Available online at www.sciencedirect.com
SCIENCE DIRECT®
JOURNAL OF GEODYNAMICS
Journal of Geodynamics 41 (2006) 119–127
http://www.elsevier.com/locate/jog

Analysis and prediction of ocean tides by the computer program VAV

Bernard Ducarme[a,*], A.P. Venedikov[b,c], J. Arnoso[b], R. Vieira[b]

[a] Research Associate NSFR, Royal Observatory of Belgium, Av. Circulaire 3, 1180 Brussels, Belgium
[b] Instituto de Astronomía y Geodesia (CSIC-UCM), Facultad de Matemáticas, Plaza de Ciencias 3, 28040 Madrid, Spain
[c] Geophysical Institute, Bulgarian Academy of Sciences, Acad. G. Bonchev Street, Block 3, Sofia 1113, Bulgaria

Accepted 30 August 2005

Abstract

The paper presents various results from the application of VAV tidal analysis program on a 36 years (7/01/1945–31/12/1980) series of ocean data from the Belgian port Ostend. The program VAV was originally developed for the processing of Earth tide data. Now, it has been supplied with specific options for the analysis of ocean tide data. One of the new options is the determination of the shallow water tides. VAV is also able to make ocean tide predictions, as well as to investigate the mean sea level and its long-term trend. Generally, useful properties of VAV are a correct application of the Method of the Least Squares, taking into account the colored character of the noise, as well as its ability to process data with gaps, without any interpolation.

Keywords: Ocean tides; Tidal analysis methods; Mean sea level

1. Introduction

The paper presents results from the application of the tidal program VAV (Venedikov et al., 2001, 2003, 2005) on the ocean tide (OT) data from the Belgian port of Ostend ($\phi = 51.23°$N, $\lambda = 2.93°$E). This series of data covers 36 years in the time interval 7/01/1945–31/12/1980 and contains 312,912 hourly ordinates.

VAV is originally designed for the processing of Earth tide (ET) data. Now, it has been supplied by specific options, corresponding to the OT characteristics and problems (Godin, 1972). We hope that the results here presented will show that VAV may be applicable to other oceanographic tidal work.

Generally, the most important features of VAV are:

(i) Model of the tidal signal allowing the application of Method of the Least Squares (MLS) with equations, taking into account all theoretical tidal waves.
(ii) Capacity to deal with data, charged by irregular drift.

* Corresponding author. Tel.: +32 2 3730248; fax: +32 2 3749822.
E-mail addresses: ducarme@oma.be (B. Ducarme), vened@geophys.bas.bg (A.P. Venedikov), jose_arnoso@mat.ucm.es (J. Arnoso), vieira@iagmat1.mat.ucm.es (R. Vieira).

0264-3707/$ – see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jog.2005.08.001

(iii) Application of MLS with estimates of the precision, taking into account the colored character of the noise (noise with frequency dependent power).
(iv) Processing of data with arbitrary gaps without interpolations.

Features of special interest for the ocean tides are:

(v) Determination of the shallow water tides (ShWT) and radiation tides.
(vi) Prediction of the ocean tides.
(vii) Study of the mean sea level.

In Section 2 we shall briefly discuss the items (i)–(iv) in light of their possible use for OT data. The remaining sections show examples from the processing of the OT data from Ostend.

2. Main features of the VAV method

2.1. Model of the tidal signal

The general model of the tidal signal for all cases of ET data is

$$S(t) = \mathrm{Re}\sum_{\omega} H_\omega \exp[\mathrm{i}(\omega t + \Phi_\omega)] = \mathrm{Re}\sum_{\omega} X(\omega) h_\omega \exp[\mathrm{i}(\omega t + \varphi_\omega)] \quad (1)$$

In this expression ω is frequency, taking a set of known discrete values, e.g. some 1200 values in the development of Tamura (1987), h_ω and φ_ω the known theoretical amplitude and phase at frequency ω, H_ω and Φ_ω the corresponding observed amplitude and phase, $X(\omega) = \delta_\omega \exp(\mathrm{i}\alpha_\omega)$ is the unknown admittance function at frequency ω, where $\delta_\omega = H_\omega/h_\omega$ is called amplitude factor and $\alpha_\omega = \Phi_\omega - \varphi_\omega$ is called phase lag. For OT data "theoretical amplitude and phase" means amplitude and phase of the equilibrium tide.

