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High-frequency P- and S-wave Attenuation in the Earth

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Abstract—Investigations of the spectral characteristics of teleseismic body waves revealed that the spectral falloff rate between 1 Hz and 10 Hz is primarily controlled by anelastic attenuation along the path. In addition, the amount of high-frequency energy in teleseismic body waves is far above the level expected on the basis of Q estimates at low frequencies, thus leading to the idea of frequency dependence in Q. Q variations in the earth's mantle can be investigated by mapping out the variations of high frequency (4–10 Hz) energy relative to the low frequency (1–3 Hz) energy in teleseismic P waves, and similar ratios at lower frequencies in teleseismic S waves. Because of the extreme sensitivity of spectral content of short-period body waves to Q variations, large uncertainties in other factors affecting spectral content can be tolerated in such studies. With the increasing number and density of broadband seismic stations recording at high sampling rates, tomographic studies of Q at high frequencies become possible.

Key words: Attenuation, body waves, seismic, high frequency.

1. Historical Overview

The measurement of seismic anelastic attenuation in the earth is a difficult undertaking. Anelastic losses must be estimated from changes in the amplitudes of seismic waves after corrections have been made for other known factors that affect them. The observed changes in the seismic wave amplitudes and spectral contents may be generated by numerous causes besides anelastic attenuation. Unfortunately, these other factors are mostly unfamiliar to us. The problem of attenuation losses, similar to other problems in seismology, seem to be more tractable when signals of very low frequencies are analyzed. Consequently, many of the early estimates of the Q structure of the earth derive from studies of the free oscillations of the earth and surface waves that repeatedly circled the earth. Q estimates for individual lines in the earth free oscillation spectra were obtained from the temporal decay rates of individual spectral lines and the sharpness of resonance peaks.

Moving towards higher frequencies, methods were developed to estimate the average Q structure of the earth from spectral amplitudes of long-period surface waves (ANDERSON and ARCHAMBEAU, 1964; ANDERSON *et al.*, 1965). The result-

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ing Q-structure estimates showed that most of the anelastic losses in the earth occurred in the upper mantle roughly corresponding to the depth of the low velocity zone. The picture that emerged from these studies is that of an earth with high anelastic losses and frequency-independent Q structures. Regardless of the exceptional progress made even at these relatively low frequencies, the influence of other factors could not be entirely eliminated. Lateral heterogeneities in the earth alter the amplitudes of free oscillations by leakage of the energy from one mode to the other by scattering. Surface waves cross major structural boundaries causing energy losses, multipathing, mode conversions, and they undergo focusing effects. These factors make the study of Q from surface waves more difficult.

Simultaneously with these studies, there was a steady increase of the body wave attenuation analyses for specific regions. Many of these studies were often qualitative in nature and provided relatively few numerical Q estimates. Most of them simply noted patterns of P- and S-wave paths of high and low amplitudes and changes in the visually detectable frequency content, and interpreted them in terms of Q changes in the mantle and crust. These studies led to dramatic progress in our understanding of the internal dynamics of the earth, the new global tectonics. They also showed that the Q in the upper mantle is highly variable, it has low Q regions under backarc basins, mid-ocean ridges and tectonically active regions (OLIVER and ISACKS, 1967; BARAZANGI *et al.*, 1975). Q values, on the other hand, were shown to be higher under old oceanic crust and shield areas. The scope of such studies of Q was limited by the limited frequency response of the photographically recording World Wide Seismic Network (WWSSN) stations, although the very existence of this network made these studies possible.

During the early 1970s progress in digital computer technology made the routine numerical simulation of seismograms possible. Such simulation studies, mostly aimed at understanding the mechanics of earthquake sources and seismic path structures, advanced our understanding of the finer features of long-period seismograms and source-path interactions. Most of the data thus analyzed initially still came from the long-period WWSSN network with only occasional inclusion of short-period seismograms. The inclusion of short-period seismograms in such studies is made more difficult by the fact that waveforms of short-period body waves are hard to model because the detailed structural information at the scale of their wavelengths, needed for accurate modeling, is generally not available. Moreover, because of the source-path tradeoffs most Q values derived from waveform modeling studies in the short-period band should be viewed with some skepticism. Studies of waveforms, amplitudes and spectra of short-period body waves at seismic arrays revealed that these vary considerably and apparently randomly over short distances, thus requiring, in their analyses, a quite different philosophy and a theoretical framework of propagation of seismic waves through random media.

It gradually became apparent that frequency independent Q models of the earth, incorporating the low Q values in the upper mantle from long-period studies, are inappropriate. Many studies indicated that the Q values in the short-period band are much lower than in the long-period band (e.g., FRASIER and FILSON, 1972; DOUGLAS *et al.*, 1972). These apparently contradictory results, alongside some similar, sporadic results from the 1960s, peacefully coexisted in the literature during these years. ARCHAMBEAU *et al.* (1969), using recording systems with better frequency response at high frequencies (the VELA Long Range Seismic Measurement, LRSM, station network), found that their data required a frequency dependent Q. SIPKIN and JORDAN (1979) and LUNDQUIST and CORMIER (1980) came to the same conclusions using WWSSN data. Support for the ideas of frequency-dependency of Q were also provided by theoretical studies of anelastic losses (e.g., MINSTER and ANDERSON, 1973; MINSTER, 1980; LUNDQUIST and CORMIER, 1980; ANDERSON and GIVEN, 1982).

The early 1970s have also produced several studies of the Q structure under North America. In the long-period band SOLOMON and TOKSÖZ (1970) found wide areal differences in Q under this region. These results were further supported by inversion of surface wave data for Q structures (LEE and SOLOMON, 1975, 1979). In the short-period band several studies of short-period attenuation in this region, using digitized analog seismic recordings, showed major variations in mantle Q (DER *et al.*, 1975; DER and MCELFRESH, 1976, 1977).

During the 1980s there was rapid progress in the understanding of short-period wave attenuation. Substantial research was fueled by the need of the U.S. Department of Defense (DoD) for accurate estimates of nuclear explosion yields by analyses of seismic P-wave amplitudes. Consequently, most of the work was concentrated on differences in mantle Q under areas where test sites were located. The ensuing debates resolved many of the apparent discrepancies among the results of various groups, and confirmed the existence of a magnitude "bias" between two of the major test sites: the Nevada test site and the Kazakh test site (LAY and HELMBERGER, 1981; DER *et al.*, 1982a; TAYLOR *et al.*, 1986; DOUGLAS, 1987, 1991; DOUGLAS and MARSHALL, 1996).

There have been several later studies of the Q structure in North America (LAY and HELMBERGER, 1981; DER *et al.*, 1982a; TAYLOR *et al.*, 1986) confirming the large variations in Q beneath the continent. A study by BACHE *et al.* (1985) showed that the mantle under most of Kazakh and Russia has a high Q. A worldwide compilation of t_p^* estimates, orginally byproducts of discrimination studies, showed that large variations in mantle Q relevant to the short-period band exist worldwide (DER *et al.*, 1982b).

Several studies were performed to delineate the ranges of frequency-dependent models compatible with the available broadband data beneath North America (DER *et al.*, 1982a; LAY and HELMBERGER, 1981; DER and LEES, 1985) and Eurasia (DER *et al.*, 1986; LEES *et al.*, 1986). A worldwide study of frequency dependence (CHAN and DER, 1988) complements the regional variations of Q_{ScS} by SIPKIN and JORDAN (1979) and shows that Q in the short-period band varies in the same sense as the variations of Q_{ScS} worldwide. Other studies support this finding (e.g., LAY and WALLACE, 1983; NAKANISHI, 1979). During the late 1980s and the 1990s there was a decrease of research activity in Q studies. Part of the reason is that the interest in seismic nuclear monitoring has shifted to regional distances and that many have felt that the quality of the then available data was inadequate for achieving rapid progress in this area of research. The latter was indeed a major difficulty in many of the studies quoted above. Often it was necessary to analyze photographic records with poor time resolution and dynamic range and resort to crude measurements, such as those of rise times, in order to put limits on Q. Such crude methods can be entirely avoided with modern digital data.

The recent decade has seen the rapid expansion of digital data networks, both for multiple use and regional networks for earthquake studies. Substantial amounts of data can be accessed through the Internet. With the increasing availability of high-quality high-frequency digital seismic data and the improved understanding of the velocity structure of the earth from tomographic studies, the time is ripe for tomographic mapping of the Q in the earth at high frequencies. At the same time, the results from velocity tomography made the classification of the regions of the earth into the old stereotypes obsolete. We now know that many of the high Q zones in the upper mantle may be remains of old slabs, suggesting that the three-dimensional Q structure of the mantle may follow similar patterns (GRAND *et al.*, 1991; VAN DER HILST *et al.*, 1997; WIDIYANTORO and ENGDAHL, 1997; VAN DER LEE and NOLET, 1997).

This review paper attempts to summarize our knowledge of short-period seismic wave attenuation, as seen through the experience of the author. This subject comprises a large volume of literature, one cannot do justice to it in this short paper. Consequently, some relevant work of considerable merit may not be quoted for the sake of brevity. Moreover, I exclude numerous studies of attenuation at regional distances where the attenuation mechanism is probably a combination of anelastic Q and scattering, and mention long-period surface wave attenuation studies in passing only.

2. Teleseismic Energy Loss in the Short-period Band

The well known standard formula for anelastic attenuation

$$A \sim \exp\left(-\pi f \frac{t}{Q}\right)$$

reveals the extreme sensitivity of seismic amplitudes to Q at high frequencies. In this formula t is the travel time, f is frequency and Q is the dimensionless quality factor the reciprocal of which is the fractional energy loss during one cycle (KNOPOFF, 1964). The quantity $t^* = t/Q$ is a commonly used parameter in attenuation studies. Because of the concentration of attenuation losses in the upper mantle low velocity zone of the earth, t^* is a weak function of distance at teleseismic distances. Ignoring the distance dependence, a t^* value of 1 sec. for P waves is commonly used in simulations of long-period seismograms. Such a value would reduce the amplitude of the 4 Hz component of a body wave by four orders of magnitude relative to the 1 Hz component, which causes such component for most events to fall below noise levels and thus renders it unobservable by most known seismic recording systems. This would rule out observation of frequencies as low as 4 Hz in the seismic data altogether. On the other hand, as we shall show below, P waves can be shown to contain energy at teleseismic distances to frequencies above 6 Hz.

Various attempts have been made to reconcile the data with the low constant Q values accepted in low frequency modeling and explain the apparent presence of such high frequencies in the data. Alternative explanations suggested and discarded included spectral leakage due to the data windows used in spectral analyses, instrument nonlinearities, noise, crustal amplification, variations in source spectra, etc. Extreme models of crustal amplification or surface reflection for explosions can be concocted to shape the spectrum such that the high frequencies would be suppressed. Such crustal models must be quite peculiar and thus they are not credible as explanations for the absence of high frequency energy for the wide variety of regions where P wave Q was studied. Likewise, cancellation of energy by surface reflection (pP) is also unlikely. This author has only seen two examples of nuclear explosions where the pP nulls were evident, after studying hundreds of events. In the real world enough high-frequency energy would escape the source region due to scattering by the heterogeneities in the earth or the roughness of the free surface such that it would be still far above the levels allowable by the extremely low Q models derived from low frequency data. It is apparent, therefore, that discrepancies of orders of magnitude in spectral amplitudes cannot be consistently explained by any combination of those suggested alternative explanations. Simply stated, the value of the high frequency Q along a path through the earth is thus tied to the presence or absence of high frequency energy (in the band 4–10 Hz) observable in P waves along that path. Because of the extreme sensitivity of high frequency energy levels to Q, all other conceivable factors play only a secondary role. This statement may appear to be too simplistic, nonetheless it is supported by considerable observational evidence.

Most of the work regarding attenuation of body waves in the earth is consistent with the notion that the anelastic energy losses are associated with losses in shear deformation. Although some studies indicated a contribution from losses in compression (TAYLOR *et al.*, 1986) these claims are contradicted by other studies. In any case, all studies agree with the assumption that anelastic losses occur *mostly* in shear deformation. Another issue is the contribution of scattering to attenuation that requires further studies. The shear loss mechanism of attenuation translates into the approximate relationship $t_s^* = 4\ t_p^*$ when the typical mantle velocities for P and S waves are considered. The approximate validity of the relationship between shear wave and P-wave attenuation makes it possible to merge and jointly interpret P- and S-wave atenuation measurements (DER *et al.*, 1986a; LEES *et al.*, 1986) and thus make use of the greater sensitivity of S waves to build attenuation models (DER *et al.*, 1986b).

