INSTITUTE OF PHYSICS PUBLISHING WAVES IN RANDOM MEDIA

Waves Random Media 13 (2003) L1–L6 PII: S0959-7174(03)58888-7

LETTER TO THE EDITOR

A practical second-order electromagnetic model in the quasi-specular regime based on the curvature of a 'good-conducting' scattering surface

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Received 28 January 2003
Published 30 April 2003
Online at stacks.iop.org/WRM/13/L1

Abstract
This letter presents an approximate second-order electromagnetic model where polarization coefficients are surface dependent up to the curvature order in the quasi-specular regime. The scattering surface is considered 'good-conducting' as opposed to the case for our previous derivation where perfect conductivity was assumed. The model reproduces dynamically, depending on the properties of the scattering surface, the tangent-plane (Kirchhoff) or the first-order small-perturbation (Bragg) limits. The convergence is assumed to be ensured by the surface curvature alone. This second-order model is shown to be consistent with the small-slope approximation of Voronovich (SSA-1 + SSA-2) for perfectly conducting surfaces. Our model differs from SSA-1 + SSA-2 in its dielectric expression, to correct for a full convergence toward the tangent-plane limit under the 'good-conducting' approximation. This new second-order formulation is simple because it involves a single integral over the scattering surface and therefore it is suitable for a vast array of analytical and numerical applications in quasi-specular applications.

1. Introduction

In previous papers [1, 2], we demonstrated that a first-order small-slope approximation (SSA-1) of Voronovich [3] can be reached in its perfectly conducting limit by a direct derivation of the surface current integral equation. In the formulation of this model, only linear orders in surface slopes and height differences were retained in the second iteration of the surface current. This derivation permitted the definition of a complementary vector that transforms the tangent-plane approximation (Kirchhoff) into the small-slope method (Bragg). In a recently submitted publication [4], which can be considered as the third in the series where the second iteration of the surface current integral equation is the starting point for our electromagnetic model derivation, we demonstrated coherence with the second-order small-slope approximation

(SSA-1 + SSA-2) by accounting for cross-terms connecting surface elevations and slopes. The main result was that the total modelled field can be written as

$$\boldsymbol{B}_s^p = \boldsymbol{P}_s^{(1)} \int \mathrm{e}^{-\mathrm{i}q_z\eta(\boldsymbol{x})}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_H\cdot\boldsymbol{x}}\,\mathrm{d}\boldsymbol{x} + \mathrm{i}\iint \boldsymbol{\mathcal{T}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})\hat{\eta}(\boldsymbol{\xi})\mathrm{e}^{-\mathrm{i}(\boldsymbol{q}_H-\boldsymbol{\xi})\cdot\boldsymbol{x}}\mathrm{e}^{-\mathrm{i}q_z\eta}\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{\xi} \tag{1}$$

where $\boldsymbol{P}_s^{(1)}$ is the Kirchhoff polarization vector and is obtained from the first iteration of the surface current integral equation. This model is consistent with SSA-1 + SSA-2 because of an arbitrary gauge function linear in $\boldsymbol{\xi}$ that can be added in the separation of the double from the single integral. The second vector under the double integral is the second-order kernel of our new bistatic model and obtained from the second iteration, and is not necessarily identical to the SSA kernel. The expressions for the two vectors in (1) are

$$\boldsymbol{P}_s^{(1)} = 2\left\{\frac{\boldsymbol{q}}{q_z}\times\hat{P}_i^p\right\}\times\hat{k}^s, \tag{2a}$$

$$\boldsymbol{\mathcal{T}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi}) = \{2(\boldsymbol{\xi}\cdot\boldsymbol{P}_H)[\boldsymbol{Q}_H(\boldsymbol{\xi})+\boldsymbol{Q}_H(\boldsymbol{q}_H)] - (\boldsymbol{\xi}\cdot[\boldsymbol{Q}'_H(\boldsymbol{\xi})+\boldsymbol{Q}'_H(\boldsymbol{q}_H)])\boldsymbol{P}_H\}\times\hat{k}^s. \tag{2b}$$

The reader is invited to read our previous papers on the bistatic model derivation to become familiar with the notation and the definitions.

