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Local and non-local curvature approximation: a new asymptotic theory for wave scattering

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Abstract

We present a new asymptotic theory for scalar and vector wave scattering from rough surfaces which federates an extended Kirchhoff approximation (EKA), such as the integral equation method (IEM), with the first and second order small slope approximations (SSA). The new development stems from the fact that any improvement of the 'high frequency' Kirchhoff or tangent plane approximation (KA) must come through surface curvature and higher order derivatives. Hence, this condition requires that the second order kernel be quadratic in its lowest order with respect to its Fourier variable or formally the gradient operator. A second important constraint which must be met is that both the Kirchhoff approximation (KA) and the first order small perturbation method (SPM-1 or Bragg) be dynamically reached, depending on the surface conditions. We derive herein this new kernel from a formal inclusion of the derivative operator in the difference between the polarization coefficients of KA and SPM-1. This new kernel is as simple as the expressions for both Kirchhoff and SPM-1 coefficients. This formal difference has the same curvature order as SSA-1 + SSA-2. It is acknowledged that even though the second order small perturbation method (SPM-2) is not enforced, as opposed to the SSA, our model should reproduce a reasonable approximation of the SPM-2 function at least up to the curvature or quadratic order. We provide three different versions of this new asymptotic theory under the local, non-local, and weighted curvature approximations. Each of these three models is demonstrated to be tilt invariant through first order in the tilting vector.

1. Introduction

In our recent publications [1–3], we demonstrated, for perfectly conducting surfaces, that the first and second order small slope approximations (SSA-1 + SSA-2) given by Voronovich [4] can be reached, in their functional form, even if the starting point of the derivation was the surface current integral equations. The SSA structure is based on the polarization coefficients of the small perturbation method (SPM) and the sum of single and double integrals. The first order SPM-1 coefficients are placed in front of the single integral while the SPM-2 coefficients, or some combination thereof, are involved in the kernel of the double integral; see equations (5.15) and (5.16) in [5]. Our demonstration in [3] suggested that the single integral can be multiplied by the Kirchhoff polarization coefficients and the kernel of the double integral is now left with a curvature or a quadratic kernel in its lowest order with respect to its Fourier variable or formally the gradient operator. This is a particular version of the extend Kirchhoff approximation (EKA) as opposed to the IEM of Fung [6]. In general, we have also demonstrated that the first and second order small slope approximations (SSA-1 + SSA-2) are defined to within an arbitrary linear kernel which for a particular choice makes the two representations compatible. This idea of quadratic second order kernel was already generalized in a recent letter [7] to the case of surfaces that are good conductors in the quasi-specular regime. We noted that the difference between the polarization coefficients of the SPM-1 and Kirchhoff approaches is quadratic in $\boldsymbol{q}_{\mathrm{H}} = \boldsymbol{k} - \boldsymbol{k}_0$ where $\boldsymbol{k}_0$ and $\boldsymbol{k}$ are the horizontal projections of the incident and scattered wavenumbers, respectively. This derivation allowed the determination of the curvature kernel only in the quasi-specular regime because of the Taylor expansion performed up to the quadratic order in $\boldsymbol{q}_{\mathrm{H}}$. This practical approximation in [7] also demonstrated that the differences in the polarization coefficients and the SSA share the same curvature order.

In the current paper we present a new asymptotic theory based on our previous advances in the electromagnetic scattering from rough surfaces. The new approach extends the domain of applicability of the practical curvature model in [7] to the most general dielectric and bistatic cases. The Neumann and the Dirichlet boundary conditions for acoustic scattering are treated as well. In fact, our new asymptotic model is as simple as just expressing the Kirchhoff and SPM-1 difference in terms of the difference of horizontal wavenumber vectors $\boldsymbol{q}_{\mathrm{H}} = \boldsymbol{k} - \boldsymbol{k}_0$ for the particular problem under study. This new generalized model converges dynamically to both Kirchhoff and SPM-1 limits, depending on the surface roughness. The curvature is shown to have a unifying effect not only on the SPM-1 and Kirchhoff limits but also on the small slope approximation (SSA) [5] and an EKA such as the IEM [6, 8].

In section 2, we define the local curvature approximation (LCA) on the basis of the Kirchhoff coefficients and a curvature kernel quadratic in its lowest order. It is shown in this same section that this kernel must possess certain fundamental properties in order to satisfy reciprocity and the proper asymptotic limits. In section 3, we derive a theory equivalent to the second order small perturbation method (SPM-2) that in turn is correct up to the curvature order. This equivalence to the SPM-2 coefficient permits a link to the local small slope approximation (SSA-1 + SSA-2). In section 4, we show how the non-local curvature approximation (NLCA) can be formulated on the basis of this equivalence to the SPM-2 function. This form is very close to the non-local small slope approximation (NLSSA) as motivated and derived in [9].

On a slightly more practical and less theoretical note, we provide in section 5 a local weighted version of our curvature approximation (WCA) to facilitate Monte Carlo simulations. This form is similar to the local weight approximation (LWA) given by Dashen and Wurmser [10] where the kernel of the single integral is surface slope dependent. Numerical and analytical comparisons with Dashen and Wurmser's as well as with the first order expansion operator model (OE-1) of Milder [11] are also given in section 5.

In section 6, we show how a tilted surface in our curvature based models generates a tilted asymptotic limit for the small perturbation method or SPM-1. This property is termed tilt invariance, and is the formal equivalent of tilted Bragg or tilted SPM-1.

Several appendices detail the expressions for the polarization kernels for the Neumann, Dirichlet, perfect conducting, and dielectric boundary conditions.

2. The local curvature approximation

In our previous paper [3], we demonstrated that the small slope approximation's (SSA-1 + SSA-2) functional form can be reached even when one starts from the surface current integral equations. Indeed, the first iteration current generates the Kirchhoff approximation while the second iteration yields a double integral similar to the one in SSA-2. However, a major difference in this EKA is that the second order kernel is quadratic in its lowest order and therefore only curvature and higher order derivatives of the surface are responsible for the extension. This is physically reasonable since the Kirchhoff approach is the 'high frequency' tangent plane approximation where the surface is considered locally flat. Hence, any improvement on the Kirchhoff approximation must come from the addition of curvature and higher order derivatives of the scattering surface. Under local scattering conditions, one can write, as suggested by Voronovich in [4] and modified by our recent developments in [3], the scattering amplitude as

$$\mathcal{S}(\boldsymbol{k},\boldsymbol{k}_0) = \frac{\mathcal{K}(\boldsymbol{k},\boldsymbol{k}_0)}{q_z}\int \mathrm{e}^{-\mathrm{i}q_z\eta(\boldsymbol{x})}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\mathrm{H}\cdot\boldsymbol{x}}\,\mathrm{d}\boldsymbol{x} - \mathrm{i}\int\!\!\int \mathcal{T}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})\hat{\eta}(\boldsymbol{\xi})\mathrm{e}^{-\mathrm{i}q_z\eta(\boldsymbol{x})}\mathrm{e}^{-\mathrm{i}(\boldsymbol{q}_\mathrm{H}-\boldsymbol{\xi})\cdot\boldsymbol{x}}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{x} \tag{1}$$

where the normalization is motivated by the Dashen and Wurmser papers (see for instance [10]). The notation adopted in the present paper is heavily influenced by Voronovich's notation along with a judicious combination of our previous notation and that of the Dashen and Wurmser papers. Most of the variables used are defined as

$$\boldsymbol{K}_\mathrm{i} = \boldsymbol{k}_0 - q_0\hat{\boldsymbol{z}} \tag{2a}$$
$$\boldsymbol{K}_\mathrm{s} = \boldsymbol{k} + q_k\hat{\boldsymbol{z}} \tag{2b}$$
$$\boldsymbol{K}_\mathrm{i}^2 = \boldsymbol{K}_\mathrm{s}^2 = K^2 \tag{2c}$$
$$q_k = \sqrt{K^2 - \boldsymbol{k}\cdot\boldsymbol{k}} \tag{2d}$$
$$q_0 = \sqrt{K^2 - \boldsymbol{k}_0\cdot\boldsymbol{k}_0} \tag{2e}$$
$$q_z = q_k + q_0 \tag{2f}$$
$$\boldsymbol{q}_\mathrm{H} = \boldsymbol{k} - \boldsymbol{k}_0 \tag{2g}$$
$$\boldsymbol{w}_\mathrm{H} = \boldsymbol{k} + \boldsymbol{k}_0 \tag{2h}$$
$$\eta(\boldsymbol{x}) \rightleftharpoons \hat{\eta}(\boldsymbol{\xi}) \tag{2i}$$

