Bulletin of the Seismological Society of America. Vol. 57, No. 1, pp. 55–81. February, 1967

SHORT-PERIOD SEISMIC NOISE

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ABSTRACT

This report consists of a summary of the studies conducted on the subject of short-period (6.0–0.3 sec period) noise over a period of approximately three years. Information from deep-hole and surface arrays was used in an attempt to determine the types of waves of which the noise is composed. The theoretical behavior of higher-mode Rayleigh waves and of body waves as measured by surface and deep-hole arrays is described. Both surface and body waves are shown to exist in the noise. Surface waves generally predominate at the longer periods (of the period range discussed) while body waves appear at the shorter periods at quiet sites. Not all the data could be interpreted to define the wave types present.

Introduction

Previous studies of short-period seismic noise have often assumed that only surface waves were present in short-period noise. However, the experimental results obtained to date cannot, in general, be explained in this fashion. The presence of random body-wave noise must also be taken into consideration. Recently, several reports have been published (Roden, 1965, Seriff *et al*, 1965) in which the problem of body waves in the noise is discussed.

Seismometers placed at depth below the surface allow examination of the amplitude-depth relationships of the waves. If only surface waves, fundamental and higher modes, are present in the noise, the amplitude-depth relationships provide definite identification of the modes present (Douze, 1964). It is only necessary that the number of seismometers operating at depth be equal to the number of modes present (further explained under deep-hole theory). When body waves are present in the noise, the identification of wave types is no longer so simple, because all the possible angles of incidence of the random body waves must be taken into account.

In general, the amplitude-depth relationships obtained from deep-hole seismographs are not sufficient to differentiate between body waves and surface waves. As an example, the amplitude-depth relationships of the vertical component of the first higher mode and of P waves at close to vertical incidence are very similar for periods around 3.0 sec. Therefore, additional information must be obtained to differentiate between the possibilities. In this report, the information was obtained by measuring phase velocities at the Wichita Mountains Seismological Observatory. In addition, the cross-correlation of surface and deep-hole noise was used to prove the presence of body-wave noise. Using all these data, a reasonably comprehensive understanding of the types of waves present was obtained. The results could only be interpreted qualitatively and not quantitatively. Despite the large amount of data available, the types of waves present could not always be identified. A possible explanation of the experimental results is given, while sufficient data are presented so that the reader can draw his own conclusion.

The results described were obtained with a deep-hole vertical-motion seismometer. The seismograph system has an amplitude and phase response similar to the short-period vertical Benioff seismograph (Benioff, 1932). The only surface waves considered are Rayleigh waves since all measurements were made with vertical seismometers. The period range discussed extends from the 6.0 sec microseisms to noise of 0.3 sec period.

The theoretical Rayleigh wave group velocities, phase velocities, and amplitude-depth relationships were obtained from the Seismic Data Laboratory in Alexandria,

TABLE 1
SITE DESCRIPTIONS

Site	Location	Depth (m)	Geologic section
Apache, Oklahoma	34°49′59″N 98°26′09″W	2917	Consists of 1500 m of high-velocity limestones (6000 m/sec) overlying volcanics of somewhat lower velocities (5500 m/sec)
Eureka, Nevada	39°12′32″N 115°42′37″W	3130	The top 1100 m is made up of sandstones, shales, thinly bedded limestones, and dolomites overlying higher-velocity limestones and dolomites
Fort Stockton, Texas	30°54′06″N 102°41′52″W	5792	Upper 3700 m consists of low-velocity shales, sand, and limestones overlying higher-velocity limestones and dolomites
Franklin, W. Virginia	38°33′02″N 79°30′47″W	3815	The upper 3060 m consists of high-velocity limestones and dolomites; below a major thrust fault at 3060 m, the section has lower velocities although the formations are repeated
Grapevine, Texas	32°53′09″N 96°59′54″W	3118	The top section consists of shales. The hole bottoms in a limestone; velocity increases steadily with depth
Pinedale, Wyoming	42°27′24″N 109°33′04″W	3022	Consists entirely of sandy shales; velocity increases slowly with depth
Wichita Mountains Seismological Observatory	34°43′05″N 98°35′21″W		Located on the same volcanics as encountered in the Apache deep hole. Measured surface velocities vary between 3000 and 4000 m/sec

Virginia. Data from both deep-hole arrays and surface arrays were used in this study. A brief description of the sites is given in Table 1.

The following sites are of particular interest: (a) Fort Stockton, Texas, because of the great depth (5790 m) of the deep hole; (b) Eureka, Nevada, because of the very low noise level; (c) Apache, Oklahoma, because the velocity section is a close approximation of a half space and because it is close (20 km) to the Wichita Mountains Seismological Observatory (WMSO) where phase velocities were measured. The site at Apache was of additional interest because the noise spectrum was similar to that at WMSO. Therefore, both the phase velocities and amplitude-depth relationship can be used. It must be noted that all the information presented was obtained from sites at some distance from the coast. Sites close to the coast typically exhibit large amplitudes at periods around 1.0 sec (Douze, 1964). There are not sufficient experimental results available to determine the waves responsible.

Theory

This section derives the theory necessary to interpret the experimental results. The theory for both body waves and Rayleigh waves will be presented. For the deep-hole measurements, the amplitude-depth relationships need to be considered, while for the surface array measurements,the phase velocities are of interest. In both cases, the phase angles and coherences yield valuable information.

The main tool in the interpretation are the spectra and cross spectra of the noise of different seismometers placed in either a horizontal or vertical plane. Therefore, the theory will be concerned with the results obtained by spectral analysis techniques. The theory of the behavior of body waves in the deep hole and across arrays is, in general, confined to the results that would be obtained in a half space. For the case of a layered media, the amplitude-depth relationships for P waves at vertical incidence has been solved (Gupta, 1965). The reason for not extending the theory to the layered case is that a convenient matrix method does not appear to be applicable when each term is an integral, as is the case when spectra are considered. In the case of fundamental and higher-mode Rayleigh waves recorded by the deep-hole seismometer, the azimuth of approach is not important because the amplitude-depth relationships are the values measured. However, in the case of body waves, the angle of incidence must be considered.

First, the theoretical amplitude-depth relationships that will be obtained from a mixture of Rayleigh modes is discussed. Then, the theory of body-wave noise at random angles of incidence is discussed in relation to the results that would be obtained from deep-hole measurements. The assumption of no P- to S-wave conversion at the free surface is made to show the mathematical procedure followed; the solution can, in this case, be obtained in closed form. Then, the formulas (not in closed form) that include P- to S-wave conversion at the free surface are derived. Next shown are the results that would be obtained in the deep hole if a mixture of Rayleigh waves were present. The results obtained from cross spectra of the vertical seismometers of a surface array are discussed for the case of surface waves and body waves.

Rayleigh Waves. The theory of higher Rayleigh modes has been extensively discussed in the literature (Ewing *et al*, 1957), and the presence of higher Rayleigh modes in earthquake surface waves has been established (Oliver and Ewing, 1958) It has previously been suggested that Rayleigh waves, both fundamental and higher modes, are responsible for seismic noise (Gutenberg, 1958).

The data used in calculating the theoretical change in amplitude with depth of the different Rayleigh modes were obtained as follows. The compressional-wave velocities were obtained from sonic logs; the shear-wave velocities were calculated from the compressional-wave velocities by assuming an appropriate Poisson's ratio (usually around 0.27); and densities were used appropriate to the lithologies encountered in the hole.

No attempt was made to model the whole crust, but the velocities were increased with depth (on the model of an average crust) sufficiently to obtain the higher modes in the desired period ranges. The oscillatory nature of the higher-mode group velocities is a basic property of even simple structures, but is accentuated when a low-velocity channel, such as a sedimentary section, is present. The maxima and minima of the group velocity curves are of interest, because they produce large amplitude arrivals from earthquakes and may be associated with peaks in the noise spectra (Gutenberg, 1958). An example of the results obtained for the change with depth of the displacements of the different Rayleigh modes (at FO-TX) is shown in Figure 1. The fundamental mode displacement decreases monotonically with depth with only slight inflections at discontinuities. Large displacements are present at depth when a lobe of a higher mode occurs in a low-velocity zone which traps a large percentage of the total energy of the wave. Low-velocity zones are present at all the holes studied. At all the sites, except AP-OK, the low-velocity zone is caused by the sediments, and at AP-OK, by the volcanics below the high-velocity limestone.

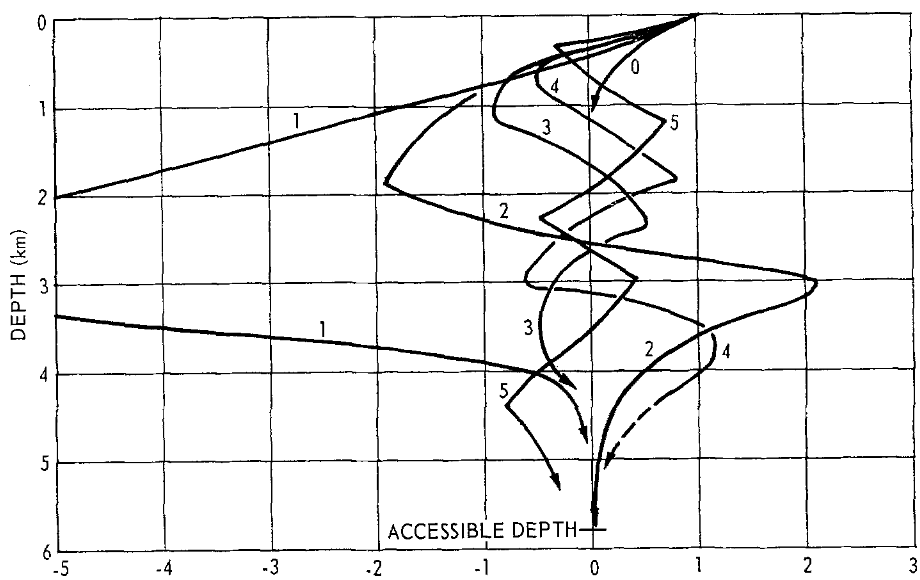

FIG. 1. Rayleigh and higher-mode amplitude (normalized to amplitude at earth's surface) as a function of depth at FO-TX. Period = 0.5 sec.