One can easily demonstrate that our second-order kernel $\boldsymbol{\mathcal{T}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})$ enjoys the following properties:

$$\boldsymbol{\mathcal{T}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{q}_H) = -q_z\boldsymbol{P}_s^{(2)}, \tag{3a}$$

$$\boldsymbol{\mathcal{T}}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{0}) = 0, \tag{3b}$$

$$\frac{\partial\boldsymbol{\mathcal{T}}}{\partial\xi_x}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{0})q_H^x + \frac{\partial\boldsymbol{\mathcal{T}}}{\partial\xi_y}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{0})q_H^y = 0 \tag{3c}$$

where $\boldsymbol{P}_s^{(2)}$ is the complementary vector needed to add to that of Kirchhoff, $\boldsymbol{P}_s^{(1)}$, in order to achieve convergence toward the first-order Bragg model (SPM-1). The expression for the complementary vector is

$$\boldsymbol{P}_s^{(2)} = 2\left\{2\left(\frac{\boldsymbol{q}_H}{q_z}\cdot\boldsymbol{P}_H\right)\boldsymbol{Q}_H(\boldsymbol{0}) - \left(\frac{\boldsymbol{q}_H}{q_z}\cdot\boldsymbol{Q}'_H(\boldsymbol{0})\right)\boldsymbol{P}_H\right\}\times\hat{k}^s, \tag{4}$$

which was derived in our previous publication [2].

The lowest-order correction brought in by the double integral in (1) to the single integral of Kirchhoff is of the order of the surface curvature. Indeed, the properties of the second-order kernel listed in (3) suggest that one must consider at least the second-order derivative of the kernel with respect to the dummy integration vector $\boldsymbol{\xi}$. The generalization to the full bistatic and full dielectric kernel is tedious and not available at this point. On the basis of a nice property of the complementary vector $\boldsymbol{P}_s^{(2)}$, we will suggest a new development which will generalize the electromagnetic model, at least in the quasi-specular regime, to 'good-conducting' surfaces. When the complementary vector is studied more closely, it becomes apparent that its lowest order in $(\boldsymbol{q}_H)$ is quadratic, where $\boldsymbol{q}_H$ is the horizontal component of the difference between the scattered and the incident wavenumbers. This observation suggests that the final field can also be interpreted as the result from the second derivative of the scattering surface because of the coincidence with the second derivative of the second-order kernel. Making the polarization surface dependent is fundamental to reaching dynamically both the Kirchhoff and Bragg limits when surface characteristics permit. This feature is crucial to enlarging the domain of applicability of the asymptotic model under consideration. Indeed, the constant-polarization coefficients, as in SSA-1, for instance, cannot guarantee convergence toward both the Kirchhoff and Bragg limits. Our objective in this letter is to derive a new quasi-specular model that extends our previous results in two directions. The first is including surface dependence into the polarization coefficients and the second is considering dielectric surfaces under the 'good-conducting' approximation. This new formulation should be comparable to the second-order dielectric model of Voronovich (SSA-2); similarities and differences will be obtained very carefully.

In the next section, we will postulate a general form for our second-order model based on surface curvature. Then the polarization coefficients will be derived under the 'good-conducting' approximation. A careful comparison with the second-order small-slope approximation SSA-2 will be made and implications of similarities or discrepancies will be addressed in detail.