where $\boldsymbol{K}_\mathrm{i}$ and $\boldsymbol{K}_\mathrm{s}$ are the three-dimensional wavenumbers of the incident and scattered waves, respectively. The scattering surface elevation is described by $\eta(\boldsymbol{x})$ and its corresponding Fourier transform $\hat{\eta}(\boldsymbol{\xi})$. The coefficient $\mathcal{K}$ is the Kirchhoff polarization matrix. The curvature kernel $\mathcal{T}$ is defined, after the change of variable in (2), as

$$\mathcal{K}(\boldsymbol{k},\boldsymbol{k}_0) \triangleq \mathcal{K}(\boldsymbol{w}_\mathrm{H};\boldsymbol{q}_\mathrm{H}) \tag{3a}$$
$$\mathcal{B}(\boldsymbol{k},\boldsymbol{k}_0) \triangleq \mathcal{B}(\boldsymbol{w}_\mathrm{H};\boldsymbol{q}_\mathrm{H}) \tag{3b}$$
$$\mathcal{T}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi}) \triangleq \mathcal{T}(\boldsymbol{w}_\mathrm{H};\boldsymbol{\xi}) = \mathcal{B}(\boldsymbol{w}_\mathrm{H};\boldsymbol{\xi}) - \mathcal{K}(\boldsymbol{w}_\mathrm{H};\boldsymbol{\xi}) \tag{3c}$$

where $\mathcal{B}$ is the polarization matrix of the first order small perturbation method (SPM-1). The kernel $\mathcal{T}$ is effectively a bivariate function as opposed to the kernel of SSA-2 where an additional variable is needed as will be demonstrated in the following section. A formal substitution is made in our curvature kernel where the difference in the wavenumbers $\boldsymbol{q}_\mathrm{H} = \boldsymbol{k} - \boldsymbol{k}_0$ is formally replaced by the ∇ operator and hence the Fourier variable ξ. This formal substitution was demonstrated in our recent letter [7] to preserve the curvature order and coincide with that of the second order SSA.

A series of three appendices are reserved for the explicit expressions of the curvature kernel and how it turns out to be quadratic for Neumann, Dirichlet, perfect conducting, dielectric boundary conditions on the scattering surface.

It is trivial to verify for the following properties of this curvature kernel, which must be satisfied for consistency with standard limits,

$$\mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \mathcal{B}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) - \mathcal{K}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) \tag{4a}$$

$$\mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \mathbf{0}) = 0 \tag{4b}$$

$$\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \mathbf{0}) = \mathbf{0}. \tag{4c}$$

The first property ensures the convergence of the LCA in (1) to SPM-1 under small roughness conditions. The second property preserves the invariance of the model under vertical translation of the scattering surface. The third property implies that the high frequency limit is the Kirchhoff approximation; equation (5.18) in Voronovich [5] defines how the high frequency limit is obtained from the first derivative of the second order kernel. Both the second and third properties come from the quadratic nature of the curvature kernel $\mathcal{T}$ as demonstrated in the appendices.

In order to preserve reciprocity under the transformations:

$$\boldsymbol{k}_0 \leftrightarrow -\boldsymbol{k} \tag{5a}$$

$$q_0 \leftrightarrow q_k \tag{5b}$$

$$\boldsymbol{w}_{\mathrm{H}} \leftrightarrow -\boldsymbol{w}_{\mathrm{H}} \tag{5c}$$

$$\boldsymbol{q}_{\mathrm{H}} \leftrightarrow \boldsymbol{q}_{\mathrm{H}}, \tag{5d}$$

the scattering amplitude as well as the other polarization coefficients exhibit the properties

$$\mathcal{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathcal{S}^t(-\boldsymbol{k}_0, -\boldsymbol{k}) \tag{6a}$$

$$\mathcal{K}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathcal{K}^t(-\boldsymbol{k}_0, -\boldsymbol{k}) \tag{6b}$$

$$\mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathcal{B}^t(-\boldsymbol{k}_0, -\boldsymbol{k}) \tag{6c}$$

$$\mathcal{K}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \mathcal{K}^t(-\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \mathcal{K}^t(\boldsymbol{w}_{\mathrm{H}}; -\boldsymbol{q}_{\mathrm{H}}) \tag{6d}$$

$$\mathcal{B}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \mathcal{B}^t(-\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \mathcal{B}^t(\boldsymbol{w}_{\mathrm{H}}; -\boldsymbol{q}_{\mathrm{H}}) \tag{6e}$$

$$\mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{\xi}) = \mathcal{T}^t(-\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{\xi}) = \mathcal{T}^t(\boldsymbol{w}_{\mathrm{H}}; -\boldsymbol{\xi}). \tag{6f}$$

Reciprocity is a fundamental principle in wave theory. Berman and Dacol [12] as well as Dashen and Wurmser [13] demonstrated that a manifestly reciprocal scattering amplitude exists from which the small slope approximation (SSA) of Voronovich [4] can be derived after straightforward approximations. This demonstrates to some extent the power of having a manifestly reciprocal theory.

We recall that the curvature kernel depends formally on the Fourier variable ξ and $\boldsymbol{w}_{\mathrm{H}} = \boldsymbol{k} + \boldsymbol{k}_0$. No other dependence on $\boldsymbol{k}$ and $\boldsymbol{k}_0$ in $\mathcal{T}$ is possible because of the formal replacement of $\boldsymbol{q}_{\mathrm{H}} = \boldsymbol{k} - \boldsymbol{k}_0$ by the Fourier variable ξ. We will show, in the next section, that extra dependence on $\boldsymbol{k}$ and $\boldsymbol{k}_0$ is possible in the SPM-2 and SSA-2 kernels.

3. Link to the local small slope approximation

The link to the SSA can be made by expanding the LCA in (1) up to the second order in powers of the surface η to obtain

$$\mathcal{S}(\boldsymbol{k},\boldsymbol{k}_0)=\frac{\mathcal{B}(\boldsymbol{w}_{\rm H};\boldsymbol{0})}{q_z}\delta(\boldsymbol{q}_{\rm H})-{\rm i}\mathcal{B}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})\hat{\eta}(\boldsymbol{q}_{\rm H})-q_z\int\tilde{\mathcal{B}}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})\hat{\eta}(\boldsymbol{k}-\boldsymbol{\xi})\hat{\eta}(\boldsymbol{\xi}-\boldsymbol{k}_0)\,{\rm d}\boldsymbol{\xi} \tag{7}$$

where

$$\mathcal{B}(\boldsymbol{w}_{\rm H};\boldsymbol{0})\equiv\mathcal{K}(\boldsymbol{w}_{\rm H};\boldsymbol{0}) \tag{8}$$

and the equivalent SPM-2 function from our curvature kernel and the Kirchhoff matrix is

$$\tilde{\mathcal{B}}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})=\frac{\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{k}-\boldsymbol{\xi})+\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{\xi}-\boldsymbol{k}_0)+\mathcal{K}(\boldsymbol{w}_{\rm H};\boldsymbol{k}-\boldsymbol{k}_0)}{2}. \tag{9}$$

The tilde over the function $\tilde{\mathcal{B}}_2$ is present to remind the reader that this function is not the formal expression of a second order SPM-2. It is rather the one obtained from our curvature approximation and therefore it ensures similar accuracy up to the curvature order only. It can be easily verified that this function has the following property:

$$\tilde{\mathcal{B}}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{k})=\tilde{\mathcal{B}}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{k}_0)=\frac{\mathcal{B}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})}{2} \tag{10}$$

which ensures the invariance of the scattering matrix $\mathcal{S}(\boldsymbol{k},\boldsymbol{k}_0)$ under vertical translation of the scattering surface. The reciprocity property of $\tilde{\mathcal{B}}_2$ is

$$\tilde{\mathcal{B}}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})=\tilde{\mathcal{B}}_2^t(-\boldsymbol{k},-\boldsymbol{k}_0;-\boldsymbol{\xi}). \tag{11}$$