Deep-Hole Theory. If the noise is assumed to consist of a mixture of Rayleigh modes that are assumed to be uncorrelated, the results of spectral analyses can be explained in the following manner. Subscript 1 will refer to the surface and subscript 2 to the deep hole.

$$X_1(t) = \sum_{n=1}^{N} X_n(t)$$

where N is the number of Rayleigh modes present in the noise. The spectrum is obtained by first autocorrelating and then taking the Fourier transform of the autocorrelation. The result is

$$\varphi_{11}(\omega) = \sum_{n=1}^{N} \varphi_n(\omega). \tag{1}$$

The spectrum of the noise recorded by the deep-hole seismograph is related to the surface noise by a transfer function $H_n(\omega) \exp(i\delta_n)$,

$$\varphi_{22}(\omega) = \sum_{n=1}^{N} | H_n(\omega)|^2 \varphi_n(\omega). \tag{2}$$

The transfer function is the theoretical change of the displacements of the Rayleigh modes with depth. According to theory, the angle δ is always 0 or 180 deg (see Figure 1); therefore, the absolute value signs are not strictly necessary in the equation.

The cross spectrum between a surface and a deep-hole noise sample is given by

$$\varphi_{12}(\omega) = \sum_{1}^{N} H_n(\omega) e^{i\delta_n} \varphi_n(\omega). \tag{3}$$

Examination of equation 3 indicates that the cross spectrum will be a real quantity if only Rayleigh waves are present, because the angle δ_n is either 0 or 180 deg. A negative cross spectrum indicates that the "cross-power" in the Rayleigh modes, 180 deg out-of-phase at depth, is larger than the cross-power of the Rayleigh modes in phase at depth. The coherence is defined as

$$\mathrm{Coh} = \frac{|\varphi_{12}|^2}{\varphi_{11} \cdot \varphi_{22}}.$$

Examination of the equations indicates that the coherence is unity at all depths if only one Rayleigh mode is present, and will always be less than unity for a mixture of modes. The coherence will be zero when equal amounts of cross-power are in phase and 180 deg out-of-phase.

The behavior of seismic noise in the frequency range between 0.5 and 5.0 cps as a function of depth suggests the possibility of body-wave noise at random angles of incidence. First, the equations for body waves at random angles of incidence are solved under somewhat restrictive assumptions that allow a solution in closed form to be obtained. The more general solution can only be solved by numerical integration.

The following assumptions are made:

(a) P-wave noise arriving independently from all angles of incidence with equal energy content;

(b) No conversion from P to S waves at the free surface;

(c) An isotropic, homogeneous half space.

The equations are derived for the noise as it would be detected by a vertical-motion seismograph. The particle displacement at the surface is taken to the

$$X_1(t) = \sum_{n=1}^{N} f_n(t) \cos\theta_n$$

where θ is the angle of incidence measured from the vertical. The autocorrelation of each independent time series $f_n(t)$ is denoted by $1/N\psi(\tau)$. The autocorrelation of $X_1(t)$ then becomes

$$\psi_{11}(\tau) = \frac{1}{N} \sum_{n=1}^{N} \psi_n(\tau) \cos^2 \theta_n .$$

By taking the Fourier transform, the power spectrum is obtained

$$\varphi_{11}(\omega) = \frac{1}{N} \sum_{n=1}^{N} \varphi_n(\omega) \cos^2 \theta_n .$$

Now letting $N \rightarrow \infty$, the power spectrum of the surface noise becomes

$$\varphi_{11}(\omega) = \varphi(\omega) \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos^2 \theta \, d\theta = \frac{\varphi(\omega)}{2} . \tag{4}$$

The spectrum of the noise at any depth is obtained as follows:

$$X_2(t) = \sum_{n=1}^{N} \left\{ \left[\frac{1}{2} f_n \left(t - \frac{\alpha}{2} \cos \theta_n \right) \right] + \left[\frac{1}{2} f_n \left(t + \frac{\alpha}{2} \cos \theta_n \right) \right] \right\} \cos \theta_n$$

where $\alpha/2$ is the vertical uphole time.

Going through the same procedure as for the surface, we obtain the spectrum at depth

$$\begin{aligned} \varphi_{22}(\omega) &= \varphi(\omega) \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \{ \cos^2 \theta + \cos (\omega\alpha \cos \theta) \cos^2 \theta \} \, d\theta \\ &= \varphi(\omega) \left\{ \frac{1}{4} + \frac{J_0(\alpha\omega)}{2} - \frac{J_1(\alpha\omega)}{2\alpha\omega} \right\} . \end{aligned} \tag{5}$$

Note that the expression in brackets approaches $\frac{1}{2}$ as $\omega \rightarrow 0$ because $J_1(\alpha\omega)/\alpha\omega \rightarrow 0.5$. The ratio of the deep-hole spectrum divided by the surface spectrum often used in the interpretation then becomes

$$R = \tfrac{1}{2} + J_0(\omega\alpha) - J_1(\omega\alpha)/(\alpha\omega)$$

which, as expected, approaches unity for very low frequencies.

The cross-spectra between the surface and the deep-hole spectra are obtained in the same way

$$\varphi_{12}(\omega) = \varphi(\omega) \left\{ J_0 \left(\frac{\alpha\omega}{2} \right) - \frac{J_1 \left(\dfrac{\alpha\omega}{2} \right)}{\dfrac{\alpha\omega}{2}} \right\} . \tag{6}$$

Notice that the cross spectrum is either positive or negative, but does not have an imaginary component, indicating that the phase changes from 0 to 180 deg. The coherence between the surface and the deep-hole noise becomes

$$\mathrm{Coh} = \frac{\left| J_0\left(\frac{\alpha\omega}{2}\right) - \frac{J_1\left(\frac{\alpha\omega}{2}\right)}{\alpha\omega/2} \right|^2}{\frac{1}{8} + \frac{J_0(\alpha\omega)}{4} - \frac{J_1(\alpha\omega)}{4\alpha\omega}}$$

which approaches unity as $\omega \rightarrow 0$.

The theory for S waves at random angles of incidence is similar. Define $\beta/2$ as the uphole time for S waves. The equations become:

$$\varphi_{11}(\omega) = \frac{1}{2}\varphi(\omega)$$

$$\varphi_{22}(\omega) = \varphi(\omega)\left\{\frac{1}{4} + \frac{J_1(\beta\omega)}{2\beta\omega}\right\}$$

$$\varphi_{12}(\omega) = \varphi(\omega)\,\frac{J_1\left(\frac{\beta\omega}{2}\right)}{\frac{\beta\omega}{2}}.$$

In the previous example, the conversion of P to S waves at the free surface was neglected. Taking the conversions into account and keeping the rest of the assumptions made before, a more realistic solution can can be obtained; however, the solution could not be obtained in closed form and numerical integration of the integrals is necessary. In this case, the time series at the surface and at depth become

$$X_1(t) = \sum_{n=1}^{N} f_n(t)\cos\theta_n + b(\theta)f_n(t)\cos\theta_n + c(\theta)f_n(t)\sin\xi_n$$

$$X_2(t) = \sum_{n=1}^{N} f_n(t - \alpha\cos\theta_n)\cos\theta_n + b(\theta)f_n(t + \alpha\cos\theta_n)\cos\theta_n + c(\theta)f_n(t + \beta\cos\xi_n)\sin\xi_n$$

where b and c are reflection coefficients for P and S waves from an incoming P wave. These coefficients are functions of the angle of incidence and can be found in the literature (Ewing et al, 1957). S waves are reflected at an angle ξ, which is connected with θ by Snell's Law.

The procedure followed in obtaining the required integrals is the same as that employed previously. Because of the large number of terms involved in the derivation, only the resulting integrals will be given.

$$\varphi_{11}(\omega) = \varphi(\omega)\frac{1}{2\theta}\int_{-\theta}^{\theta} [\cos^2\theta + b^2\cos^2\theta + c^2\sin^2\xi + 2b\cos^2\theta + 2c\cos\theta\sin\xi + 2bc\cos\theta\sin\xi]\,d\theta \tag{7}$$

$$
\begin{aligned}
\varphi_{22}(\omega) = \varphi(\omega) \frac{1}{2\theta} \int_{-\theta}^{\theta} \Big[& \cos^2 \theta + b \cos^2 \theta + c^2 \sin^2 \xi + 2b \cos (\omega\alpha \cos \theta) \cos^2 \theta \\
& + 2c \cos \omega \left(\frac{\alpha}{2} \cos \theta + \frac{\beta}{2} \cos \xi \right) \cos \theta \sin \xi \\
& + 2bc \cos \omega \left(\frac{\beta}{2} \cos \xi - \frac{\alpha}{2} \cos \theta \right) \cos \theta \sin \xi \Big] \, d\theta
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\varphi_{12}(\omega) = \varphi(\omega) \frac{1}{2\theta} \int_{-\theta}^{\theta} \Big[& \cos \left(\omega \frac{\alpha}{2} \cos \theta \right) \cos^2 \theta + b^2 \cos \left(\omega \frac{\alpha}{2} \cos \theta \right) \cos^2 \theta \\
& + c^2 \cos \left(\omega \frac{\beta}{2} \cos \xi \right) \sin^2 \xi + 2b \cos \left(\omega \frac{\alpha}{2} \cos \theta \right) \cos^2 \theta \\
& + ic \left\{ \sin \left(\omega \frac{\alpha}{2} \cos \theta \right) - \sin \left(\omega \frac{\beta}{2} \cos \xi \right) \right\} \cos \theta \sin \xi \\
& + ibc \left\{ \sin \left(\omega \frac{\alpha}{2} \cos \theta \right) + \sin \left(\omega \frac{\beta}{2} \cos \xi \right) \right\} \cos \theta \sin \xi \Big] \, d\theta
\end{aligned} \tag{9}
$$

Examination of these formulas indicates that if the numerical integration is carried out between even limits, a number of the terms disappear because they are odd functions. As expected, the spectra are real quantities; however, the cross spectrum is a complex quantity, so that the phase is no longer either 0 or 180 deg but attains intermediate values. In cases of practical interest the phase angles are so close to 0 or 180 deg that spectral analyses are not sufficiently accurate to detect the difference.