2. The postulated general form

In our previous derivation [1, 2] of the bistatic model under the perfectly conducting condition, two polarization vectors were obtained; these are $\boldsymbol{P}_s^{(1)}$ which represents the Kirchhoff polarization and $\boldsymbol{P}_s^{(2)}$ which represents the complementary polarization needed to turn the Kirchhoff polarization into the small-perturbation polarization. This formulation is consistent with the small-slope approximation SSA-1 of Voronovich [3]. If one performs an expansion in $\boldsymbol{q}_H$, one notices that the complementary polarization vector has no constant and no linear order in $\boldsymbol{q}_H$. Furthermore, the Kirchhoff polarization vector $\boldsymbol{P}_s^{(1)}$ represents with high fidelity the tangent-plane approximation, and therefore any correction must transit through surface curvature or higher-order derivatives of the scattering surface. Indeed, even though not fully complete in linear slopes, the Kirchhoff polarization collected just enough slope to be correct and therefore physically complete under the local tangent-plane approximation. Those two observations suggest that one can postulate a general form for a surface scattering model of second order as follows:

$$\mathcal{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{2\sqrt{q_k q_0}}{q_k + q_0} \int \left\{ \mathcal{K}(\boldsymbol{k}, \boldsymbol{k}_0) + \frac{\mathrm{i}}{4}\left[\frac{1}{2}\mathcal{C}_{xx}\eta_{xx} + \mathcal{C}_{xy}\eta_{xy} + \frac{1}{2}\mathcal{C}_{yy}\eta_{yy}\right]\right\} \mathrm{e}^{\mathrm{i}q_z\eta(\boldsymbol{x})}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_H\cdot\boldsymbol{x}}\,\mathrm{d}\boldsymbol{x} \quad (5)$$

where the scattering matrix is given in Voronovich's notation when appropriate, in order to simplify the comparison with his second-order model. The polarization matrices are constant as regards the integration vector $\boldsymbol{x}$ and can be derived from the second-order derivative of the second-order kernel $\boldsymbol{\mathcal{T}}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi})$ from our model (1) or that of Voronovich's $\boldsymbol{\mathcal{M}}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi})$ matrix. We will demonstrate that a simpler procedure is to derive the kernels in (5) from the complementary $\boldsymbol{P}_s^{(2)}$-vector by simple factorization of $\boldsymbol{q}_H^2$. The difficult part though, is that these curvature polarization coefficients may still contain infinite orders of $\boldsymbol{q}_H$. Only a finite-order expansion will be derived here, and given up to the third order.

In the current derivation, we consider the dielectric property of the scattering surface under the 'good-conducting' approximation and therefore we derive the polarization coefficients in (5) by using full standard dielectric polarization coefficients of the Kirchhoff and Bragg models, instead of using our perfectly conducting polarization vector $\boldsymbol{P}_s^{(2)}$. In this case, the $\mathcal{K}$-matrix in (5) can be provided by the stationary phase approximation in the full dielectric case, as derived by Stogryn [5]. It is obvious from this intuitive formulation that the tangent-plane limit of Kirchhoff is preserved whether the second-order derivatives of the surface vanish or the electromagnetic frequency tends to infinity (the $\mathcal{C}$-coefficients are inversely proportional to the electromagnetic wavenumber, as will be show later). The first-order small-perturbation limit of Bragg is reached if and only if the curvature terms can turn the Kirchhoff polarization matrix $\mathcal{K}$ into the Bragg matrix $\mathcal{B}$, whose expression can be obtained by standard small-perturbation methods. This requirement places a constraint on the curvature polarization matrices in the following form:

$$4q_z(\mathcal{K}-\mathcal{B}) = \tfrac{1}{2}\mathcal{C}_{xx}\boldsymbol{q}_{Hx}^2 + \mathcal{C}_{xy}\boldsymbol{q}_{Hx}\boldsymbol{q}_{Hy} + \tfrac{1}{2}\mathcal{C}_{yy}\boldsymbol{q}_{Hy}^2. \qquad (6)$$