The unknowns, estimated by the classical harmonic analysis, are the observed H_ω and Φ_ω. The problem of using these unknowns in MLS is that there are too many tides with very close frequencies ω. When all of them are included in the MLS equations, they become linearly dependent. The equations become independent if we take into account only a small number of the most important tides. Then, since a great number of tides are neglected, the equations are not correctly created, producing results with low precision. In addition, this precision cannot be correctly estimated.

Since Venedikov (1966) the ET methods of analysis started to deal with equations whose unknowns are the even and odd components of the admittance $X(\omega) = \delta_\omega \exp(\mathrm{i}\alpha_\omega)$, which are theoretically and practically slowly varying functions of ω.

By using this, it is accepted that for some narrow frequency intervals $\Omega_1, \Omega_2, \ldots, \Omega_m$, which shape the so-called tidal groups, the admittance $X(\omega)$ can be represented as

$$X(\omega) = x_j = \text{constant}, \quad \text{for}\, \omega \in \Omega_j,\ j = 1, 2, \ldots, m \quad (2)$$

The groups are formed around the main tidal constituents taking into account the Rayleigh criterion to ensure a correct separation of these main constituents. For example the S_2 and K_2 waves are only separable from a six month tidal record.

Then, the model of the tidal signal can be represented as

$$S(t) = \mathrm{Re}\sum_{j=1}^{m} x_j \sum_{\omega \in \Omega_j} h_\omega \exp[\mathrm{i}(\omega t + \varphi_\omega)] \quad (3)$$

This model includes a moderate number of unknowns x_j and, in the same time, it can take into account all theoretical tides, without creating linear dependences. This made possible the use of MLS with correctly created equations.

It is easy to show that the model of Munk and Cartwright (1966), designed for OT analysis, is equivalent to (1) as it also accepts that the admittance $X(\omega)$ does not change very rapidly with ω. Due to this, we believe that the model (3) can be successfully applied on OT data.

The response method (Munk and Cartwright, 1966; Yaramanci, 1978) assumes the linear tidal admittance is a slowly varying function of frequency but allows isolated exceptions to include anomalous tides such as non-linear tides. Namely, the response method allows the separation of linear and non-linear (shallow water) components as well as radiation tides even if they are at the same frequency. The method was also successfully applied to ET data (Merriam, 2000).

An example of a similar capacity of our model is the way we deal with the Ter-Diurnal (TD) and Quarter-Diurnal (QD) tides. When all TD tides are in one group, the corresponding Ω_{TD} is the interval (2.753 cpd, 3.082 cpd), which includes the tide S_3. Since S_3 is meteorologically affected, we use an additional Ω_{S_3} = (3 cpd, 3 cpd), including only the radiation tide S_3.

In the same way for the QD tides we can use Ω_{TD} = (3.79 cpd, 4.00 cpd), which includes S_4, with a special Ω_{S_4} = (4 cpd, 4 cpd) that includes only the radiation tide S_4.

The similarity with the model of Munk and Cartwright does not mean a perfect coincidence. They use a continuous admittance function, whose variability from a tidal band to a neighboring band depends on the number of the cross-regression coefficients used. Our admittance $X(\omega)$ in (2) is a stepwise function, remaining a constant in a given interval Ω_i, which however, may have arbitrary jumps from one to another Ω_i. In such a way the variability of $X(\omega)$ depends on how well the set of Ω_i is constructed. The idea of VAV is to experiment different variants for every case and choose among them according to a suitable statistical criterion.

2.2. Output parameters

The main output of the ET analysis are the δ and α, estimated for the frequency intervals (the tidal groups) Ω_j, while the output of the OT analyses are the observed amplitudes and phases, with a specific definition of the phases for oceanographic purposes. When VAV is applied on ocean data, the output phases are in agreement with general ocean tides practice, that is, the phases are referred to the Greenwich meridian and the phase lags are counted positive. In the present version of VAV, when OT data are processed, the ET output α is replaced by the output of the observed phase α_o according to the oceanographic convention, i.e. the Greenwich phase α_G, changed of sign:

$$\alpha_o = -\alpha_G = -(\alpha + d_1\lambda) \tag{4}$$

where λ is the longitude (East positive) and d_1 is the first argument number of Doodson (Doodson, 1922; Melchior, 1983). For long period (LP) it is necessary to add 180° to the phase.

