Comparison of Ocean-Wave and Microseism Spectrums As Recorded at Barbados, West Indies

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Abstract. Power spectrums computed from recordings of ocean waves and microseisms made at Barbados, West Indies, are compared. The ocean-wave spectrums show abrupt onsets of waves having energy peaks in the low-frequency end of the spectrum which can be compared with the onset of energy peaks in the low-frequency end of the microseism spectrums, but at a frequency double that of the ocean waves. A delay of 18 to 24 hours is observed between the onset of the ocean waves and the corresponding development of energy in the microseism spectrums which may be attributable to the time required to develop an interfering wave pattern in the vicinity of the island, this interference giving rise to the generation of microseisms in accordance with the Longuet-Higgins theory. A linear increase with time in the frequency of the peak energy in the ocean-wave spectrums is observed; it is consistent with the increase expected by the dispersive propagation of ocean waves over the ocean.

Introduction. Many studies relating to the nature and origin of microseisms have led to general agreement that the oceans or large bodies of water provide a coupling medium which transfers energy from the atmosphere to the earth in the form of microseisms. Numerous studies show wind-generated waves and swell to be involved in the energy transfer; however, there is not general agreement on the precise mechanism or mechanisms by which this transfer is accomplished.

Darbyshire [1950] presented ocean-wave and microseism spectrums showing a 1 to 2 relationship in frequency between the waves and microseisms. *Longuet-Higgins* [1950] proposed a theory whereby interfering ocean waves introduce microseisms into the ocean bottom at a frequency double that of the ocean-wave frequency. *Dinger and Fisher* [1955] obtained data on microseisms and ocean waves on Guam which were consistent with the Longuet-Higgins theory. *Oliver and Ewing* [1957] presented data in which longer-period microseisms are attributed to ocean swell of identical period in the vicinity of the seacoast near the seismograph. *Haubrich et al.* [1963] present comparative spectrums of ocean waves and microseisms which show correspondence at the primary frequency of the ocean waves as well as at double the frequency of the ocean waves.

Studies at Guam as reported by *Dinger and Fisher* [1955] gave evidence that microseisms recorded there during periods of typhoon activity in the Pacific Ocean were generated by ocean-wave activity in the vicinity of Guam, these waves having propagated from the storm area. These tentative conclusions prompted further work on Barbados in order to study the relation of time changes in ocean-wave activity as recorded off shore at Barbados and the intensity and location of storms in the North Atlantic. During one phase of this study simultaneous recordings of ocean waves and microseisms were made from which comparative power spectrums were computed.

Description of installation. Figure 1 is a map of the island of Barbados showing the location of the wave gage and the seismograph. Barbados is located at the eastern end of the Caribbean Sea, and its northeastern coast provides excellent exposure to incident ocean waves from the North Atlantic. The wave gage design follows that of *Snodgrass* [1956]. The pressure transducer of the gage was mounted on the bottom in water thirty feet deep. Analog ink-on-paper recordings were made on a strip recorder.

The seismometer, a horizontal-component instrument having a typical coil and magnet transducer, had a natural undamped period of 13.3 seconds (frequency 75 cycles/ksec). The amplified signal was recorded on a pen-and-ink strip recorder. The over-all response of the system was maximum at about 250 cycles/ksec, and it was down about 6 db at about 800 and 100