Under 'good-conducting' conditions, the difference between the Kirchhoff and the Bragg matrices, similar to our previous $\boldsymbol{P}_s^{(2)}$-vector, can be shown to be quadratic to lowest order in $\boldsymbol{q}_H$ that is quadratic. In this case, the curvature polarization matrices $\mathcal{C}$ can also be expanded in orders of $\boldsymbol{q}_H$:

$$\mathcal{C}_{xx} = \mathcal{C}_{xx}^{00} + \mathcal{C}_{xx}^{10}\boldsymbol{q}_{Hx} + \mathcal{C}_{xx}^{01}\boldsymbol{q}_{Hy} + \tfrac{1}{2}\mathcal{C}_{xx}^{20}\boldsymbol{q}_{Hx}^2 + \mathcal{C}_{xx}^{11}\boldsymbol{q}_{Hx}\boldsymbol{q}_{Hy} + \tfrac{1}{2}\mathcal{C}_{xx}^{02}\boldsymbol{q}_{Hy}^2 + \cdots, \qquad (7a)$$

$$\mathcal{C}_{xy} = \mathcal{C}_{xy}^{00} + \mathcal{C}_{xy}^{10}\boldsymbol{q}_{Hx} + \mathcal{C}_{xy}^{01}\boldsymbol{q}_{Hy} + \tfrac{1}{2}\mathcal{C}_{xy}^{20}\boldsymbol{q}_{Hx}^2 + \mathcal{C}_{xy}^{11}\boldsymbol{q}_{Hx}\boldsymbol{q}_{Hy} + \tfrac{1}{2}\mathcal{C}_{xy}^{02}\boldsymbol{q}_{Hy}^2 + \cdots, \qquad (7b)$$

$$\mathcal{C}_{yy} = \mathcal{C}_{yy}^{00} + \mathcal{C}_{yy}^{10}\boldsymbol{q}_{Hx} + \mathcal{C}_{yy}^{01}\boldsymbol{q}_{Hy} + \tfrac{1}{2}\mathcal{C}_{yy}^{20}\boldsymbol{q}_{Hx}^2 + \mathcal{C}_{yy}^{11}\boldsymbol{q}_{Hx}\boldsymbol{q}_{Hy} + \tfrac{1}{2}\mathcal{C}_{yy}^{02}\boldsymbol{q}_{Hy}^2 + \cdots. \qquad (7c)$$

We will seek these curvature polarization matrices up to the linear order in q_H, which is equivalent to third order in the total polarization.

After some tedious algebraic manipulations, the constant-curvature matrices, with respect to $\boldsymbol{q}_H$, can be put into the following simple form:

$$\mathcal{C}_{xx}^{00} = \frac{-4\operatorname{cosec}^3\gamma}{K}\begin{pmatrix} 1-\frac{(7+\cos 2\gamma)\operatorname{cosec}\gamma}{2\sqrt{\varepsilon}} & 0 \\ 0 & 1-\frac{(2+\sin^2\gamma)\sin\gamma}{\sqrt{\varepsilon}} \end{pmatrix} \qquad (8a)$$

$$\mathcal{C}_{xy}^{00} = \frac{4\operatorname{cosec}^2\gamma}{K}\begin{pmatrix} 0 & 1-\frac{(3+2\operatorname{cosec}^2\gamma)\sin\gamma}{\sqrt{\varepsilon}} \\ -1+\frac{(3+2\operatorname{cosec}^2\gamma)\sin\gamma}{\sqrt{\varepsilon}} & 0 \end{pmatrix} \qquad (8b)$$

$$\mathcal{C}_{yy}^{00} = \frac{4\operatorname{cosec}\gamma}{K}\begin{pmatrix} 1-\frac{3\operatorname{cosec}\gamma}{\sqrt{\varepsilon}} & 0 \\ 0 & 1-\frac{(1+2\operatorname{cosec}^2\gamma)\sin\gamma}{\sqrt{\varepsilon}} \end{pmatrix} \qquad (8c)$$

where γ is the grazing angle, K is the electromagnetic wavenumber and ε is the relative permittivity of the scattering surface. The dielectric dependence of the polarization is inversely proportional to $\sqrt{\varepsilon}$. In this form the perfectly conducting limit is trivially obtained.