Body waves at vertical incidence result on a standing wave pattern. Consequently, the phase angle at depth can only be 0 or 180 deg. The cross spectrum given in equation (9) indicates that is no longer the case for body waves at angles other than the vertical. In general, over a narrow band of frequencies, the amplitude-depth relationships of P waves at close to vertical incidence are often similar to one of the Rayleigh-mode amplitude-depth relationships. For example, the third higher Rayleigh mode and P waves at close to vertical incidence are almost identical for a frequency of 2 cps at AP-OK.

Narrow-band filtering together with cross-correlation offers a possibility (although not a unique one) of distinguishing between surface waves and body waves. Because surface waves in the noise produce standing wave patterns the cross-correlation between noise samples from any depth will have a maximum at zero lag. The maximum will be positive or negative, depending on whether the surface waves at depth in phase or 180 deg out-of-phase with the surface. In the case of body waves the maximum in the cross-correlation can be at a lag equal to the travel time between seismometers under certain conditions. To illustrate this phenomenon consider a seismometer at depth and one at the surface and neglect the S wave reflection for simplicity. The cross-correlation is:

$$\psi_{12} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \{ f(t - \alpha) + f(t + \alpha) \} \cdot f(t + \tau) \cdot dt$$

where α is the uphole time for the appropriate angle of incidence.

The shape of the correlation will depend on $f(t)$; if $f(t)$ is a unit impulse, a cross-correlation with maxima at each side of $t = 0$ will be obtained. If $f(t)$ approximates a sine wave this behavior is no longer obtained. Averaging over a large number of angles incidence does not change the general conclusion. From the above discussion, it is apparent that if a cross-correlation is obtained, with a maximum at other than zero lag, body waves are present. However, a correlation with a maximum at zero lag can be either body waves or surface waves. It must be noted that the cross-correlation is still even (except for the small S wave contribution) and the cross spectra is real. If very narrow-band filtering were employed, i.e., examining essentially one frequency, it would be possible to interpact the results quantitatively. However, the cross-correlations over a finite band width used here can only be interpreted qualitatively to distinguish between surface waves and body waves.

If the origin of the body waves was at the surface close to a deep-hole site and the waves were traveling downwards, the cross correlation would not be even and the cross spectrum would show a linear change of phase with frequency. This behavior was not detected at any of the sites.

Surface Array Theory. In the case of surface waves arriving at the array from a given direction (or a given direction and angle of incidence in the case of body waves), the equations can be obtained easily. The spectrum of the noise at two seismometers (a and b) will be the same (ignoring seismometer-to-ground coupling problems):

$$\varphi_{aa} = \varphi_{bb} = \sum_{n=1}^{N} \varphi_n(\omega).$$

The cross spectra will be:

$$\varphi_{ab} = \sum_{n=1}^{N} \varphi_n(\omega) \exp\left[-i\omega\left(\frac{\Delta x}{v_n}\cos\beta\right)\right] \tag{10}$$

where

N = number of wave types present in the noise
Δx = distance between the seismometers
v = phase velocity of the waves
β = angle between the direction connecting the seismometers and the direction of arrival.

The cross spectra between two seismometers give only apparent phase velocities, and two cross spectra are necessary to obtain the real phase velocities and directions of arrival. The theoretical results indicate that coherence will be unity if only one wave type is present, and less than unity in all other cases. Experimental data from arrays usually indicates that the noise often appears to be omnidirectional (isotropic), i.e., arriving with approximately equal energy content from all directions. For the case of surface waves, the solution that would be obtained has been solved by Backus *et al* (1964). The spectrum of each seismometer is the same, $\varphi(\omega)$, and the cross spectra become

$$\varphi_{ab}(\omega) = \varphi(\omega) J_0\left(\frac{\omega\Delta x}{v}\right) \tag{11}$$

where the Δx and v are, respectively, separation between seismometers and phase velocity.

Because of the possible presence of body-wave noise, it is necessary to also consider the case of body waves from random directions and random angles of incidence. For P waves arriving at the surface seismometers with equal energy content from random directions and all angles of incidence, the time series for a vertical-motion seismometer a is

$$X_a(t) = \sum_1^N f_n(t) \cos \theta_n .$$

At seismometer b, the time series becomes

$$X_b(t) = \sum_1^N f_n \left(t - \frac{\Delta x}{v} \sin \theta_n \cos \beta_n\right) \cos \theta_n$$

where θ refers to the angle of incidence and β to the direction of the waves. Proceeding in exactly the same fashion as described in the previous section, the spectrum of each seismometer becomes

$$\varphi_{aa}(\omega) = \varphi_{bb}(\omega) = \tfrac{1}{2}\varphi(\omega)$$

and the cross spectrum becomes

$$\varphi_{ab}(\omega) = \varphi(\omega) \cdot \frac{1}{2\pi} \cdot \frac{1}{\pi} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \exp\left[-i\omega\left(\frac{\Delta x}{v} \sin \theta \cos \beta\right)\right] \cos^2 \theta \cdot d\beta\, d\theta.$$

By performing the last integration, the expression reduces to

$$\varphi_{ab}(\omega) = \varphi(\omega) \frac{1}{2\pi} \int_0^{2\pi} \frac{J_1\left(\dfrac{\omega \Delta x}{v} \cos \beta\right)}{\dfrac{\omega \Delta x}{v} \cos \beta} \cdot d\beta. \tag{12}$$

Notice that under the assumptions made, both formulas 11 and 12 have no imaginary parts and the phase angle is either 0 or 180 deg. Therefore, velocities can only be obtained from the coherence values.

Experimental Results

The methods used in spectral analysis are briefly discussed in order to acquaint the reader with the reliability of the results. In order to facilitate discussion of the experimental results, the passband of the short-period Benioff was divided somewhat arbitrarily into three period ranges: 5.0 to 2.0 sec, 2.0 to 0.8 sec, and 0.8 to 0.3 sec. These divisions were chosen partly for convenience, and partly because somewhat different wave types appear to predominate in the different period ranges.

The 6.0 sec microseisms are not discussed here, because the analyses indicated clearly that, as measured by a vertical-motion seismometer, the fundamental mode Rayleigh wave is the only wave type present. This conclusion has been previously reached by a number of authors (see for example, Gutenberg, 1958). The examples discussed in this section are only a small part of the large amount of information that lead the author to the stated conclusions.

Spectral Analysis. The principal tool used in the interpretation of the data consisted of obtaining spectra and cross spectra, and the associated auto- and cross-correlations of long-noise samples. The techniques used to obtain spectra, and the accuracy and resolution that is obtained have been extensively discussed in the literature (e.g., Blackman and Tukey, 1958).

The length of the noise sample used varied between 180 and 450 sec. As a compromise between accuracy and resolution, a lag of 8 per cent of the sample was usually used; however, either smaller or greater lags were sometimes employed to increase either the accuracy or the resolution of the results. A hanning smoothing function was used in all cases. In this section of the report, each figure will give the length of sample and the lag used, to allow the reader to determine the reliability of the results. The sampling rate used (usually 25 samples/sec) insured that the folding frequency was well outside of the frequency range of interest. Theoretical studies on the accuracy of cross spectra (e.g., Amos *et al*, 1963) indicate that the experimental coherences are a complex function of the actual coherence, the smoothing function and the lag window, and that considerable errors are to be expected when actual coherences are close to zero.

The magnifications at 1 cps were used to calibrate the power spectrum; therefore, only the values at 1 cps are correct ground motion values. Because of the identical responses of the seismographs used, the deep-hole-divided-by-surface ratios used in the interpretation are correct at all frequencies. The ratios are obtained by dividing the deep-hole noise spectrum by the surface spectrum, and will be called power ratios in the body of the report. The square root of the power ratio will be referred to as the amplitude ratio.

Microseisms, 5.0 to 2.0 Sec. The ratio of deep-hole-divided-by-surface noise spectra at all sites investigated indicates that fundamental mode Rayleigh waves is not the only wave present in the period range between 5.0 and 2.0 sec. Figure 2 shows the experimental power ratio, and Figure 3, the phase angle and coherence obtained from noise samples when the deep-hole seismometer was located at a depth of 5200 m at FO-TX. The theoretical curves for the first three Rayleigh modes and for P waves at vertical incidence are also shown. These analyses were made at a time when a storm in the Atlantic was the cause of large microseisms in the period range of 2.0 to 6.0 sec. The lowest value of the power ratio occurs at a period of 4.0 sec; however, the coherence is not zero until a period of 3.4 sec is reached. This behavior indicates that between 3.4 and 4.0 sec most of the power was in phase as is also shown by the phase angle (Figure 3); therefore, the fundamental mode, which is the only wave in phase, was larger than the other waves at these periods. To explain the power ratio, the rest of the energy must be in the first higher mode. Examination of the Figures (2 and 3) indicates that for periods less than 3.4 sec, only a small amount of fundamental mode Rayleigh waves can be present in the noise. The experimental values lie between the theoretical first higher Rayleigh mode and P-wave curves. The experimental data can be explained by the presence of the first higher mode mixed with either appreciable P-wave noise or a small amount of fundamental mode noise.