2.3. Model of the drift and frequency dependent estimation of the precision

The ET data have a drift, which is, at least partly, instrumental. The OT data have not an instrumental drift. In the same time there are incessant non-tidal variations of the water level, which may be considered as an irregular drift (see Sections 5 and 6).

VAV uses a flexible model of the drift, based on the hypothesis that the drift has many discontinuities and changes of its behavior. In our opinion, this is really the model, which may well meet the capricious variations of the water level.

The data are partitioned into intervals of fixed length ΔT. In every interval of length ΔT the drift is separately represented by polynomials of selected low power K_d (Venedikov et al., 2003). Originally ΔT was fixed to 48 h. Now VAV can use much shorter ΔT, even $\Delta T < 24$ h, which improves the approximation of a strongly variable drift.

Due to this model of the drift, the first stage of the application of MLS appeared to be a transformation of the hourly data from the time domain into filtered data with a time step ΔT hours in a time/frequency domain.

The final stage of the analysis is an application of MLS on the data in the time/frequency domain. This allows getting frequency dependent estimates of the precision. The usual applications of MLS provide a single mean square deviation (MSD) as an estimate of the variance of the data, which is reasonable only in the case of white noise. Unlike this practice, VAV provides different MSD as estimates of the variances at the basic frequencies of 1, 2, ..., n cycles per day (cpd), corresponding to the colored character of the noise.

2.4. The problem of the data gaps

Due to the use of MLS the problem of the data gaps is solved in a simple and efficient way: we create equations for the really existing data and we do not use non-existing data in the gaps. Thus, it is not necessary to apply interpolations that can introduce a noise with extremely bad quality. This happened to be particularly important for the data series of Ostend, where exist multiple cases of missing data, as well as eight cases of gaps over 100 h, the largest one being of 805 h, i.e. over one month.

3. Results of the analysis of the data from Ostend

Table 1 gives the results of the analysis for the most important tidal waves, obtained by the application of the VAV program. The results are obtained through multiple variants of the drift parameters ΔT and K_{d}, until getting the lowest MSD.

The VAV results are compared with the analysis of Melchior et al. (1967) of the first 14 years of the same data. At that time the ET analysis used to follow the convention of the astronomers using West positive longitudes, i.e. Ostend had a negative longitude. Due to this it seems necessary to add $2d_1\lambda$, where d_1 is the first argument number of Doodson and λ is the East longitude in degrees.

A large value of the ratio δ_ω is an indication that there is a resonance at that frequency. The semi-diurnal family (SD) as a whole is amplified in the North Atlantic. But in the LP family terms as Msm and Msf are even more amplified. They are examples of Shallow Water Tide (ShWT). M_4 is indeed more resonant.

4. Determination of the non-linear tides by VAV

Besides the tidal constituents deriving directly from the tidal potential, some additional "tides" are present in the ocean tides signal. Some of them are due to the non-linear behavior of the tides in the coastal areas. They are called shallow water terms and their arguments are a combination of the arguments of the main tidal constituents. There are also radiation tides, which are periodic variations due to the effect of the solar radiation on the atmosphere and the oceans. Besides the harmonics of the solar day (S_1) one can observe seasonal terms. These seasonal variations will