It appears that frequency dependent Q is inherent in the commonly accepted mechanism of attenuation in the earth's mantle. Anelastic media can be modeled with the following mathematical form of the general stress-strain relationship

$$\sigma + \tau_\sigma \dot{\sigma} = M_R(\varepsilon + \tau_\varepsilon \dot{\varepsilon})$$

where τ_σ and τ_ε are relaxation times for stress and strain and M_R is an elastic modulus and the dot denotes a time derivative. Thus both the stresses and strains are also dependent on the rates of stress and strain. This relation assumes a dynamic loss as the material goes through a stress and strain cycle. The stress-strain relationship above gives rise to a frequency dependent Q of the form

$$\frac{1}{Q} = \frac{\omega(\tau_\varepsilon - \tau_\sigma)}{1 + \omega^2 \tau_\varepsilon \tau_\sigma}$$

where $\omega = 2\pi f$. Relaxation times are typically dependent on the temperature in the fashion

$$\tau = \tau_0 \exp\left(\frac{(E^* + PV^*)}{RT}\right)$$

and E^*, V^* and τ_0 are material constants, P is pressure and T is temperature, R is the gas constant. Thus knowing the pressures and the material constants, the attenuation measurements will provide information with regard to the temperatures in the earth. Most anomalously high attenuation zones for high frequency seismic waves in the earth seem to be associated with high temperatures, conductivities and heat flow. High temperatures thus will give rise to higher attenuation (SOLOMON, 1972). Superposing simple absorption band models such as that described above one can construct more general frequency versus Q relationships. A model of a broad absorption band often used has the form (e.g., LUNDQUIST and CORMIER, 1980)

$$|F(\omega)| = \exp\left[-\frac{\omega t_m^*}{\pi} \arctan \frac{\omega(\tau_1 - \tau_2)}{1 - \omega^2 \tau_1 \tau_2}\right]$$

where t_m^*, τ_1 and τ_2 are appropriately chosen constants with the dimension of time. In the time domain the attenuation-related dispersion associated with such Q models can be described in terms of a minimum phase filter with the appropriate amplitude response (DER and LEES, 1985; CHOY and CORMIER, 1986). Given the strong likelihood of the frequency dependence of Q in the earth, Q estimates in limited frequency bands cannot be extrapolated to other bands.

If Q is frequency dependent, appropriate ways of measuring it must be applied. In the case of constant Q the simple method of fitting linear functions to spectral ratios in a linear frequency vs. the logarithm of the spectral ratio can be used. The slope of such line will be inversely proportional to the value of Q. If Q is frequency dependent, the averaged slope of such line has little meaning by itself. On the left side of the following equation we define apparent t^*, $\bar{t}^*$. This quantity can be related to the actual value of $t^*(f)$ by the following formula,

$$\bar{t}^* = t^* + f\left(\frac{dt^*}{df}\right) = \frac{-1}{\pi}\frac{d(\ln A)}{df},$$

where f is frequency and A is amplitude. The formula states that in the frequency-dependent case a spectral ratio used to measure t^* will be a biased; the bias consisting of the gradient term $f(dt^*/df)$. The question is how much error is caused by Q estimation by this term. As we shall show below this error is slight, because according to most evidence Q (or t^*) changes only slowly with frequency.

If t^* varies slowly with frequency this formula could be used to derive t^* by solving this differential equation although there will still be an unknown integration constant. Absolute values of t^* can be obtained from some types of measurements that include the decay rates of multiple *ScS* and free oscillations of the earth and can be used, in principle, to estimate the unknown integration constant.

3. Estimation Methodology Issues

Most studies of the mechanisms of individual earthquakes and earth structures on a larger scale employ waveform modeling as a tool. This popular methodology attempts to reproduce the observed seismograms by incorporating the earth structures along the paths involved, assuming the source mechanisms and time functions and Q (or t^*) values. All these parameters are adjusted until a good visual or RMS fit between the observed waveforms and the synthetics is achieved. Despite the fact that in most studies typically t^* values near 1 sec. are used for P waves and $t^* = 4$ sec. for S waves, and these seem to be unacceptable based on spectral evidence, no adjustments are usually made to these t^* values. The primary reason is that even large t^* values do not affect the low frequency waveforms being modeled, especially those of the relatively narrow-band short-period seismograms from the old WWSSN system. Relatively small adjustments to the source time functions and structures could always be made to improve the fits between the synthetic and observed waveforms. Moreover, good waveform fits can also be obtained by inflexibly using the fixed t^* values mentioned above even though other data indicated sizeable regional variations in the Q values along the teleseismic paths involved. The problems are associated with the low sensitivity of the waveform fitting procedure at low frequencies ($f < 1$ Hz) to Q variations. The predominantly low frequency character of raw teleseismic body wave seismograms is due not only to Q but to the source spectra, and it is relatively easy to match the overall waveform of a teleseismic body wave by matching the low frequency amplitude and phase only, while ignoring the relatively higher frequency energy riding over it. The station-averaged coherences between the synthetic and actual seismograms become lower at higher frequencies and the actual spectral shapes of the two differ also (DER and SHUMWAY, 1988).

Part of the historical controversy between waveform modelers and spectral modelers was based on differences in philosophical outlook. Waveform modeling is inherently a totally deterministic procedure which clearly breaks down at frequencies above 1 Hz. In some waveform modeling work the waveforms are actually low-pass filtered to "improve" the fit. If, instead, high-pass filtering above 1.5 Hz would be applied to teleseismic body wave seismograms, the waveform modeling task would become utterly impossible. At high frequencies the wave amplitudes and waveforms vary considerably over distances of a few kilometers as evidenced by numerous studies at various seismic arrays. Even though the amplitude variations can be large, they do not amount to orders of magnitude. The appropriate models applicable here are random media, a concept alien to the deterministic mindset used in seismic modeling. Thus the attenuation estimates at high frequencies must, instead, be based on statistical and spectral methods rather than on waveform modeling. Waveform fitting is a procedure that is more appropriate and useful for source mechanism studies at low frequencies and for large events. The problems described above are not inevitable in time domain waveform modeling studies. If spectral fitting is performed simultaneously with the time domain waveform fitting and care is taken that key time domain features of the seismograms, such as rise times, also fit, results more consistent with the Q values derived from frequency domain estimation procedures can be obtained (CHOY and CORMIER, 1986).

The observed properties of short-period body waves support the statements made above. As expected for heterogeneous random media, amplitudes of teleseismic body waves at seismic arrays vary considerably over short distances. Many of these variations are systematic for sources at the same location, but vary with azimuth and slowness (CHANG and VON SEGGERN, 1980). Such variations in waveforms can also be effectively modeled by treating the problem statistically, using the theories developed for modeling random media (CAPON and BERTEUSSEN, 1974; AKI *et al.*, 1985). The amplitude variations in such cases may be regarded as products of the focusing and defocusing of the energy related to small-scale heterogeneities in the crust and the mantle. Moreover, there may be strong variations in amplitude levels, although lesser variations in spectral shapes, among various stations are obviously not related to Q but caused by near-surface layering (DER *et al.*, 1980). Thus body wave amplitude variations among some individual sites may tell us less about the Q in the underlying mantle than spectral variations, although *on the average* both are diagnostic.

Analogously to amplitude variations the spectral shapes also vary slightly across seismic arrays. Taking spectral ratios between all pairs of sensors at NORSAR for a set of 10 teleseismic events, the variations in the apparent t_p^* thus obtained have a standard deviation of about 0.06 sec., presumably due to the waveform fluctuations and not actual variations in attenuation. Such variations put a limit on the accuracy of relative t^* measurements. Nevertheless, these statistical fluctuations are smaller than the regional differences in t_p^* found and the more pronounced differences are found in the long-period band. It appears therefore that the magnitudes of the errors in absolute and relative t^* and relative amplitude measurements due to the heterogeneity of the media can be reduced by averaging multiple spectral measurements with different paths.

Frequency dependence introduces another complication into the methodology of attenuation measurements in the short-period band. Theoretically, it should be possible to detect frequency dependence by comparing observed spectra with well-known source spectra. For the frequency-dependent case their ratio should have a noticeable curvature compared to the ratio associated with a constant Q. BACHE (1980) and WALCK (1988) found that certain instrument corrected spectral ratios in the short-period band had such curvature. It appears, however, that the curvature was detected at the steeply descending portion of the short-period instrument response curve that applied in theses cases. It is somewhat doubtful that the actual short-period instrument response at the frequencies, where it falls off rapidly is known well enough to make such a claim. Therefore it is preferable to tie together independent short and long period t^* estimates; each derived from the appropriate instrument or broadband instruments. At this point the question of frequency dependence cannot be decided convincingly based solely on data in the short-period band data (DOUGLAS, 1991; DER and LEES, 1985), nonetheless it is an inescapable conclusion when short- and long-period Q measurements are compared.

Relative t^* measurements between two stations must also depend on spectral shapes more than on amplitude data. In Figure 1 histograms of relative t^* and $\log_{10}$(amplitude) $\sim$ magnitude differences are shown for the station pair OB2NV (at the Nevada test site) and RKON (Red Lake Ontario) for a large number of events. Note that the t^* measurements group tightly while magnitude differences have a large scatter. By regression analyses of a large data set we have found the relation $\Delta m_b \sim 1.35\ \Delta t^*$ (DER *et al.*, 1979), and this apparently is not valid for the individual event pair measurements.

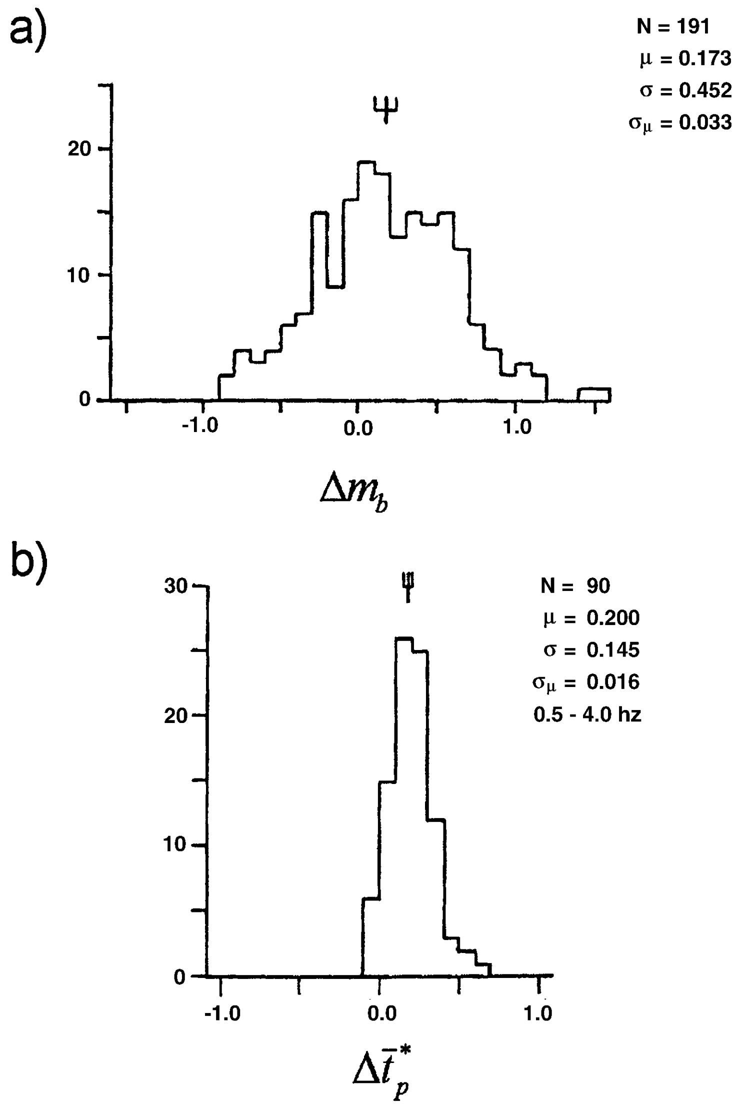

Figure 1
Histograms of the relative P wave t^* differentials and m_b differentials for a suite of common teleseismic events between the stations RKON (Red Lake, Ontario) and the station OB2NV at the Nevada test site. Both stations were located on granite, thus near-surface layering effect is minimized. Note the small scatter in the relative t^* values and the large scatter in relative m_b. Assuming a dominant frequency of 1 Hz, the attenuation could be described in terms of t_p^* differentials as $\Delta m_b = 1.35\ \Delta t^*$. Part of the scatter in m_b differences may be due to radiation patterns, although most must be due to the lateral heterogeneities under the two stations. This figure should be indicative of the relative importance of spectral t^* and amplitude measurements in attenuation studies.

Table 1

Typical time domain measurements to be fitted to differential $t^*(f)$ models (shield-to-shield ~ deep earthquake-to-shield).