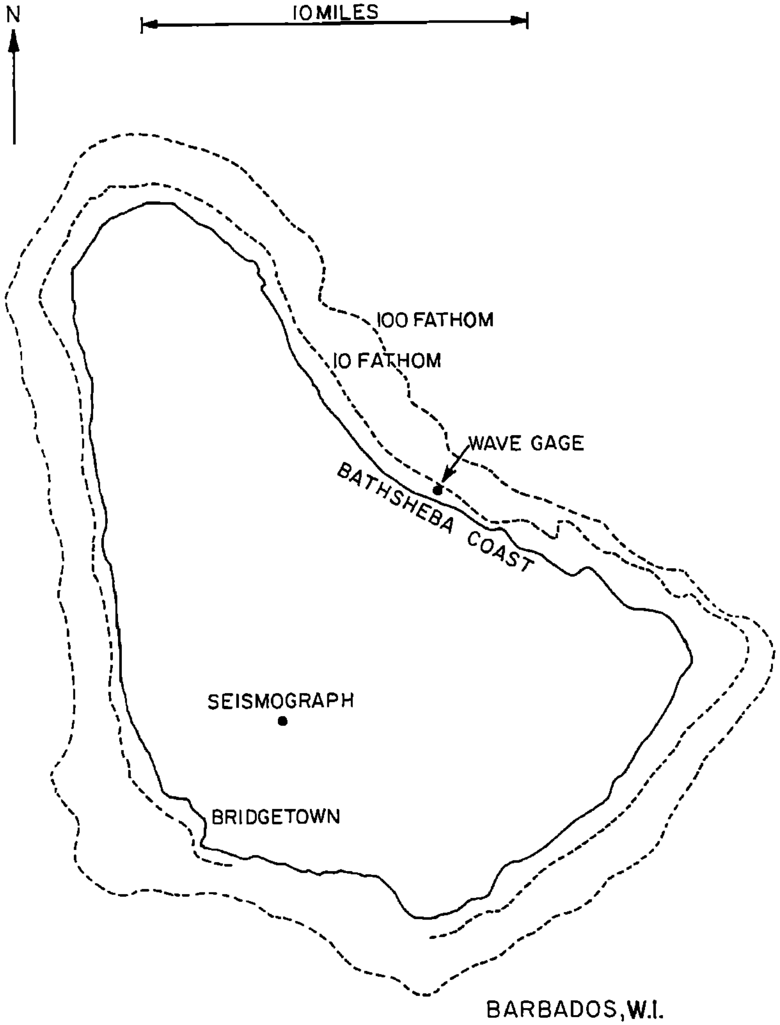

Fig. 1. Map of Barbados showing locations of seismograph and wave gage.

cycles/ksec. Figure 2 shows a sample recording of the wave gage and the seismograph.

Data analysis. Power-spectrum analyses of digital data taken from the analog records have been computed by *Tukey*'s [1949] method, which has been applied and discussed by *Pierson* [1952] and *Munk et al.* [1959].

In applying this method, we let $x_1, x_2, \cdots, x_N$ be the values of the pressure as measured by the wave gage and the seismograph response at the times $t_1, t_1 + \Delta t, \cdots [t_1 + (N - 1)\Delta t]$. The following quantities were computed:

$$r_k = \frac{\sum_{i=1}^{N-k} x_i x_{i+k}}{N - k} - \frac{\sum_{i=1}^{N-k} x_i \sum_{i=1}^{N-k} x_{i+k}}{(N - k)^2},$$

$$(k = 1, 2, \cdots, m) \quad (i = 1, 2, \cdots, N) \tag{1}$$

$$P_h = \frac{1}{m} + \frac{2}{m} \sum_{k=1}^{m-1} \frac{r_k}{r_0} \cos \frac{\pi k h}{m} + \frac{r_m}{m r_0} \cos \pi h, \quad (h = 1, 2, \cdots, m) \tag{2}$$

$$P_h' = 0.23 P_{h-1} + 0.54 P_h + 0.23 P_{h+1}, \quad (h = 2, 3, \cdots, (m - 1)) \tag{3}$$

and

$$f = h/(2m\Delta t) \tag{4}$$

where f is wave frequency and h is an integer which assigns a definite value to f, as defined in