The curvature coefficients up to first order in $\boldsymbol{q}_H$ are

$$3\mathcal{C}_{xx}^{10} = \frac{-18\cot\gamma\operatorname{cosec}^4\gamma}{K^2}\begin{pmatrix} 1-\frac{(15+\cos 2\gamma)\operatorname{cosec}\gamma}{3\sqrt{\varepsilon}} & 0 \\ 0 & 1-\frac{4\sin\gamma}{3\sqrt{\varepsilon}} \end{pmatrix} \qquad (9a)$$

$$\mathcal{C}_{xx}^{01} + 2\mathcal{C}_{xy}^{10} = \frac{2\operatorname{cosec}^4\gamma\sec\gamma}{K^2}\begin{pmatrix} 0 & 1+3\cos 2\gamma-\frac{(39+104\cos 2\gamma+15\cos 4\gamma)\operatorname{cosec}\gamma}{8\sqrt{\varepsilon}} \\ -1-3\cos 2\gamma+\frac{(1+72\cos 2\gamma-9\cos 4\gamma)\operatorname{cosec}\gamma}{8\sqrt{\varepsilon}} & 0 \end{pmatrix} \qquad (9b)$$

$$2\mathcal{C}_{xy}^{01} + \mathcal{C}_{yy}^{10} = -\frac{\operatorname{cosec}^3\gamma\sec\gamma}{K^2}\begin{pmatrix} 5-3\cos 2\gamma-\frac{2(7-5\cos 2\gamma)\operatorname{cosec}\gamma}{\sqrt{\varepsilon}} & 0 \\ 0 & 5-3\cos 2\gamma-\frac{(7-16\cos 2\gamma+\cos 4\gamma)\operatorname{cosec}\gamma}{\sqrt{\varepsilon}} \end{pmatrix} \qquad (9c)$$

$$3\mathcal{C}_{yy}^{01} = \frac{12\operatorname{cosec}^2\gamma\sec\gamma}{K^2}\begin{pmatrix} 0 & 1-\frac{15(3-\cos 2\gamma)\operatorname{cosec}\gamma}{12\sqrt{\varepsilon}} \\ -1+\frac{(13-7\cos 2\gamma)\operatorname{cosec}\gamma}{4\sqrt{\varepsilon}} & 0 \end{pmatrix}. \qquad (9d)$$

The second-order matrices will not be given here, to keep the presentation simple and comprehensible.

One can easily show that the perfectly conducting coefficients in (8) and (9) are identical to the evaluation of the second-order derivative of $\boldsymbol{\mathcal{T}}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi})$. Most of the curvature polarization matrices are fully determined in (8) and (9). It is interesting to note, however, that $\mathcal{C}_{xx}^{01}$, $\mathcal{C}_{xy}^{10}$, $\mathcal{C}_{xy}^{01}$, and $\mathcal{C}_{yy}^{10}$ are determined by two constraints in (9). This leaves some degree of freedom in defining these polarization matrices. To resolve this ambiguity, two solutions are possible. The first simple solution is to impose two extra constraints and construct a model which can reproduce both Kirchhoff and Bragg limits. The second solution is to impose a third limit, which could be the second-order Bragg limit as in the SSA model.

3. Comparison with the second-order small-slope approximation (SSA-2)

The full expression of the second-order small-slope approximation is given by

$$\mathcal{S}(\boldsymbol{k},\boldsymbol{k}_0)=\frac{2\sqrt{q_k q_0}}{q_k+q_0}\int\left\{\mathcal{B}(\boldsymbol{k},\boldsymbol{k}_0)-\frac{\mathrm{i}}{4}\int\mathcal{M}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})\hat{\eta}(\boldsymbol{\xi})\mathrm{e}^{\mathrm{i}\boldsymbol{\xi}\cdot\boldsymbol{x}}\mathrm{d}\boldsymbol{\xi}\right\}\mathrm{e}^{\mathrm{i}q_z\eta(\boldsymbol{x})}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_H\cdot\boldsymbol{x}}\,\mathrm{d}\boldsymbol{x} \qquad (10)$$

