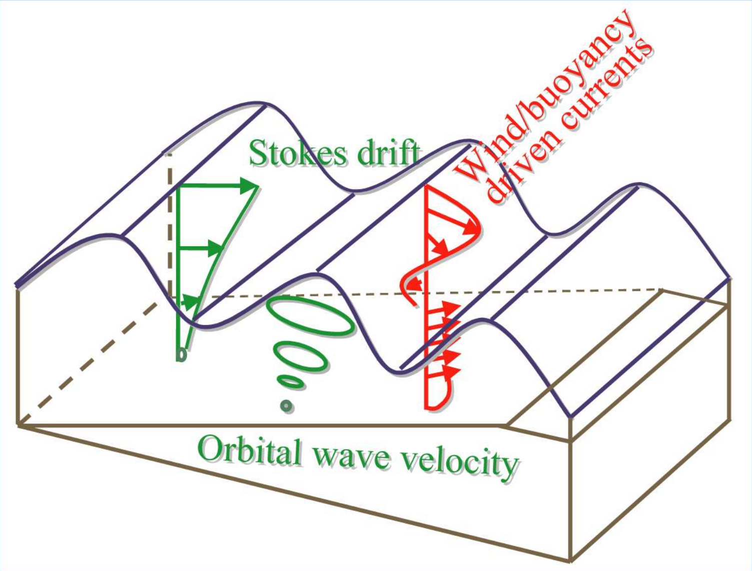
Three - Dimensionnal coupled modeling of wave- and wind/buoyancy- driven currents Example from the Gulf of Aigues - Mortes (NW Mediterranean sea, France)

C.DENAMIEL (1)*, F.BOUCHETTE (1), Y.LEREDDE (1), P.MARSALEIX (2)

(1) Dynamique de la Lithosphere, ISTEEM, Universite Montpellier 2,CC60, France
(2) Laboratoire d'Aerologie de Toulouse, Toulouse, France

* e - mail: denamiel@dstu.univ-montp2.fr

Rationale



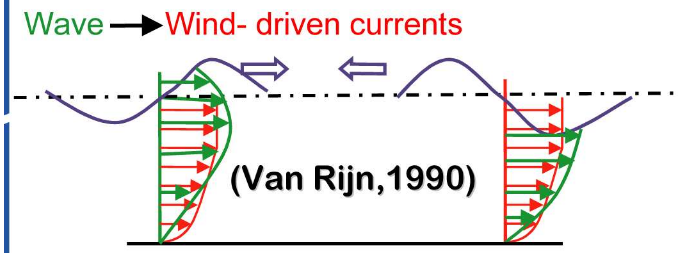
Hydrodynamics of nearshore tidalless continental shelf depends on two main processes:

- **wind/buoyancy- driven currents**
- **wave-driven currents**

Simulations on the Gulf of Aigues-Mortes (GAM, NW Mediterranean sea, France), a prototypic area, show that these two types of currents:

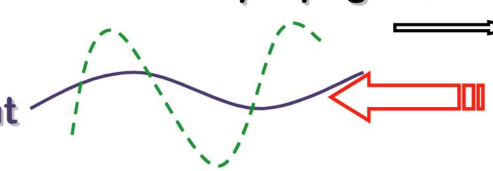
- present **strong vertical variations** in **intensity** and/or **direction**
- have an intensity of the **same order of magnitude**
- **interract**

It is thus necessary to take into account the **two processes** as well as their **three-dimensionnal coupled effects**.