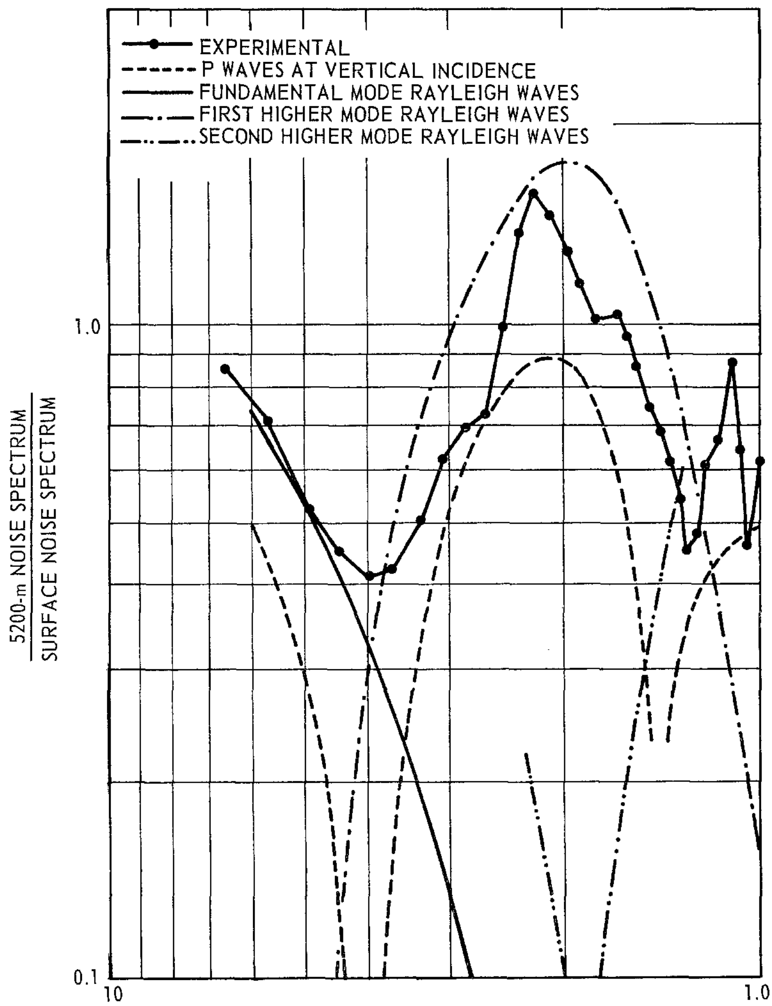

FIG. 2. Deep-hole (5200 m) vertical noise spectrum divided by surface noise spectrum. Theoretical amplitudes are included. FO-TX 300 sec sample, 10 samples/sec, 5 per cent lags.

The deep-hole results at FO-TX indicated that either P waves or first higher mode Rayleigh waves predominated in the noise but the results could not be used to conclusively distinguish between the two waves. In an attempt to distinguish between these two possibilities, information from WMSO was used from the same time as the FO-TX analyses. The amplitudes in the period range under discussion, increased at both WMSO and FO-TX at the same time indicating the same source and thus the same wave types. Cross spectra between noise samples from three seismometers located in a 3 km tripartite were obtained. The cross spectra of the

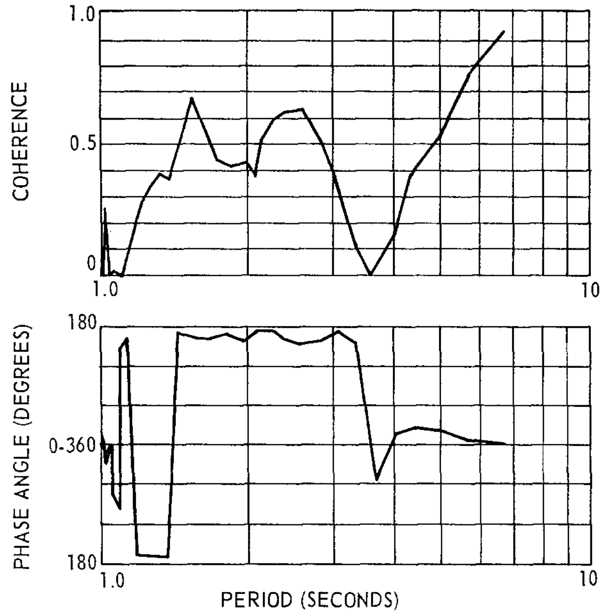

FIG. 3. Phase angle and coherence of the noise between the surface and 5200 m, FO-TX. 300 sec sample, 10 samples/sec, 5 per cent lags.

noise from two seismometers are sufficient to specify an apparent velocity. Two pairs of seismometers are sufficient to specify phase velocity and angle of arrival. The cross spectra between the noise samples from all three noise samples were used to check if consistent results were obtained from all combinations of pairs. The results obtained are given in Table 2. As indicated in the theoretical results, if the noise field is isotropic, the phase angle is 0 or 180 deg. The phase angles obtained

TABLE 2

PHASE VELOCITY MEASUREMENTS FROM CROSS SPECTRA, WMSO

Period (sec)	Azimuth (deg)	Velocity (km/sec)	Average* (coherence)^2	Spectral Amplitude ($m\mu^2$/cps)
6.667	213	2.85	.88	51.0
5.714	215	3.16	.88	91.1
5.000	219	3.49	.86	92.2
4.444	226	3.68	.83	67.0
4.000	229	3.63	.76	39.8
3.636	228	3.83	.70	26.3
3.333	228	4.26	.69	22.4
3.077	229	4.33	.64	23.6
2.857	226	4.41	.54	21.8
2.667	226	4.24	.46	18.5
2.500	228	4.51	.46	16.1
2.353	229	4.87	.41	15.1
2.222	231	4.23	.22	13.2
2.105	237	4.69	.23	10.6
2.000	235	5.70	.25	9.12

* A plot of one of the coherences is shown in Figure 8.

from the experimental results indicated clearly that part of the noise at periods greater than 2.0 sec was directional. The directions obtained indicated that the storm in the Atlantic was responsible for the directional part of the noise.

Examinations of combinations of directional and nondirectional noise (from equation 10, 11, and 12) indicates that the phase velocity obtained for periods greater than approximately 2.0 sec by these measurements is too large. This behavior is caused by the isotropic noise staying either in phase or 180 deg out-of-phase. In the period range under examination here, this causes the phase angle from the cross spectra to be intermediate between the actual value for the directional noise and 0 deg value of the omnidirectional noise. As an example, assume 50 per cent isotropic 3.0 sec period waves at 4 km/sec and 50 per cent directional waves at 4 km/sec arriving along the line connecting two seismometers 3 km apart; in the case, the phase angle will be 64 deg. If only the unidirectional waves were present, the phase angle would be 88 deg. The above presented argument indicates that the real phase velocities of the directional noise are less than those obtained from the cross spectra. Therefore, the phase velocities in Table 2 indicate that body waves are excluded for anything except very shallow angles of emergence. The choice, therefore, lies between fundamental and first higher mode Rayleigh waves; however, results from FO-TX during the same time indicate that only small amounts of fundamental mode Rayleigh waves were present in the noise at these periods. Therefore, the first higher mode Rayleigh waves must predominate.

The arguments given above are meant to show the existence of the first higher Rayleigh mode, and are not intended to prove that this mode predominates at all times and at all locations. However, phase angles from all deep-hole measurements always show the same behavior; the phase angle changes from 0 to 180 deg at a period that can be explained by *P* waves or higher modes. The fundamental mode never predominates at periods less than approximately 4.0 sec. It is of interest that when the first higher mode can be shown to predominate, a small high in the spectra usually appears at 3.0 sec (see Table 2). Particle motion diagrams of the 3.0 sec microseisms produced ambiguous results; almost all sizes and shapes of ellipses were obtained. This failure was possibly caused by the presence of different wave types in closely adjacent period ranges. The possible presence of Love waves in the noise could also contribute to the failure to obtain reproducible results.

At the sites where thick sections of low-velocity rock are present (usually shales), the results obtained are more difficult to interpret than at the sites where predominantly high-velocity rocks are present. Figure 4 shows an example of the results obtained at the Pinedale, Wyoming, site. The section at this location is composed entirely of shales. The results can probably be best explained by a combination of *P* waves and fundamental Rayleigh waves if the location of the nodal point is taken as the main criteria for interpretation. However, there is some doubt as to the validity of this interpretation. Figure 4 also shows the results obtained from spectral analysis of a surface wave of an earthquake from Baja California. The group velocity of these waves was about 3.2 km/sec, indicating that they are surface waves, probably higher mode Rayleigh waves as recorded by the vertical-motion seismographs. It is noticeable from the figure that the behavior of the amplitude-depth relationships of the noise and these surface waves is very similar, especially in regard to the location of the nodal point. This behavior suggests that the theoretical Rayleigh wave curves may be in error. The theoretical Rayleigh wave computer program used does not take into account the well-known velocity

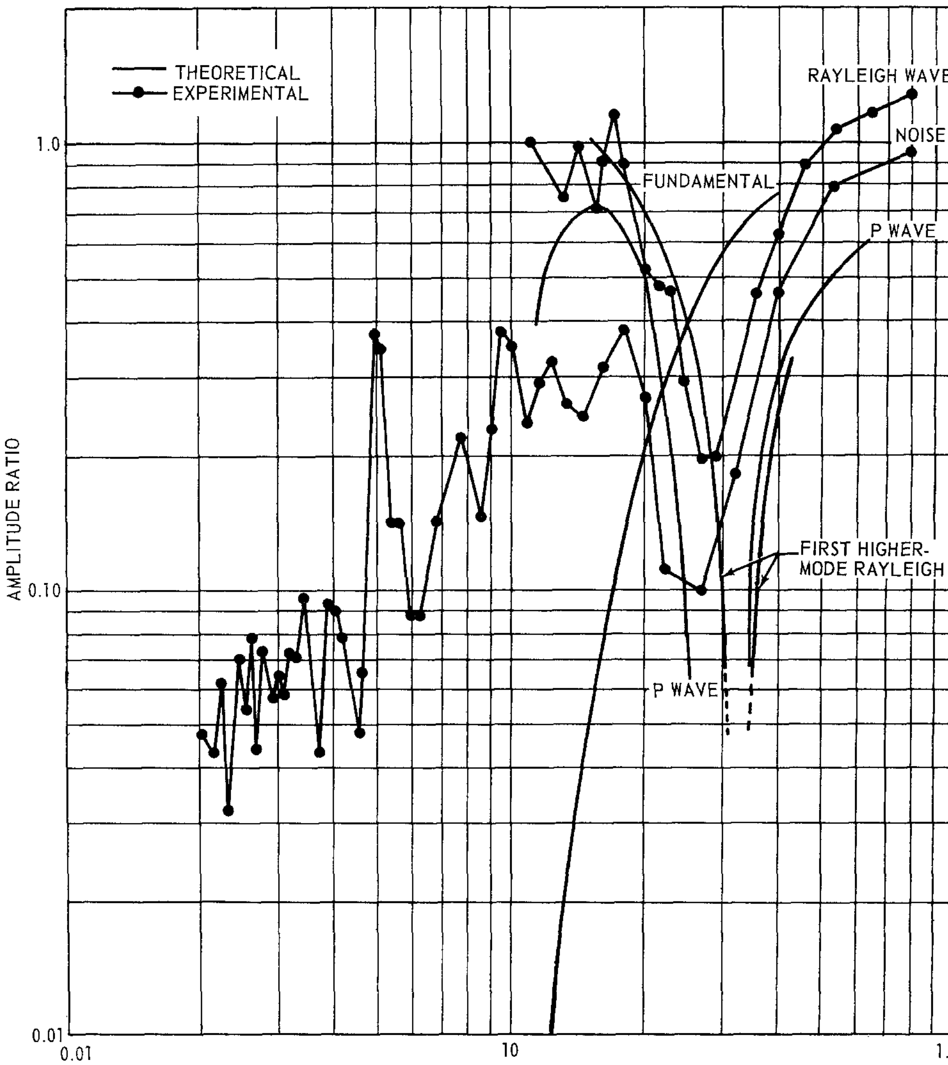

FIG. 4. Amplitude ratio of the noise and a Rayleigh mode from Baja California, as recorded at 3060 m at Pinedale, Wyoming. Theoretical curves for P waves and Rayleigh waves are included.

anisotropy of shales. Preliminary results from an anisotropic Rayleigh wave program suggest that the discrepancies between theoretical and experimental results can be explained in this way.