S-wave periods (sec.) shield-to-shield	S-wave periods (sec.) shield-to-tectonic	Corresponding amplitude ratios shield/tectonic
1.3	2.5	8
1.8	3	5
2.3	3.4	3.5
P-wave period (sec.) shield-to-shield	P-wave period (sec.) shield-to-tectonic	
0.7	0.9	2–3

With frequency independent Q the spectral ratio and related measurements can define the absolute Q unambiguously. If Q is frequency dependent then the measurement of $t^*(f)$ becomes less direct. Slopes of spectral ratio estimates, which comprise the bulk of attenuation-related data, are mostly influenced by the apparent t^*, $\bar{t}^*$. Over limited frequency ranges any possible gradual changes in these slopes cannot be detected because of the inherent errors in such measurements. Thus the function $t^*(f)$ must be constructed for each path by piecing together various types of information consisting of spectral ratios between assumed source spectra and absolute Q estimates derived from source independent amplitude decay measurements. Additional constraints may be provided from S-wave measurements, making the assumption that most losses occur in shear deformation (DER and LEES, 1985; DER *et al.*, 1986a).

The observed short-period seismic body wave data characteristics from deep earthquakes in South America and in two broad regions of North America (designated as tectonic-T and shield-S) which must be matched by modeling, are summarized in Table 1. Because the S waves observed had varying waveforms in both U.S. regions, and the differential in periods increased with decreased period, this phenomenon by itself is an indication that the change in period was due to attenuation (Table 1). Each pair of models had to account for the observed differential in apparent t^* values, amplitude and period changes for both P and S waves (DER *et al.*, 1980).

When the differential attenuation between two paths, or average differential attenuation between paths involving two types of tectonic regimes must be estimated, then there will be two functions of frequency $t^*(f)$ that must be estimated. Spectral differences expressed as spectral ratios are not sufficient to constrain any of these. It is possible, however, to constrain the relative spacing of such curves by fitting amplitude, spectral content and time domain characteristics for both P and S waves over the two types of paths. An example of such a procedure is described by DER and LEES (1985) for constraining the relative differences in $t^*(f)$, involving the passage body waves through the upper mantle under the "tectonic" western U.S. mantle versus the "shield" type central U.S. mantle. As stated above, the available measurements consisted of station-averaged time domain amplitude levels, spectral ratios over the short-period band, mostly from high frequency analog stations (LRSM) and time domain average wave periods from WWSSN network for both P and S waves. The issue was to decide which of three types of pairs of $t^*(f)$, each pair corresponding to a shield-to-shield (S-S) path and for a shield-to-tectonic (S-T) type path, describes the properties of the data observed. Since the paths from deep earthquakes cross the upper mantle only once, the missing leg of the path is considered 'shield' type implicitly. Two of the three pairs of $t^*(f)$ that have been examined converge at low frequencies and have either strong (CS) or weak (CW) convergence above 1 Hz, the third pair prescribed two curves that were quasi-parallel and did not converge at low frequencies. The high t^* curve for all models was arbitrarily made to converge to 0.8 sec., a value that is not critical for the argument regarding the *relative* run of the curves in each pair. These models are shown in Figure 2.

The issue of such models arose because of the possibility that some pairs of *divergent* $t^*(f)$ models could be consistent with both the observed spectral differences between the Kazakh and Nevada test sites and small differences in explosion magnitude estimates between the two test sites, the so-called 'magnitude bias'. In this case the central United States may be considered as an analog of the Kazakh test site. It was also proposed, on geological grounds, that a better analog of the Kazakh test site would be the northeastern United States (Maine). On the other hand, it is clear, based on geophysical observations, that this cannot be true. Studies of teleseismic body-wave spectra indicate that the northeastern United States is underlain by a mantle of moderate attenuation, resulting in the severe loss of high frequency energy in P and S waves (SOLOMON AND TOKSÖZ, 1970; DER *et al.*, 1982a). In contrast, the P waves from Kazakh explosions typically contain a considerable amount of high-frequency energy, indicating a high Q mantle underneath. Thus despite any geological similarities between Kazakh and the Northeastern United States, the attenuative properties of the mantle underlying them are quite different.

Simulations of the consequences of the three types of models assuming various source pulse shapes (a combined result of sources and attenuation crossing the downgoing leg of the paths) are shown in Figures 3a–c. Source pulses of various duration were convolved with the WWSSN instrument response and the causal minimum phase attenuation operators associated with the frequency-dependent t^* models in Figure 2. Despite the limited bandwidth, the three pairs of $t(f)$ models have quite noticeable consequences with regards to differentials in wave amplitudes and periods, associated with the differences in the two types of upper mantle under North America, the upgoing leg of the ray paths. The QP and CS models both

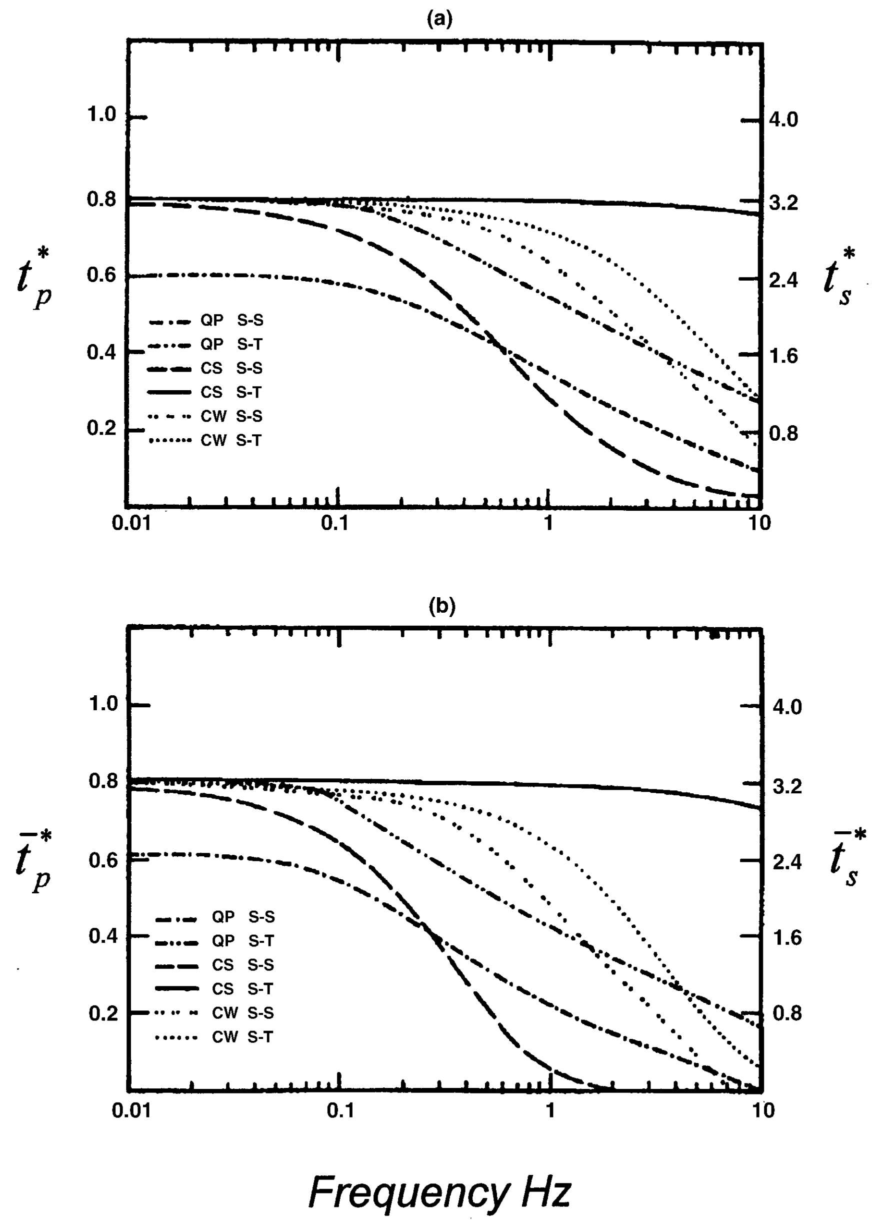

Figure 2
Three pairs of $t^*(f)$ models; a weakly convergent (CW) pair, a strongly convergent (CS) pair and a quasi-parallel (QP) pair. These are to be matched to spectral and waveform data in Table 1 derived from the central and western United States short-period stations.

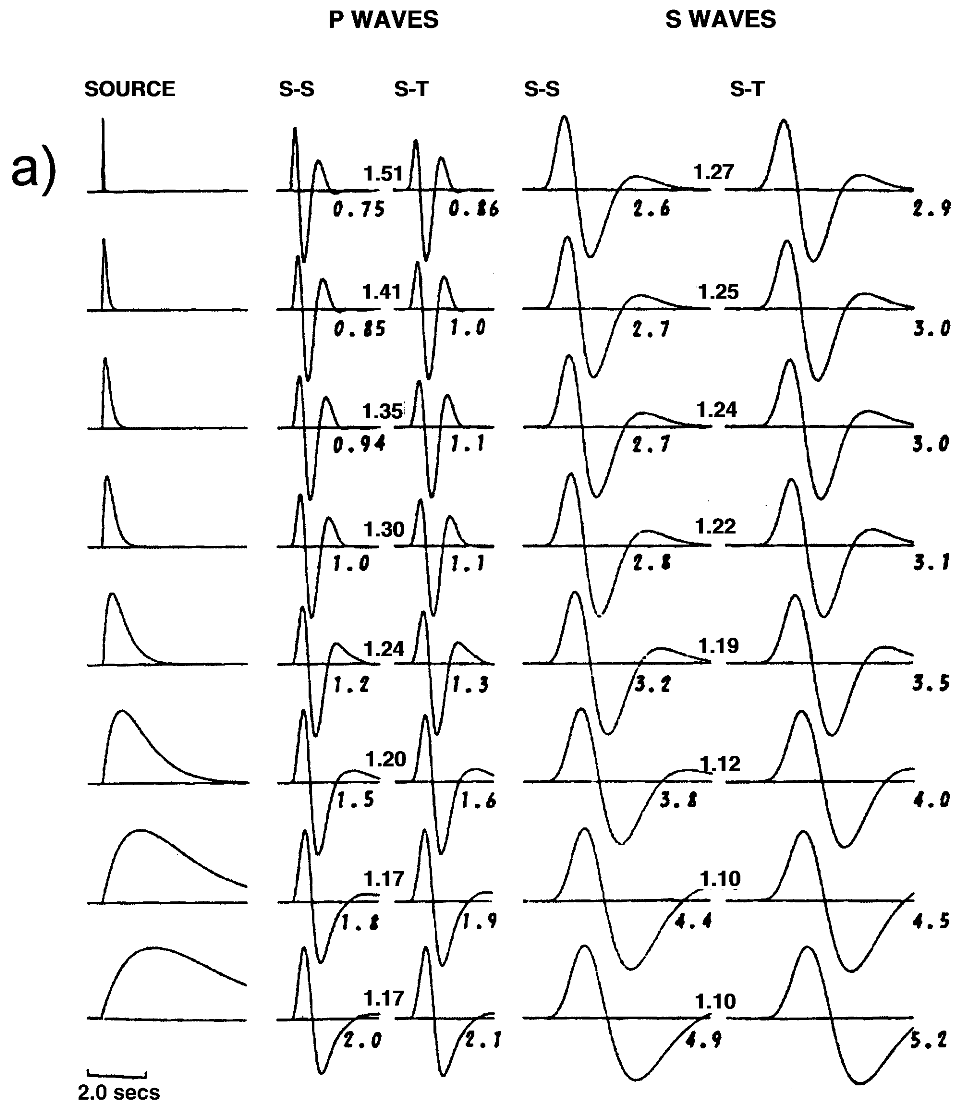

Figure 3(a)

reproduce the observed periods for the waves. Conversely, the observed apparent t^* values, derived from slopes of spectral ratios near 1 Hz, are near 0.2 sec. for *P* waves and 0.8 sec. for *S* waves and are correct only for the *QP* model. The *CS* model, because of its strong frequency dependence near 1 Hz, overstates these by a factor of 3.5. The *CS* model also yields amplitude differentials that are too large. The *CW* model is also totally unacceptable, because it causes excessively small changes in both amplitudes and wave periods.

