, which operated from 1968 to 1971, has been described by *Patterson* [1969], *Ward* [1974], and *Hamilton and Ward* [1974]. Measurements made by the ODGP in Camille were discussed by *Hamilton and Steere* [1969] and *Patterson* [1974]. Measurements in Felice were described by *Hamilton* [1970]. The Ocean Current Measuring Program (OCMP), described by *Hall* [1972], has been operating from 1972 to the present. *Forristall et al.* [1977] discussed the meteorology of Delia and current measurements made in that storm. In all, 116 hours of wave staff data comprising 55,319 individual waves were studied. Values of m_0 ranged from 3.92 to 118.59 ft² (0.36 to 11.02 m²).

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TABLE 1. Data Sources

Storm	Location	Time, CDT	Date
Camille	ODGP station 1	0230–1620	Aug. 17, 1969
Camille	ODGP station 2	0200–1400	Aug. 17, 1969
Felice	ODGP station 3	0400–1600	Sept. 15, 1970
Delia	OCMP station 1	0600–2400	Sept. 4, 1973
Carmen	OCMP station 2	0600–2400	Sept. 7, 1974
Carmen	OCMP station 3	0600–2400	Sept. 7, 1974
Eloise	OCMP station 2	1400–0100	Sept. 22–23, 1975
Eloise	OCMP station 3	1900–0800	Sept. 22–23, 1975

The analyses were done for consecutive 30-min segments of data. The mean sea level for each segment was calculated and defined as the zero level. A wave was then defined as the part of the record between two consecutive passages of the trace down across the zero level. This definition is sometimes referred to as the zero-downcrossing method. The 30-min records were thus divided into a number of consecutive sections, each of which constitutes a wave. The period of each wave is then the time between downcrossings, and the height of the wave is the difference between the highest and lowest elevations during the wave, that is, the distance between the crest and the preceding trough. The quantity m_0 in (1) can be found by integrating the wave power spectrum or by the much simpler means of taking the mean square of the wave profile. Both methods yield the variance of the record.

Waves from different records were combined with one another and the measured distribution function extended to quite low levels of probability by normalizing the individual wave heights by $(m_0)^{1/2}$, the true rms of the wave profile. The fraction of the total number of waves exceeding given values of $H_0/(m_0)^{1/2}$ were then plotted as the triangles in Figure 2. The solid curve in Figure 2 is the Rayleigh distribution plotted from (1). Equation (1) overpredicts the probabilities of the highest waves, and the error grows worse toward the low-probability tail of the distribution, which is particularly important for design purposes. The results in Figure 2 are substantially the same as those given by *Haring et al.* [1976].

Although the overprediction of the Rayleigh distribution is worst for the highest waves, it is still substantial for moderate normalized wave heights, and this is one reason for the discrepancies between previous comparisons. For example, the average of the heights of the highest one third of the normalized wave heights in the data set, denoted by $H_{(1/3)}$, was found to be

$$H_{(1/3)} = 3.77(m_0)^{1/2} \tag{2}$$

However, manipulation of (1) gives the prediction that

$$H_{(1/3)} = 4.005(m_0)^{1/2} \tag{3}$$

Goda [1974] found the factor in (2) equal to 3.79 for the data from Nagoya port, which he considered to be his only deep-water waves. For his measurements of shallow water waves, (3) gave a good fit.

The error in the prediction given by (3) is of particular interest, since $H_{(1/3)}$ has been defined to be the 'significant wave height.' Since the factor 4.005 in (3) is so close to exactly 4, it is almost always replaced by exactly 4, the relationship thus being given an air of exactitude that it really does not possess. Indeed, significant wave heights reported in the literature have often been calculated by (3) rather than the original definition. When both methods are used, the results are invariably different.

