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Technical note

Wavelet bicoherence analysis of wind–wave interaction

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Abstract

A new technique in the analysis of wind–wave interaction, wavelet bicoherence, will be applied in this article. Wavelet bicohence has the ability to detect phase coupling and nonlinear interactions of the quadratic order with time resolution. It is used in this study to analyze wind–wave interaction during wave growth in a Mistral event. A selected record of simultaneously measured wind and wave data during Mistral is divided into five segments and the computations of the wavelet bicoherence are conducted for the whole record and for all divided segments. The results show that the phase coupling occurs between wind speed and wave height over a certain range of frequencies and that the range is different from one segment to another due to the non-stationary feature of the time series.

Keywords: Morlet wavelet; Cross-spectrum; Cross-bispectrum; Wave growth; Phase coupling

1. Introduction

Several studies were conducted to analyze wind–wave interaction. Among those are (Phillips, 1957, 1958; Miles, 1957, 1959a,b, 1962, 1967; Lighthill, 1962; Kraus and Businger, 1994). The studies analyzed the generation of waves due to the wind action and the mechanism of primary transfer of energy from wind to water, but still the mechanism by which wind generates the waves is not yet understood.

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Wavelet transform is a recently developed mathematical technique with enormous applications in the fields of science and engineering. It overcomes the shortcomings of the Fourier analysis by adding the time resolution and giving temporal characteristics of the signals.

The purpose of this paper is to introduce wavelet bicoherence as a new tool for analyzing wind–wave interaction. This tool was first introduced by Milligen et al. (1995) in their analysis of turbulence. Wavelet bicoherence is a measure of phase coupling that occurs in a signal or between two signals. Phase coupling is defined as occurring when two frequencies, $f1$ and $f2$, are simultaneously present in the signal(s) along with their sum (or difference) frequencies, and the sum of the phases of these frequency components remains constant. The bicoherence measures this quantity and is a function of two frequencies $f1$ and $f2$ which is close to 1 when the signal contains three frequencies $f1, f2, f$ that satisfy the relation $f = f1 + f2$; if this relation is not satisfied, it is close to zero (Milligen et al., 1995).

2. Ocean data

The data used in this work was collected during FETCH experiment (Drennan et al., 2003). An ASIS buoy was deployed on 18th March, 1998, and remained until 10th April, 1998. This period corresponds to a Mistral event, which is a regional wind occurring

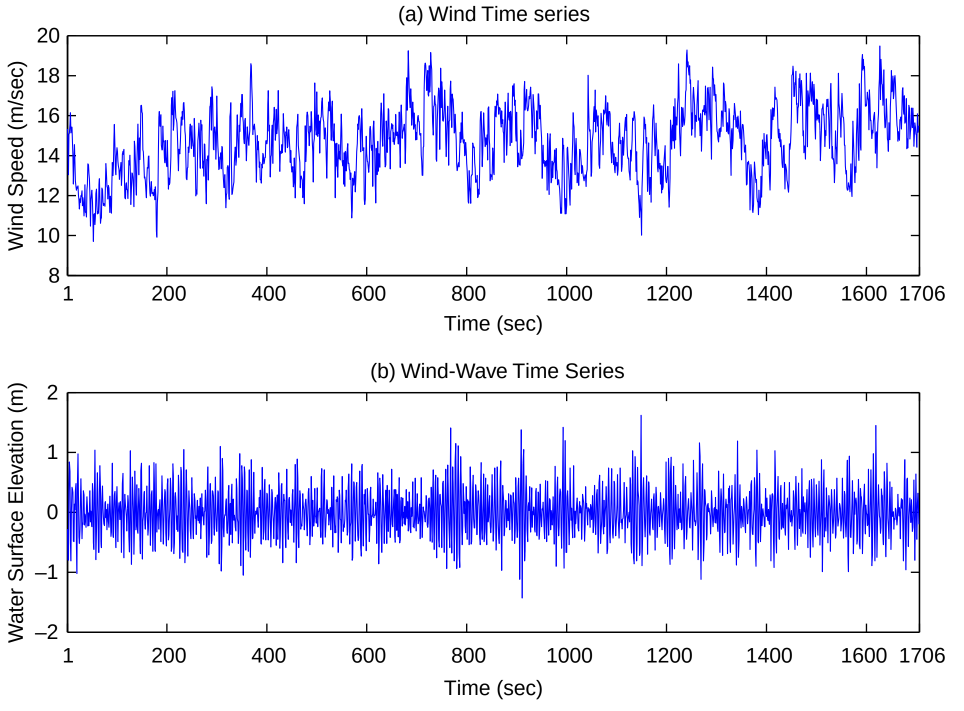

Fig. 1. Time Series of wind speed and wind–wave during Mistral event (whole record).