The local SSA has the same functional form as in (1) but with the SPM-1 coefficient $\mathcal{B}$ in front of the first single integral and $\tilde{\mathcal{M}}$ as the kernel of the double integral and related to the SPM-2 function $\mathcal{B}_2$. The formal expression for the SSA-equivalent scattering matrix is then

$$\begin{aligned}\mathcal{S}(\boldsymbol{k},\boldsymbol{k}_0)=&\frac{\mathcal{B}(\boldsymbol{k},\boldsymbol{k}_0)}{q_z}\int{\rm e}^{-{\rm i}q_z\eta(\boldsymbol{x})}{\rm e}^{-{\rm i}\boldsymbol{q}_{\rm H}\cdot\boldsymbol{x}}\,{\rm d}\boldsymbol{x}\\&-{\rm i}\int\!\!\int\tilde{\mathcal{M}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})\hat{\eta}(\boldsymbol{\xi}){\rm e}^{-{\rm i}q_z\eta(\boldsymbol{x})}{\rm e}^{-{\rm i}(\boldsymbol{q}_{\rm H}-\boldsymbol{\xi})\cdot\boldsymbol{x}}\,{\rm d}\boldsymbol{\xi}\,{\rm d}\boldsymbol{x}\end{aligned} \tag{12}$$

where the relation between the kernel $\tilde{\mathcal{M}}$ and the function $\tilde{\mathcal{B}}_2$ is obtained by Taylor expansion of (12) up to the second order surface elevation and by element by element comparison with (7):

$$\tilde{\mathcal{M}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})=\frac{\tilde{\mathcal{B}}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{k}-\boldsymbol{\xi})+\tilde{\mathcal{B}}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{k}_0+\boldsymbol{\xi})-\mathcal{B}(\boldsymbol{k},\boldsymbol{k}_0)}{2} \tag{13}$$

which yields the following relation to the curvature kernel:

$$\tilde{\mathcal{M}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})=\frac{\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{\xi})+\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}-\boldsymbol{\xi})-\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})}{2}. \tag{14}$$

This kernel has the following properties:

$$\tilde{\mathcal{M}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{q}_{\rm H})=0 \tag{15a}$$

$$\tilde{\mathcal{M}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{0})=0 \tag{15b}$$

$$\boldsymbol{\nabla}\tilde{\mathcal{M}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{0})=\tfrac{1}{2}\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}) \tag{15c}$$

$$\boldsymbol{\nabla}\tilde{\mathcal{M}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{0})\cdot\boldsymbol{q}_{\rm H}\approx\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}). \tag{15d}$$

The first property preserves the SPM-1 limit. The second is for the translation invariance and the third and the fourth ensure the Kirchhoff approximation under the high frequency limit, up to the lowest quadratic order in $\boldsymbol{q}_{\rm H}$. It will be shown in section 6 that the last two identities also play a major role in the tilt invariance property.

Finally this modified kernel of the SSA-2 model obeys the following reciprocity property:

$$\tilde{\mathcal{M}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})=\tilde{\mathcal{M}}^t(-\boldsymbol{k},-\boldsymbol{k}_0;\boldsymbol{\xi}). \tag{16}$$

The original SSA-2 kernel and the one in (14) are equivalent at least up to the curvature order of the scattering surface; see [7] in the quasi-specular regime.

4. The non-local curvature approximation

It is possible to express our model in the form of the non-local small slope approximation (NLSSA) given in [9]. We term this form the non-local curvature approximation (NLCA). The non-local aspect refers to multiple scattering at the surface. For more motivation of this non-local model the reader is referred to [9]. We have

$$\mathcal{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{\mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0)}{q_z} \int \mathrm{e}^{-\mathrm{i}q_z \eta(\boldsymbol{x})} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\mathrm{H}\cdot\boldsymbol{x}} \,\mathrm{d}\boldsymbol{x} + \frac{q_k + q_0}{2 q_k q_0} \times \int\int\int \tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) \mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{\xi})\cdot\boldsymbol{x}_1 - \mathrm{i}q_k \eta(\boldsymbol{x}_1)} \mathrm{e}^{\mathrm{i}(\boldsymbol{k}_0-\boldsymbol{\xi})\cdot\boldsymbol{x}_2 - \mathrm{i}q_0 \eta(\boldsymbol{x}_2)} \,\mathrm{d}\boldsymbol{\xi} \,\mathrm{d}\boldsymbol{x}_1 \,\mathrm{d}\boldsymbol{x}_2 \tag{17}$$

where the triple integral replaces the double integral in the local SSA. This triple integral includes the multiple scattering up to double bounces on the scattering surface. In order to find the non-local kernel, one searches for solutions linear in $\tilde{\mathcal{B}}_2$ in the form

$$\tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) = \alpha + \beta \tilde{\mathcal{B}}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) + \gamma \tilde{\mathcal{B}}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{w}_\mathrm{H} - \boldsymbol{\xi}) \tag{18}$$

which constitutes the simplest form of the final solution. The third term was not enforced in the derivation of the original NLSSA [9]. After inserting (18) into (17) and expansion in powers of η up to the second order one finds that the non-local kernel is related to $\tilde{\mathcal{B}}_2$ in (7) by

$$\tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) = \tilde{\mathcal{B}}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) + \tilde{\mathcal{B}}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{w}_\mathrm{H} - \boldsymbol{\xi}) - \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) \tag{19}$$

where, if the expression for $\tilde{\mathcal{B}}_2$ in (9) is used, one finds this simple relation:

$$\tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) = \mathcal{T}(\boldsymbol{w}_\mathrm{H}; \boldsymbol{k} - \boldsymbol{\xi}) + \mathcal{T}(\boldsymbol{w}_\mathrm{H}; \boldsymbol{\xi} - \boldsymbol{k}_0) - \mathcal{T}(\boldsymbol{w}_\mathrm{H}; \boldsymbol{k} - \boldsymbol{k}_0). \tag{20}$$

This non-local kernel exhibits the following property:

$$\tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{k}) = \tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{k}_0) = 0 \tag{21}$$

which of course preserves the SPM-1 limit of the NLCA. The high frequency limit can also be checked by linearization of the kernel with respect to the Fourier variable. With the gradient property

$$\boldsymbol{\nabla}\tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{k}) \cdot \boldsymbol{q}_\mathrm{H} = -\boldsymbol{\nabla}\tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{k}_0) \cdot \boldsymbol{q}_\mathrm{H} = \boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_\mathrm{H}; \boldsymbol{q}_\mathrm{H}) \cdot \boldsymbol{q}_\mathrm{H} \approx 2\mathcal{T}(\boldsymbol{w}_\mathrm{H}; \boldsymbol{q}_\mathrm{H}) \tag{22}$$

one can demonstrate that the high frequency limit is indeed the Kirchhoff approximation. The reciprocity property of $\tilde{\phi}$ is

$$\tilde{\phi}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) = \tilde{\phi}^t(-\boldsymbol{k}, -\boldsymbol{k}_0; -\boldsymbol{\xi}). \tag{23}$$

Even though the SPM-2 limit is not formally attained by this non-local form, it is consistent with the NLSSA at least up to the curvature order in the scattering surface.

It should be noted that the polarization matrix of the single integral in the NLSSA and NLCA is that of SPM-1. The surface dependence of the triple integral is only in the phases and not in the kernel. Hence, the NLSSA and NLCA functional forms seem to be more 'universal' than that of SSA-1 + SSA-2 or LCA in view of the formal phase factor representation given by Tatarskii [14].

5. The weighted curvature approximation

5.1. Formulation of the model

There is an interesting reduced form of our LCA in (1) where a single integral can be written with an integrand dependent on the surface slope. This reduced form is

$$\mathcal{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{1}{q_z} \int \{\mathcal{B}(\boldsymbol{w}_\mathrm{H}; \boldsymbol{q}_\mathrm{H}) - \mathcal{T}(\boldsymbol{w}_\mathrm{H}; -q_z \boldsymbol{\nabla}\eta)\} \mathrm{e}^{-\mathrm{i}q_z \eta(\boldsymbol{x})} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\mathrm{H}\cdot\boldsymbol{x}} \,\mathrm{d}\boldsymbol{x} \tag{24}$$

which still retains the proper SPM-1 and Kirchhoff limits. When one compares the LCA of (1) with the expression in (31), the argument of the kernel is not necessarily a formal replacement of $\boldsymbol{\xi}$ by $-q_z\nabla\eta$ simply because the constant coefficient $\mathcal{K}$ in front of the first integral in (1) is now replaced by $\mathcal{B}$ in addition to the reduction of the integral dimensions.