Table 1
Results of the tidal analysis of the data from Ostend

Wave	Theoretical amplitude, H_ω (cm)	VAV results			Melchior et al. (1967)	
		Observed amplitude, H_ω (cm)	Ratio, δ_ω	Observed phase (°)	Observed amplitude, H_ω (cm)	Observed phase[a] (°)
Long period (LP) waves						
Msm	0.17	12.6 ± 1.7	73.3	76.8 ± 7.8		
Mm	0.90	1.7 ± 1.5	1.9	351.8 ± 52.2	0.7 ± 2.1	352.6 ± 234.4
Msf	0.15	12.10 ± 0.82	81.2	128.2 ± 3.9	3.1 ± 1.4	51.4 ± 24.2
Mf	1.70	3.76 ± 0.68	2.2	183.6 ± 10.3	1.1 ± 2.0	157.0 ± 109.9
Diurnal (D) waves						
O_1	9.85	8.80 ± 0.09	0.89	171.8 ± 0.6	9.48 ± 0.57	173.4 ± 2.5
P_1	4.59	2.48 ± 0.09	0.54	332.4 ± 2.1	2.34 ± 0.54	332.2 ± 11.9
K_1	13.85	5.25 ± 0.09	0.38	355.8 ± 1.0	5.51 ± 0.41	355.1 ± 5.9
Semi-diurnal (SD) waves						
N_2	1.84	29.72 ± 0.29	16.17	351.4 ± 0.6	30.55 ± 0.89	352.9 ± 1.5
M_2	9.60	175.52 ± 0.30	18.29	15.9 ± 0.1	179.56 ± 0.17	17.0 ± 0.7
S_2	4.47	51.03 ± 0.29	11.43	68.8 ± 0.3	52.42 ± 0.61	69.4 ± 0.8
Ter-Diurnal (TD) wave						
M_3	0.08	0.84 ± .04	10.7	90.8 ± 2.8	0.73 ± 0.52	85.9 ± 30.1
Quarter-Diurnal (QD) wave						
M_4	0.007	9.69 ± 0.05	1384	355.2 ± 0.3	10.48 ± 0.43	358.8 ± 3.2

[a] With the correction $2d_1\lambda$.

Table 2
Representation of the annual modulation of M_2 at Ostend

Wave	N	Amplitude (cm)	MSD (cm)	δ_ω	Phase (°)	MSD (°)
(a) Separation in different tidal groups						
α_2	4	2.51	0.28	76	67.5	6.5
M_2	19	175.66	0.28	18	15.8	0.1
β_2	8	1.06	0.28	36	204.3	15.3
(b) Introduction of ShWT's						
H_1	1	2.19	0.29	–	−21.7	7.7
M_2	56	175.52	0.30	18	15.9	0.1
H_1	1	1.69	0.28	–	122.8	9.9

N: number of tidal waves in the group; δ_ω: ratio between the observed amplitude and the corresponding equilibrium tide.

appear as additional long period terms. Periods longer than one year can be studied in the tidal residues after subtraction of the predicted tide (see Section 5).

The determination of the ShWT is an option of VAV, especially developed for the OT case. As shown in Venedikov et al. (2005), VAV includes a standard set of ShWT. It is also possible to experiment with any set of ShWT, defined in one of the following ways: (i) as linear combinations of selected tides, (ii) directly by the frequencies and (iii) directly by chosen argument number of Doodson.

The determination of ShWT requires the following expansion of the model (3), which also corresponds to the capacity of the response method, mentioned in Section 2.1.

Let $w = w_1, w_2, \ldots, w_\mu$ be shallow water frequencies and let H_w and Φ_w be the unknown observed amplitude and phase at frequency w. The expansion of the model (3) for the determination of H_w and Φ_w, used by VAV, is

$$S(t) = \mathrm{Re}\sum_{j=1}^{m} x_j \sum_{\omega \in \Omega_j} h_\omega \exp[\mathrm{i}(\omega t + \varphi_\omega)] + \mathrm{Re}\sum_{w=w_1}^{w_\mu} X_w \exp(\mathrm{i}wt), \tag{5}$$

where $X_w = H_w \exp(-\mathrm{i}\Phi_w)$ represents a pair of unknowns.

In principle, a frequency w may be outside the intervals Ω, as well as inside a given interval. It can even coincide with a main tidal frequency ω as, for instance, $ALP_1 \equiv \sigma Q_1$, $UPS_1 \equiv \nu_1$, $MP_1 \equiv \tau_1$, $MNS_2 \equiv \varepsilon_2$, $KJ_2 \equiv \eta_2$, $H_1 \equiv \alpha_2$, $H_2 \equiv \beta_2$, λ_2, $SKM_2 \equiv 2S_2$. This coincidence may produce a tendency towards linear dependence of the equations and thus unstable results with very low precision. Therefore, some care is necessary when considering these ShWT. If the corresponding ET group is separated, e.g. α_2 and β_2 which represent an annual modulation of M_2 (Table 2), it could be dangerous to introduce the ShWT's, here H_1 and H_2. It is seen in Table 2a that the waves α_2 and β_2 are amplified as their amplitude ratio δ_ω is more than twice the corresponding value for M_2. It indicates that a non-linear tide is superimposed on the tide derived from the tidal potential. However, if we consider only the complete M_2 group, including α_2 and β_2, these waves will get implicitly the same amplitude factor as the main wave, i.e. M_2 and the ShWT's H_1 and H_2 will take the remaining energy, without any loss of precision. The stability of the solution is not affected. The final result will be the same for what concerns ocean tide prediction.