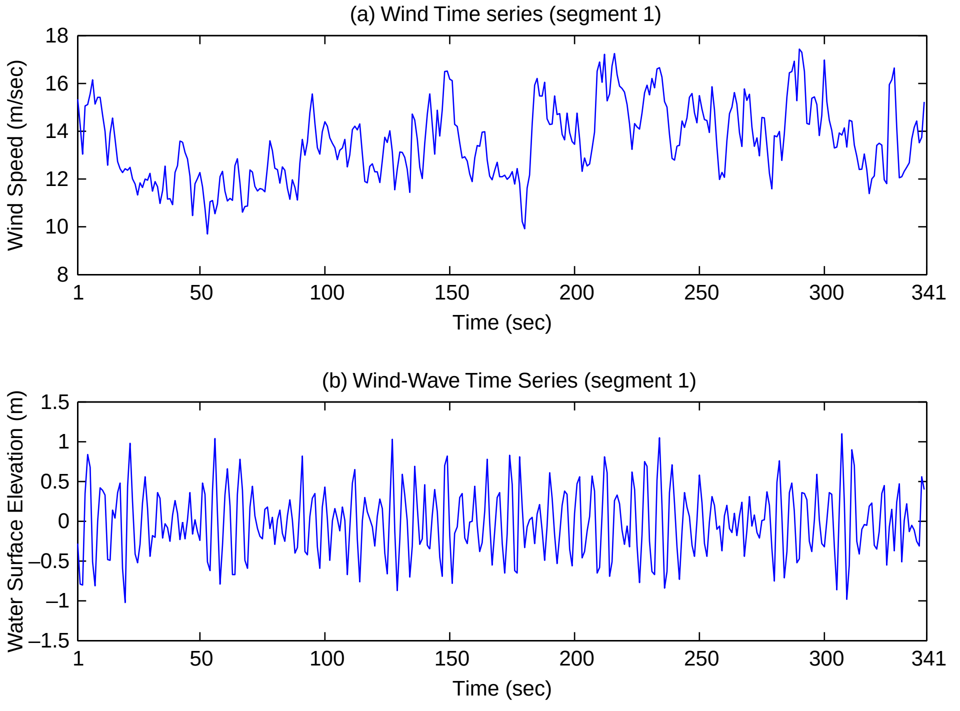

Fig. 2. Time Series of wind speed and wind–wave during Mistral event (segment one).

during conditions of high pressure over western Europe/Bay of Biscay, combined with a low-pressure system south of the Alps. The Mistral is characterized by a strong, steady and cold flow with near surface wind speeds frequently in excess of 30 m/s (Drennan et al., 2003). The selected record contains the maximum wind speed and the maximum wave height observed during 6 h of wave generation. The length of the record is 1706 s of continuous measurements of wind speed and wave height (Fig. 1). The record is divided sequentially into five segments each one with a length of 341 s as shown in Figs. 2–6, respectively.

3. Methodology

3.1. Continuous wavelet analysis

The continuous wavelet transform $W(a,\tau)$ of a function $h(t)$, is defined as:

$$W(a,\tau) = \int_{-\infty}^{+\infty} h(t)\psi^{*}_{a,\tau}(t)\mathrm{d}t \tag{1}$$

where a and τ are scale and time variables respectively, and $\Psi_{\alpha,\tau}$ represents the wavelet family generated by continuous translations and dilations of the mother wavelet Ψt.