A further source of confusion stems from the combination of (1) and (3) to produce the distribution

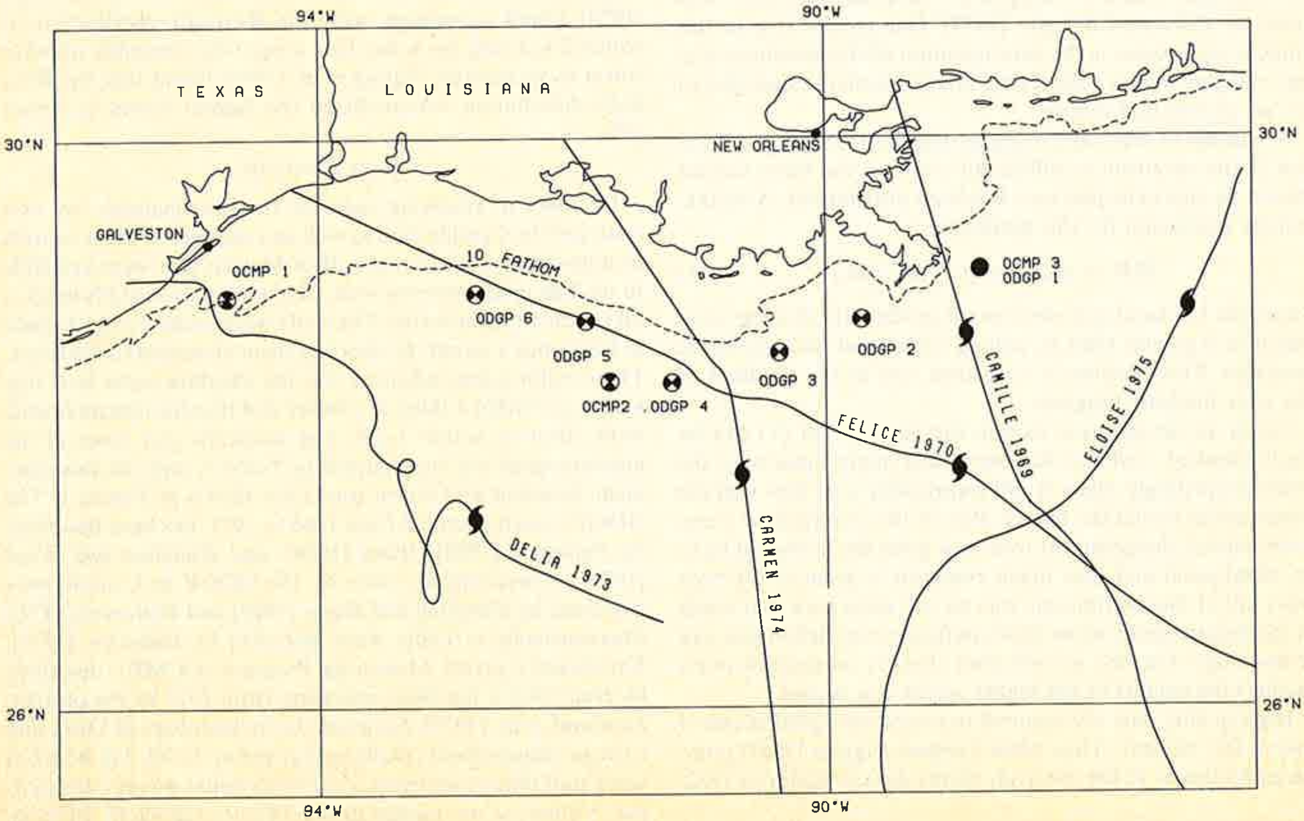

Fig. 1. Storm tracks and measuring station locations.