The results for the QD tides are given in Table 3 and compared with the results of Melchior et al. (1967), indicated by M-P. The phases of the M-P results are corrected by $8\lambda = 23.44°$ in order to fit the VAV determination, i.e. the oceanographic definition of the phases.

The estimated amplitudes of ShWT's at the even frequency domains 4, 6 and 8 cpd are shown in Fig. 1. These ShWT's appeared to be much larger than the ShWT's at the odd frequencies, not shown here.

The ShWT's at the highest frequencies are shown in Fig. 2. For hourly data the Nyquist frequency is 12 cpd. Fig. 2 shows that 12th-diurnal tides, with frequencies close but under 12 cpd, are accessible. Namely, we have got significant amplitudes of M_{12} (the 6th sub-harmonic of M_2) and ST_{34}, whose amplitudes are larger than one millimetre.

5. Prediction of the ocean tides

VAV has the following options for the prediction of the OT: (i) by using a complete set of the estimated amplitudes and phases at given epoch, (ii) by using the amplitude factors δ and the phase lags α for a set of tidal groups and (iii)

Table 3
Results for the ShWT's at 4 cpd

ShWT name	Analysis by	Amplitude ± MSD (cm)	Phase ± MSD (°)
MN_4	VAV	3.274 ± 0.055	330.15 ± 0.96
	M-P	3.39 ± 0.49	335.3 ± 8.5
M_4	VAV	9.691 ± 0.055	355.19 ± 0.33
	M-P	10.48 ± 0.43	358.8 ± 3.2
SN_4	VAV	0.637 ± 0.055	102.72 ± 4.97
	M-P	0.48 ± 0.34	104.1 ± 40.7
MS_4	VAV	6.098 ± 0.055	58.49 ± 0.52
	M-P	6.43 ± 0.44	60.8 ± 4.0
MK_4	VAV	1.849 ± 0.055	55.23 ± 1.71
	M-P	1.89 ± 0.39	62.0 ± 12.0
S_4	VAV	0.212 ± 0.055	172.93 ± 15.03
	M-P	0.36 ± 0.19	186.5 ± 31.0
SK_4	VAV	0.241 ± 0.055	154.54 ± 13.22
	M-P	0.26 ± 0.37	162.3 ± 86.1

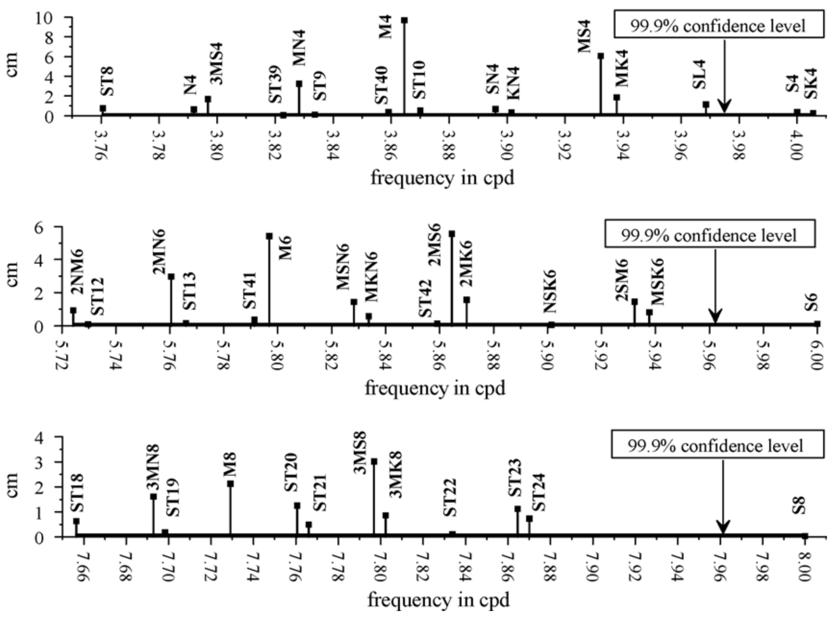

Fig. 1. Amplitudes of the ShWT's obtained by VAV at 4, 6 and 8 cpd frequency domains.