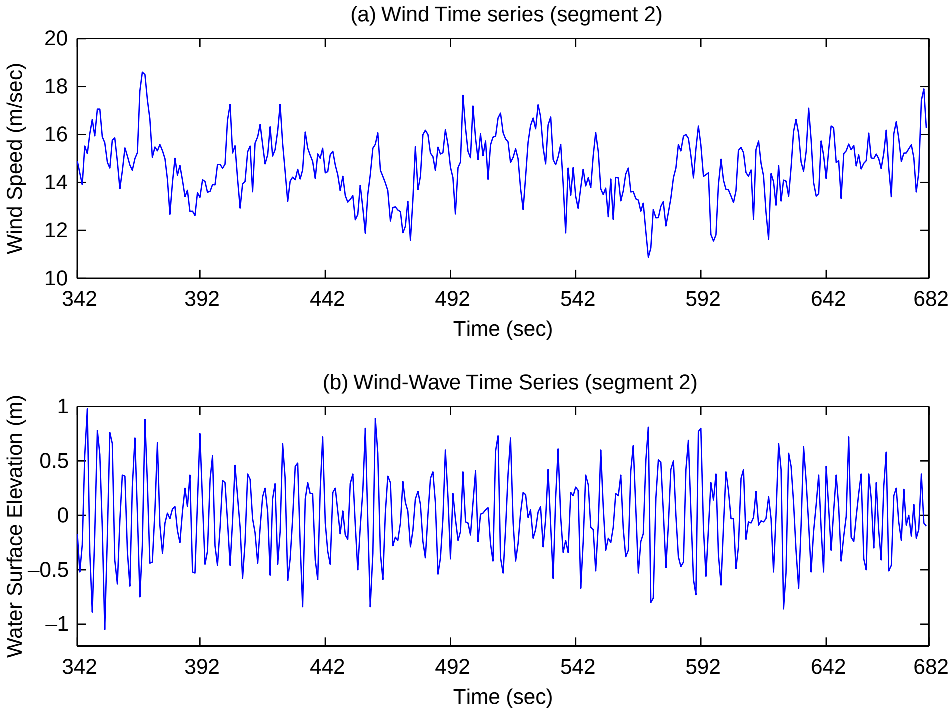

Fig. 3. Time Series of wind speed and wind–wave during Mistral event (segment two).

These translations and dilations are obtained by

$$\psi_{a,\tau} = \frac{1}{\sqrt{a}}\psi\left(\frac{t-\tau}{a}\right) \tag{2}$$

Following Torrence and Compo (1998) and Addison (2002), the complex Morlet wavelet (Morlet, 1981) to be implemented in this study is defined as:

$$\psi(t) = \Pi^{-1/4}\mathrm{e}^{\mathrm{i}w_o t}\mathrm{e}^{\frac{-t^2}{2}} \tag{3}$$

In this definition, w_o is chosen to be 6.0 to approximately satisfy the wavelet admissibility condition (Farge, 1992).

Torrence and Compo (1998) developed a code for computing the continuous wavelet transform of the time series of the signal. In this work the code was modified to compute the wavelet cross-bispectrum and the wavelet cross-bicoherence.

3.2. Power-spectrum

In order to have statistical stability the wavelet power spectrum is integrated over a finite time interval (Milligen et al., 1995, 1997). The wavelet power spectrum:

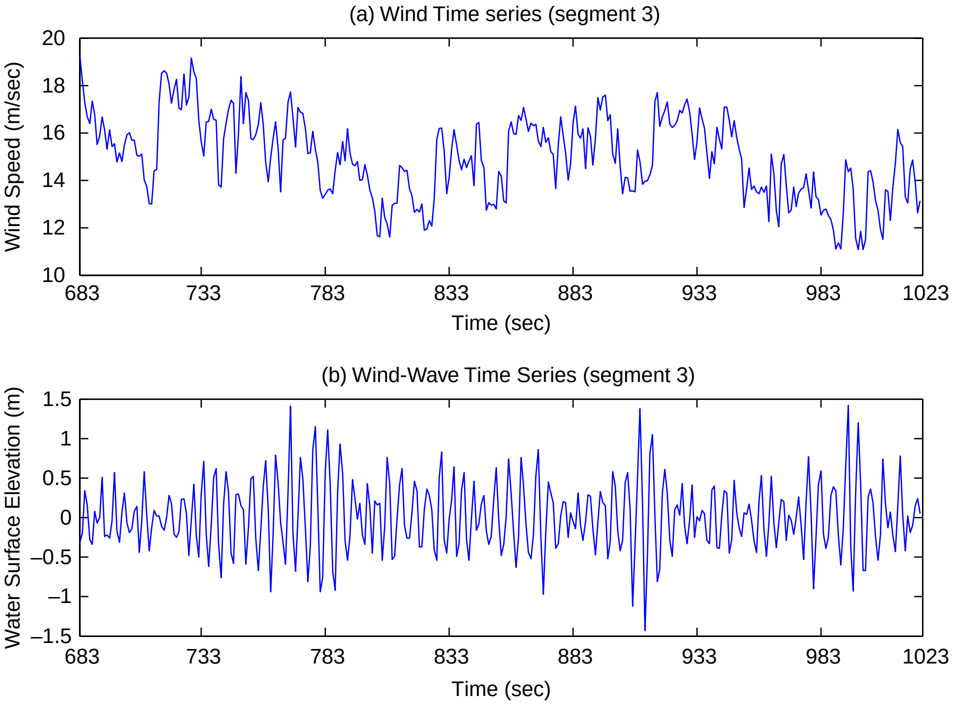

Fig. 4. Time Series of wind speed and wind–wave during Mistral event (segment three).

$$P_{xx} = \int_T W_x^*(a,\tau)W_x(a,\tau)\mathrm{d}\tau \tag{4}$$

$W_x(a,\tau)$: wavelet transform of the time series
$W_x^*(a,\tau)$: the complex conjugate of wavelet transform of the time series
T: a finite time interval

3.3. Cross-spectrum

The wavelet cross-spectrum is defined as follows:

$$P_{xy} = \int_T W_x^*(a,\tau)W_y(a,\tau)\mathrm{d}\tau \tag{5}$$

$W_x^*(a,\tau)$: the complex conjugate of wavelet transform of the first time series
$W_y(a,\tau)$: the wavelet transform of the second time series

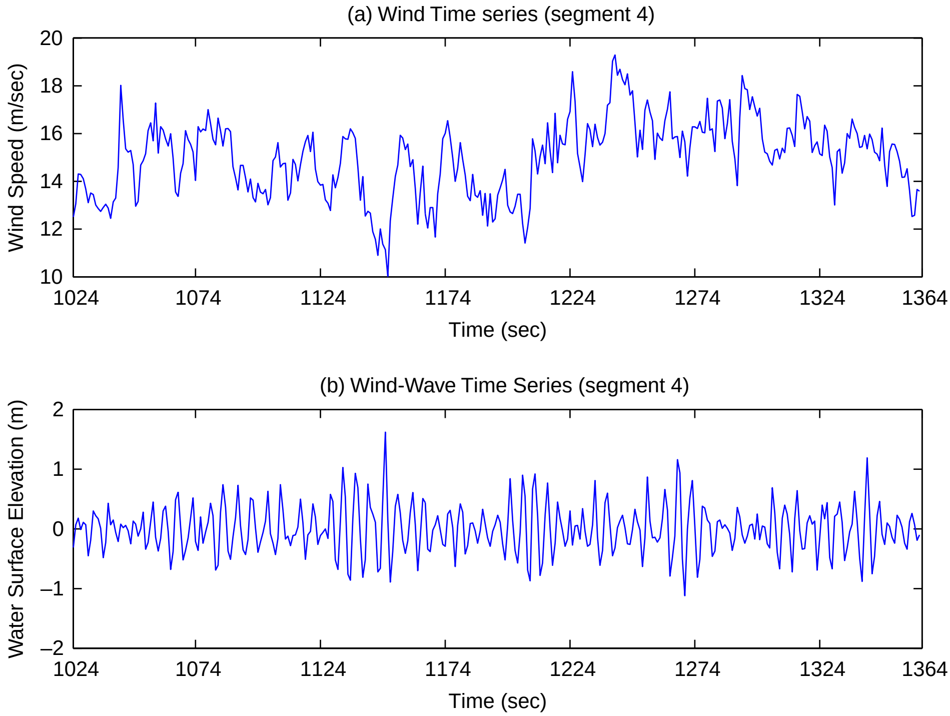

Fig. 5. Time Series of wind speed and wind–wave during Mistral event (segment four).