Since $\mathcal{T}$ is at least quadratic, the SPM-1 limit is trivially reached because $\mathcal{T}$ does not contribute to the linearized limit in η in (24). The high frequency limit is obtained by noticing that in this case the phase factor oscillates too rapidly and therefore the integrand can be evaluated at the stationary point of the phase. This is traditionally termed the stationary phase approximation. When the stationary phase approximation (or equivalently the high frequency limit) is applied one imposes that the variable

$$\boldsymbol{\zeta} = \boldsymbol{q}_{\mathrm{H}} + q_z\nabla\eta \tag{25}$$

be zero. This means that the surface slope is locked at $-\boldsymbol{q}_{\mathrm{H}}/q_z$. For this value of the slope it can be checked by simple inspection that the form in (24) reduces to the Kirchhoff approximation. This model will be termed the weighted curvature approximation (WCA) and will be demonstrated to be accurate and practical for, in particular, numerical simulations.

5.2. The local weight approximation

This local WCA is already in the form that Dashen and Wurmser investigated in their paper [10]. In order to reach this form Dashen and Wurmser started from a composite model based on a local expression of the SSA-1 and then iterated over the integrand through a differential equation (E1) in [10] which ensures accuracy up to the curvature order in the scattering surface as well as invariance under arbitrary tilt. Their resulting model is termed the local weight approximation (LWA), and their differential equation is solved after the following change of variable:

$$-q_z\nabla\eta = \boldsymbol{q}_{\mathrm{H}} - \boldsymbol{\zeta} = \boldsymbol{q}_{\mathrm{H}} - \hat{\boldsymbol{\zeta}}\zeta \tag{26}$$

is made. The LWA was demonstrated to be complete up to the first order in surface curvature. We have checked the high frequency limit of LWA and found that the Kirchhoff limit is formally reached under the stationary phase approximation. This means that the LWA as formulated for Dirichlet, Neumann, and perfect conducting boundary conditions has a wide range of applicability. LWA is then the only single integral model with a slope dependent integrand which reaches both the high frequency Kirchhoff and the small perturbation method SPM-1 limits. We will demonstrate that our WCA is a practical generalization of the LWA to the full dielectric case.

5.3. Analytical comparison

There are two possibilities when analytically comparing our model with the LWA of Dashen and Wurmser. The first possibility is that our WCA in (24) is of the same accuracy as the LWA and therefore the differential equation may be used to degrade the kernel to find that from which Dashen and Wurmser should have started in order to find our approximation. This degraded model is then

$$\mathcal{S}(\boldsymbol{k},\boldsymbol{k}_0) = \frac{1}{q_z}\int\left\{\mathcal{B}(\boldsymbol{w}_{\mathrm{H}};\boldsymbol{q}_{\mathrm{H}}) - \mathcal{T}(\boldsymbol{w}_{\mathrm{H}};-q_z\nabla\eta) + \zeta\frac{\partial\mathcal{T}}{\partial\zeta}(\boldsymbol{w}_{\mathrm{H}};\boldsymbol{q}_{\mathrm{H}} - \hat{\boldsymbol{\zeta}}\zeta)\right\}\mathrm{e}^{-\mathrm{i}(q_z\eta(\boldsymbol{x})+\boldsymbol{q}_{\mathrm{H}}\cdot\boldsymbol{x})}\,\mathrm{d}\boldsymbol{x} \tag{27}$$

and should be compared with the composite model from which Dashen and Wurmser started their iteration.

The second possibility is that our WCA in (24) may have lost some accuracy in terms of surface curvature during the process of reduction from the original LCA in (1). In this case the differential equation in Dashen and Wurmser [10] can be used to improve the integrand. The differential equation can be recast in the following primitive form:

$$\delta\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}-\hat{\boldsymbol{\zeta}}\zeta)=-\zeta\int\frac{\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}-\hat{\boldsymbol{\zeta}}\zeta)}{\zeta^2}\,{\rm d}\zeta-\zeta\mathcal{C}(\hat{\boldsymbol{\zeta}}) \tag{28}$$

where $\mathcal{C}(\hat{\boldsymbol{\zeta}})$ is an arbitrary function. Note that a constant term in $\mathcal{T}$ remains unchanged by the integral. When the quadratic kernel is inserted into (28), a closed form solution is, in general, difficult or even impossible to find. For this reason, we concentrate on a Taylor expansion of the kernel in powers of ζ:

$$\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}-\hat{\boldsymbol{\zeta}}\zeta)\approx\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})+\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})\cdot\hat{\boldsymbol{\zeta}}\zeta+\tfrac{1}{2}\hat{\boldsymbol{\zeta}}\boldsymbol{\nabla\nabla}\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})\hat{\boldsymbol{\zeta}}\zeta^2+\cdots. \tag{29}$$

This expansion is equivalent to limiting the study to the quasi-specular regime.

For this particular case, the improved kernel resulting from the primitive in (28) can be written as

$$\delta\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}-\hat{\boldsymbol{\zeta}}\zeta)\approx\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})-\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})\cdot\boldsymbol{\zeta}\log\zeta-\tfrac{1}{2}\boldsymbol{\zeta}\boldsymbol{\nabla\nabla}\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})\boldsymbol{\zeta}+\cdots. \tag{30}$$

Hence, the scattering matrix is

$$\mathcal{S}(\boldsymbol{k},\boldsymbol{k}_0)=\frac{1}{q_z}\int\{\mathcal{K}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})+\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H})\cdot\boldsymbol{\zeta}\log\zeta+\cdots\}{\rm e}^{-{\rm i}(q_z\eta(\boldsymbol{x})+\boldsymbol{q}_{\rm H}\cdot\boldsymbol{x})}\,{\rm d}\boldsymbol{x} \tag{31}$$

where the improvement on the Kirchhoff model is now apparent in the form of local tilting for the quasi-specular regime.

5.4. Numerical evaluation

In order to check numerically our WCA, we have implemented a Monte Carlo simulation of surface scattering from perfectly conducting and dielectric ocean-like surfaces. For such surfaces, all of the cited models are hardly differentiable if the spectrum imposed is a power law proportional to k^{-4} and Gaussian statistics. We have therefore chosen to generate non-linear ocean-like surfaces according to the Creamer *et al* [15] scheme. Figure 1 shows one realization of such a linear surface along with its non-linear counterpart after a Creamer *et al* iteration. Figure 2 shows the corresponding surface slope for both cases. A set of 100 such surfaces has been used to compute the average scattered power. The incident electromagnetic field is chosen to be the Thorsos Gaussian-tapered wave [16]. One should note that for these surfaces the Rayleigh criterion ($K\sigma$) is greater than 1 where σ is the root mean square (rms) of the surface elevation. For this high value of the Rayleigh criterion, the small perturbation SPM-1 (i.e. simple Bragg scattering) is not valid. Figure 3 shows good agreement between the WCA in (24) and Voronovich's first order small slope approximation (SSA-1) [5], Dashen and Wurmser's LWA [10], and Milder's first order operator expansion (OE-1) [11]. The LWA and OE-1 where particularly chosen for their accuracy in surface curvature. The good agreement suggests that our model, the WCA in (24), can be considered as an improved SSA-1 as it is an extension of Dashen and Wurmser's model to dielectric surfaces. The WCA model seems to perform better near grazing angles than LWA. Figure 4 shows a numerical result obtained using the WCA for a dielectric surface.

A more extensive numerical comparison will be made in the future in order to compare our LCA (1) and NLCA (17) with local and non-local SSA-1 + SSA-2 as well as higher iteration of the operator expansion method (OE-1 + OE-2).

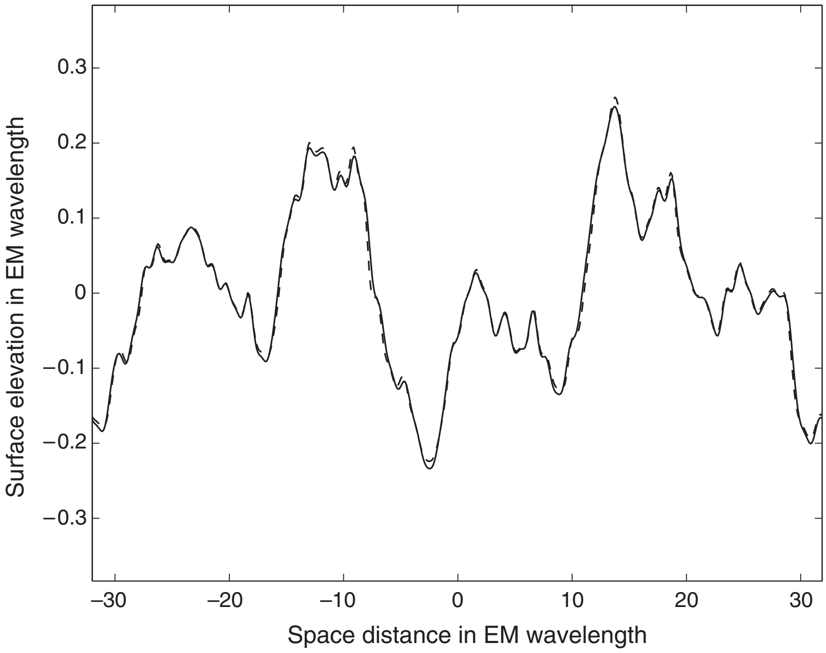

Figure 1. One elevation realization out of 100 of linear (solid) and non-linear (dashed) surfaces for a power law spectrum in k^{-4}.