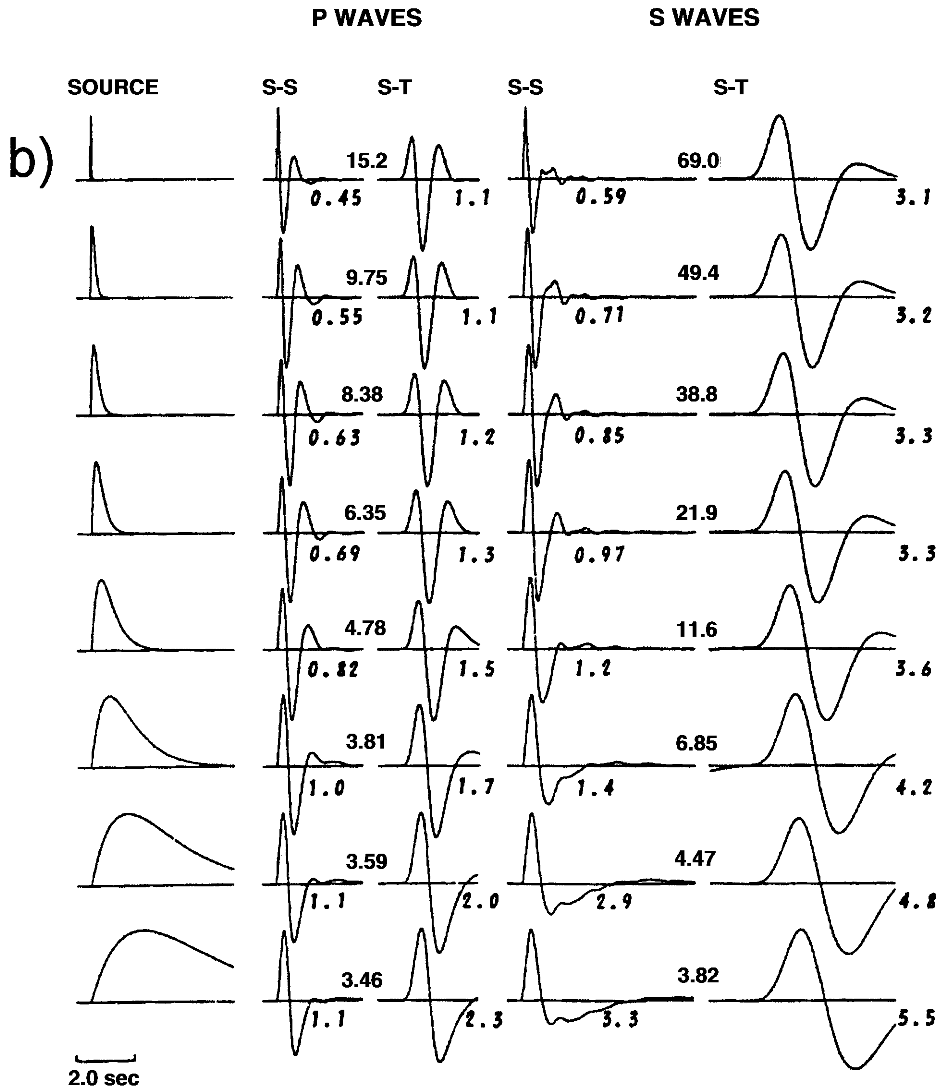
P WAVES
S WAVES
SOURCE
S-S
S-T
S-S
S-T
b)
15.2
0.45
1.1
0.59
69.0
3.1
9.75
0.55
1.1
0.71
49.4
3.2
8.38
0.63
1.2
0.85
38.8
3.3
6.35
0.69
1.3
0.97
21.9
3.3
4.78
0.82
1.5
1.2
11.6
3.6
3.81
1.0
1.7
1.4
6.85
4.2
3.59
1.1
2.0
2.9
4.47
4.8
3.46
1.1
2.3
3.3
3.82
5.5
2.0 sec

Figure 3(b)

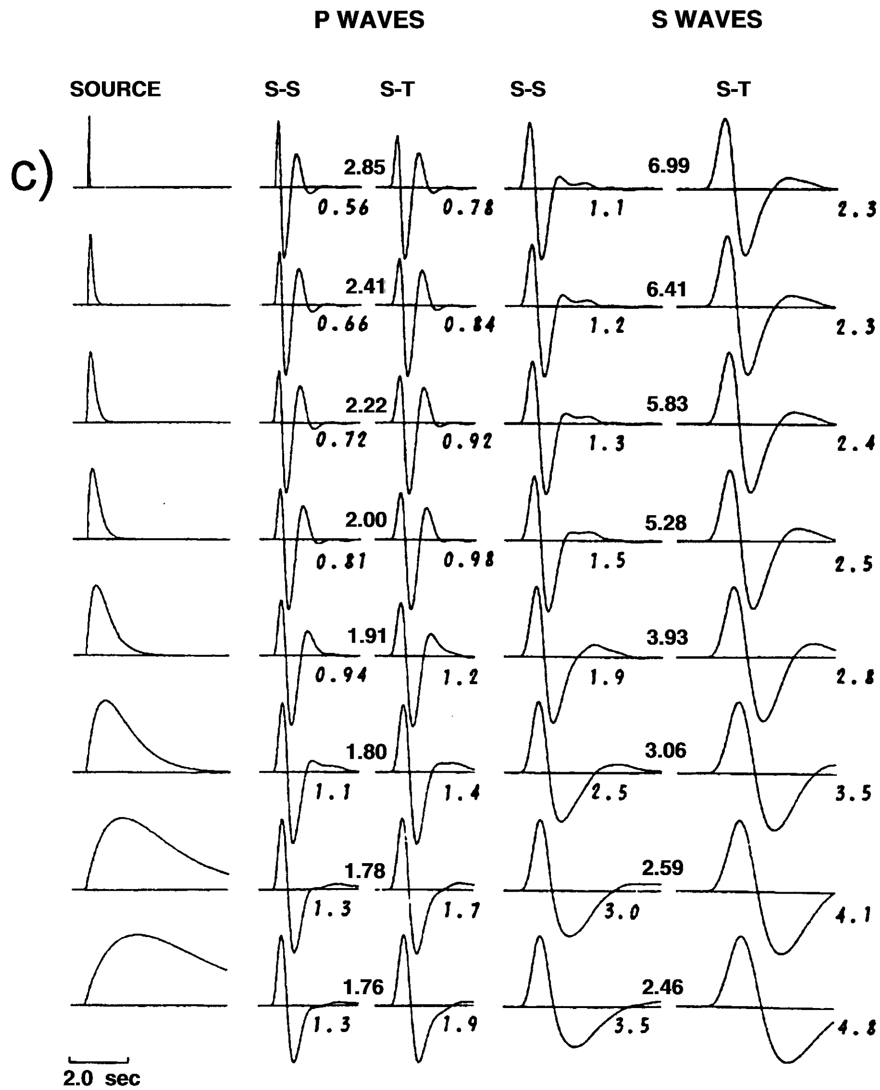

Figure 3(c)

Figures 3 a–c. Synthetic P and S waveforms corresponding to the shield-to-shield (S-S) and shield-to-tectonic (S-T) paths for the (a) weakly convergent (*CW*), (b) strongly convergent (*CS*), and (c) quasi-parallel (*QP*) pair of $t^*(f)$ models computed by using the short-period WWSSN response. All waveforms were normalized to the same amplitude in the plot in order to better observe them. The source pulses are shown on the left. The peak-to-peak amplitude ratios are shown between each pair of waveforms, the dominant periods are shown below each trace in italics. Only the *QP* pair of frequency dependencies matches the observations described in Table 1.

Although these results do not apply directly to the test sites, they drastically reduce the likelihood of the kind of regional variations that could result in such scenarios. Despite the limitations of the data in the standard WWSSN short-period band (0.5–5 Hz), these three models could be easily differentiated by simple simulations, although these results by themselves do not prove frequency dependence. The common trends in the QP model could be removed and the differentials in amplitude and frequency contents in the data would still be satisfied. This exercise demonstrates, however, that large divergences in regional $t^*(f)$ in the short-period band are not likely.

The question of the 'magnitude bias', the prime motivator for this work, was finally resolved by a direct experiment (Joint Verification Experiment–JVE) which established that nuclear explosions of identical size at the Nevada and Kazakh test sites indeed would result in a difference in magnitude estimates at teleseismic distances and that the size of this difference was in good agreement with that predicted by the QP type models (DOUGLAS and MARSHALL, 1996; SYKES and EKSTRÖM, 1989) or a constant t^* differential. This finding also removes the need for arguments concerning Kazakh test site 'analogs' and models very different from the QP types. Further evidence for the inadmissibility of strongly frequency-dependent models in the short-period band was provided by DOUGLAS (1991), who demonstrated that Yellowknife seismograms of the Alma Ata earthquake deconvolved using such models which resulted in physically improbable displacement histories where forward and reversed displacements followed in quick succession.

In order to measure attenuation differences among various paths for common events, ratios of body-wave spectra associated with two paths have been commonly utilized. If the shape and magnitude of the source spectrum can be estimated theoretically, the spectral ratios between an observed spectrum and some assumed source spectrum can be utilized to estimate the attenuation. Obviously, in either case, the fit cannot extend to the frequency range where the noise approaches the signal in amplitude. Thus t^* for paths with high attenuation cannot be measured very accurately because only a small, low frequency, portion of the spectrum is observable above the background noise. Moreover, in such cases relatively small uncertainties in the assumed source functions, which mostly affect the spectral details at low frequencies but not the asymptotic falloff rates, can cause sizeable variations in the t^* derived. This problem is demonstrated by the debate between DOUGLAS *et al.* (1993) and ZHU *et al.* (1993) which involved t^* estimates for a path between Tuamotu and Yellowknife. The apparently high t^* value for this path is apparently due to high mantle temperatures under the Tuamotu region. Nevertheless, such arguments over difference in t_p^* of less than 0.2 sec. (rather than between 0.5 and 1 sec.!) demonstrate the recent progress made in this field.

Some researchers question the appropriateness of a least-squares fitting procedure to obtain slopes of spectral ratios. Clearly, the distribution of the estimated spectral ratio around the theoretical best fit is not normal. Nevertheless, it is unlikely that major errors in the t^* estimated have resulted from such procedures. The other question asks for the best way to compute the spectrum of body wave transients. In most early studies some tapering window function was used and the fitting was done in the frequency range where the S/N ratio was high. For strongly varying spectra the errors of spectral leakage are of some concern. Multi-taper methods (PERCIVAL and WALDEN, 1993; ZHU *et al.*, 1989) reduce such problems if very short (~ 3 sec.) waveform segments are used in computing spectra. Oppositely, using such short segments was avoided in practically all earlier studies and the general results produced by this approach (ZHU *et al.*, 1989) seem to differ little from those of earlier studies. Nevertheless, multi-taper methods are generally preferable to single taper analyses.

Another approach, running the waveforms through a bank of sharply tuned band-bass filters and measuring the energy in various bands, can also reduce the problems with spectral leakage. It was discovered however, that this method produces results very similar to the windowed spectra if tapers with low leakage, such as the Parzen taper, were used (SHARROCK *et al.*, 1995). Even though some of the earlier methods may have biases, estimated much below 0.1 sec., they were certainly adequate to support the most important conclusions drawn from them. These conclusions involve the absolute sizes of t^* in the short-period band (much less than 1 sec. for P waves), the sizes of relative variations across North America 0.2–0.3 sec.), and the general need for frequency dependence. The main limitations in such studies are the variability and scatter in spectral and amplitude data at high frequencies and the uncertainties in the assumed source spectra, not the methodology in computing spectra.

In view of new tomograpic studies of the earth's velocity and Q structure (ROMANOVICZ *et al.*, 1987; ROMANOVICZ and MONTAGNER, 1990; ROMANOVICZ, 1990), the crude regional subdivisions of the upper mantle into generic tectonic regimes used in past studies appear to be obsolete and require re-evaluation. With the increasing number of broadband digital seismic stations and regional networks, true tomographic studies of Q at high frequencies are now feasible. Due to the extreme sensitivity of the spectra of body waves to Q at high frequencies, the Q effects dominate. Thus one can map out the absorbency of the earth in three-dimensional grids using simple tools of spectral analysis. Whenever applicable, variations due to other factors, such as near-surface layering may be corrected, but such effects are of minimal importance.

In computing the apparent t^* from spectral ratios of body waves and theoretical source spectra errors will occur due to the uncertainties in assumed source spectra. There are several models for source spectra from nuclear explosions. Most assume a ω^{-2} falloff rate of spectral amplitude of the source spectra at high frequencies (VON SEGGERN and BLANDFORD, 1972; MUELLER and MURPHY, 1971). Some assume a ω^{-3} falloff rate at high frequencies adjoining a transitory ω^{-2} rate at lower frequencies (HELMBERGER and HADLEY, 1981). All these have different scaling laws and spectral overshoots. None of these models are well enough defined to rule out others. Typically the reduced displacement potentials (RDPs) for above may fit the data optimally, on occasion, for individual explosions. Consequently there will be unavoidable uncertainties in Q (or t^*) depending on the models. Most seismologists prefer a ω^{-2} falloff rate for earthquakes (e.g., AKI, 1967). A more complex earthquake source spectral model that includes ω^{-1}, ω^{-2} and ω^{-3} falloff rates was proposed by GELLER (1976). Assuming that t^* is derived from spectral ratios at the high frequency end beyond the corner frequencies in the short-period band up to 10 Hz, and the ω^{-2} and ω^{-3} falloff rates translate into a difference of t^* of about 0.1 sec., with the *lower* value associated with the ω^{-3} falloff. In any case despite the uncertainties the t^* are fairly well constrained at high frequencies.