It should be noted that the results of the cross-spectrum depend heavily on the fluctuations of the individual signals because if one of the signals is fluctuated less than the other the effect of the more fluctuating signal will dominate in the resultant cross-spectrum (Hajj et al., 1998).

3.4. Linear-coherence

The normalized wavelet cross-spectrum (to have values between 0 and 1) gives the linear coherence as follows:

$$Coh_{xy}(a) = \frac{|P_{xy}(a)|^2}{P_{xx}(a)P_{yy}(a)} \tag{6}$$

$Coh_{xy}(a)$: linear coherence between two time series
$P_{xx}(a)$: power spectrum of first time series

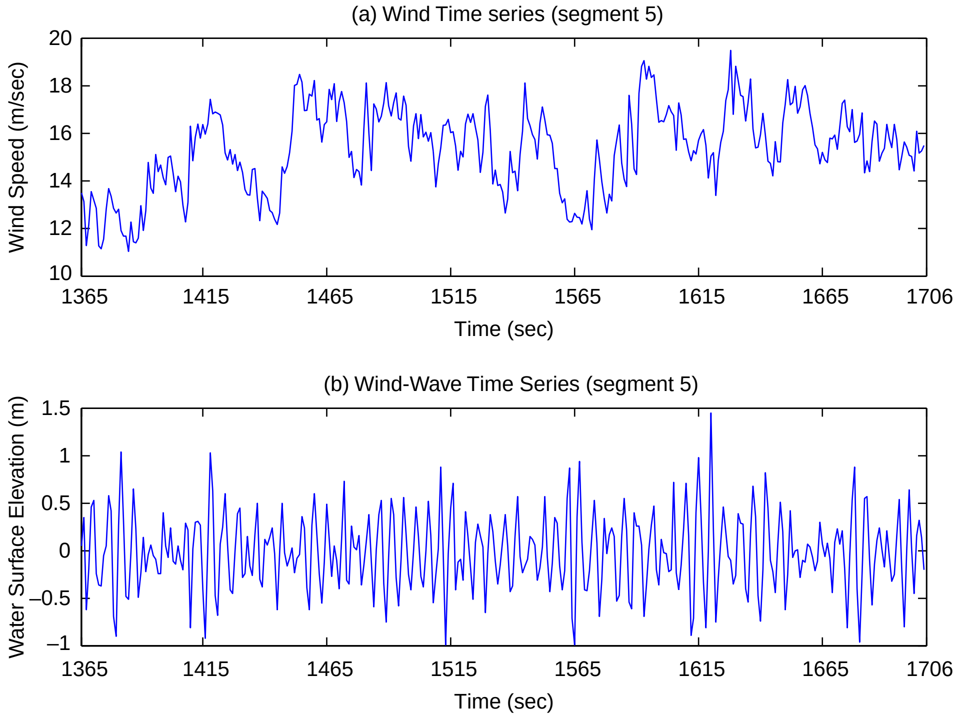

Fig. 6. Time Series of wind speed and wind–wave during Mistral event (segment five).

$P_{yy}(a)$: power spectrum of second time series
$P_{xy}(a)$: cross-spectrum of the first and the second time series

3.5. Cross-bispectrum

The wavelet cross-bispectrum is defined as follows:

$$B_{yxx}(a1, a2) = \int_T W_y^*(a, \tau) W_x(a1, \tau) W_x(a2, \tau) \mathrm{d}\tau \tag{7}$$

where $\frac{1}{a} = \frac{1}{a1} + \frac{1}{a2}$ (frequency sum rule). The wavelet cross-bispectrum measures the amount of phase coupling in the time interval T that occurs between wavelet components of scale lengths $a1$ and $a2$ of $x(t)$ and wavelet component a of $y(t)$ such that the sum rule is satisfied (Milligen et al., 1995). The normalized squared wavelet cross-bicoherence is defined as:

$$[b_{yxx}(a1,a2)]^2 = \frac{|B_{yxx}(a1,a2)|^2}{\left[\int_T |W_x(a1,\tau)W_x(a2,\tau)|^2 \mathrm{d}\tau\right]\left[\int_T |W_y(a,\tau)|^2 \mathrm{d}\tau\right]} \tag{8}$$

4. Results and discussions

To examine the phase coupling and the non-linearity between wind and wave, wavelet bicoherence (wind & wind & wave) is computed for the whole record of simultaneously measured wind speed and wave height given in Fig. 1. The integration is conducted over the time ranges between 1 and 1706 s (the whole window), and the results are given in Fig. 7. The distribution of wavelet bicoherence levels shows the different levels of phase coupling and non-linear interaction between wind and wave with a high value at $f1$ and $f2$ equal to 0.002.