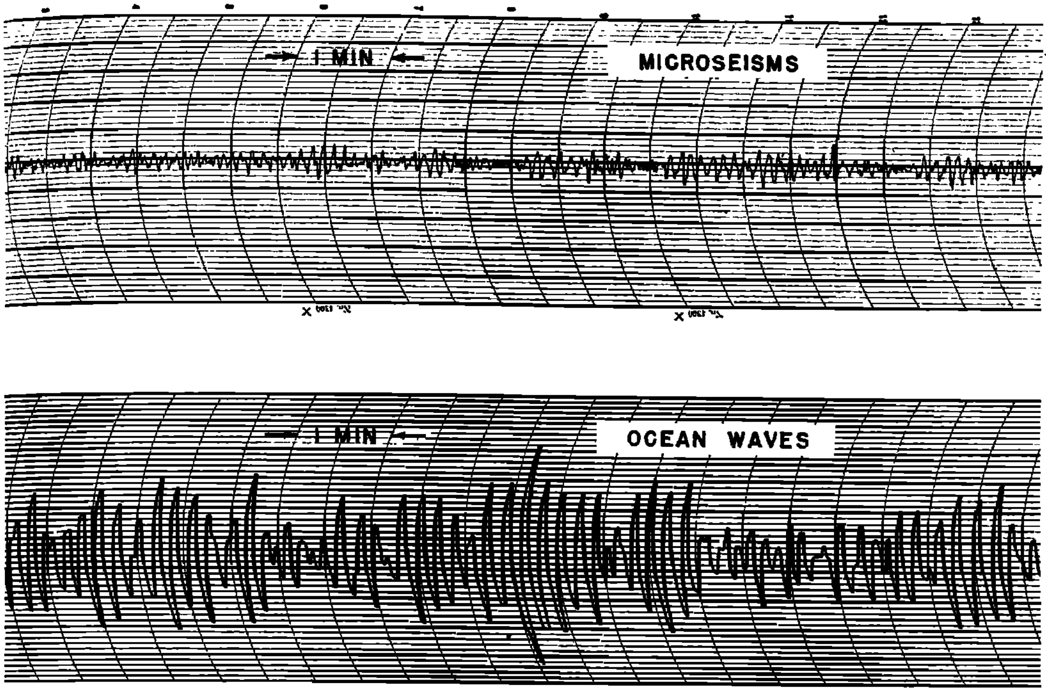

Fig. 2. Typical recordings taken at 1200 on Jan. 7, 1955.

(4). The quantities i and k are integers, N is the number of data points used in computing the power-spectrum estimate P_k, and the values P_k' represent the smoothed distribution of the P_k values.

In computing these quantities, 600 N data points were read from the analog ink-on-paper recordings. Readings were taken every 2 seconds from the ocean-wave records, so that a 20-minute sample of record was used. On seismograph records $\Delta t = 1$ second was used; thus a 10-minute sample was analyzed. In all computations the value of m was 60.

The power-spectrum estimates P_k were normalized by the division of all terms in (2) by the factor r_0. Setting k equal to zero in (1), we have

$$r_0 = \frac{\sum_{i=1}^{N} x_i^2}{N} - \left[\frac{\sum_{i=1}^{N} x_i}{N}\right]^2$$

This normalization gives each P_k (and hence each P_k') a nondimensional value which is proportional to the power-spectrum estimate for that value of h. When P_k' is plotted as a function of h (or of f as given by equation 4), a normalized power spectrum is obtained such that the area under each plot is a fixed value.

Because the area is fixed by this normalization, it must be recognized in the presentation of time sequences of spectrums that as energy develops in one portion of a spectrum the relative value at some other part of the spectrum must decrease, even though the absolute value may not have decreased from the previous spectrum in the sequence. The portions of the spectrums that are of interest in the following discussion deal with those portions in which there appears a peak relative to the nearby part of the spectrum; therefore, by plotting the normalized values of the ordinates, all plots stay within the same range, and the presentation of a time sequence in the same figure is simplified.

$$E_h = \frac{r_0 P_h' \, \Delta t m C^2}{\pi K_h^2} \qquad (5)$$

is used to obtain a dimensional value of the ocean-wave spectral estimates. C is a calibration factor, the value of which is determined by the characteristics of the wave gage and recording instrumentation. K_h is a factor which corrects the pressure data for the pressure attenuation resulting from the depth of the water in which the gage head is placed. This correction is a function of h. The units of Δt and C are here chosen to give E_h the units cm^2 ksec. These units can be converted to ft^2 sec by multiplying by 0.93.

E_h is therefore an estimate of the energy per unit interval of h and is centered on each integral value of h. Because in the plotting of the power spectrums in this report the normalized values of P_h' are used, the value of E_h at the peak values of the ocean-wave spectrums is given on each graph, thus providing a means for relative comparison between spectrums. The normalization factor in the case of the seismic records is r_0, and it must be considered to be in arbitrary units.