$$P(H > H_0) = \exp(-2.005H_0^2/H_{(1/3)}^2) \quad (4)$$

Now if $H_{(1/3)}$ is calculated from (3), equations (1) and (4) describe precisely the same distribution. If, however, $H_{(1/3)}$ is calculated by actually averaging the highest one third of the waves, as was done by *Earle* [1975], the equations are significantly different. The latter calculation will naturally produce a distribution which fits the data better, since it is guaranteed to produce the correct value for $H_{(1/3)}$. This is the major reason that *Earle* [1975] finds a better fit to the Camille data than do others who have studied it. *Thompson* [1974] also normalized with respect to the measured $H_{(1/3)}$, while *Wu* [1973] normalized with respect to the average wave height. *Chakrabarti and Cooley* [1977] normalized with respect to the rms wave height, as opposed to the rms of the wave profile. For precise comparisons between data sets it is important that the same form of the Rayleigh distribution be used. Furthermore, since m_0 is the fundamental quantity produced by modern wave hindcasting programs, the wave heights should be normalized with respect to $(m_0)^{1/2}$ rather than some derived quantity such as $H_{(1/3)}$.

The calculation of the maximum wave in a data segment presents further problems. First, the probability distribution of the maximum is calculated from (1), and then, various functions of the distribution are calculated. As is typical in mathematics, these functions are precisely defined but labeled with words that are loaded with colloquial meanings in the English language.

The expected value or ensemble average of the maximum wave height in a record of given length is called the 'expected' maximum, while the modal value of the extreme value distribution is called the 'most probable' maximum. For a sample of 200 waves the expected maximum from the Rayleigh distribution is about 4% higher than the most probable maximum. *Earle* [1975] compared the observed maxima from Camille with the predicted most probable maxima, although he actually estimated the ensemble average from the data. This 4% difference plus the 5% difference from the different definitions of significant wave height makes his results perfectly consistent with those of *Haring et al.* [1976].

AN EMPIRICAL DISTRIBUTION

It is thus evident that the Rayleigh distribution overpredicts the maximum waves in a record by a significant amount. It is natural to suspect that some of the discrepancy is due to the effect of spectral width, since the Rayleigh distribution was derived with the assumption of a narrow spectrum. *Cartwright and Longuet-Higgins* [1956] were able to eliminate this assumption in constructing distributions for crest heights which were dependent on a spectral width parameter. In a study of 10,000 waves recorded by a shipborne wave recorder, *Cartwright* [1958] found good agreement with the crest height theory, although the effects of the spectral width were negligible in his data.

Unfortunately, the crest height theory cannot be extended to deal with zero-crossing wave heights. In fact, the theory does not give the distribution of zero-crossing crest heights but rather the distribution of local maxima, some of which may be negative. As the spectral width parameter increases to unity, the number of local maxima becomes much greater than the number of zero crossings, and their distribution approaches a normal distribution. But as was pointed out by *Goda* [1974], the true width of wave spectra approaches unity and is reduced to other values only by the filtering that always accompanies