In order to analyze the non-stationary feature of the measured record of wind and wave, the record is divided into five segments. The wavelet bicoherence (wind & wind & wave) is computed for both wind and wave measurements in the five segments given in Figs. 2–6. The integration for each segment is conducted over the whole window of each segment and the results are shown in Figs. 8 through 12. Fig. 8 shows that wavelet bicoherence for

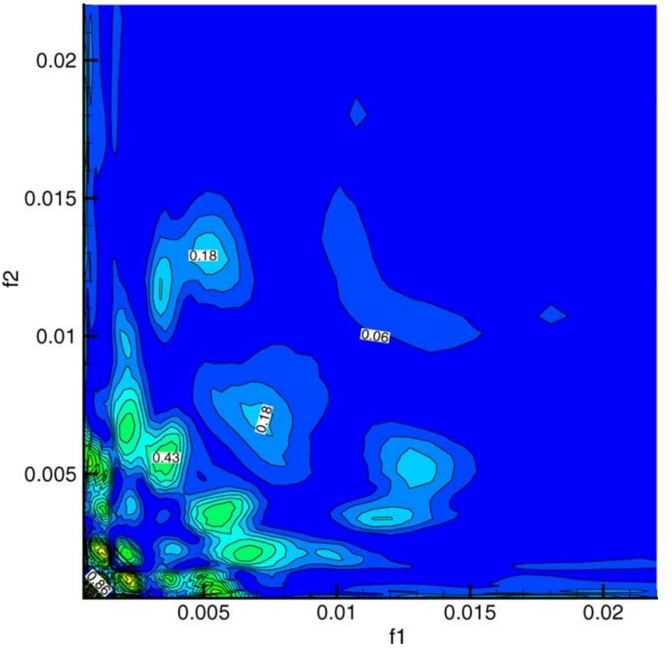

Fig. 7. Wavelet bicoherence (wind & wind & wave) for the whole record given in Fig. 1.

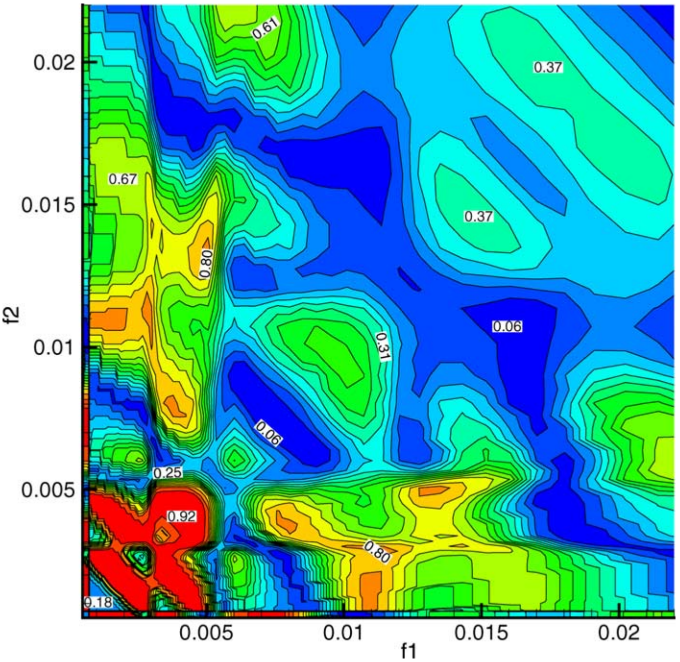

Fig. 8. Wavelet bicoherence (wind & wind & wave) for segment one.