Wind/buoyancy- driven current profiles :

- In red: without wave
- In green: with waves



Coupling of the circulation model SYMPHONIE and the wave model REF/DIF

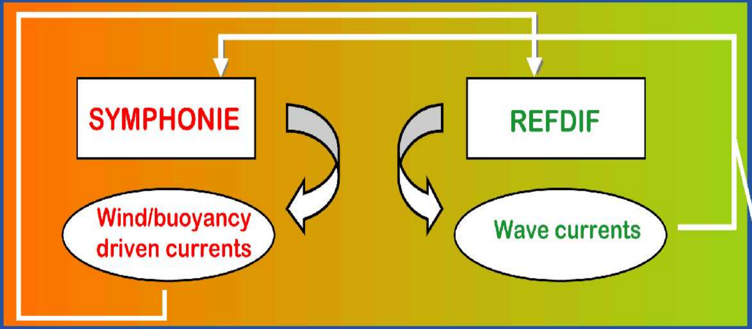
The **SYMPHONIE model**, adapted from the Johns *et al.* (1991) model, has been developed for ten years by the Laboratoire d'Aerologie (Toulouse, France) and is now largely used by the French community to compute the **oceanic circulation** on continental shelves, and specically on the Gulf of Lion. This model is based on the **three-dimensional primitive equations** and is widely described in Estournel et al. (1997). The coupling of SYMPHONIE with the REFDIF model allows to impose the **wave driving terms**, TX and TY, as forcing in the **Navier-Stokes equations**. These terms are composed of the **radiation stress** gradients and the wave **overpressure terms**.

REF/DIF is a weakly non linear combined **refraction** and **diffraction** model based on the parabolized "**mild-slope equation**" (Berkhoff equation, 1972) and the Mei and Tuck **parabolic diffraction model** (1980).This model takes into account the influence of a Doppler velocity which is usually a vertically integrated general current (Booij,1981 ; Kirby, 1984).
The coupled wind- and wave- driven currents model allows to include the "**Doppler velocity**" U_D from Symphonie model, through Kirby and Chen (1989).

$$\begin{cases} TX = -\frac{\partial S_{xx}}{\partial x} - \frac{\partial S_{xy}}{\partial y} + \frac{\partial \langle s_{w x} P_w \rangle}{\partial \sigma} \\ TY = -\frac{\partial S_{yx}}{\partial x} - \frac{\partial S_{yy}}{\partial y} + \frac{\partial \langle s_{w y} P_w \rangle}{\partial \sigma} \end{cases}$$

$$\begin{cases} S_{xy} = D\langle U_{orb} V_{orb}\rangle + \delta_{xy}\langle s_{w\sigma} P_w\rangle \\ S_{xx} = D\langle U_{orb} U_{orb}\rangle + \delta_{xx}\langle s_{w\sigma} P_w\rangle \\ S_{yy} = D\langle V_{orb} V_{orb}\rangle + \delta_{yy}\langle s_{w\sigma} P_w\rangle \end{cases}$$

with $\delta_{xy} = 0$ and $\delta_{xx} = \delta_{yy} = 1$



$$\begin{cases} U_{Dx}(x,y) = \int_0^1 u_{cx}(x,y,\sigma)\frac{kDcosh(2kD\sigma)}{sinh(2kD)}d\sigma \\ U_{Dy}(x,y) = \int_0^1 u_{cy}(x,y,\sigma)\frac{kDcosh(2kD\sigma)}{sinh(2kD)}d\sigma \end{cases}$$

Described in this poster

The equations of motion are those given by Mellor (2003) and Denamiel (2003):

- $\frac{\partial DU*}{\partial x} + \frac{\partial DV*}{\partial y} + \frac{\partial \Omega}{\partial \sigma} + \frac{\partial \zeta_c}{\partial t} = 0$
- $$\begin{cases} \frac{\partial DU*}{\partial t} + \frac{\partial DU*U*}{\partial x} + \frac{\partial DU*V*}{\partial y} + \frac{\partial(\Omega U*)}{\partial \sigma} - fDV* + \frac{\partial DP_c}{\partial x} - \frac{\partial}{\partial \sigma}(\sigma \frac{\partial D}{\partial x} P_c) + gD\frac{\zeta_c}{\partial x} = TX - \frac{\partial \overline{w'u'}}{\partial \sigma} \\ \frac{\partial DV*}{\partial t} + \frac{\partial DV*U*}{\partial x} + \frac{\partial DV*V*}{\partial y} + \frac{\partial(\Omega V*)}{\partial \sigma} + fDU* + \frac{\partial DP_c}{\partial y} - \frac{\partial}{\partial \sigma}(\sigma \frac{\partial D}{\partial y} P_c) + gD\frac{\zeta_c}{\partial y} = TY - \frac{\partial \overline{w'v'}}{\partial \sigma} \\ \frac{\partial P_c}{\partial \sigma} = -Dg\frac{\rho_c}{\rho_0} \end{cases}$$