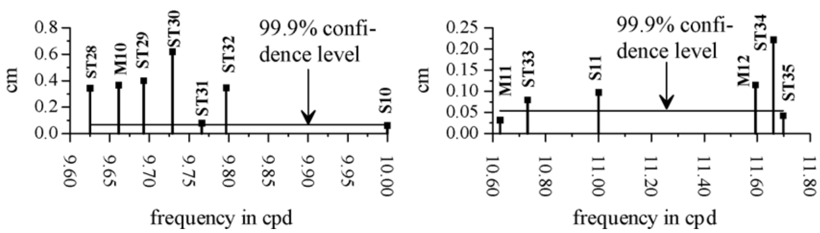

Fig. 2. Amplitudes of the ShWT's obtained by VAV at the highest frequencies 10, 11 and 12 cpd (but under 12 cpd).

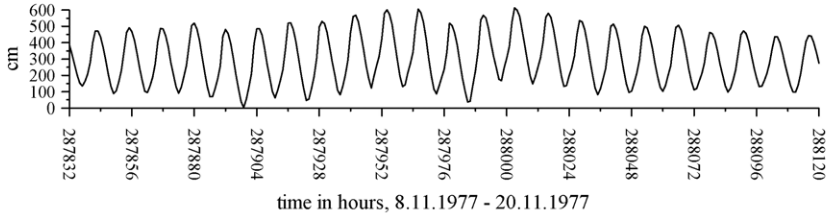

Fig. 3. Existing data in the predicted interval.

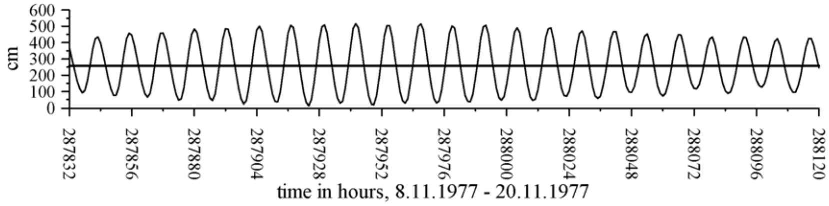

Fig. 4. Predicted tidal curve by a VAV analysis with a constant drift, i.e. constant mean sea level (the thick line).

through the analysis of the data in whatever interval, preceding the prediction interval. Here, only two variants of the option (iii) are shown. Let us consider the prediction of existing data, displayed in Fig. 3.

The predicted curve in Fig. 4 is obtained by the analysis of one year interval from 0 h, 1/11/1976, to 0 h, 1/11/1977. The differences with Fig. 3 are due to the fact that the non-tidal variations of the water level are not predicted.

Fig. 5 shows a prediction, which can be called “prediction in real time”. The data used for the prediction are with the same start as above, i.e. at 0 h, 1/11/1976. Unlike the case exhibited in Fig. 4, the last time of the interval is just 1 h before the predicted value. For the prediction of the tides at 1 h, 8/11/1977, we use the real data till 0 h, 8/11/1977, then, for the prediction of the tides at 2 h, 8/11/1977, we use the real data till 1 h, 8/11/1977, etc., till the whole predicted interval is covered. In this way we are able to predict the drift 1 h after the last observed value, by using the estimated drift polynomial till this time, together with the tidal signal. The comparison with Fig. 3 shows that we get a much more reliable prediction, including the prediction of the non-tidal variation of the water level.