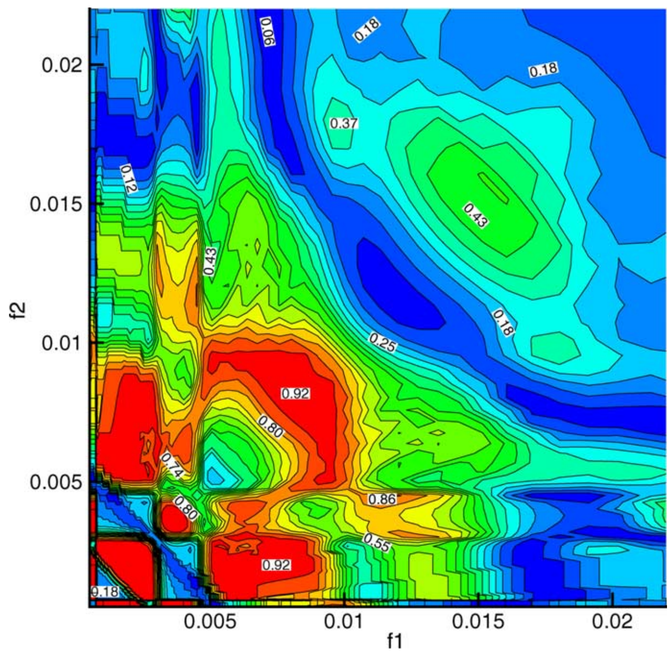

Fig. 9. Wavelet bicoherence (wind & wind & wave) for segment two.

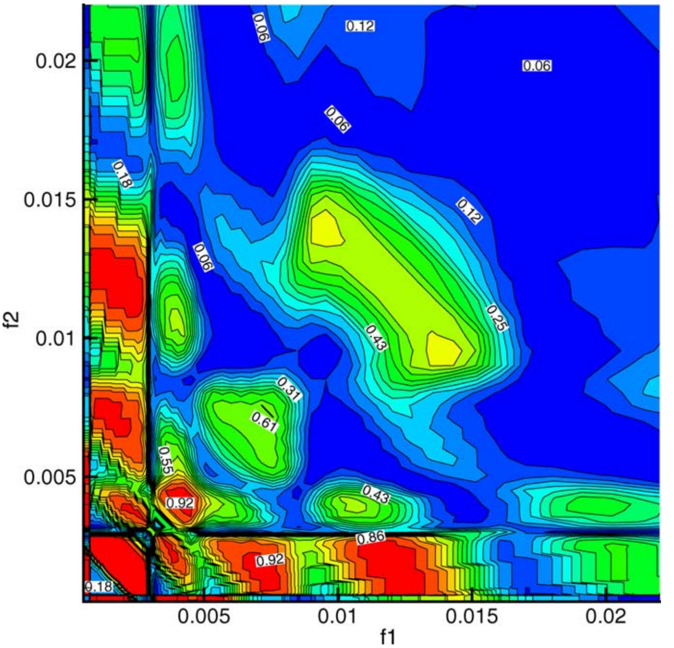

Fig. 10. Wavelet bicoherence (wind & wind & wave) for segment three.

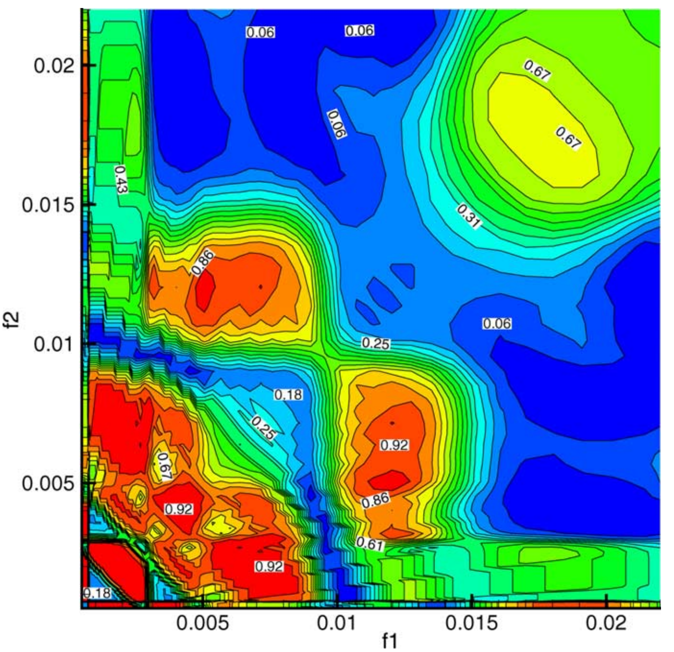

Fig. 11. Wavelet bicoherence (wind & wind & wave) for segment four.