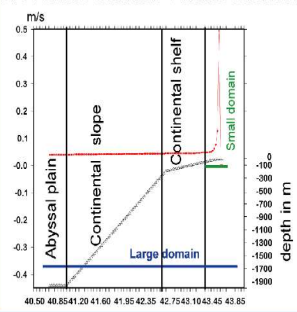
where
- $\langle\rangle = \frac{1}{2\pi}\int_0^{2\pi}()d\psi$ is the wave phase averaging operator with $\psi = \sigma t' - \vec{k}\vec{x}$ the wave phase
- $s(x,y,\sigma,t) = \sigma D - h + s_w$ is the sigma vertical coordinate with $D = h + \zeta_c$ the mean water column depht, $h(x,y)$ the bottom depth, $\zeta_c(x,y)$ the mean elevation due to wind and buoyancy forcing and $s_w(x,y,\sigma,t) = a\frac{sinh(kD\sigma)}{sinh(kD)}cos\psi$ is the instantaneous elevation due to wave
- $\overrightarrow{U*} = (U*, V*, \Omega)$ is the total current : $\overrightarrow{U*} = \overrightarrow{U_{stokes}} + \vec{u}_c$ and $\overrightarrow{U_{stokes}}(U_{stokes}, V_{stokes}) = \begin{cases} U_{stokes} = \frac{(ak)^2}{2}Ccos\theta_{wave}\frac{cosh(2kh\sigma)}{sinh^2(kh)} \\ V_{stokes} = \frac{(ak)^2}{2}Csin\theta_{wave}\frac{cosh(2kh\sigma)}{sinh^2(kh)} \end{cases}$
- k is the wave number, C is the intrinsic phase speed ($C^2 = \frac{g}{k}tanh(kh + \frac{a}{kh})$) and a is the wave amplitude
- $P = P_w + P_c$ is the total pressure with $P_w + gz = ga\frac{cosh(kh\sigma)}{cosh(kh)}cos(\psi)$ the wave pressure and P_c the hydrostatic pressure
- $w'u'$ and $w'v'$ are the Reynolds stresses
- $\overrightarrow{u_c(x,y,\sigma)}$ is the wind/buoyancy- driven velocity

Academic case study

This study presents the introduction of the **wave forcing terms** into the SYMPHONIE model on linear and regular bathymetry. In order to optimize computations, a **grid nesting** was realized with a large domain including an abyssal plain from 2000 m water depth, a continental 1% slope and a 0.2% continental shelf.
The wave conditions were supposed to be constant except on the small domain (h > -70 m)

These conditions are:
- **first order** and **monochromatic Stokes wave**
- **amplitude: 2m**
- **period: 10s**
- **direction: southern towards northern**