where $\mathcal{B}$ is the standard first-order Bragg polarization. The kernel under the double integral $\mathcal{M}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})$ is suggested by Voronovich [6] to be related to first- and second-order Bragg limits. The explicit form of the second-order kernel is

$$\mathcal{M}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})=\mathcal{B}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{k}-\boldsymbol{\xi})+\mathcal{B}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi}+\boldsymbol{k}_0)+2(q_k+q_0)\mathcal{B}(\boldsymbol{k},\boldsymbol{k}_0) \qquad (11)$$

where the expression for the second-order Bragg limit $\mathcal{B}_2(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})$ is given in [6].

The objective now is to compare SSA-2 in (10) to our model in (5). The second-order kernel of SSA-2, $\mathcal{M}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})$, will be expanded to quadratic order in $\boldsymbol{\xi}$ in the quasi-specular direction $q_H\approx 0$ and under the 'good-conducting' limit. Only after these expansions can our model be checked against SSA-2. We noticed that all the perfectly conducting coefficients are identical whether we go through (6), $\mathcal{T}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})$, or $\mathcal{M}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})$. In the dielectric case, however, we found some differences. The similarities and differences can be listed as follows:

$$\mathcal{C}^{00}_{xx}\equiv\mathcal{M}^{00}_{xx}, \qquad (12a)$$
$$\mathcal{C}^{00}_{xy}\neq\mathcal{M}^{00}_{xy}, \qquad (12b)$$
$$\mathcal{C}^{00}_{yy}\equiv\mathcal{C}^{00}_{yy} \qquad (12c)$$

in quadratic order and

$$\mathcal{C}^{10}_{xx}\equiv\mathcal{M}^{10}_{xx}, \qquad (13a)$$
$$\mathcal{C}^{01}_{xx}+2\mathcal{C}^{10}_{xy}\neq\mathcal{M}^{01}_{xx}+2\mathcal{M}^{10}_{xy}, \qquad (13b)$$
$$2\mathcal{C}^{01}_{xy}+\mathcal{C}^{10}_{yy}\equiv 2\mathcal{M}^{01}_{xy}+\mathcal{M}^{10}_{yy}, \qquad (13c)$$
$$\mathcal{C}^{01}_{yy}\neq\mathcal{M}^{01}_{yy} \qquad (13d)$$

in cubic order. These equalities demonstrate that SSA-$(1+2)$ does converge toward the Kirchhoff approximation in the high-frequency limit but with small discrepancies for some polarizations. In quadratic order, the difference is as small as replacing the factor 3 in (8) by 1 in the dielectric formula. The good agreement for perfect conductivity with both our second-order model, $\mathcal{T}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})$, and that of SSA, $\mathcal{M}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})$, validates our new derivations. The small differences in the dielectric case can be interpreted as the necessary corrections needed in order to make SSA-2 consistent with the Kirchhoff limit.

4. Conclusions

We have presented a simplified second-order model for quasi-specular scattering where the surface curvature extends the Kirchhoff model. The latter is well known to be an accurate model for surface scattering when the surface can be considered locally flat. Any correction to this model reflects the fact that the scattering surface is not locally flat and hence has a finite local radius of curvature. We noticed that factoring out a quadratic term in $\boldsymbol{q}_H=\boldsymbol{k}-\boldsymbol{k}_0$ in the complementary polarization vector is equivalent to taking the second-order derivative of the second-order kernel from our model or from that of the small-slope approximation SSA. This coincidence is then exploited to derive, under the 'good-conduction' assumption, new polarization matrices to be used with surface curvature. This model contains only a simple single integral and can be implemented numerically in a very efficient manner. This new model can explain depolarizations encountered in quasi-specular situations such as by traditional radar altimeters or by novel systems such as the reflected GPS signals.

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