It is, of course, entirely possible that the noise does consist of fundamental mode Rayleigh waves and P waves, and that the first higher mode was not present at the time of the experiments. With the limited depth of the hole and with no surface array information, the problem cannot be solved. With holes of the usual depth of approximately 3000 m, it is not possible to distinguish between the two possibilities of P waves or first higher mode Rayleigh waves. Figure 5 shows the amplitude-depth relationship of the 2.0 sec noise at AP-OK. Either theoretical curve will explain the experimental data.

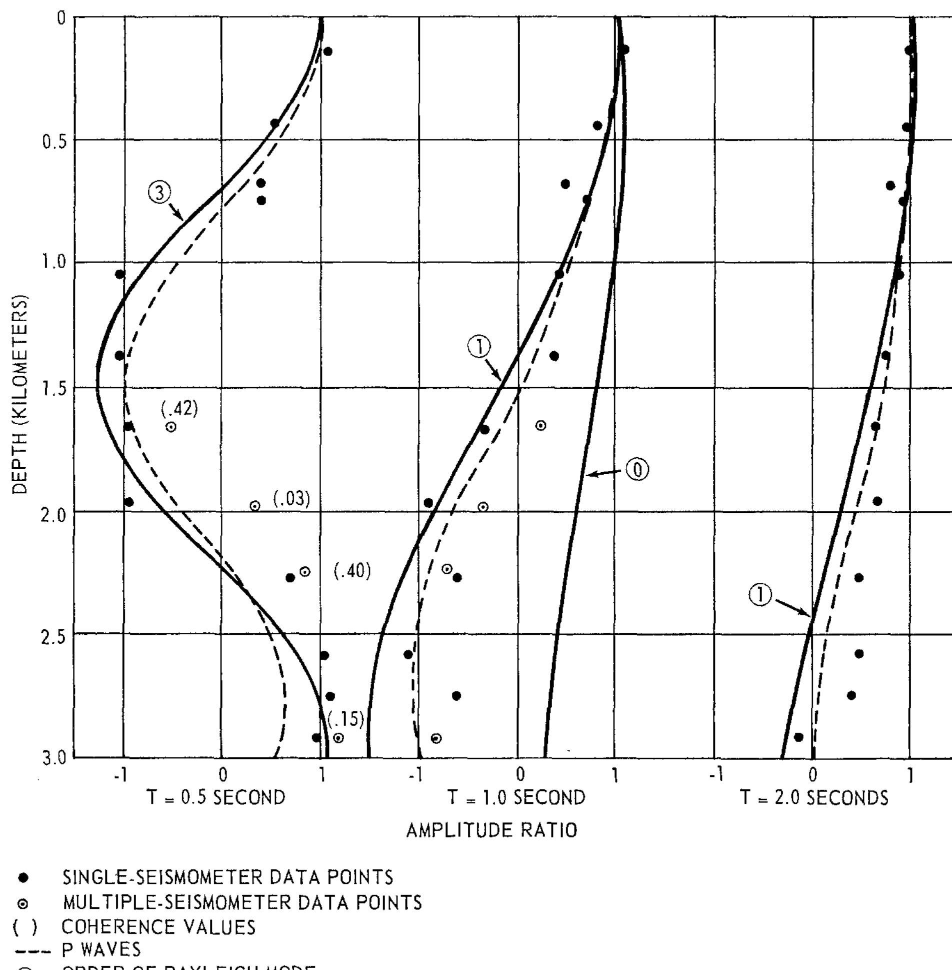

FIG. 5. Observed and theoretical amplitude ratio with depth for periods of 0.5, 1.0, and 2.0 sec. 180 sec sample, 25 samples/sec, 5 per cent lags.

Noise in the Period Range of 2.0 and 0.8 Sec. The noise in this period range predominates in the spectra at sites close to the coast; at quiet sites, distant from the coast, the noise at these periods has usually been attenuated to very small values.

Considerable difficulty has been encountered in interpreting the data at these periods. The amplitude-depth relationships agree quite well with the theoretically predicted P-wave amplitude-depth relationships. As an example, Figure 6 shows the

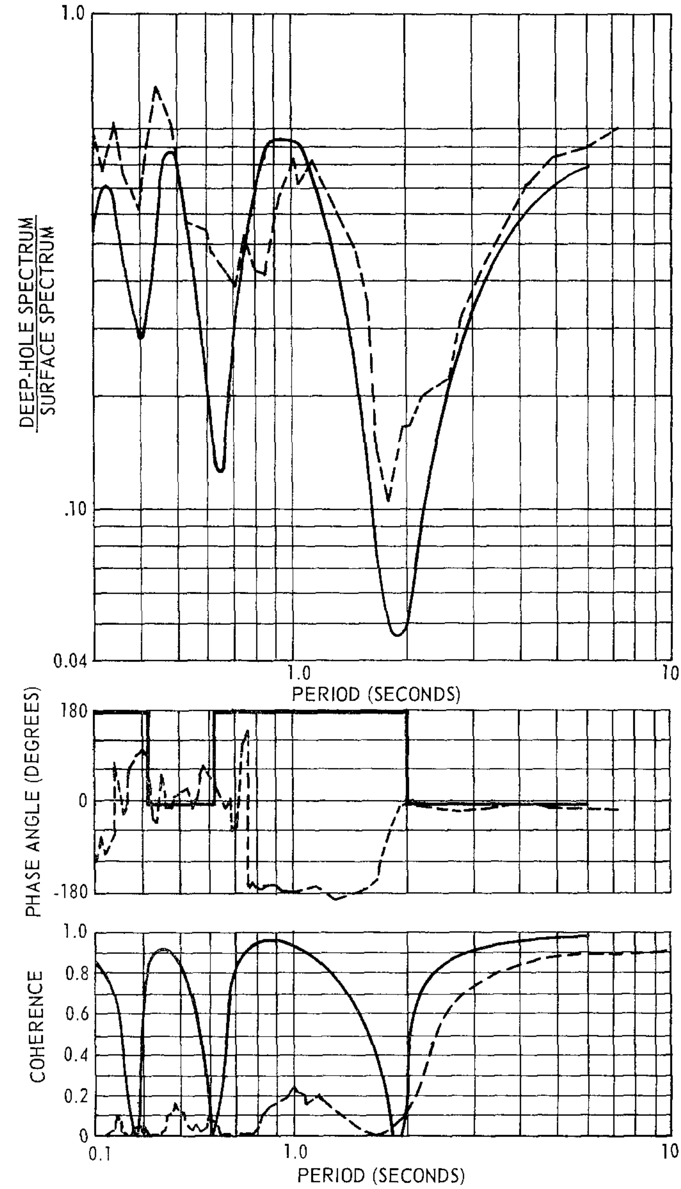

FIG. 6. Theoretical and experimental power ratios, phase angles, and coherences theory for P waves arriving randomly -45 to $+45$ deg from the vertical. Uphole 0.5 sec. 180 sec sample, 25 samples/sec, 8 per cent lags.

power ratio obtained at AP-OK from the noise at the surface and 2917 m. The theoretically predicted amplitude-depth relationship for P wave at random angles of incidence between -45 and $+45$ deg from the vertical, and the experimental and theoretical phase angles and coherences are also shown in the same figure. The agreement between theory and experiment is quite good, and could be improved even further by assuming the presence of some S-wave noise. Figure 5 shows