An additional complication factor for nuclear explosion is that anelastic losses seem to occur in the near explosion environment, in the range where linearity was assumed previously (TRULIO, 1978; MINSTER, 1978a,b; DAY and MINSTER, 1984). Such losses were not accounted for by the source models mentioned above. These losses explain some apparent discrepancies between absolute amplitudes of the elastic waves leaving explosion sources at close distances and those observed teleseismically that apparently required large values of t^*.

Despite the fact that digitized old photographic recordings from narrow-band systems are generally unsuitable for spectral analyses, some time domain features of these seismograms can be used to delimit $\bar{t}^*$. Assuming various limiting values of source time functions (such as delta functions, or double delta functions with opposite polarities), it can be shown that dominant periods or rise times measured from the time domain records are limited by the Q along the path (STEWART, 1984). Such arguments were used to put lower limits on Q in several studies (DER *et al.*, 1982a, 1986). The study by CHOY and CORMIER (1986) has shown how such constraints and fitting of spectra can contribute to better Q estimates in time domain studies.

4. Regional Variations of Seismic Body-wave Attenuation at High Frequencies

Large-scale regional variations of body-wave attenuation have been discovered early during the development of plate tectonic theory in areas of subduction near Fiji (OLIVER and ISACKS, 1967) and were found later in other areas as well. Although some values were assigned to Q_p and Q_s, most of these studies relied on qualitative observations, noting the variations in the dominant periods and amplitudes of body waves. Although such variations could be explained by other means for individual events, such as source directivities, the authors of such studies presented convincing evidence that such variations can be explained consistently only by regional variations in Q in the upper mantle. The variations in Q as postulated from these anomalies in wave amplitudes and frequency contents generally coincide with corresponding variations in mantle velocities, low Q values being accompanied with the lowering of both the body-wave and shear-wave velocities by increased temperatures. Later studies which used surface reflections also confirmed the existence of low Q zones under the backarc basins of many subduction zones (BARAZANGI *et al.*, 1975). Manifestations of Q variations in the short-period band are quite obvious in the visible variations of the amplitudes and dominant periods of body waves, especially of S waves. These findings correlate well with other indications of mantle temperature variations such as heat flow and mantle conductivity.

Figure 4 presents examples of 3-component seismograms recorded on the WWSSN short-period system for two stations in the Tonga-Fiji region. The station NIU (top) shows high-frequency waveforms associated with a high Q path, the lower set of seismograms is the same event seen at VUN along a low Q path with considerably lower dominant frequency of waveforms. Despite narrow bandwidth of the WWSSN system, the contrast is quite visible. These are just two examples of a large-scale study of waveform frequency contents and amplitudes that found large lateral variations in the upper mantle Q associated with a subducting slab. Equivalent variations have also been confirmed in other subduction regions.

Another example of visible Q effects is from a study of the upper mantle Q in areas behind island arcs using pP waves from deep earthquakes (BARAZANGI *et al.*, 1975). The top four traces in Figure 5 display teleseismic seismograms where the pP phase crossed high Q upper mantle (slab), and two at the bottom where the pP

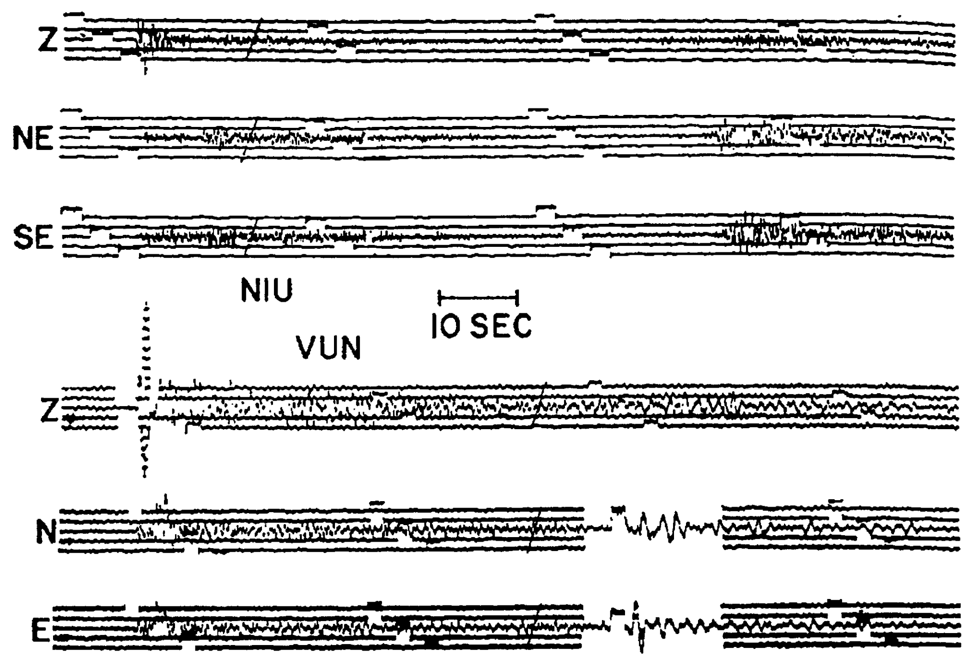

Figure 4
Short-period seismograms recorded at the station NIU, Tonga, involving a high Q path, and at the station VUN, Fiji, along a low Q path from an event of December 10, 1965 at 18.1°S, 179.3°W, depth 624 km. Note the contrast between the high frequency nature of both the P and S phases at NIU as contrasted by the much lower frequencies for the two phases at VUN (after OLIVER and ISACKS, 1967).

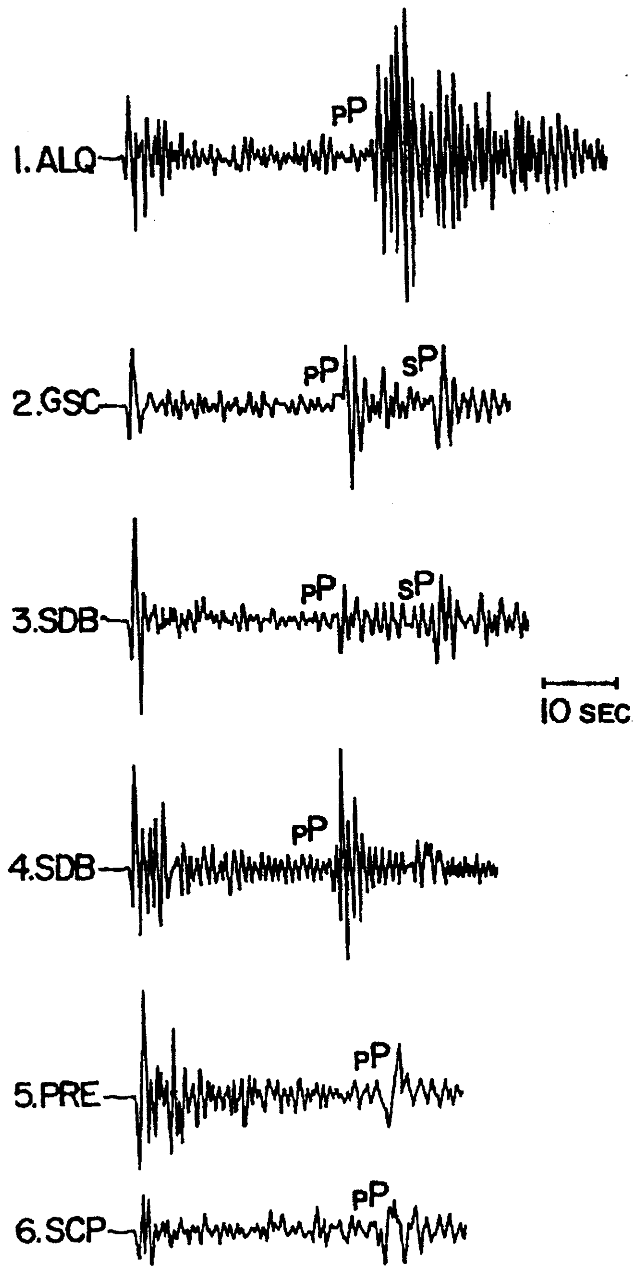

Figure 5
Examples of WWSSN seismograms containing pP phases from deep earthquakes that passed through portions of upper mantle with low attenuation (1, 2, 3, 4) and high attenuation (5 and 6). (After BARAZANGI *et al.*, 1975). Note the differences in the frequency content of pP.

phase traveled through low Q mantle (backarc basin). The differences in the sizes and frequency contents of these arrivals are quite obvious. Thus Q effects in the short-period band are often not subtle, but may be detected by visual inspection.

Another interesting region with respect to lateral variations in high frequency Q already mentioned is North America. The regional variations in this continent were first outlined by SOLOMON and TOKSÖZ (1970) who found, by the analyses of P and S waves from deep earthquakes recorded over North America, a low Q region west of the Rocky Mountains, high Q areas in central North America and the Pacific Coast and some lower Q under the eastern part of the United States. Even though the manifestations of this variation in Q are subtle at low frequencies, they are quite evident when short-period P- and S-body wave data are analyzed. Firstly the short-period P- and S-wave amplitudes vary in a geographical pattern consistent with the picture provided by SOLOMON and TOKSÖZ (1970), NORTH (1977), BOOTH *et al.* (1974), and DER *et al.* (1980). A simple montage of S waveforms from deep earthquakes as observed across North America shows the radical changes in their

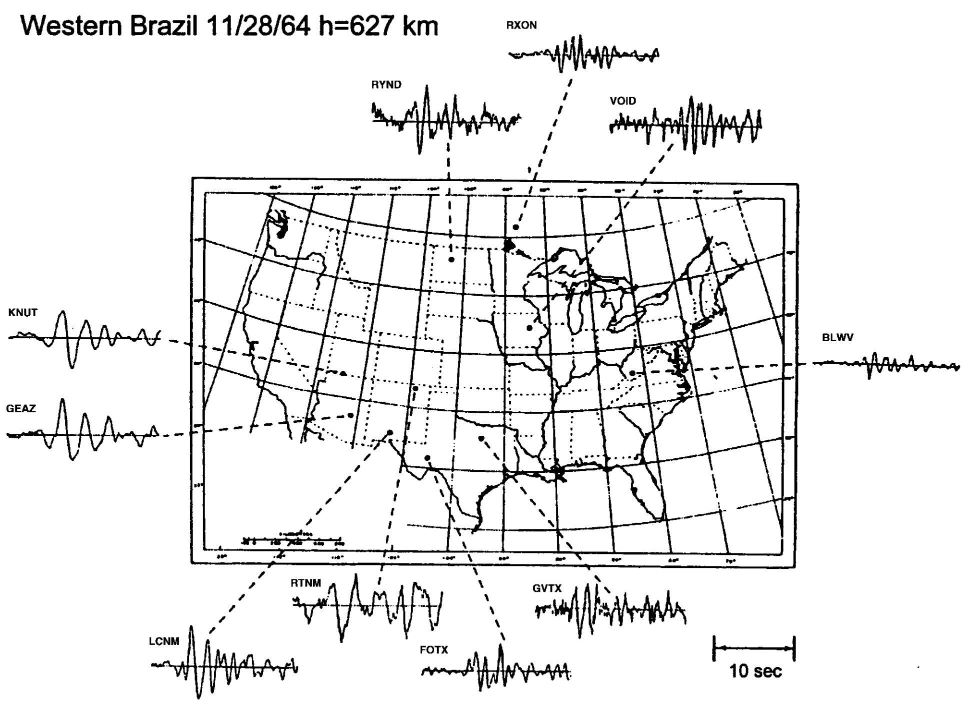

Figure 6
Teleseismic S waveforms from a deep earthquake across the United States. These waveforms are tracings from film and were not corrected for the scalar instrument magnification but the instruments match in the shapes of frequency responses. This figure indicates comparable attenuation-induced variability in frequency contents as those encountered in the studies of backarc basins, high frequency waveforms east of the Rocky Mountains and low frequency waveforms west of it (after DER *et al.*, 1980).