The LWA of Dashen and Wurmser [10] is a tilt invariant model by construction. In fact, the differential equation solved by Dashen and Wurmser ensures that the tilted SPM-1 limit is reached since a composite or tilted Bragg model was used to define it. The good agreement between the LWA and our WCA suggests that our model is also tilt invariant up to the curvature order. This observation will be proved in the next section with the help of previous developments on the SSA by Voronovich [17].

6. Tilt invariance

A very stringent condition that an asymptotic model must satisfy is tilt invariance. We describe as 'tilt invariance' the feature that the tilted small perturbation method can be reached by simply tilting the surface explicitly present in the formulation of the asymptotic model. The LWA and the SSA are tilt invariant models. By construction, the LWA reproduces SPM-1 or tilted Bragg limits for arbitrary tilt. Also, the SSA has been proven recently by Voronovich [17] to reproduce the tilted Bragg form up to the linear order in the tilt. Here, we utilize some of Voronovich's mathematics to demonstrate that our curvature asymptotic models are also tilt invariant. Two equations from [17] (equations (25) and (26) in that work) are reproduced in Voronovich's notation below:

$$q_z \boldsymbol{\nabla} M(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{0}) \cdot \vec{a} = M(\boldsymbol{k}, \boldsymbol{k}_0; q_z\vec{a}) + \mathrm{O}(a^2) \tag{32}$$

and

$$B_{\mathrm{tilted}} = q_k q_0 B(\boldsymbol{k}, \boldsymbol{k}_0) - \frac{q_k q_0}{2q_z} M(\boldsymbol{k}, \boldsymbol{k}_0; q_z\vec{a}) + \mathrm{O}(a^2) \tag{33}$$

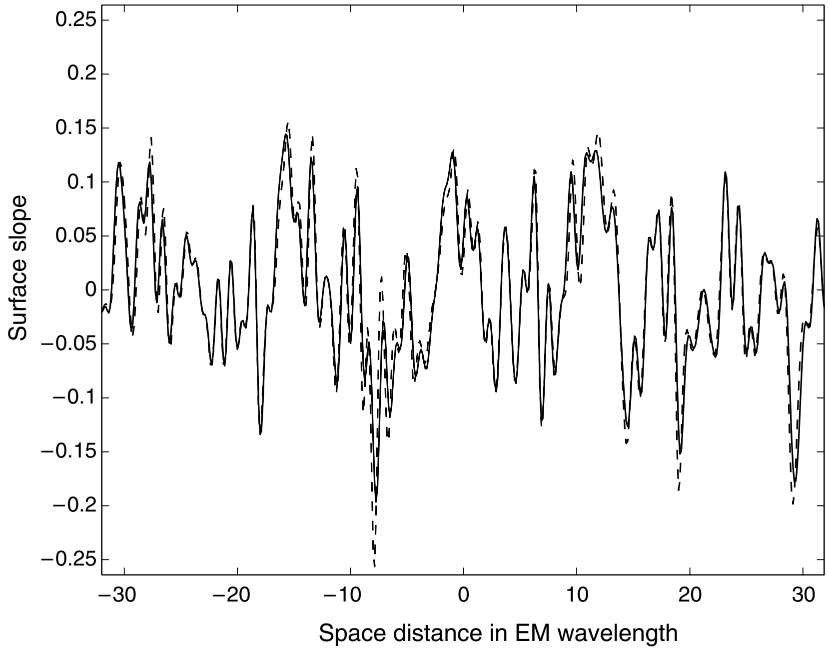

Figure 2. One slope realization out of 100 of linear (solid) and non-linear (dashed) surfaces.

where $\vec{a}$ is a three-dimensional tilt vector and M is the SSA-2 kernel. Equation (33) was shown numerically to hold up to the linear order in the tilting vector. Let us rewrite (33) in our notation and normalization as

$$\mathcal{B}_{\text{tilted}} = \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) - 2\mathcal{M}(\boldsymbol{k}, \boldsymbol{k}_0; q_z\vec{a}) + \mathrm{O}(a^2) \tag{34}$$

or equivalently

$$\mathcal{B}_{\text{tilted}} = \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) - 2q_z\boldsymbol{\nabla}\mathcal{M}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{0}) \cdot \vec{a} + \mathrm{O}(a^2). \tag{35}$$

Since the Kirchhoff limit is also attained by the SSA for the acoustic and perfect conducting cases, the following property can be observed:

$$\boldsymbol{\nabla}\mathcal{M}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{0}) \cdot \boldsymbol{q}_{\mathrm{H}} = \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) - \mathcal{K}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}). \tag{36}$$

In the case of dielectric surfaces, the high frequency limit of SSA nearly reproduces the Kirchhoff amplitude. The discrepancy occurs mainly in relatively small cross-polarization terms as demonstrated in [7] in the quasi-specular directions. This property of the SSA kernel yields the approximations

$$\boldsymbol{\nabla}\mathcal{M}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{0}) \approx \tfrac{1}{2}\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) + \mathrm{O}(q_{\mathrm{H}}^2) \tag{37a}$$

$$\mathcal{M}(\boldsymbol{k}, \boldsymbol{k}_0; q_z\vec{a}) \approx \tfrac{1}{2}q_z\boldsymbol{q}_{\mathrm{H}} \cdot \boldsymbol{\nabla}\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{0}) \cdot \vec{a} + \mathrm{O}(q_{\mathrm{H}}^2). \tag{37b}$$

These approximations hold because of the quadratic nature of the curvature kernel. The tilt invariance in (35) is then

$$\mathcal{B}_{\text{tilted}} \approx \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) - q_z\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) \cdot \vec{a} + \mathrm{O}(a^2) \tag{38a}$$

$$\mathcal{B}_{\text{tilted}} \approx \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) - q_z\boldsymbol{q}_{\mathrm{H}} \cdot \boldsymbol{\nabla}\boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{0}) \cdot \vec{a} + \mathrm{O}(a^2). \tag{38b}$$

With these last properties we can demonstrate that LCA, NLCA, and WCA are approximately tilt invariant up to the first order in the tilting vector.

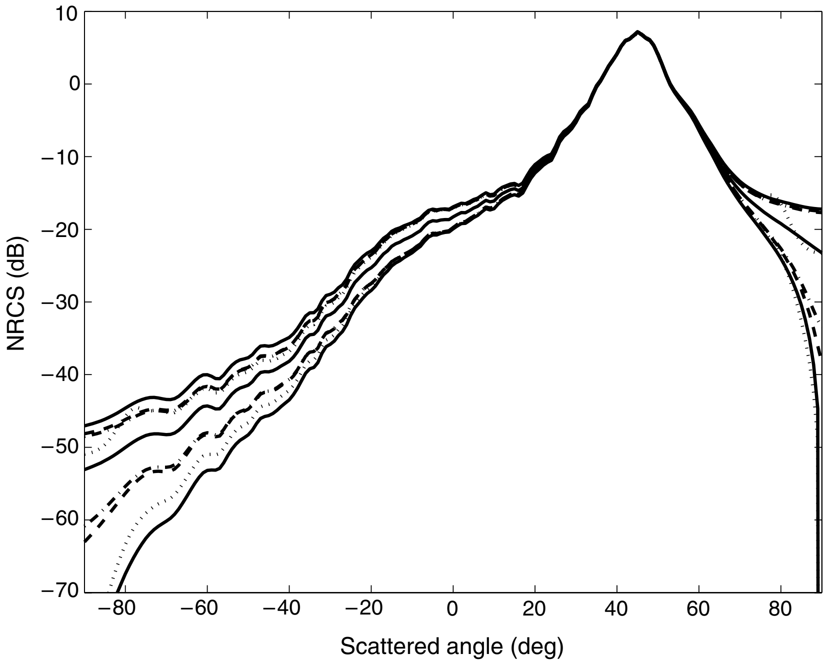

Figure 3. Monte Carlo comparison between our WCA (dashed), the Voronovich first order small slope approximation (SSA-1; the solid lines: for VV, the higher curve; for HH, the lower curve), Dashen and Wurmser's LWA (dashed–dotted), and Milder's first order operator expansion OE-1 (dotted). The Kirchhoff model is shown by the solid curve in the middle. The incidence angle is 45°.