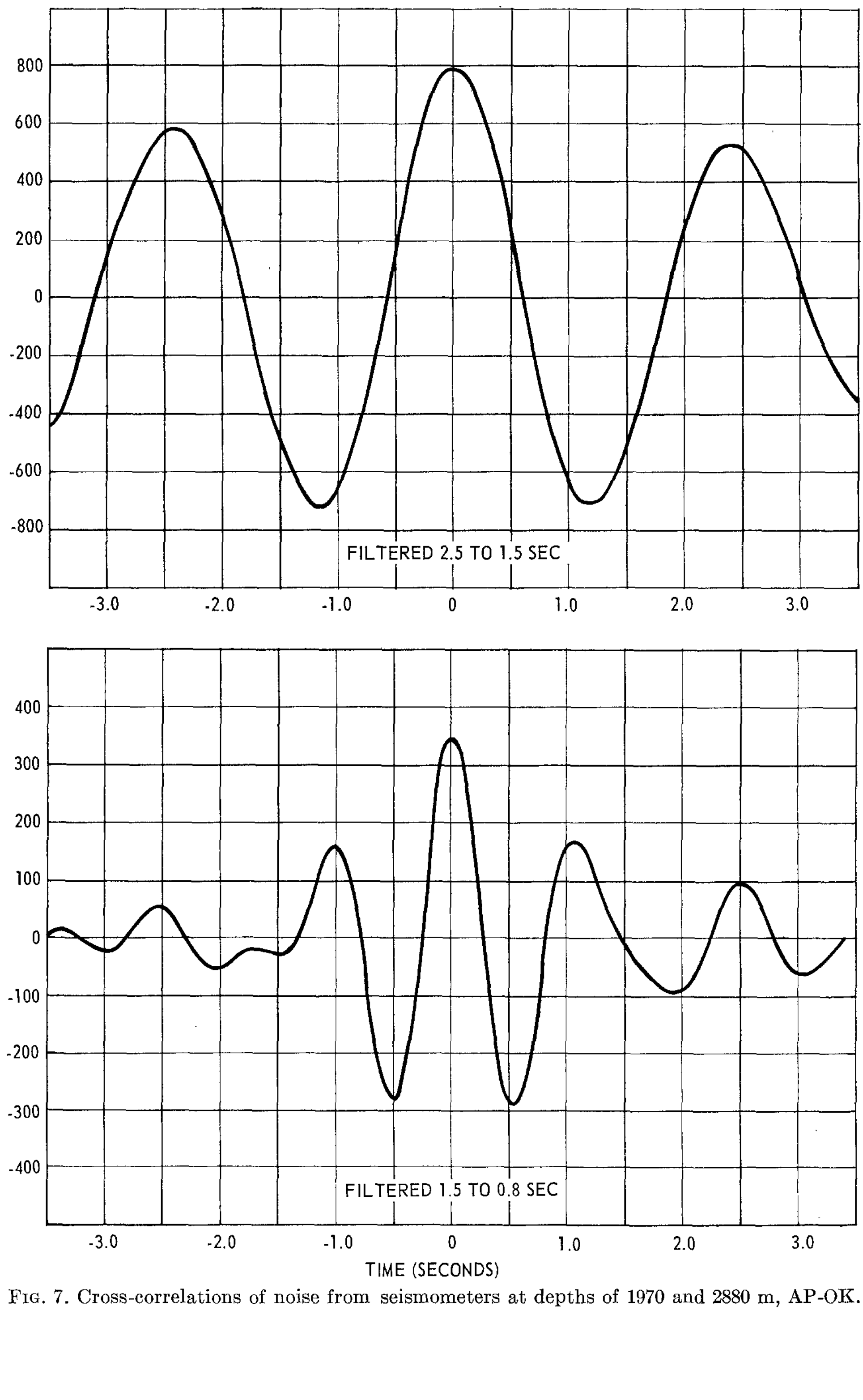

FIG. 7. Cross-correlations of noise from seismometers at depths of 1970 and 2880 m, AP-OK.

the amplitude-depth relationships for the 1.0 sec noise as measured at AP-OK; the theoretical curve for P waves at vertical incidence fits the experimental data quite well. However, the first higher mode theory is also quite close to the experimental data. There exists some doubt that P waves are the correct explanation of the noise at these periods. In an attempt to distinguish between the two possibilities (surface or body waves), cross-correlations were obtained after digital filtering. The digital filters had extremely sharp cut-offs, and only noise in the period range of interest passed through. Figure 7 shows the results obtained in the period ranges of 2.5 to 1.5 sec and 1.5 to 0.8 sec.

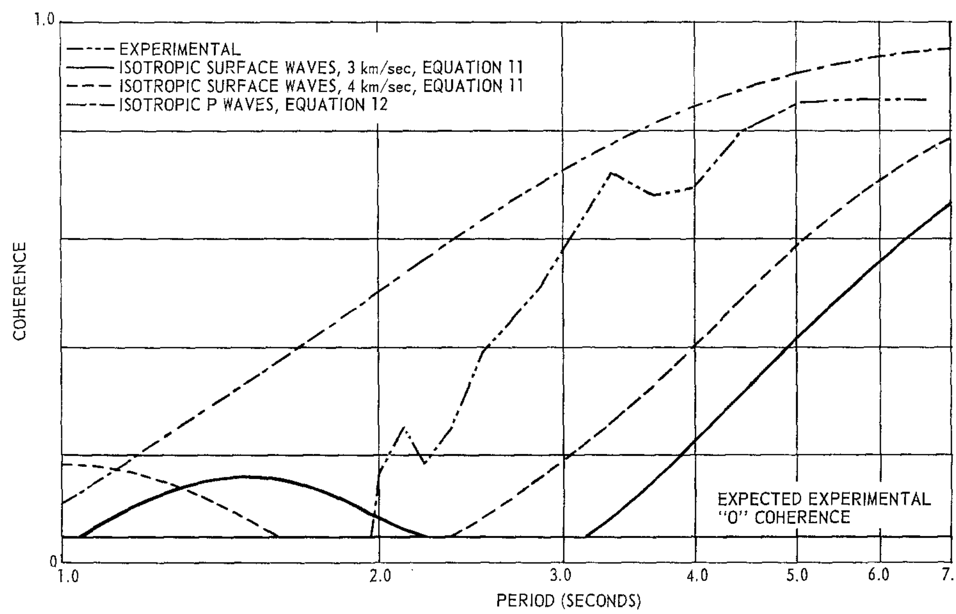

FIG. 8. Experimental and theoretical coherences of noise from seismometers 3 km apart, WMSO. 180 sec sample, 25 samples/sec, 8 per cent lags.

The experimental evidence shown in Figure 7 indicates that surface waves are responsible for the noise in these two period ranges. As mentioned in the section on deep-hole theory, P waves at close to vertical incidence can set up a standing wave pattern that will result in cross-correlations of the type shown in Figure 7. However, it appears unlikely that noise, the statistics of which indicate that it is a completely random phenomenon, will act in this fashion. It must be noted at this time that the cross-correlation of signals will result in the highest value at a lag equal to the uphole time. Furthermore, as will be shown in the next section, random P waves, when present, will give maximum in the cross-correlation at a lag other than zero.

Figure 8 shows the coherence between noise samples from seismometers 3 km apart at WMSO. The high coherences at periods greater than 2.0 sec were caused by directional noise. For periods between 1.0 and 2.0 sec, the phase angles indicated

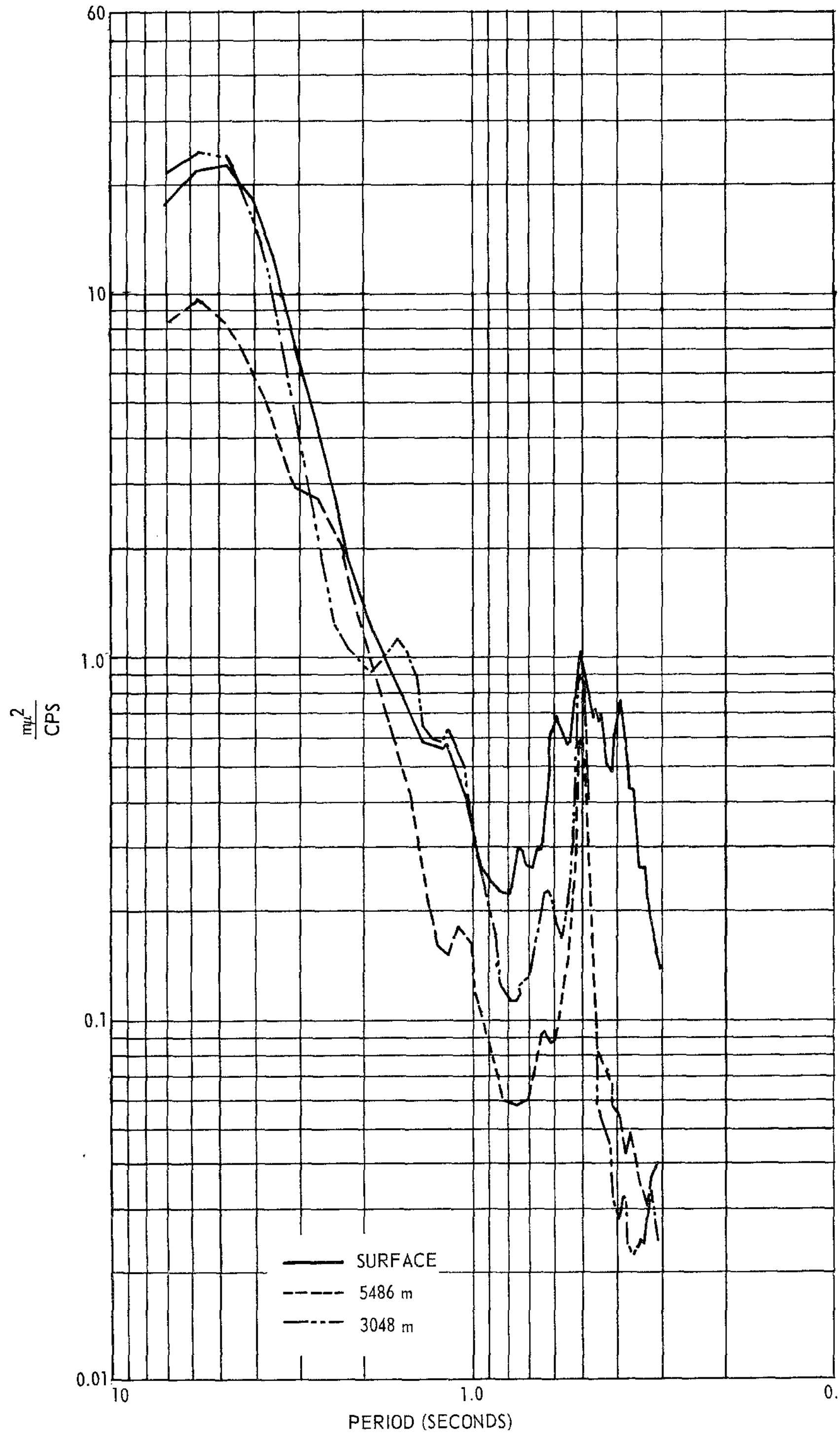

FIG. 9. Spectra of the noise at the surface, and at 1370 and 2890 m, GV-TX. 180 sec sample, 25 samples/sec, 8 per cent lags.

the presence of essentially isotropic noise. The experimental results can best be explained by waves traveling at velocities of 3.0 to 4.0 km/sec.