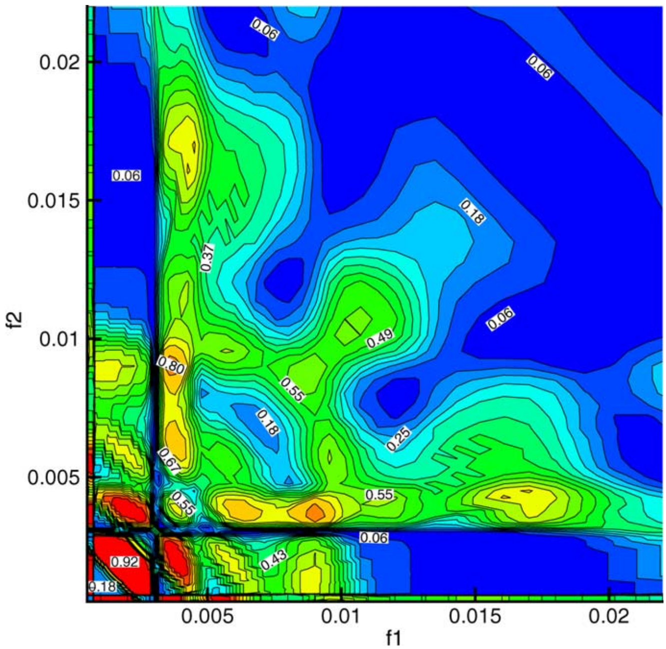

Fig. 12. Wavelet bicoherence (wind & wind & wave) for segment five.

segment one has different levels, and that there are zones of high levels of phase coupling and nonlinear interaction between wind speed and wave height. At $f1$ ranges from 0.002 to 0.005 and $f2$ ranges from 0.002 to 0.005, these ranges represent the zone of highest level of phase coupling between wind speed and wave height.

For segment two (Fig. 9) there is phase coupling at $f1$ and $f2$ range between 0.005 and 0.009. Also, at $f1$ ranges between 0.006 to 0.01 and $f2$ ranges from 0.005 to 0.01 there is another zone of high level of bicoherence. In segments three, four and five it can be observed that the phase coupling and non-linear wind–wave interaction occur at different levels, which are different from one segment to another as shown in Figs. 10–12, respectively.

The results obtained so far show that there is a phase coupling in all computed wavelet bicoherence for the combination of wind & wind & wave for the whole record and the divided segments. Furthermore, the non-stationary feature of the measured data set is clearly shown in the computed wavelet bicoherence for each segment. An averaging over time for each segment shows that the phase coupling and non-linear interaction between wind and wave occurs with high values over different ranges of frequencies.

The above results show that the use of wavelet analysis with time resolution adds valuable information through the use of wavelet bicoherence in detecting the phase coupling between simultaneously measured signals of wind speed and wave height, hence overcoming the shortcomings of Fourier based analysis.

5. Conclusions

A novel technique in analyzing wind–wave interaction during wave growth is introduced, based on wavelet analysis and bispectral analysis. Wavelet bicoherence detects phase coupling between short-lived wavelets and shows that there is nonlinear interaction between wind speed and wave height. The non-linearity in the interaction between wind and wave during wave growth occurred over a certain range of frequencies, which is different from one segment to another due to the non-stationary feature of the examined data set.

The results obtained in this study show important features in wind–wave interaction during wave growth. Furthermore, this study clearly strengthens the usefulness of wavelet analysis and bispectrum analysis in detecting features that were hidden by the assumption of stationary in Fourier-based analysis. Furthermore, wavelet bicoherence can be used to study wave & wave non-linear interactions because these wave–wave interactions cause energy transfer from shorter to longer period waves as the spectrum grows.

6. Acknowledgments

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