6. Analysis of the long-term variations in connection with the mean sea level

In connection with the mean sea level VAV can approximate the non-tidal components as a part of the tidal analysis. The approximation is made by an annual component $A(t)$, composed by frequencies 1, 2, . . ., K_A cycles/year (cpy) and polynomials $P(t)$ of the time t of power K_P. It is possible to use a single polynomial over the whole data interval (Fig. 6). It is clear that the tide gauge behavior was abnormal at the beginning of the records. Another option is to find points of discontinuities, which partition the data in a number of segments and in which different polynomials $P(t)$ are used. Figs. 7 and 8 are given to illustrate the flexibility of VAV. The “potential” discontinuities should be cross-checked against the tide gauge log book.

In the examples, demonstrated by Figs. 6–8, the order of the annual component is always $K_A = 2$. The gray filled curve is the drift, i.e. the observed non-tidal variations of the sea level, the thick black line represents the approximation by $A(t) + P(t)$ and the white transparent line is the curve of $P(t)$, which is actually the estimated mean sea level.

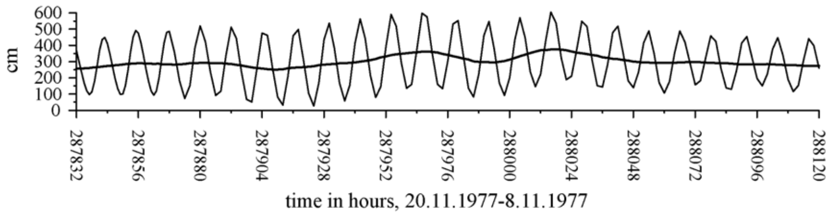

Fig. 5. Predicted data and predicted drift (thick line), hour by hour, by the option: “prediction in real time”.

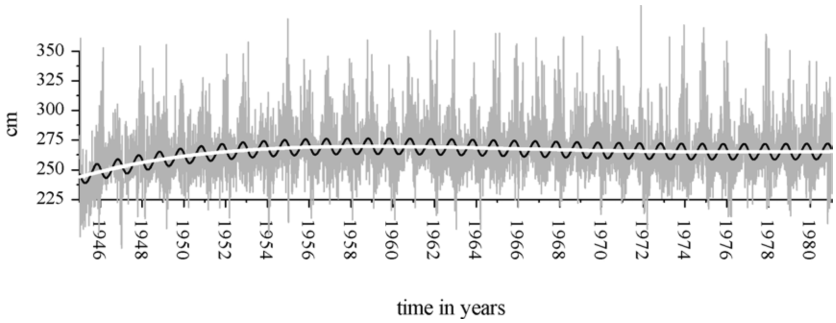

Fig. 6. Approximation of the non-tidal variations of the sea level by an annual modulation $A(t)$ and a single polynomial $P(t)$ of power $K_P = 4$ in a single segment, without discontinuities.

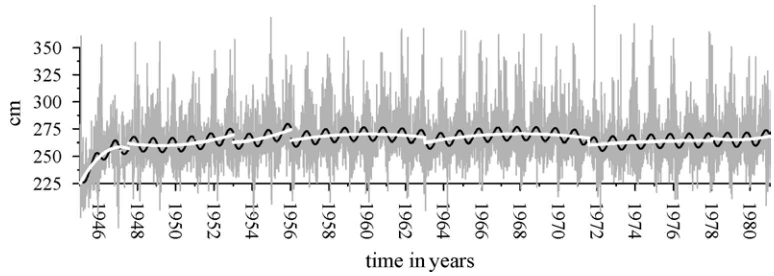

Fig. 7. Approximation of the non-tidal variations of the sea level by $A(t)$ and $P(t)$ with power $K_P = 2$ and 6 points of discontinuities.

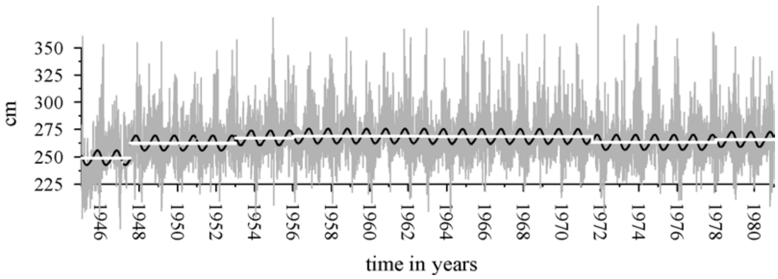

Fig. 8. Approximation of the non-tidal variations by $A(t)$ and $P(t)$ with power $K_P = 0$, i.e. by a stepwise function with six points of discontinuities.