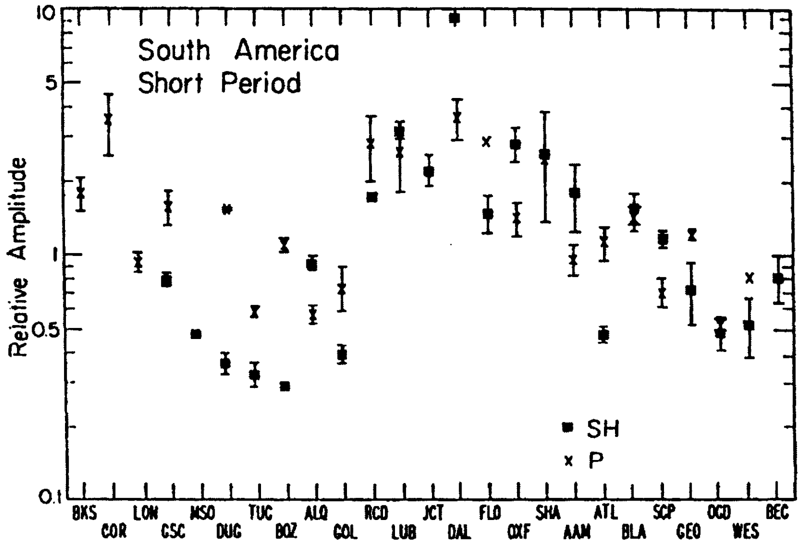

Figure 7
Relative P- wave and SH-amplitude variations across North America (after LAY and HELMBERGER, 1981) derived from deep earthquake data. The amplitudes were corrected for the appropriate double-couple radiation patterns. The seismic stations are arranged to form a west-to-east cross section. The P and SH variations track each other, the variations in the latter being only somewhat larger because typically the P waves have more high frequency content than the SH. Note the low amplitudes in the western mountainous regions (DUG, TUC, ALQ, GOL, BOZ) and the high amplitudes in central North America (RCD, LUB, JCT, DAL, FLOW and OXF) and the West Coast (BKS and COR). Intermediate and low values are found on the East Coast (ATL, BLA, SCP, GEO, OGD and WES).

spectral contents (Fig. 6), the S waves observed in the southwestern United States have considerably longer periods than those elsewhere. Amplitudes of both P and SH waves from deep earthquakes disclose a remarkable pattern (Fig. 7). First of all, the amplitude patterns of P and SH waves from a later study track each other however, the contrast for SH waves is larger (LAY and HELMBERGER, 1981). They also follow the pattern of SOLOMON and TOKSÖZ (1970) and later workers; low amplitudes in the Basin and Range, high amplitudes in central North America and the Pacific Coast. The P-wave spectral and amplitude differentials between the Basin and Range and the central United States found in the studies of DER and MCELFRESH (1976, 1977) can be explained by two models of upper mantle shown in Figure 8. The western U.S. model (WUS) in this figure is derived by a reinterpretation of the findings of ARCHAMBEAU *et al.* (1969) in which the Q losses are reassigned mostly to the western end of their SW-NE profile, which extended from the Nevada Test Site to the north central U.S.

In contrast to the old narrow-band WWSSN data the differences in mantle Q_p can be easily seen when modern new data from the U.S. National Seismic Network are analyzed. We have selected six stations of this network with identical amplitude responses. Two from the central U.S (MIAR–Mt. Ida, AR, AAM–Ann Arbor MI), two from the eastern seaboard (BINY–Binghamton, NY, BLA–Blacksburg, VA) and two from the Basin and Range (ALQ–Albuquerque, NM, TPNV–Topopah, NV). According to previous studies (e.g., SOLOMON and TOKSÖZ, 1970) these three pairs are listed in the order of increasing attenuation. Unlike the P waves from the WWSSN network, the decrease in high frequency contents in P waveforms are immediately visible from top to bottom of Figure 9. Passing a pair of these seismograms for AAM and ALQ through a bank of band-pass filters also evidences the contrast (Fig. 10). Finally, the spectra for eight stations computed by averaging five eigenspectra which utilize the multi-taper method (Fig. 11) clearly show the contrast between the spectra observed in the central United States (Fig. 11a) with those of the stations west of the Rocky Mountains (Fig. 11b). While the former have spectra that decrease gradually and, where the background noise level allows it, remain above the noise

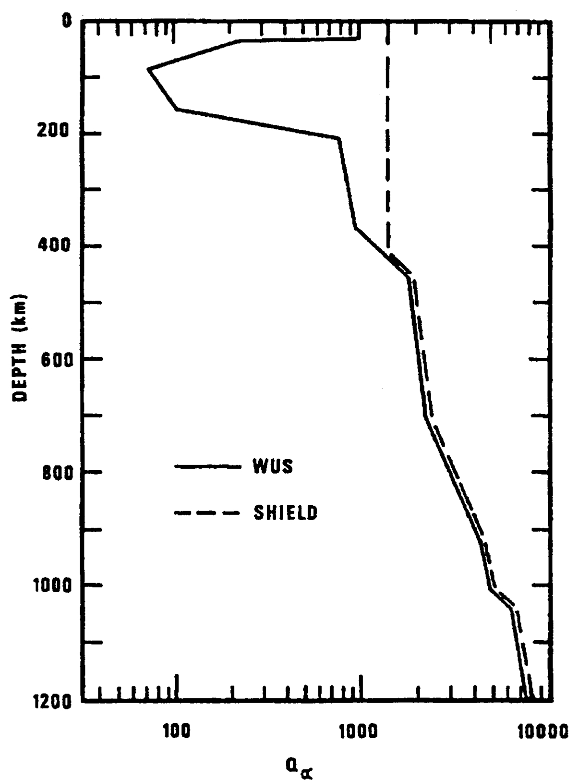

Figure 8
Proposed Q_p models for short-period P waves under the tectonically active western United States (WUS) versus the north central United States (shield). The WUS model is a modified version of the model of ARCHAMBEAU *et al.* (1969).

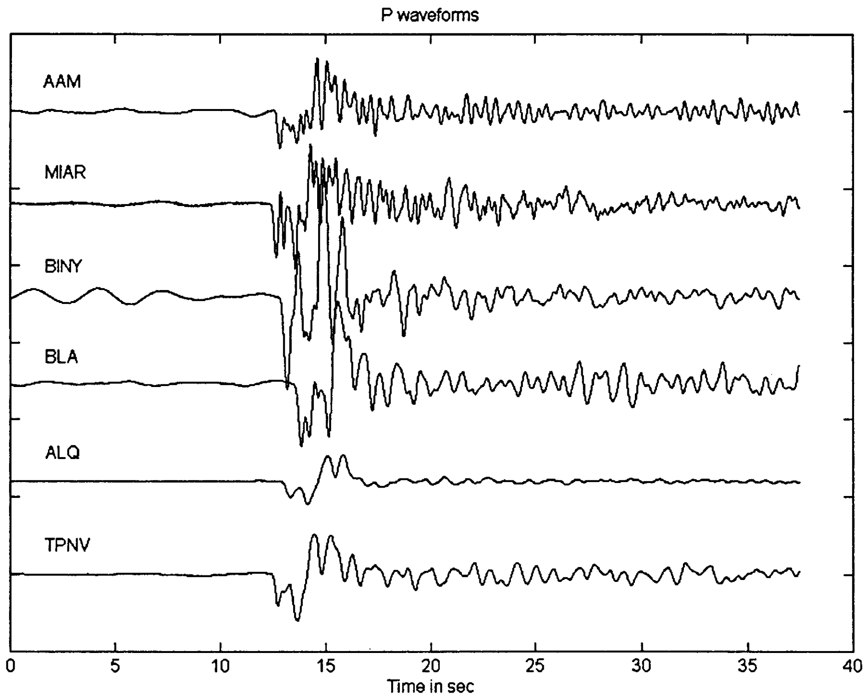

Figure 9
P-wave waveforms from a deep earthquake in Jujuy Province, Argentina (July 20, 1997, 22.69S, 66.02W, depth = 256.5 km) as recorded at stations of the National Seismic Station Network. Since the frequency responses of these stations increase and flatten out towards the higher frequencies, the visible differences in the frequency contents of these waveforms at the stations in the central (left) and western (right) United States are quite dramatic. This phenomenon is considerably harder to see with seismic system responses employed earlier.

at frequencies up to 7–10 Hz, the spectra associated with the latter rapidly decrease with frequency and reach the noise level below 6 Hz. This happens even though the overall signal-to-noise ratio is lower at the western stations. The results are in agreement with the picture of the regional variations from other studies presented above. It is noteworthy that the relative spectral contents of seismograms, the *S* waveforms in several studies quoted above (DER *et al.*, 1980, 1982a; LAY and HELMBERGER, 1981) and the *P*-wave spectral results by DER *et al.* (1980) correlate very well with the upper mantle velocity inversion results which exhibit a pronounced upper mantle low velocity layer in the western part of the continent (VAN DER LEE and NOLET, 1997). The features of their map show very low velocities in the Basin and Range, higher velocities on the eastern seaboard and California; details that correlate well with attenuation measurements. Besides the broad regional features similarities include the sudden transition from Texas to New Mexico (DER and MCELFRESH, 1977) and the higher velocities (lower attenuation) in the Montana and Idaho area.

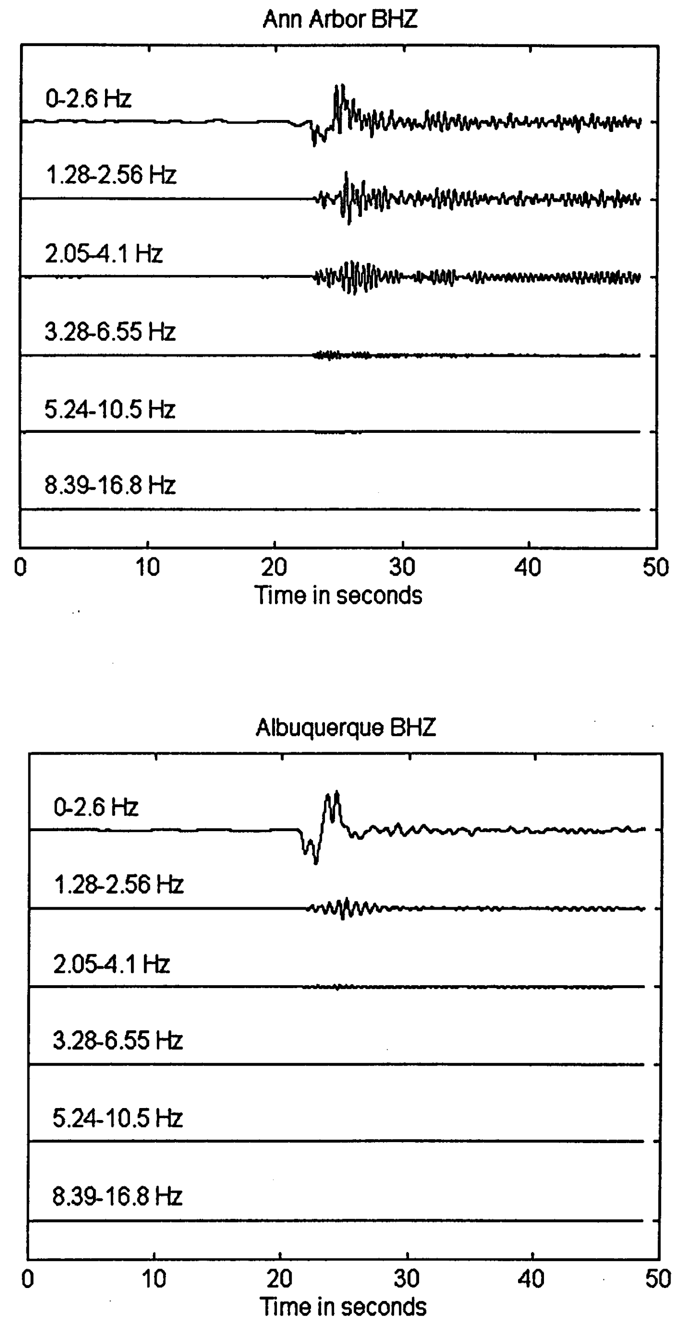

Figure 10
Band-pass filter results for a pair of short-period vertical seismograms for two stations (Ann Arbor, MI and Albuquerque, NM). Note the differences in spectral contents and the diminished high frequency content of the lower example.