The most probable solution is that the noise consists of a mixture of Rayleigh modes, possibly on the basis of equipartition of energy as proposed by Sax and Hartenberger (1964). Some evidence for the presence of the second higher mode is obtained from Figure 2, where at a period of 1.1 sec, the experimental data can only be explained by the presence of the second higher mode. However, this peak in the power ratio did not appear at all times; therefore, while it may be present, the second higher mode does not always predominate. It is apparent from the results that the fundamental mode Rayleigh wave does not exist with appreciable energy

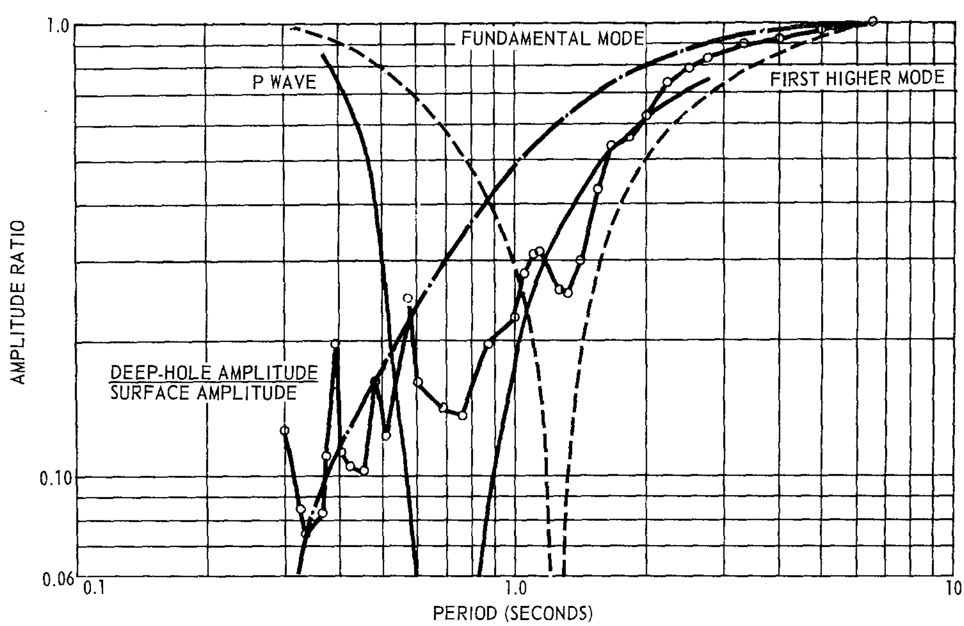

FIG. 10. Amplitude ratio of deep-hole noise amplitude to surface noise amplitude as a function of period. Also shown are the theoretical fundamental and first higher Rayleigh modes and the theoretical *P*-wave amplitudes. Depth 668 m, GV-TX.

content at the quiet sites. Close to the coast, however, a considerable percentage of the noise must consist of the fundamental mode to explain results obtained during previously reported experiments (Douze, 1964).

Noise in the Period Range of 0.8 to 0.3 Sec. The noise amplitudes in the period range between 0.8 and 0.3 sec varies considerably from site to site. The sites with large noise amplitudes are close to centers of population, and the noise is usually connected with cultural activity. The noise at this period range can logically be divided into three parts: the cultural noise, the sharp spectral peaks, and the residual noise when the other two noise types are not present. Each of the three parts is characterized by different wave types.

Figure 9 shows the spectra of the noise at the surface, at 1370 and 2890 m at GV-TX. Because of the close proximity of the site to Dallas, Texas, the cultural noise background is extremely large at the surface. The spectra in Figure 9 indicate clearly that the amplitude (on the average) of the noise decreased very rapidly from the surface down to 1370 m and that the level only decreased slowly below this depth. The only wave type that decreases in amplitude with depth sufficiently rapidly to account for experimental results is the fundamental mode Rayleigh wave. Figure 10 shows the experimental power ratio between the 668 m depth and

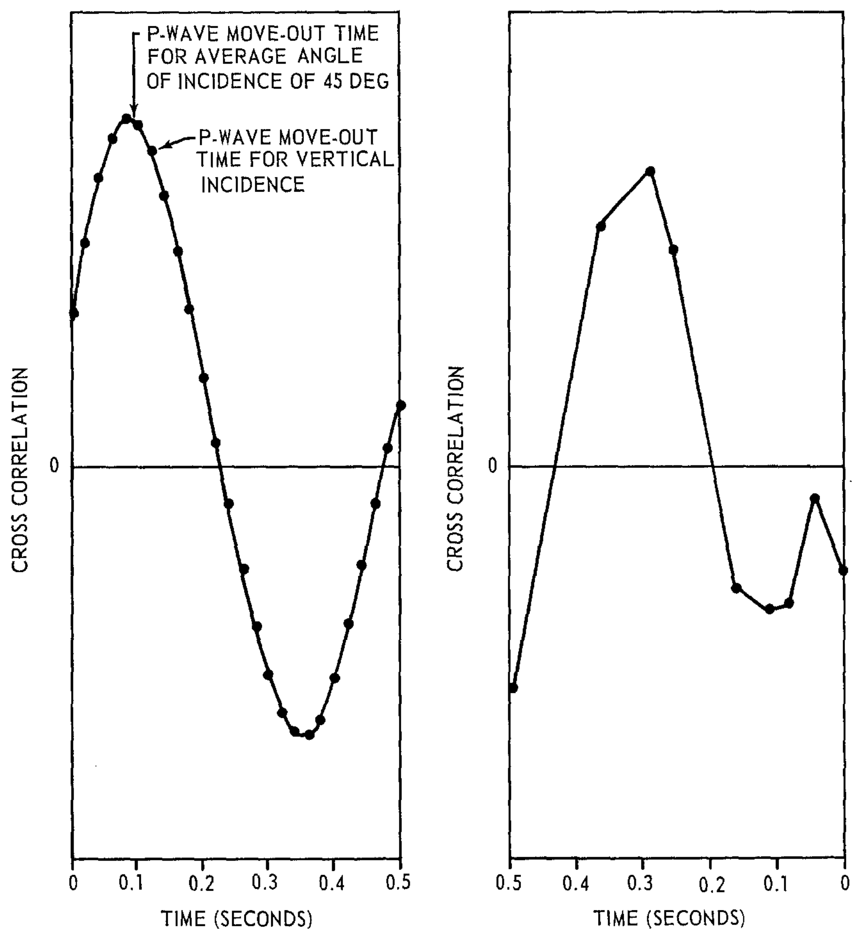

FIG. 11. Cross-correlation between seismometers at depths of 2890 and 2570 m at GV-TX, on left, and at EKN on right.

the surface, together with the theoretical fundamental mode, first higher mode, and P waves. The results clearly indicate, with minor discrepancies, that the fundamental mode accounts for the rapid decrease in noise amplitudes in the first 668 m.

At depths where the amplitude of the fundamental mode has become negligible the amplitude-depth relationships can be explained by either a combination of higher-mode Rayleigh waves, by body waves, or by a mixture of both. The power ratio in Figure 10 shows lows at both the nodal points of the first higher mode and the P waves indicating the possible presence of these waves. In an attempt to distinguish between the possibilities, the noise from seismometers at depths of 2570 and 2890 m were cross-correlated after narrow-band analog filtering (low pass and high pass at 3 cps, 24 dB/oct). Noise samples from closely adjacent seismometers were used because the coherences were high (0.4–0.6). The result (Figure 11) shows that the cross-correlation peaks at 0.1 sec, indicating that the noise consists of body waves and not of surface waves. The cross-correlation was approximately even (negative lags not shown). If only *P* waves were present, the measured uphole time would indicate that the average angle of incidence is 45 deg from the vertical. However, it is likely that *S* waves also contributed to the average uphole time measured. Cross-correlation between deep-hole noise samples from EK-NV, a very

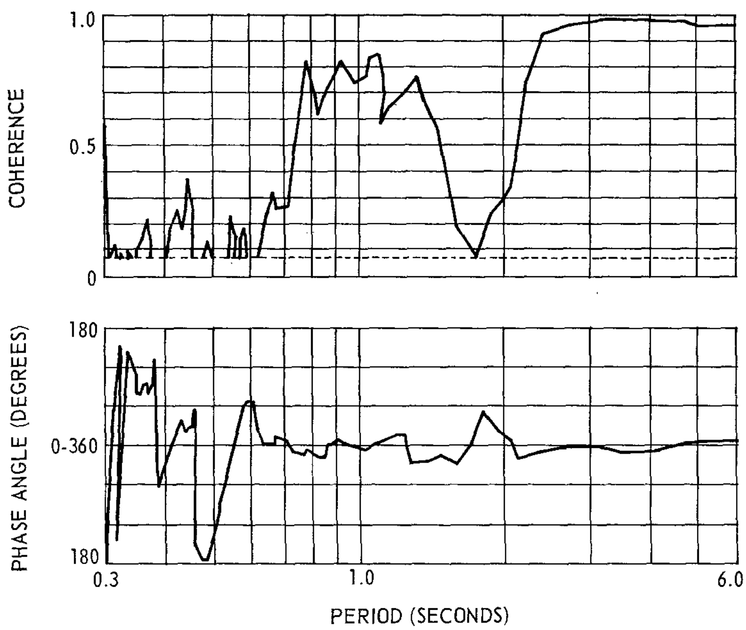

FIG. 12. Coherence and phase angle of noise samples from seismometers at depths of 1980 and 1370 m, GV-TX. 180 sec samples, 25 samples/sec, 8 per cent lags.

quiet site, showed that the noise at approximately 3.0 cps also consisted of body waves.

The coherences were typically high at all sites where multiple seismometers were placed in close proximity (< 600 m). The coherence was a complex function of the distance between seismometers. Figure 12 shows the coherence and phase angle of the noise from seismometers at depths of 1370 and 1980 m. Notice that the phase angle departs from 0 deg at 0.5 sec period; as will be discussed later, the peak at this period probably consists of another wave type. All sites investigated, with the exception of Eureka, Nevada, showed the presence of a sharp peak at 0.49 sec period. Often it was hidden by cultural noise at the surface; however, at depth it was always clearly visible in the spectra. In discussing the 0.49 sec peak, the data will be used from AP-OK, FO-TX, and WMSO, where the peak is very prominent.