The graphs of the microseism spectrums have a frequency scale which is double that of the scale on the graphs of the ocean-wave spectrums.

Comparative spectrums. Late in Decmber 1954 a tropical storm moved north of Barbados, and early in January 1955 several low-pressure areas moved across the North Atlantic. The winds associated with these systems are considered to be the source of the waves incident on Barbados during this time. Figures 3, 4, and 5 show the comparisons of 6-hour sequences of ocean-wave and seismic spectrums.

Attention is called to the following points in these comparative spectrums:

1. Peak energies in the microseism spectrums occur at a frequency approximately twice that at which the corresponding ocean-wave spectrums show a peak *after* the microseisms have fully developed a response to the incident waves. As is discussed in point 3 below, there is an apparent time lag in the development of microseisms corresponding to the relatively sudden onset of waves from a new source. During the interval of the time lag the 2-to-1 relation does not exist; see, for example, the spectrums at 2400 on 1–2–55 and at 0600 on 1–3–55 (Figure 3).

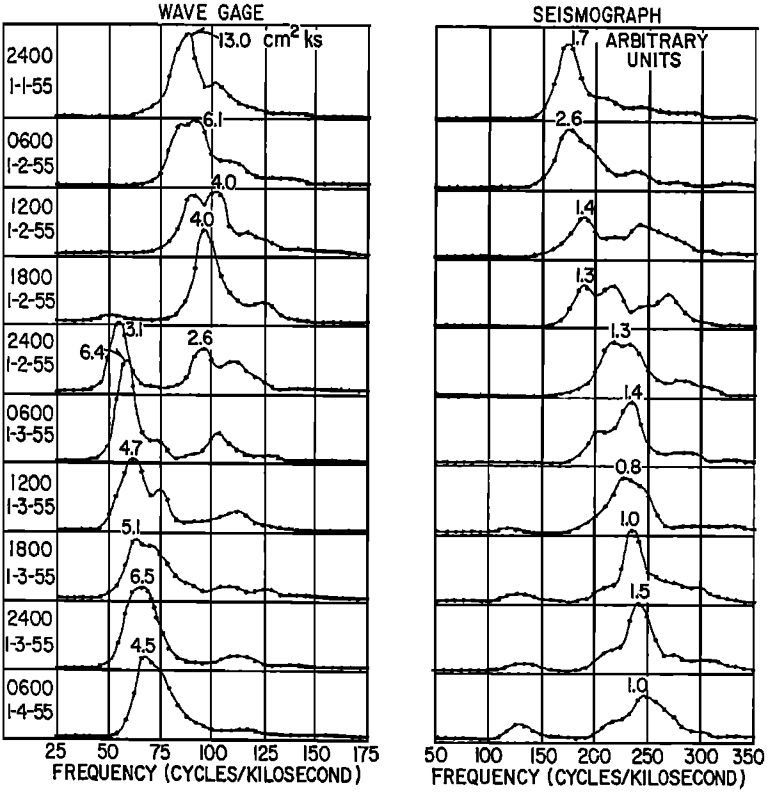

Fig. 3. A 6-hour sequence of ocean-wave (*left*) and microseism (*right*) spectrums computed from simultaneous recordings made during the period Jan. 1 to Jan. 4, 1955.