In order to demonstrate this tilt invariant feature, we make the following substitutions in the expression for the LCA in (1):

$$\eta(\boldsymbol{x}) \Rightarrow \eta(\boldsymbol{x}) + \vec{a} \cdot \boldsymbol{x} \tag{39a}$$

$$\hat{\eta}(\boldsymbol{\xi}) \Rightarrow \hat{\eta}(\boldsymbol{\xi}) + \mathrm{i}\vec{a} \cdot \boldsymbol{\nabla}\delta(\boldsymbol{\xi}). \tag{39b}$$

The evaluation of the tilted LCA, after linearization in surface elevation, then gives

$$\mathcal{B}_{\text{tilted}} \doteq \mathcal{K}(\boldsymbol{k}, \boldsymbol{k}_0) - q_z \boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\text{H}}; \boldsymbol{0}) \cdot \vec{a} + \mathcal{T}(\boldsymbol{w}_{\text{H}}; \boldsymbol{q}_{\text{H}} - q_z\vec{a}) \tag{40}$$

where the second term in the right-hand side is identically zero because the curvature kernel is quadratic in its lowest order. The third term can be expanded up to the linear order in the tilt vector as

$$\mathcal{B}_{\text{tilted}} \doteq \mathcal{K}(\boldsymbol{k}, \boldsymbol{k}_0) + \mathcal{T}(\boldsymbol{w}_{\text{H}}; \boldsymbol{q}_{\text{H}}) - q_z \boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\text{H}}; \boldsymbol{q}_{\text{H}}) \cdot \vec{a} \tag{41}$$

and after use of the first property of the curvature kernel in (4) one finds

$$\mathcal{B}_{\text{tilted}} \doteq \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) - q_z \boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\text{H}}; \boldsymbol{q}_{\text{H}}) \cdot \vec{a}. \tag{42}$$

This final equality holds owing to the approximation in (38). Therefore, the LCA in (1) is tilt invariant up to the curvature order. A similar check can be made for the non-local curvature approximation (NLCA) in (17).

Let us examine the case of the WCA in (31). Making the replacements (39) in (31) one finds, after linearization in the surface elevation, that

$$\mathcal{B}_{\text{tilted}} \doteq \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) - \mathcal{T}(\boldsymbol{w}_{\text{H}}; -q_z\vec{a}) + (\boldsymbol{q}_{\text{H}} + q_z\vec{a}) \cdot \boldsymbol{\nabla}\mathcal{T}(\boldsymbol{w}_{\text{H}}; -q_z\vec{a}). \tag{43}$$

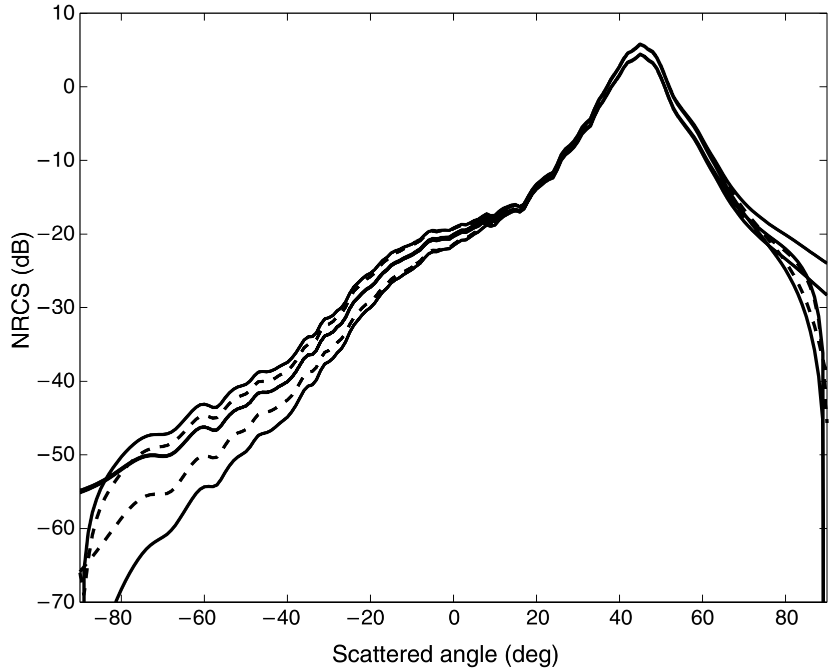

Figure 4. VV and HH polarization comparison between SSA-1 (solid) and WCA (dashed) for a dielectric surface. The Kirchhoff model is shown by the solid curve in the middle. The relative permittivity constant is $\epsilon = 70 - \mathrm{i}36$ for a 3 GHz electromagnetic frequency.

This equation can also be seen as the differential equation that Dashen and Wurmser solved in their equation E1 [10]; however, in this case the kernel $\mathcal{T}$ becomes the unknown. Dashen and Wurmser solved the differential equation without linearization in the tilt vector and therefore their model is invariant under arbitrary tilt. Expansion of (43) up to the linear order in the tilt vector gives

$$\mathcal{B}_{\mathrm{tilted}} \doteq \mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) - q_z \boldsymbol{q}_{\mathrm{H}} \cdot \boldsymbol{\nabla\nabla} \mathcal{T}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{0}) \cdot \vec{a}. \tag{44}$$

When this equation is compared with (38) one finds that the SPM limit of the tilted WCA is indeed the tilted SPM.

7. Conclusions

A new asymptotic theory for scalar and vector wave scattering of rough surfaces is presented. This new development federates an extended Kirchhoff approximation (EKA) such as the IEM by [6, 8] with the first and second order SSA given by [4]. Both the Kirchhoff approximation (KA) and the first order small perturbation method (SPM-1 or Bragg) are reached dynamically as a function of the surface conditions. We derived a second order kernel from a formal inclusion of the derivative operator in the difference between the polarization coefficients of KA and SPM-1. This kernel is termed the curvature kernel since it is quadratic at lowest order. The curvature kernel is as simple as the expressions for both Kirchhoff and SPM-1 coefficients. We have already demonstrated in our previous letter [7] that the formal difference has the same curvature order as SSA-1 + SSA-2. Unlike the case for SSA, the second order small perturbation method (SPM-2) is not enforced in our development. However, our curvature kernel should reproduce a reasonable approximation of the SPM-2 function at least up to the curvature or quadratic order. Three different versions of this new asymptotic theory are given. The first is the LCA in (1), similar to the first and second order small slope approximation (SSA-1 + SSA-2) given by Voronovich [5]. The second is the non-local curvature approximation (NLCA) given in (17), which is functionally similar to the non-local small slope approximation (NLSSA) also given by Voronovich [9] and includes multiple scattering up to double bounces on the scattering surface. The third version is the WCA given in (31), to be compared with the LWA of Dashen and Wurmser [10]. This reduced version is meant to be convenient for Monte Carlo simulations. A satisfactory numerical comparison was made between our WCA and the LWA as well as with the first order operator expansion (OE-1) of Milder [11]. The good agreement reached between our WCA and the LWA suggests that our model can be seen as a simple extension of the LWA to dielectric surfaces where accuracy in surface curvature is reasonable even after the reduction of our more exact forms in the LCA (1) and NLCA (17). We have also shown in the last section that our curvature based models are tilt invariant in the sense that the SPM limit of a tilted asymptotic model, LCA, NLCA, or WCA is indeed the tilted SPM. This property holds for linear order in the tilt vector and for the curvature order in the scattering surface.