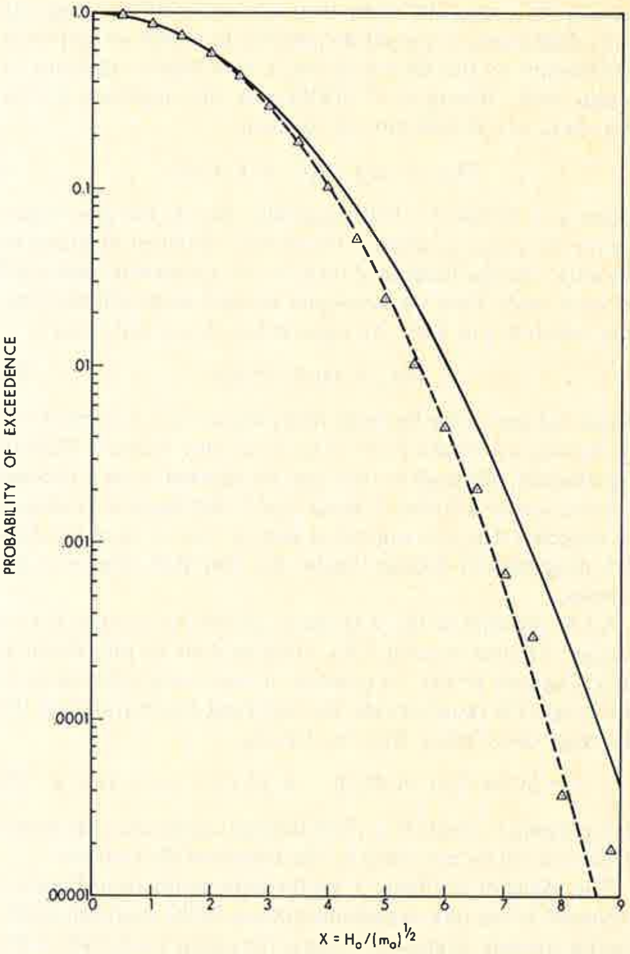

Fig. 2. Probability of exceeding a given normalized wave height. The triangles are data, the solid line is the Rayleigh distribution, and the dashed line is the empirical Weibull distribution.

recording and processing. It seems undesirable for a parameter to be so dependent on the details of the data processing. Fortunately, the many local maxima due to the high-frequency components in the spectrum do not greatly influence the zero-crossing wave heights. *Jahns and Wheeler* [1973] noted that as long as the number of waves was estimated on the basis of zero crossings, narrow spectrum statistics could be used.

A far more likely source of the discrepancy between theory and observation is the failure of the assumption of normality of the wave profile. Our data showed a definite excess of high crest points and a lack of low trough points. Thus the crest heights might well fit the theoretical distribution better than the wave heights for reasons unrelated to the width of the spectrum. This skewness of the wave profile distribution is associated with the nonlinearity of the waves. *Longuet-Higgins* [1963] has also attacked the nonlinear problem and produced complicated distributions for the wave profile, but there seems to be no way to extend these results in a wave height distribution. If the wave height distribution does depend on the nonlinearity of the waves, it may be found empirically to be a function of such things as water depth and wave steepness. However, both our data and other published data indicate that except for very shallow water or very low waves, any such dependence is weak.

It does not appear likely that our theoretical understanding of the distribution function for wave heights will improve greatly in the near future. However, since all the available data seem quite consistent when they are analyzed by using the same definitions, it should be possible to match an empirical distribution to the data and use it with high confidence in design work. *Haring et al.* [1976] took this approach, fitting their data to a distribution of the form

$$H(x) = \exp\,[x^2(C_1 + C_2x)/8] \tag{5}$$

where $x = H_0/(m_0)^{1/2}$. Unfortunately, they do not give values for the constants C_1 and C_2, which were obtained by fitting to the data, and the functional form in (5) is somewhat awkward to work with. Thus we developed another empirical distribution which fit our data. An appropriate functional form is

$$E(x) = \exp\,(-x^\alpha/\beta) \tag{6}$$

which reduces to the Rayleigh distribution for $\alpha = 2$ and $\beta = 8$. A distribution of the form of (6) is usually called a Weibull distribution, although it may also be referred to as a Fréchet distribution or a Fisher-Tippett type 2 distribution. It should be expected that α is somewhat greater than 2, since the data fall progressively further below the Rayleigh line as x increases.

To fit the data to (6), a transformation that reduces it to a straight line can be used. This could be done by plotting ln ln $[E(x)]$ against ln (x). In practice, it was more convenient to work with the ratio between the empirical distribution and the Rayleigh distribution $R(x)$ by defining

$$y = \ln\,[\ln E(x)/\ln R(x)] = \ln\,(8/\beta) + (\alpha - 2)\ln x \tag{7}$$

and plotting y versus ln x. This display accentuates the errors which should be corrected by the empirical distribution.

The triangles in Figure 3 are the data points from Figure 2 replotted in the new coordinate system. If the Rayleigh distribution correctly matched the data, the points would fall on the x axis. The family of distributions suggested by *Haring et al.* [1976] plot as steadily steepening rising curves.