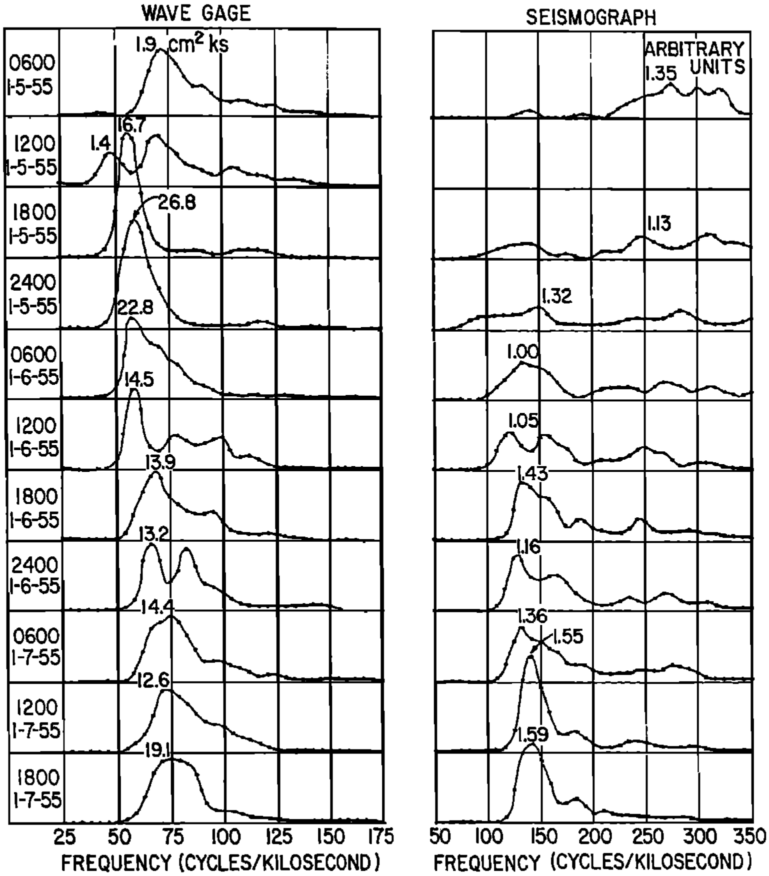

Fig. 4. A 6-hour sequence of ocean-wave (*left*) and microseism (*right*) spectrums computed from simultaneous recordings made during the period Jan. 5 to 7, 1955.

2. A linear increase with time occurs in the frequency of the peak energies of the ocean-wave spectrums, this increase being consistent with the dispersive propagation of waves in the ocean. For example, the increase in frequency in the ocean-wave peak at 50 cycles/kilosecond at 1200 hours on January 5, 1955, to a peak at the frequency of 75 cycles/kilosecond at 1200 hours January 7, 1955 (Figure 4), is the increase expected by waves propagating from a source about 2500 miles distant. These shifts in frequency are also reflected in the microseism spectrums by corresponding increases in the frequency of the peak energies.

3. An onset of new ocean-wave activity, as evidenced by a new spectral peak appearing at the low-frequency end of the spectrum, is accompanied by an onset in the low-frequency end of the microseismic spectrum but it lags in time by 18 to 24 hours. This observation is most evident in Figure 3. The onset of ocean waves with a frequency around 50 cycles/ksec is seen at 1800 hours on January 2, 1955, and 6 hours later the energy appearing at about this frequency has greatly increased. It is not until 1200 hours on January 3 that a corresponding spectral content is apparent in the microseism spectrum.

Another illustration of this time lag is shown in Figure 4. At 1200 hours on January 5 an onset of waves is seen at approximately a frequency of 50 cycles/ksec. This onset develops so that 6 hours later the wave spectrum is dominated by this onset of waves. It is not until the 1200 hour of January 6 that a well-defined spectral peak is apparent in the microseism spectrum which corresponds to this onset of wave energy in the wave spectrum.