In a previous publication (Douze, 1964), the 0.49 sec noise was attributed to the presence of the third higher Rayleigh mode. Figure 13 shows the spectra of the noise

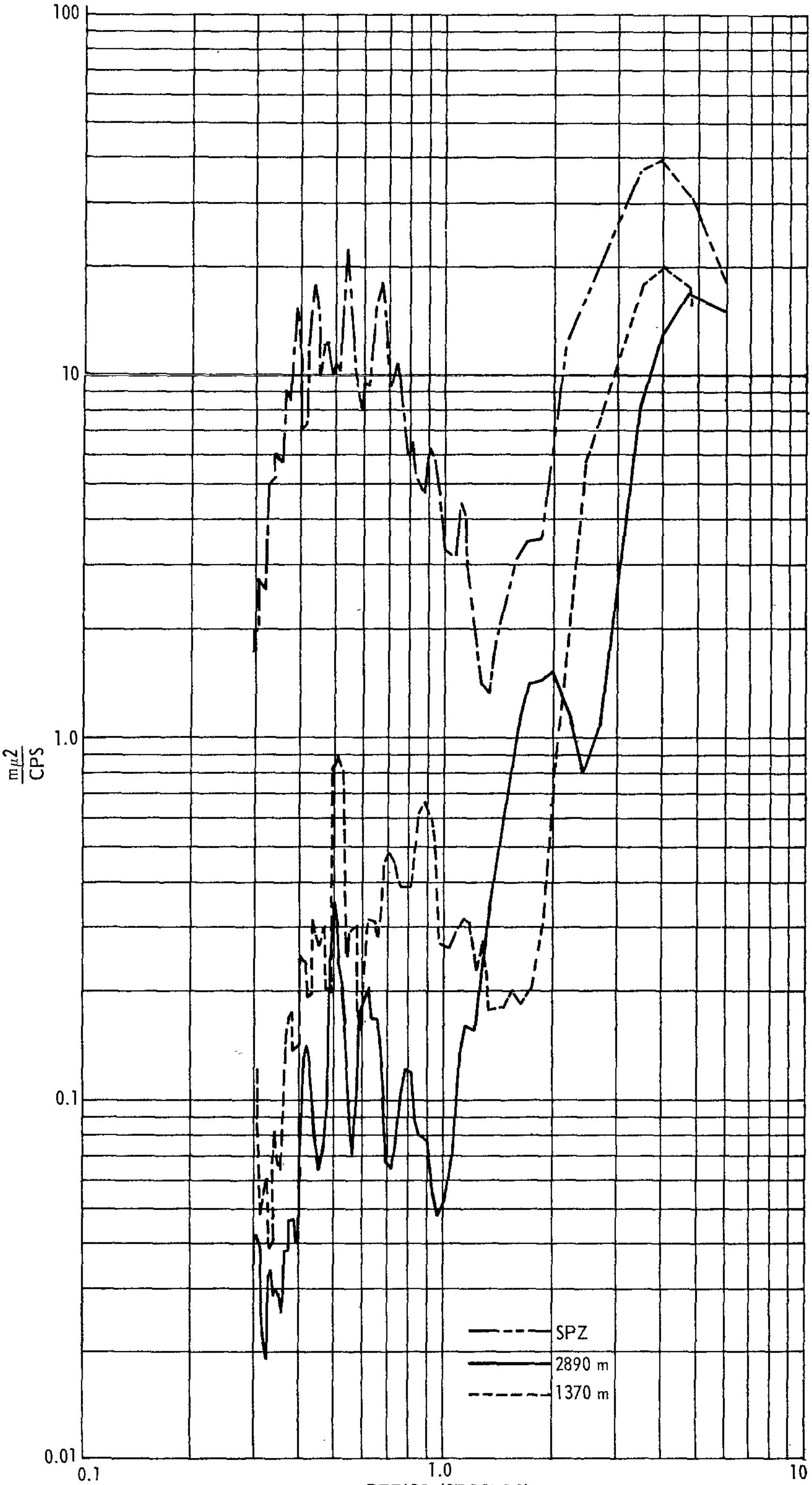

FIG. 13. Spectra of the noise at the surface, and at 3048 and 5486 m, FO-TX, 180 sec sample, 10 samples/sec, 8 per cent lags.

at the surface, 3048 and 5486 m at FO-TX. The sharp peak at 0.49 sec was still present at the bottom of the hole; comparison with the theoretical amplitude-depth relationships (Figure 1) indicates that the third higher mode cannot be the cause of the peak at this period unless the theoretical results are greatly in error. Theoretical investigations indicated that the amplitude-depth relationships can also be explained by P waves arriving at close to vertical incidence. Figure 5 shows the results obtained with a single seismometer in the deep hole and an array of four deep-hole seismometers at AP-OK. The data from the array does not fit either of the theoretical curves closely. The experimental second nodal point appears to

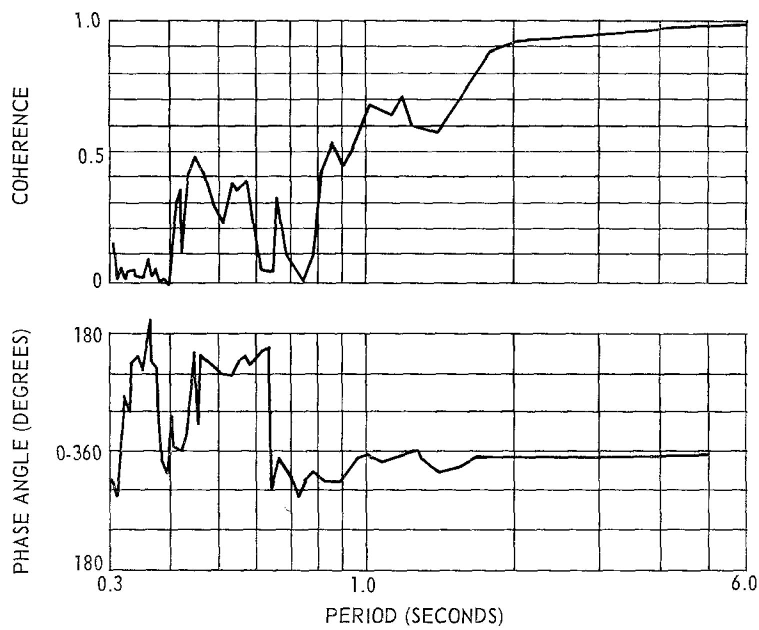

FIG. 14. Phase angle and coherence of noise from seismometers 1 km apart WMSO, 180 sec samples, 25 samples/sec, 8 per cent lags.

occur at a shallower depth than indicated by either theory suggesting the presence of an even higher Rayleigh mode. However, the amplitude-depth relationships of the Rayleigh modes depend on the assumptions made on the velocity section below the hole, and the third mode could probably be made to fit by changing the velocities. It must be noted that the P-wave theory does not depend on the velocity section below the hole.

Surface and body waves can, in theory, be identified by their phase velocities. The phase velocity of the 0.49 sec noise is approximately 3.0 km/sec as measured by WMSO personnel (personal communication, George Gray). Figure 14 shows the cross spectrum between seismometers 1 km apart at the observatory. In general, the noise is isotropic as indicated by the tendency of the phase angle to remain at either 0 or 180 deg. If the 0.49 sec noise is assumed to be isotropic, the coherence (0.29) and the theoretical results (formulas 11 and 12) indicate that the phase velocity is 3.2 km/sec. This velocity indicated that the noise consists of surface waves. However, several features of the experimental data are difficult to explain by the presence of one higher Rayleigh mode alone. Despite numerous attempts to locate a seismometer at a nodal point, no such depth could be found. The presence of mixture of wave types could explain this behavior. High resolution spectra often indicate the presence of another peak at approximately 0.51 sec period. If these two adjacent peaks were caused by different wave types, the results from spectral analysis can be expected to be inconclusive because of lack of resolution.

The coherences (see Figure 5) gave values that would be expected if the 0.49 sec peak had very high coherence and the noise at the same period apart from the peak was incoherent like the noise at adjacent periods. The low coherence at 1950 m is typical of results close to theoretical nodal points, indicating the presence of approximately equal power in and out of phase. In conclusion, the experimental results do not indicate which type of wave is responsible for 0.49 sec noise.

A discussion of the 0.49 sec noise is incomplete without a discussion of the reason for the existence of the sharp peak. Either there exists a very widespread source of this peak, or the earth in some fashion acts as a filter. Despite considerable effort, no common surface source has been found that can explain the sharp peak at this period. If the waves are body waves, some subsurface source must be hypothesized; earthquake records show that the earth does not act as a filter which preferentially passes 0.49 sec noise for body waves. If the waves are surface waves, some filtering mechanism presently not understood must exist. It is quite possible that further investigation of this phenomenon will result in some fundamental discovery on wave transmission in the crust.

Conclusions

Some of the waves present in the noise have been identified. The evidence presented shows that, apart from the fundamental mode, the first higher mode is present in the noise at periods around 3.0 sec. Because of the similarity between body waves and surface waves, it could not be established that the first higher Rayleigh mode always predominates at periods between 4.0 and 2.0 sec. However, it has been established that, at sites some distance from the coast, the fundamental Rayleigh mode is not present with appreciable amplitudes.

In the period range of 2.0 to 0.8 sec, the experimental data were not conclusive. The amplitude-depth relationships can be explained by either a mixture of higher modes or by a predominance of P waves. Both cross-correlations and coherences across surface arrays indicated that surface waves are the preferred interpretation.

In the period range of 0.8 to 0.3 sec, the cultural noise has been shown to consist principally of fundamental mode Rayleigh waves. At depths where the fundamental mode has decreased to negligible values, the remainder of the noise consists of random body waves. The sharp peak commonly present at 0.49 sec consists of Rayleigh mode or modes of order higher than third, or of body waves.

Acknowledgements

The surface-array studies were carried out under Contract AF 49(638)-1150, under the direction of the Air Force Office of Scientific Research, Office of Aerospace Research. The deep-hole studies were carried out under Contract AF 33(600)-43369 and Contract AF 33(657)-13668 under the direction of the Air Force Technical Applications Center, HQ USAF. Both contracts were supported by the Department of Defense under the VELA UNIFORM program of the Advanced Research Projects Agency.

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Manuscript received May 16, 1966.