For $K_P = 0$, $P(t)$ represents the global mean sea level. However, the significant variations of the sea level show that finding the global mean sea level is not an elementary problem. It needs a very careful choice of the interval of data through which it will be defined.

7. Conclusions

The application of VAV on the long OT series of Ostend shows that VAV can be useful for oceanography. We have obviously a successful analysis, estimating the amplitudes and the phases (as they are defined in oceanography) of the main tidal constituents. In parallel with the waves derived from the luni-solar tidal potential it is possible to estimate the resonant ShWT's. Very important in the determination of both the luni-solar tides and the ShWT's is the estimation of the precision, allowing to check the significance of the amplitudes.

We believe that the prediction problem is also well solved by VAV. Of course, the idea of the "prediction in real time" is not yet possible, due to the fact that the data become available with a considerable time delay. Nevertheless, it is not impossible in a near future to reorganize the procedures, in order to be able in real time to predict not only the tides, but also the fast changes of the sea level, important in the circumstances of disastrous meteorological conditions. Finally, we also believe that the way of computing the mean sea level may be useful, for instance, for some fundamental geodetic tasks.

It should be noticed that there are options of VAV, which may be helpful but which are not demonstrated here. Such are the options of automatic search of anomalies, study of the time variations of the tidal parameters and so on.

Acknowledgments

The work of A. Venedikov in the Institute of Astronomy and Geodesy in Madrid on the paper during December 2002–December 2003 has been supported by the State Secretariat for Education and Universities at the Spanish Ministry of Education and Culture for which we express our thankfulness to the Spanish Authorities. His stay in Brussels for the completion of this paper has been supported by the Royal Observatory of Belgium and the International Centre for Earth Tides. The author J. Arnoso has been supported by program I3P of European Social Fund. Projects REN2002-00544/RIES and REN2001-2271/RIES of Spanish CICYT have partially supported this work. The authors are very thankful to Prof. J.B. Merriam, one anonymous referee and Prof. T. Baker for their valuable suggestions that helped in improving the manuscript.

References

Doodson, A.T., 1922. The harmonic development of the tide generating potential. Proc. R. Soc. Lond. A100, 305–329.

Godin, G., 1972. The Analysis of Tides. Liverpool University Press, 263 pp.

Melchior, P., 1983. The Tides of the Planet Earth, second ed. Pergamon Press, Oxford, 609 pp.

Melchior, P., Pâquet, P., Van Cauwenberghe C., 1967. Analyse harmonique de vingt années d'enregistrements de marées océaniques à Ostende. Observatoire Royal de Belgique, Communications, Série B, 20, 8 pp. Bull. Classe des Sciences, Acad. Royale de Belgique LIII (2) 123–130.

Merriam, J.B., 2000. The response method applied to the analysis of superconducting gravimeter data. Phys. Earth Planet. Interiors 121, 289–299.

Munk, W.H., Cartwright, D.F., 1966. Tidal spectroscopy and prediction. Philos. Trans. R. Soc. Lond. A259 (1105), 533–581.

Tamura, Y., 1987. An harmonic development of the tide-generating potential. Bulletin d'Informations des Marées Terrestres 99, 6813–6855.

Venedikov, A.P., 1966. Une méthode d'analyse des marées terrestres à partir d'enregistrements de longueurs arbitraires. Academie R. de Belgique, Bull. Cl. Sci. LII (3), 463–485.

Venedikov, A.P., Arnoso, J., Vieira, R., 2001. Program VAV/2000 for tidal analysis of unevenly spaced data with irregular drift and colored noise. J. Geodetic Soc. Jpn. 47 (1), 281–286.

Venedikov, A.P., Arnoso, J., Vieira, R., 2003. VAV: a program for tidal data processing. Comput. Geosci. 29, 487–502.

Venedikov, A.P., Arnoso, J., Vieira, R., 2005. New version of the program VAV for tidal data processing. Comput. Geosci. 31, 667–669.

Yaramanci, Y., 1978. The principles of the response method. Bulletin d'Informations des. Marées Terrestres 78, 4649–4656.