Figure 5 provides still another example of this

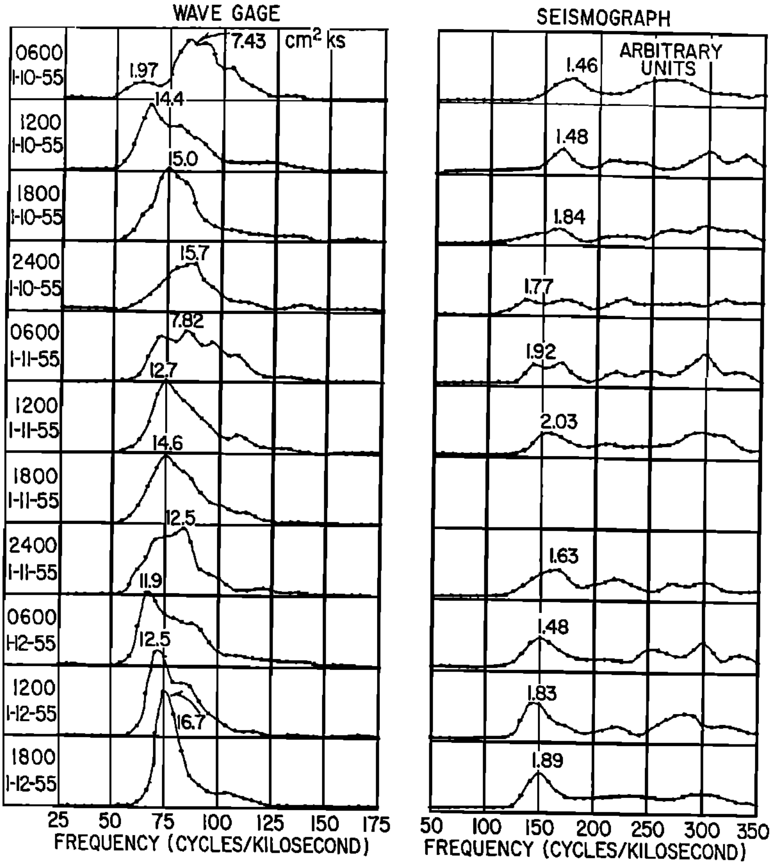

Fig. 5. A 6-hour sequence of ocean-wave (*left*) and microseism (*right*) spectrums computed from simultaneous recordings made during the period Jan. 10 to 12, 1955.

lag. At 0600 hours on January 10, 1955, we observe the onset of waves with an energy peak at a frequency of about 60 cycles/ksec; however, maximum energy still peaks near a frequency of 80 cycles/ksec. The microseisms show a peak near 160 cycles/ksec. At 1200 hours the lower-frequency waves become the predominant part of the ocean-wave spectral graph. It is not until 2400 hours that an energy peak shows in the microseism spectrum at a frequency corresponding to the low-frequency onset of ocean waves 12 to 18 hours earlier. As time goes on through January 11 and 12, the continued incidence of ocean waves at the lower frequencies results in the development of a well-defined peak on the microseism spectrums showing the 2-to-1 relationship in frequency between microseisms and waves.

Discussion. The 2-to-1 relationship between the frequencies of the microseism and ocean-wave spectrums can be interpreted as giving support to the *Longuet-Higgins* [1950] theory of the generation of microseisms. The fact that the onset of new ocean-wave energy in the low-frequency end of the spectrums preceded the corresponding onset of microseism energy at a corresponding frequency would indicate that the transfer of energy from ocean waves to seismic energy occurred near the island; otherwise the higher speed of propagation of microseismic energy from a distant location would precede the incidence of ocean waves or perhaps even be present most of the time from a variety of locations and thus show no particular correspondence to the ocean-wave activity in the localized area of Barbados.

It is interesting to speculate as to a cause of the delay in the onset of an energy peak in the microseism spectrums as compared with the time of onset for the corresponding half-frequency peak in the ocean-wave spectrums. If the Longuet-Higgins mechanism is the effective mechanism for the generation of microseisms, the delay may be associated with the time required for an interfering ocean-wave pattern to develop in the area around Barbados. The time required for waves arriving from the north to travel from Barbados to the South American coast and return is approximately equal to the observed time of the delay.

Energy peaks in the microseism spectrums at the same frequency as the ocean-wave frequency are not observed in these comparative spectrums. The frequency response of the seismograph is such that any low-frequency microseisms (50 to 60 cycles/ksec) could not be recorded. In the light of the recent results of *Haubrich et al.* [1963] and *Oliver and Page* [1963] it now appears unfortunate that the seismograph response did not permit the recording of microseismic frequencies over the primary frequency range of the ocean waves. It could be important to know whether energy in the microseism spectrums at the primary frequency of the ocean waves appeared simultaneously with the arrival of the ocean waves or whether a delay was experienced, as in the double frequency component in the microseism spectrums. Such data could provide information as to whether the mechanisms of energy transfer for the primary and first harmonic frequencies are closely related, as suggested by *Oliver and Page* [1963], or are independent mechanisms if the buildup in the microseismic energy at the two frequencies is not simultaneous.

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