Except for the data points where x is less than 2, the data actually show a reasonably linear trend. The fit for $x < 2$ is relatively unimportant; i.e., the data will be closely matched by (6) in this range even for wide variations in α and β. The poor fit for $x < 2$ in Figure 3 is partly due to the logarithmic transformations. Thus those points were ignored in fitting the dashed regression line in Figure 3. The parameters of the line are

$$\alpha = 2.126 \qquad \beta = 8.42 \tag{8}$$

With these parameters, (6) plots as the dashed line in Figure 2. The fit to the data points is good everywhere, although there is naturally some sampling scatter around the low-probability tail of the curve. Detailed analysis of additional storm wave data would undoubtedly result in values of the parameters slightly different from those given in (8), but those values are consistent with other published data sets to within plotting accuracy.

Statistics Derived From the Empirical Distribution

One of the advantages of the functional form of (6) is that it can be conveniently manipulated to give the same sort of statistics that *Longuet-Higgins* [1952] derived from the Rayleigh distribution. Equation (6) gives the probability that the normalized wave height will exceed x. The probability distribution function $F_X(x)$ is defined as the probability that X will be less than x, so

$$F_X(x) = 1 - E(x) = 1 - \exp\,(-x^\alpha/\beta) \tag{9}$$

Then the probability density function is

$$f_X(x) = \frac{d}{dx}F_X(x) = \frac{\alpha}{\beta}x^{\alpha-1}\exp\,(-x^\alpha/\beta) \tag{10}$$

For some purposes it is useful to know the average normalized height of the highest $1/p$ waves, $H_{(1/p)}$. First, note that the proportion $1/p$ of heights which exceed a certain value x_0 is given from (6) as

$$1/p = \exp\,(-x_0^{\,\alpha}/\beta) \tag{11}$$

from which

$$x_0 = (\beta \ln p)^{1/\alpha} \tag{12}$$

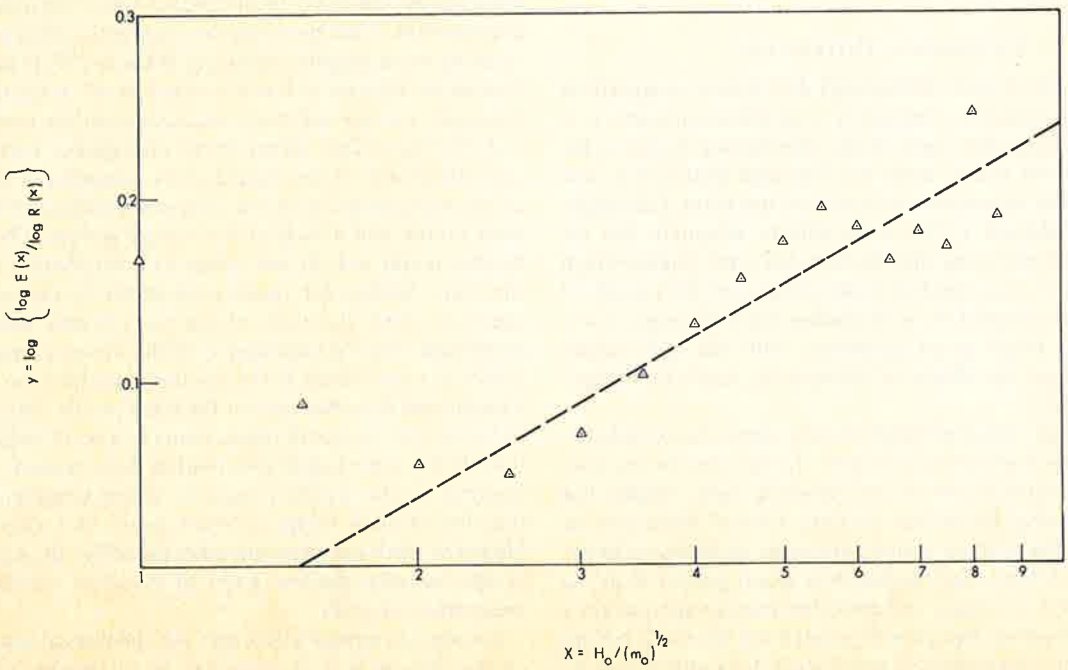

Fig. 3. Error in the Rayleigh distribution. The triangles are data; the dashed line is the empirical Weibull distribution.