Acknowledgments

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Appendix A. Neumann and Dirichlet boundary conditions

From [4, 10], one can write the Kirchhoff coefficients for both the Neumann and Dirichlet boundary conditions as

$$\mathcal{K}_{\rm ND}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}) = (K^2 - \boldsymbol{k}\cdot\boldsymbol{k}_0 + q_k q_0, -[K^2 - \boldsymbol{k}\cdot\boldsymbol{k}_0 + q_k q_0]) \tag{A.1}$$

and the first order small perturbation method (SPM-1) coefficients as

$$\mathcal{B}_{\rm ND}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}) = (2[K^2 - \boldsymbol{k}\cdot\boldsymbol{k}_0], -2q_k q_0) \tag{A.2}$$

where the difference gives the curvature kernel evaluated at $\boldsymbol{w}_{\rm H}$ and $\boldsymbol{q}_{\rm H}$,

$$\mathcal{T}_{\rm ND}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}) = \mathcal{B}_{\rm ND}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}) - \mathcal{K}_{\rm ND}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}). \tag{A.3}$$

Explicitly, this kernel can be written as

$$\mathcal{T}_{\rm ND}(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}) = (K^2 - \boldsymbol{k}\cdot\boldsymbol{k}_0 - q_k q_0, K^2 - \boldsymbol{k}\cdot\boldsymbol{k}_0 - q_k q_0) \tag{A.4}$$

where one notices that both Neumann and Dirichlet boundary conditions have the same curvature kernel

$$\mathcal{T}_N(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}) = \mathcal{T}_D(\boldsymbol{w}_{\rm H};\boldsymbol{q}_{\rm H}) = K^2 - \boldsymbol{k}\cdot\boldsymbol{k}_0 - q_k q_0. \tag{A.5}$$

Now comes the fundamental question: is this difference quadratic in $\boldsymbol{q}_{\rm H} = \boldsymbol{k} - \boldsymbol{k}_0$ to its lowest order as already demonstrated for a particular case in our letter [7] ? The particular case treated in [7] is quasi-specular regime for surfaces showing good conduction.

In order to demonstrate this quadratic feature, we must make the following change of variables:

$$\begin{aligned} \boldsymbol{q}_{\rm H} &= \boldsymbol{k} - \boldsymbol{k}_0 \qquad & \boldsymbol{k} &= (\boldsymbol{w}_{\rm H} + \boldsymbol{q}_{\rm H})/2 \\ \boldsymbol{w}_{\rm H} &= \boldsymbol{k} + \boldsymbol{k}_0 \qquad & \boldsymbol{k}_0 &= (\boldsymbol{w}_{\rm H} - \boldsymbol{q}_{\rm H})/2 \end{aligned} \tag{A.6}$$

and then note that the products $\boldsymbol{k} \cdot \boldsymbol{k}_0$ and $q_k q_0$ are even functions on $\boldsymbol{q}_{\mathrm{H}}$:

$$\boldsymbol{k} \cdot \boldsymbol{k}_0 = \frac{\boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{q}_{\mathrm{H}}^2}{4} \tag{A.7a}$$

$$q_k q_0 = \tfrac{1}{4}\sqrt{(4K^2 - \boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{q}_{\mathrm{H}}^2)^2 - (2\boldsymbol{w}_{\mathrm{H}} \cdot \boldsymbol{q}_{\mathrm{H}})^2}. \tag{A.7b}$$

Hence, the kernel is

$$\mathcal{T}_N(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = K^2 - \tfrac{1}{4}\left(\boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{q}_{\mathrm{H}}^2 - \sqrt{(4K^2 - \boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{q}_{\mathrm{H}}^2)^2 - (2\boldsymbol{w}_{\mathrm{H}} \cdot \boldsymbol{q}_{\mathrm{H}})^2}\right). \tag{A.8}$$

Finally, the curvature kernel can be obtained by the formal replacement of $\boldsymbol{q}_{\mathrm{H}}$ by $\boldsymbol{\xi}$:

$$\mathcal{T}_N(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{\xi}) = K^2 - \tfrac{1}{4}\left(\boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2 - \sqrt{(4K^2 - \boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2)^2 - (2\boldsymbol{w}_{\mathrm{H}} \cdot \boldsymbol{\xi})^2}\right). \tag{A.9}$$

This kernel is even in $\boldsymbol{\xi}$ and by simple inspection one finds that the constant of zeroth order in $\boldsymbol{\xi}$ is zero. Therefore the curvature kernel has its lowest order quadratic in $\boldsymbol{\xi}$.

Appendix B. Perfectly conducting boundary conditions

From [4], one can simply write the Kirchhoff coefficients for the perfectly conducting boundary conditions as

$$\mathcal{K}_\infty(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \begin{pmatrix} (K^2 + q_k q_0)\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0 - k k_0 & K(q_k + q_0)(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}_0) \cdot \hat{z} \\ K(q_k + q_0)(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}}) \cdot \hat{z} & -[(K^2 + q_k q_0)\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0 - k k_0] \end{pmatrix} \tag{B.1}$$

and those for SPM-1 as

$$\mathcal{B}_\infty(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = 2\begin{pmatrix} K^2\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0 - k k_0 & K q_0(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}_0) \cdot \hat{z} \\ K q_k(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}}) \cdot \hat{z} & -q_k q_0 \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0 \end{pmatrix}. \tag{B.2}$$

Hence, the difference is

$$\mathcal{T}_\infty(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{\xi}) = \begin{pmatrix} (K^2 - q_k q_0)\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0 - k k_0 & K(q_k - q_0)(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}}) \cdot \hat{z} \\ K(q_k - q_0)(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}}) \cdot \hat{z} & (K^2 - q_k q_0)\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0 - k k_0 \end{pmatrix}. \tag{B.3}$$

The quadratic form is identified by making the following substitutions:

$$q_k q_0 \Rightarrow \tfrac{1}{4}\sqrt{(4K^2 - \boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2)^2 - (2\boldsymbol{w}_{\mathrm{H}} \cdot \boldsymbol{\xi})^2} \tag{B.4a}$$

$$\boldsymbol{k} \cdot \boldsymbol{k}_0 \Rightarrow \frac{\boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2}{4} \tag{B.4b}$$

$$k k_0 \Rightarrow \tfrac{1}{4}|\boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2| \tag{B.4c}$$

$$\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0 \Rightarrow \frac{\boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2}{|\boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2|} \tag{B.4d}$$

$$\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}_0 \Rightarrow 2\frac{\boldsymbol{w}_{\mathrm{H}} \times \boldsymbol{\xi}}{|\boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2|} \tag{B.4e}$$

$$q_k - q_0 \Rightarrow \sqrt{K^2 - \left(\frac{\boldsymbol{w}_{\mathrm{H}} + \boldsymbol{\xi}}{2}\right)^2} - \sqrt{K^2 - \left(\frac{\boldsymbol{w}_{\mathrm{H}} - \boldsymbol{\xi}}{2}\right)^2}. \tag{B.4f}$$

The last two terms are not quadratic in $\boldsymbol{\xi}$ by themselves. Actually each term is linear in $\boldsymbol{\xi}$ to its first order. Their product in the polarization matrix in (B.3) is therefore quadratic in its lowest order.

Appendix C. Fully dielectric boundary conditions

The dielectric Kirchhoff polarization coefficients (see [18, 19]) can be written in the dyadic form as

$$\underline{\mathcal{K}} = \frac{q^2}{2}\frac{R_{\mathrm{VV}}(q/2)\hat{\boldsymbol{K}}_{\mathrm{s}}\hat{\boldsymbol{K}}_{\mathrm{i}} + R_{\mathrm{HH}}(q/2)(\hat{\boldsymbol{K}}_{\mathrm{s}} \times \hat{\boldsymbol{K}}_{\mathrm{i}})(\hat{\boldsymbol{K}}_{\mathrm{s}} \times \hat{\boldsymbol{K}}_{\mathrm{i}})}{(\hat{\boldsymbol{K}}_{\mathrm{s}} \times \hat{\boldsymbol{K}}_{\mathrm{i}})^2}. \tag{C.1}$$

All of the dielectric dependence is entering through the Fresnel coefficients R_{VV} and R_{HH}. This dyadic equation can also be rewritten as

$$\underline{\mathcal{K}} = K^4\frac{R_{\mathrm{VV}}(q/2)\hat{\boldsymbol{K}}_{\mathrm{s}}\hat{\boldsymbol{K}}_{\mathrm{i}} + R_{\mathrm{HH}}(q/2)(\hat{\boldsymbol{K}}_{\mathrm{s}} \times \hat{\boldsymbol{K}}_{\mathrm{i}})(\hat{\boldsymbol{K}}_{\mathrm{s}} \times \hat{\boldsymbol{K}}_{\mathrm{i}})}{K^2 + \boldsymbol{k}\cdot\boldsymbol{k}_0 - q_k q_0}. \tag{C.2}$$