The differences in the mantle Q under various former nuclear test sites can be detected *reciprocally*, noting spectral differences at common high Q recording sites. All P waveforms from the Nevada test site appear to be deficient in high frequency energy when compared to test sites in high Q areas such as the Kazakh test site as shown in Figure 12. In this figure we compare spectra of Nevada (Yucca Flats and Pahute Mesa) and Kazakh (Degelen and Shagan) nuclear explosions at the common sensor A0 of NORSAR which were arranged in the order of decreasing estimated yields (body-wave magnitudes) from top to bottom. The Kazakh explosion spectra fall off less rapidly with increasing frequency and remain above the noise level to higher frequencies (DER *et al.*, 1985). DOUGLAS (1987) and BACHE *et al.* (1985) reported the corresponding phenomenon at the British array sites. Because of explosion source scaling effects, this effect is seen better at low yields in the figure. The same is true for earthquakes; intraplate earthquakes inside areas underlain by high Q mantle contain more high frequency energy. Another study that confirmed the low Q nature of the mantle under much of western North America consisted of

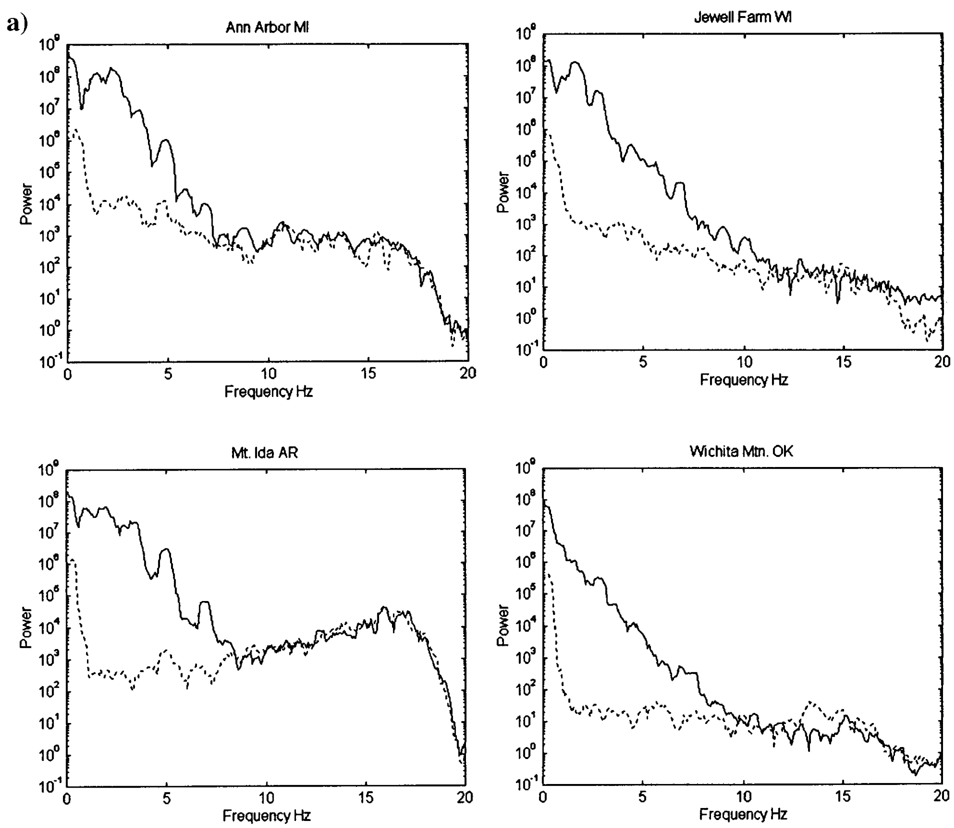

Figure 11(a)

b)

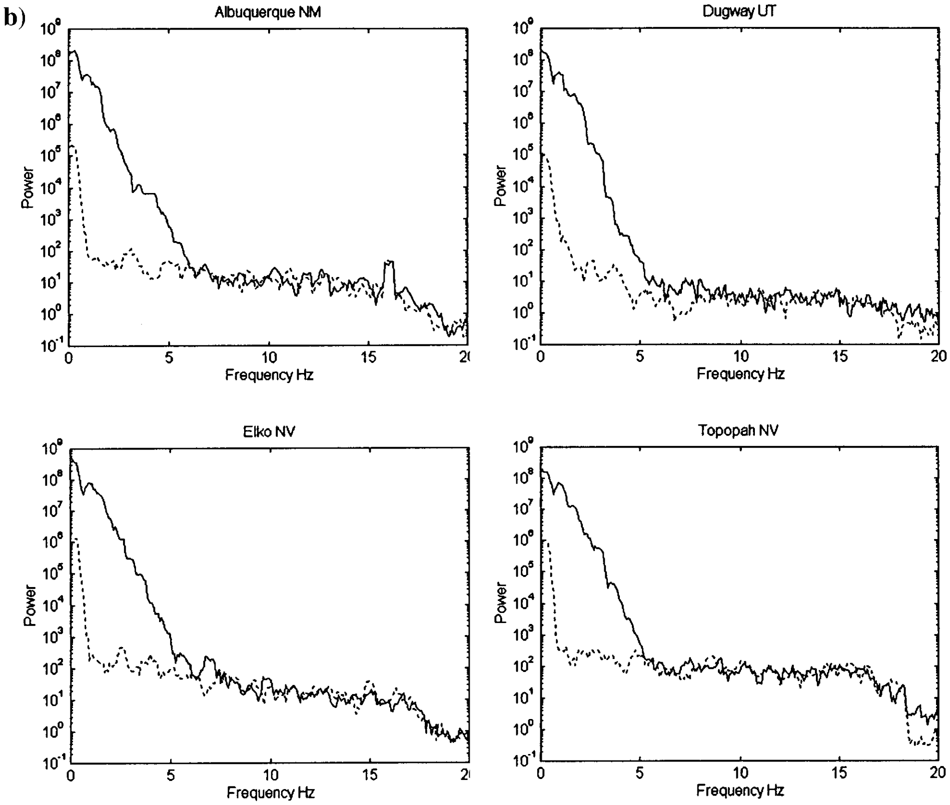

Figure 11(b)

Figure 11a & b. (a) Spectral of P waves of the Argentina events as recorded in the central United States, (b) as recorded west of the Rocky Mountains. Dotted lines are noise spectra. These spectra were computed by the multi-taper method averaging the first five eigenspectra and were not corrected for the identical instrument responses. Note that while the spectra fall off gradually and are above the noise in the 6–8 Hz range in the central United States, those in the west fall off sharply and show no significant energy above the noise above 5 Hz.

the spectral analyses of P waveforms from the Salmon nuclear explosion in Mississippi (DER and MCELFRESH, 1977). The P waves from this explosion contain

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Figure 12

Comparison of P-wave power spectra of nuclear explosions from the Nevada test sites (Yucca Flats and Pahute Mesa) and the Kazakh test sites (Shagan and Degelen) computed for the A0 array site of the Norwegian Seismic Array (NORSAR). The estimated yields increase from top to bottom. The spectra were not corrected for instrument and were terminated as the S/N ratio fell below 2. Note that all spectra from the Kazakh test sites fall off slowly with increasing frequency and indicate the presence of high frequency signal energy reaching 9 Hz. In comparison with these, the spectra of Nevada explosions fall of faster and do not show energy above the noise beyond 3.5 Hz. These spectral differences indicate high mantle Q under Kazakh and low mantle Q beneath the Nevada test site.

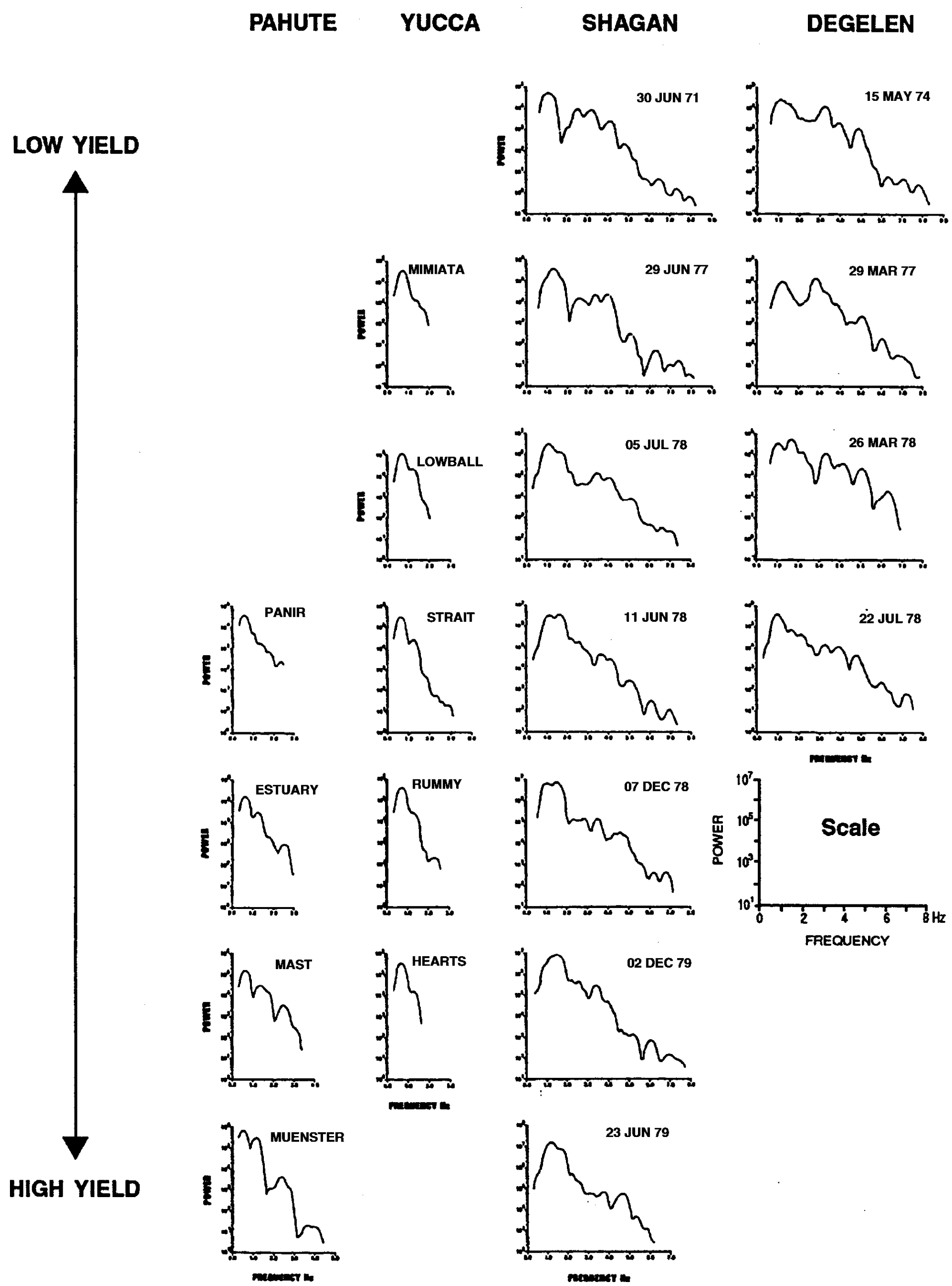
PAHUTE
YUCCA
SHAGAN
DEGELEN
LOW YIELD
HIGH YIELD
30 JUN 71
15 MAY 74
MIMIATA
29 JUN 77
29 MAR 77
LOWBALL
05 JUL 78
26 MAR 78
PANIR
STRAIT
11 JUN 78
22 JUL 78
ESTUARY
RUMMY
07 DEC 78
MAST
HEARTS
02 DEC 79
MUENSTER
23 JUN 79
POWER
Scale
FREQUENCY
8 Hz

Figure 12

more high frequency energy at the seismic stations located in the eastern United States than those located in the Colorado Plateau and the Basin and Range.

Assembling worldwide t_p^* estimates derived from high frequency falloff rates of spectra from both earthquakes and explosions and sorting them according to generic types of paths (Fig. 13) reveals a large variability in t_p^* (DER *et al.*, 1982b). The means vary from 0.15 sec. to 0.4 sec. and the variance for the mixed and tectonic path categories is larger. This must be a reflection of the heterogeneity of the 'tectonic' population. Even though these t^* values are crude, it is revealing that only a single estimate approximated 1 sec. A similar observation was made by SHARROCK *et al.* (1995) for teleseismic observations of P waves from Pacific subduction zone events. It appears that high frequency attenuation varies considerably worldwide, but most t_p^* values remain in the 0–0.5 sec. range and values close to 1 sec. must be rare.