TABLE 2. Average Heights of the Highest $1/p$ Waves, Normalized by $(m_0)^{1/2}$

p	$H_{(1/p)}$ Empirical	$H_{(1/p)}$ Rayleigh	Empirical/Rayleigh
100.000	6.108	6.672	0.915
20.000	5.192	5.617	0.924
10.000	4.733	5.091	0.930
5.000	4.214	4.500	0.936
4.000	4.029	4.291	0.939
3.333	3.870	4.113	0.941
3.000	3.774	4.005	0.942
2.500	3.599	3.810	0.945
2.000	3.370	3.553	0.949
1.667	3.165	3.326	0.952
1.428	2.974	3.117	0.954
1.250	2.792	2.916	0.957
1.111	2.610	2.718	0.960
1.000	2.413	2.506	0.963

The mean value of the heights $H_{(1/p)}$ greater than x_0 is given by

$$\exp(-x_0^\alpha/\beta)H_{(1/p)} = \int_{x_0}^{\infty} x f_X(x)\, dx$$

$$= x_0 \exp(-x_0^\alpha/\beta) + \int_{x_0}^{\infty} \exp(-x^\alpha/\beta)\, dx \qquad (13)$$

from which

$$H_{(1/p)} = (\beta \ln p)^{1/\alpha} + p \int_{x_0}^{\infty} \exp(-x^\alpha/\beta)\, dx$$

$$= (\beta \ln p)^{1/\alpha} + p\left\{\frac{1}{\alpha}\beta^{1/\alpha}\Gamma\left(\frac{1}{\alpha}\right) - \int_0^{x_0} \exp(-x^\alpha/\beta)\, dx\right\} \qquad (14)$$

The remaining integral in (14) is easily evaluated numerically, and the results for selected values of p are displayed in Table 2.

Table 2 also shows the values of $H_{(1/p)}$ derived from the Rayleigh distribution and the ratio of the empirical values to the Rayleigh values. Thus, for example, the empirical value of the normalized significant wave height $H_{(1/3)}$ is 3.774, which is 0.942 times the value calculated from the Rayleigh distribution.

Again, following *Longuet-Higgins* [1952], the expected value of the maximum normalized wave height in N waves is

$$E(x_{\max}) = \int_0^{\infty} [1 - (1 - \exp(-x^\alpha/\beta))^N]\, dx \qquad (15)$$

Now, let $\theta = x^\alpha/\beta$, and

$$E(x_{\max}) = \frac{1}{\alpha}\beta^{1/\alpha} \int_0^{\infty} [1 - (1 - e^{-\theta})^N]\theta^{1/\alpha - 1}\, d\theta \qquad (16)$$

When the binomial theorem is used,

$$E(x_{\max}) = \frac{1}{\alpha}\beta^{1/\alpha}\Gamma(1/\alpha)\left[N\frac{1^{1/\alpha}}{1} - \frac{N(N-1)}{2!}\left(\frac{1}{2}\right)^{1/\alpha} + \cdots + (-1)^{N+1}\left(\frac{1}{N}\right)^{1/\alpha}\right] \qquad (17)$$