In order to get the polarization matrix one should wrap the dyadic form with the incident and scattered polarization vectors:

$$\hat{\boldsymbol{V}}_{\mathrm{i}} = \frac{q_0\hat{\boldsymbol{k}}_0 + k_0\hat{\boldsymbol{z}}}{K} \qquad \text{and} \qquad \hat{\boldsymbol{H}}_{\mathrm{i}} = \hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{z}} \tag{C.3a}$$

$$\hat{\boldsymbol{V}}_{\mathrm{s}} = \frac{q_k\hat{\boldsymbol{k}} - k\hat{\boldsymbol{z}}}{K} \qquad \text{and} \qquad \hat{\boldsymbol{H}}_{\mathrm{s}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{z}}. \tag{C.3b}$$

Hence, these identities apply:

$$\hat{\boldsymbol{V}}_{\mathrm{i}}\cdot\hat{\boldsymbol{K}}_{\mathrm{s}} = \hat{\boldsymbol{H}}_{\mathrm{i}}\cdot(\hat{\boldsymbol{K}}_{\mathrm{i}} \times \hat{\boldsymbol{K}}_{\mathrm{s}}) = \frac{k_0 q_k + k q_0\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}_0}{K^2} \tag{C.4a}$$

$$\hat{\boldsymbol{H}}_{\mathrm{i}}\cdot\hat{\boldsymbol{K}}_{\mathrm{s}} = \hat{\boldsymbol{V}}_{\mathrm{i}}\cdot(\hat{\boldsymbol{K}}_{\mathrm{s}} \times \hat{\boldsymbol{K}}_{\mathrm{i}}) = \frac{k(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}_0)\cdot\hat{\boldsymbol{z}}}{K} \tag{C.4b}$$

$$\hat{\boldsymbol{V}}_{\mathrm{s}}\cdot\hat{\boldsymbol{K}}_{\mathrm{i}} = \hat{\boldsymbol{H}}_{\mathrm{s}}\cdot(\hat{\boldsymbol{K}}_{\mathrm{i}} \times \hat{\boldsymbol{K}}_{\mathrm{s}}) = \frac{k q_0 + k_0 q_k\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}_0}{K^2} \tag{C.4c}$$

$$\hat{\boldsymbol{H}}_{\mathrm{s}}\cdot\hat{\boldsymbol{K}}_{\mathrm{i}} = \hat{\boldsymbol{V}}_{\mathrm{s}}\cdot(\hat{\boldsymbol{K}}_{\mathrm{s}} \times \hat{\boldsymbol{K}}_{\mathrm{i}}) = \frac{k_0(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}})\cdot\hat{\boldsymbol{z}}}{K}. \tag{C.4d}$$

It can be shown that this fully dielectric Kirchhoff formulation is quadratic in its lowest order in $\boldsymbol{q}_{\mathrm{H}} = \boldsymbol{k} - \boldsymbol{k}_0$.

For simplicity, we use the Kirchhoff coefficients as simplified by [6, 8] and put in this approximate form:

$$\mathcal{K}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \mathcal{K}_\infty(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) * \mathcal{R}(q/2) \tag{C.5}$$

where the operator '$*$' is the element by element product of the two matrices. The $\mathcal{R}$ matrix is based on the Fresnel coefficients and can be written as

$$\mathcal{R}(q) = \begin{pmatrix} R_{\mathrm{VV}}(q) & R_{\mathrm{VH}}(q) \\ R_{\mathrm{HV}}(q) & R_{\mathrm{HH}}(q) \end{pmatrix} \tag{C.6}$$

where

$$R_{\mathrm{VV}}(q) = \frac{\epsilon q - \sqrt{(\epsilon-1)K^2 + q^2}}{\epsilon q + \sqrt{(\epsilon-1)K^2 + q^2}} \tag{C.7a}$$

$$R_{\mathrm{VH}}(q) = R_{\mathrm{HV}}(q) = \frac{R_{\mathrm{VV}} + R_{\mathrm{HH}}}{2} \tag{C.7b}$$

$$R_{\mathrm{HH}}(q) = -\frac{q - \sqrt{(\epsilon-1)K^2 + q^2}}{q + \sqrt{(\epsilon-1)K^2 + q^2}} \tag{C.7c}$$

where ϵ is the relative permittivity. This approximate Kirchhoff model is valid for surfaces showing good conduction and away from grazing angles since $(R_{\mathrm{VV}} - R_{\mathrm{HH}})$ is assumed to be small.

The variable q is the norm of the three dimensional vector which is defined as the difference between the scattered and the incident wavenumbers as

$$q = |\vec{q}| = |\boldsymbol{K}_{\mathrm{s}} - \boldsymbol{K}_{\mathrm{i}}| = \sqrt{\boldsymbol{q}_{\mathrm{H}}^2 + q_z^2}. \tag{C.8}$$

Now one operates the following substitutions:

$$\boldsymbol{q}_{\mathrm{H}} \Rightarrow \boldsymbol{\xi} \tag{C.9a}$$

$$q \Rightarrow \sqrt{\boldsymbol{\xi}^2 + (q_k^2 + q_0^2) + 2q_k q_0} \tag{C.9b}$$

$$(q_k^2 + q_0^2) \Rightarrow 2\left(K^2 - \frac{\boldsymbol{w}_{\mathrm{H}}^2}{4} - \frac{\boldsymbol{\xi}^2}{4}\right) \tag{C.9c}$$

$$2q_k q_0 \Rightarrow \tfrac{1}{2}\sqrt{\left(4K^2 - \boldsymbol{w}_{\mathrm{H}}^2 - \boldsymbol{\xi}^2\right)^2 - (2\boldsymbol{w}_{\mathrm{H}} \cdot \boldsymbol{\xi})^2} \tag{C.9d}$$

and notices that the resulting Kirchhoff coefficients are even in $\boldsymbol{\xi}$.

The SPM-1 coefficients for dielectric boundary conditions are taken from the appendix of [17]:

$$\mathcal{B}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \begin{pmatrix} B_{\mathrm{VV}}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) & B_{\mathrm{VH}}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) \\ B_{\mathrm{HV}}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) & B_{\mathrm{HH}}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) \end{pmatrix} \tag{C.10}$$

where

$$B_{\mathrm{VV}}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \frac{2q_k q_0(\epsilon - 1)(q_k' q_0' \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0 - \epsilon k k_0)}{(\epsilon q_k + q_k')(\epsilon q_0 + q_0')} \tag{C.11a}$$

$$B_{\mathrm{VH}}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \frac{2q_k q_0(\epsilon - 1)K q_k' (\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}_0) \cdot \hat{z}}{(\epsilon q_k + q_k')(q_0 + q_0')} \tag{C.11b}$$

$$B_{\mathrm{HV}}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = \frac{2q_k q_0(\epsilon - 1)K q_0' (\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}}) \cdot \hat{z}}{(q_k + q_k')(\epsilon q_0 + q_0')} \tag{C.11c}$$

$$B_{\mathrm{HH}}(\boldsymbol{w}_{\mathrm{H}}; \boldsymbol{q}_{\mathrm{H}}) = -\frac{2q_k q_0(\epsilon - 1)K^2 \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_0}{(q_k + q_k')(q_0 + q_0')} \tag{C.11d}$$

and the primed variables are defined as

$$q_k' = \sqrt{\epsilon K^2 - \boldsymbol{k} \cdot \boldsymbol{k}} \tag{C.12a}$$

$$q_0' = \sqrt{\epsilon K^2 - \boldsymbol{k}_0 \cdot \boldsymbol{k}_0}. \tag{C.12b}$$

The following additional substitutions:

$$q_k' \Rightarrow \sqrt{\epsilon K^2 - \left(\frac{\boldsymbol{w}_{\mathrm{H}} + \boldsymbol{\xi}}{2}\right)^2} \tag{C.13a}$$

$$q_0' \Rightarrow \sqrt{\epsilon K^2 - \left(\frac{\boldsymbol{w}_{\mathrm{H}} - \boldsymbol{\xi}}{2}\right)^2} \tag{C.13b}$$

are needed to finally demonstrate that the curvature kernel in this case is indeed also quadratic to its lowest order in $\boldsymbol{\xi}$.

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