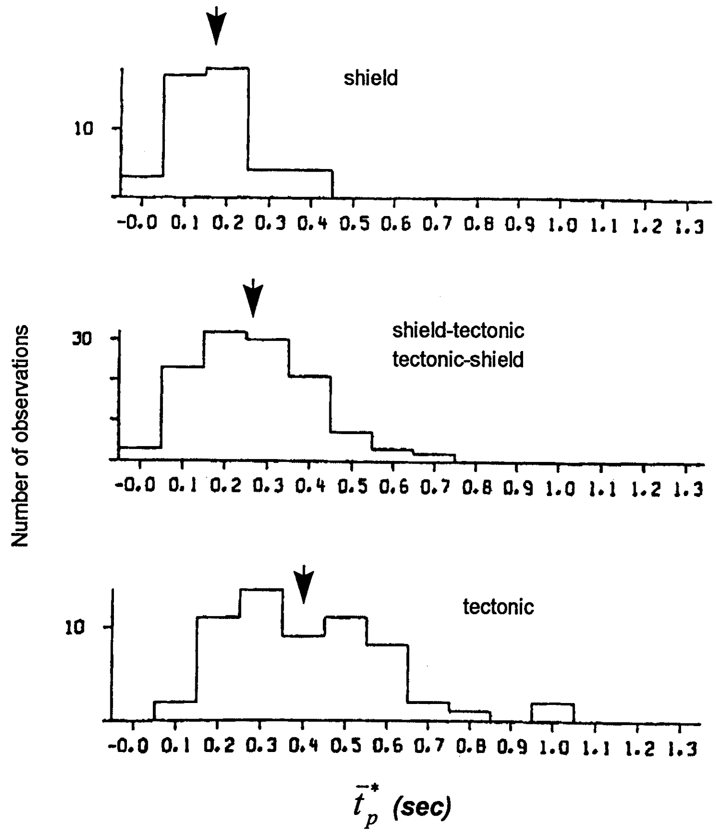

Figure 13

Histograms of worldwide P-wave t^* estimates sorted with respect to generic path types. The mean t_p^* increases from top to bottom (arrows) for each of these populations, indicating that t_p^* varies considerably on the global scale and increases with increasing 'tectonic' component in the propagation path.

5. *Frequency Dependence*

It has been pointed out by many investigators that it is generally impossible to reconcile the Q values derived from long-period data with the spectral content of teleseismic body waves (SIPKIN and JORDAN, 1979; LUNDQUIST and CORMIER, 1980; DER *et al.*, 1982a; CLEMENTS, 1982). If teleseismic body-wave spectra are corrected for a constant t^* then the high frequency end of the spectrum will increase exponentially in such a way that the result is incompatible with source proposed spectra of either earthquakes or explosions (AKI, 1967; MUELLER and MURPHY, 1971; HELMBERGER and HADLEY, 1981).

Unfortunately, most absolute Q figures derived from long-period data are also subject to unknown errors due to various lateral heterogeneities in the earth. The highly heterogeneous nature of the core mantle boundary and the earth's near surface will have an effect on the multiple *ScS* measurement. Similarly, lateral heterogeneities will cause a loss of energy in the various free oscillation modes through scattering and intermode conversions. If most of the measured losses in such measurements result from anelastic Q then frequency dependence must exist. To derive the actual function $t^*(f)$ one must reconcile both absolute Q estimates and relative measurements of $\bar{t}^*$ for a given region. This was attempted by DER *et al.* (1986c) for the shield areas of Eurasia and by DER and LEES (1985) for the continental United States roughly subdivided into central and western regions. In both of these studies we attempted to match a variety of frequency domain and time domain characteristics of seismograms for both P and S waves to models of frequency dependence.

The first study fitted a complex Q model to a large set of diverse data. As was typical for the mid-1980s we had only digital array data (NORSAR), some digital Seismic Research Observatory (SRO) data and WWSSN photographic data. For the latter only waveform characteristics such as dominant periods and rise times could be fitted. Hand-digitized WWSSN data were found to be too unreliable for analyses of high frequencies. The array and SRO data provided digital data for multiple *ScS* and body-wave (P and S) surface reflections, a profile consisting of Russian PNE (peaceful nuclear explosions). Added to these were some PP/P spectral ratios from NORSAR for constraining Q_p. Collecting all these we have derived a frequency-dependent Q model summarized in Figure 14. The $t^*(f)$ and $\bar{t}^*(f)$ curves appropriate to these data are shown with the constraints (shaded boxes) imposed by various kind of measurements in Figure 15. It appears, however, that in spite of the uncertainties in these constraints, frequency dependence is required.

The latter study also demonstrated the difficulties in reconciling a variety of observations and measurements to arrive at some satisfactory solution for a $t^*(f)$ for two generalized regions. In the latter study we matched the relative averaged time domain amplitudes, spectral ratios, dominant periods of short period P and S waves from deep earthquakes across the United States to three pairs of frequency-dependent

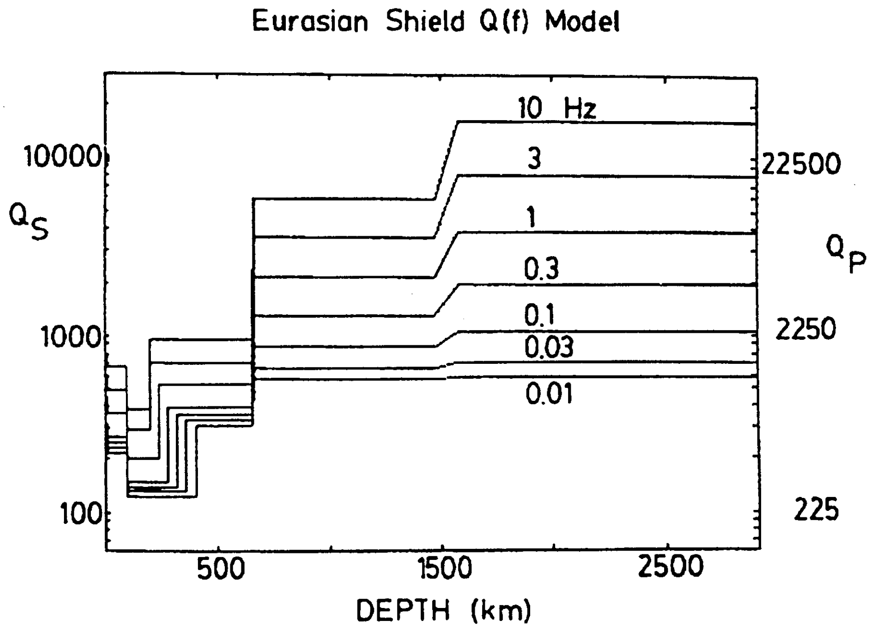

Figure 14
The EURS Q model. Each line is the plot of Q versus depth for a different frequency. This is essentially the same generic kind of model as those proposed by MINSTER and ANDERSON (1973) and LUNDQUIST and CORMIER (1980), but adapted to a high mantle Q 'shield' environment. (After DER *et al.* 1986b.)

models. The procedures followed and the results are discussed as a demonstrative example in the methodology section above.

Worldwide it was found that the values of low frequency attenuation measurements from multiple *ScS* and the overall frequency characteristics of long-period body waves as interpreted in terms of Q crossing the upper mantle, varied in the same sense (CHAN and DER, 1988; SIPKIN and JORDAN, 1980; LAY and WALLACE, 1983; NAKANISHI, 1979). Typically relatively low attenuation (t_s^* values in the range 1.5–2.5 for two-way travel through the upper mantle) was found under old continental shields and old oceans, while higher values (t^*s in the range 3.5–5) are characteristic for ocean ridges (SCHLUE, 1981; SHEEHAN and SOLOMON, 1992), tectonically active areas of Eurasia (DER *et al.*, 1986a), Mexico (LAY and WALLACE, 1983). The picture that emerges from worldwide studies defines an earth with large three-dimensional variations in frequency-dependent Q which have been outlined only in some interesting, highly anomalous areas. Much of Eurasia outside of tectonically stable areas is still poorly explored. The forms of frequency dependence are still not understood well.

There has been considerable recent progress in the understanding of the earth's three-dimensional velocity structure recently based on results of tomographic studies. Therefore, several of the old categories such as 'shield' and 'tectonic' lost

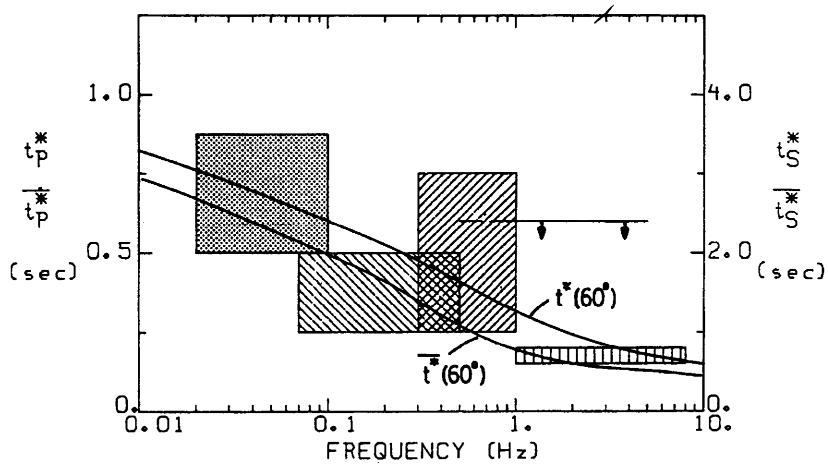

Figure 15
Plot of $t^*(f)$ and $\bar{t}^*(f)$ for direct S and P at 60° as predicted by the EURS Q model. The curves are superposed on constraints imposed by various $\bar{t}^*$ observations (boxes); among them spectral constraints in the short-period band (rightmost box), short-period waveform constraints (downward arrows). In the long-period range these include *S-SS-SSS* and multiple *ScS* constraints (boxes on the left). Note the differences between the actual and apparent t^*. This is an illustration of how various types of measurements can be fitted together to construct a frequency dependent t^* model (after DER *et al.*, 1986; LEES *et al.*, 1986).

their original meaning as laterally homogeneous provinces of the earth. The emerging picture of the velocity structure of the earth requires the reevaluation and reinterpretation of the Q structure as well. With the increasing number of digital seismic stations recording at high sampling rates (see the report by the NATIONAL RESEARCH COUNCIL, 1995) and broad frequency bands, the time has come to explore global variations of Q by mapping the absorbency of the upper mantle to high frequency P and S waves. As can be seen from the discussions above, such mapping can be accomplished by applying quite simple, broadband frequency domain analysis methods to high frequency seismograms. The determination of the three-dimensional Q structure of the earth will provide useful constraints on the temperature distribution and thus on the deductions of the internal dynamics of the earth.

6. Conclusions

Due to the spatial variability of high frequency teleseismic P and S waveforms, and to a lesser degree, of their seismic spectra, deterministic waveform modeling is not an effective method for estimating anelastic attenuation at high frequencies.

The reasons are that spectra of synthetic seismograms generally do not match the spectra of the data at higher frequencies, i.e., waveform fitting becomes increasingly ineffective for higher frequencies.

The issue is, essentially, the absorbency of the lithospheric and upper mantle structures to high frequency seismic body waves. Owing to the extreme sensitivity of the spectral shapes to Q variations, as opposed to other factors, the Q at high frequencies can be easily measured by noting the variations in the spectral shapes along various paths. The regional variations derived from spectral variations correlate well with gross patterns of amplitude variations of both P and S waves at high frequencies, despite the substantially greater scatter in amplitude data occasioned by other factors. Most data support anelastic losses in shear deformation in the mantle and no evidence has been found which favors significant losses in compression at high frequencies.

Q in the earth appears to be frequency dependent, based on the consistent pattern of lower Q estimates from long-period data than for high frequency data for the same general regions. Nevertheless, no evidence has been found for rapid variations of teleseismic t^* values with frequency. The available data can be easily reconciled with slow, gradual variations of Q throughout the observable seismic band. Gross regional variations in t_p^* are in good agreement with t_s^* variations after the latter are divided by the factor of four in both the short- and long-period bands, while the absolute values of these appear higher in the long-period band. This supports the idea of quasi-parallelism of regional $t^*(f)$ curves.

The degree of anelastic attenuation of high frequency body waves varies considerably over the mantle. Large variations have been shown to exist in many areas of the world where subduction takes place. While the downgoing slabs are characterized by low attenuation, the upper mantle Q values under backarc extensional basins are generally low. The mantle attenuation beneath the oceans seems to decrease with the increasing age of the overlying crust, starting with extremely strong attenuation near mid-ocean ridges. There is a large range of variation in the attenuative properties of the mantle under continental North America. The Basin and Range province is characterized by high mantle attenuation, central North America with low attenuation while the eastern seaboard has a moderate degree of mantle attenuation. Many areas of tectonically active parts of Eurasia are only poorly explored in this respect but tend to have lower mantle Q than the 'shield' areas. South America is characterized by pockets of low mantle Q in some areas, such as the Altiplano.

With the increasing number of digital seismic stations recording at high sampling rates and broad frequency bands, there is an opportunity to explore and reevaluate the global variations in the absorbency of the upper mantle to high frequency P and S waves and thus refine the broad picture outlined above. Regional seismic networks and portable seismometer configurations are especially suitable to map small-scale variations in mantle Q. Many of the difficulties encountered in earlier studies due to photographic recordings can be avoided by using broadband data. Such data can be applied to the direct estimation of broadband attenuation operators. The information derived from Q tomography, complementary to velocity tomography, will result in a better understanding of the temperature variations within the earth and the dynamics of the upper mantle.

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