Using double-precision arithmetic, it is possible to evaluate (17) explicitly up to $N = 50$. However, we are also interested in much larger values of N, for which the binomial coefficients become unmanageable and for which an asymptotic expression must be developed. For large N, write

$$\theta_0 = \ln N \qquad (18)$$

and let

$$\theta = \theta_0 + \theta' \qquad (19)$$

Then,

$$(1 - e^{-\theta})^N = \left(1 - \frac{e^{-\theta'}}{N}\right)^N \simeq \exp(-e^{-\theta'}) \qquad (20)$$

with errors of order $1/N$. Then, (16) becomes

$$E(x_{\max}) = \frac{1}{\alpha}\beta^{1/\alpha}\int_{-\theta_0}^{\infty} [1 - \exp(-e^{-\theta'})]\,(\theta_0 + \theta')^{1/\alpha - 1}\, d\theta'$$

$$= \frac{1}{\alpha}\beta^{1/\alpha}\left\{\int_{-\theta_0}^{0} (\theta_0 + \theta')^{1/\alpha-1}\, d\theta' - \theta_0^{1/\alpha - 1}\int_{-\theta_0}^{0} \exp(-e^{-\theta'})(1 + \theta'/\theta_0)^{1/\alpha - 1}\, d\theta' + \theta_0^{1/\alpha-1}\int_0^{\infty} [1 - \exp(-e^{-\theta'})](1 + \theta'/\theta_0)^{1/\alpha-1}\, d\theta'\right\} \qquad (21)$$

TABLE 3. Expected Maximum Wave Heights, Normalized by $(m_0)^{1/2}$

	Empirical		Rayleigh		
N	Exact	Asymptotic	Exact	Asymptotic	Empirical/Rayleigh
1	2.413		2.506		0.963
2	3.084		3.241		0.952
5	3.887		4.135		0.940
10	4.422	4.508	4.740	4.831	0.933
20	4.904	4.978	5.289	5.368	0.927
50	5.475	5.534		6.008	0.921
100		5.917		6.449	0.918
200		6.274		6.862	0.914
500		6.714		7.379	0.910
1000		7.027		7.744	0.907
2000		7.325		8.095	0.905
5000		7.699		8.533	0.902
10,000		7.969		8.853	0.900
20,000		8.229		9.161	0.898
50,000		8.560		9.552	0.896
100,000		8.801		9.837	0.895

The first integral in (21) may be evaluated explicitly. The integrands in the second and third integrals are small except near $\theta' = 0$. Thus all except the first term in the binomial expansions may be neglected, leading to errors of order $1/\theta_0$ at most. Then, when the substitution $z = e^{-\theta'}$ is made,

$$E(x_{\max}) = \frac{1}{\alpha}\beta^{1/\alpha}\left\{\alpha\theta_0^{1/\alpha} + \theta_0^{1/\alpha - 1}\left[-\int_1^\infty e^{-z}\frac{dz}{z} + \int_0^1 (1 - e^{-z})\frac{dz}{z}\right]\right\} = (\beta\theta_0)^{1/\alpha}(1 + \gamma/\alpha\theta_0) \qquad (22)$$

where $\gamma = 0.5772$ is Euler's constant. By using the values of α and β from (8), equation (22) was evaluated for several values of N, and the results displayed in Table 3. Shown also are the expected maxima of the Rayleigh distribution and the ratios between the predictions of the empirical and Rayleigh distributions. Thus, for example, the expected value of the maximum wave in 1000 waves is $7.027(m_0)^{1/2}$, which is 0.907 times the prediction from the Rayleigh distribution.

CONCLUSIONS

The Rayleigh distribution substantially overpredicts the heights of the highest waves in a record. This conclusion has been supported here by the detailed analysis of 116 hours of hurricane wave data from the Gulf of Mexico. Once some confusion in definitions is considered, it is also consistent with a large body of published wave data. It does not seem feasible to produce theoretically a distribution which fits the data better, but it has been possible to fit the data with an empirical function of simple form. Statistics developed from the empirical distribution show that the significant wave height is 0.942 times that calculated from the Rayleigh distribution and the expected value of the maximum wave in 1000 is 0.907 times the height calculated from the Rayleigh distribution. The empirical distribution should be used whenever precise predictions of maximum wave heights are needed.

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