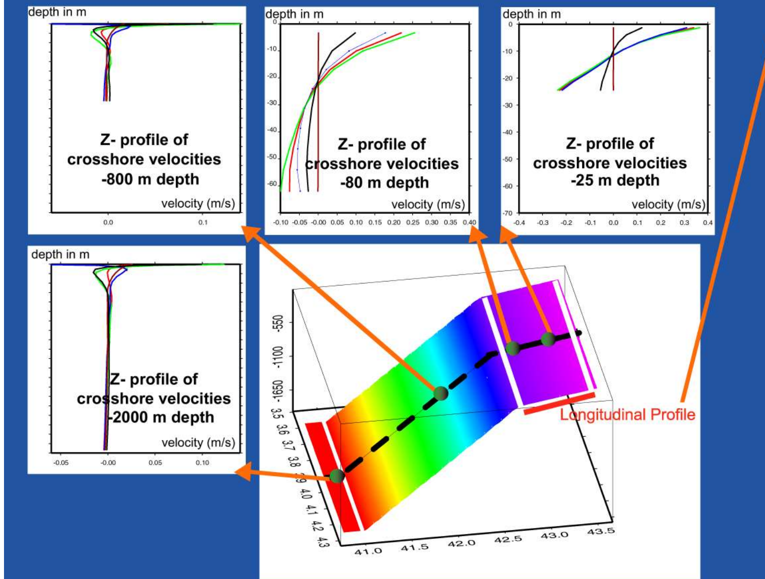
Crosshore surface Stokes current



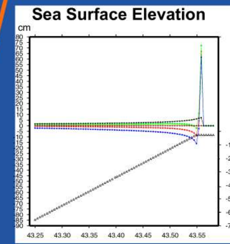
On the small domain, the wave propagation is computed thanks to the REF/DIF model. The results are then interpolated and introduced into the SYMPHONIE model in form **TX, TY** and the circulation model is initialized with the **Stokes currents.**

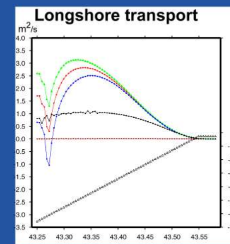
Caution:
- **periodic conditions** are imposed on the eastern and western boundaries of the domains
- **water height conservation conditions** are imposed on the offshore open boundary

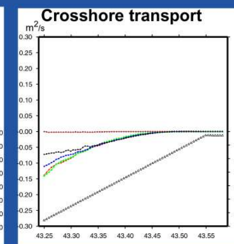
Results

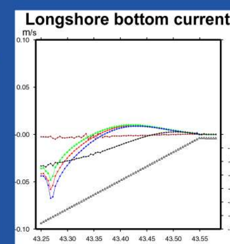


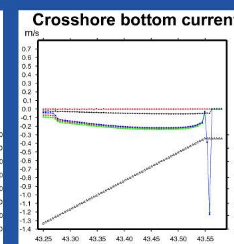
Longitudinal Profiles on the small domain

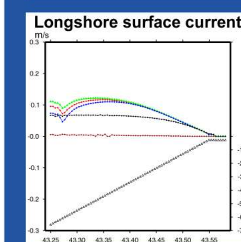


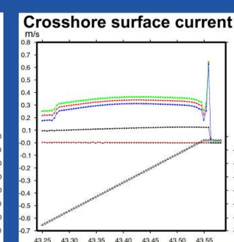












Southern waves without wind
Southern waves and wind
Southern waves and northern wind
Waves without wave forcing terms and wind
Southern wind without waves

Duration of simulation : 17 hours

In the particular case of a linear and regular bathymetry the study of one longitudinal profile is sufficient.

Southern wind only (black feature): pilot simulation showing classical results obtained without wave interaction.

Southern wave intialization without wave forcing terms (brown feature): the final state fall at rest. Indeed, after the domain initialization, there is not any more forcing (wave nor wind).

1) sea surface elevation:

- **h < -20 m**: the elevation is equal to 0 cm.
- **- 10 m < h < - 5 m:** set-down which reaches **-10 cm** for the southern wave simulation **(red feature)**. It increases up to **- 5 cm** with a southern wind **(green feature)** and decreases up to **- 18 cm** with a northen wind **(blue feature).**
- **h > -5 m**: set-up which reaches **+ 60 cm** for the southern wave simulation **(red feature)**. It increases up to + **70 cm** with a southern wind **(green feature)** and remains identical with a northen wind **(blue feature).**

2) transports: mainly eastward longshore transports (longshore transports mean intensity: 2 m^2/s and crosshore transports mean intensity: 0.10 m^2/s)

- **h < - 65 m**: transports decrease for all the simulations because of the offshore boundary condition
- **- 65 m < h < - 30 m** : longshore transports increase and reach **2.8 m^2/s** for the southern wave simulation **(red feature)**, **3.0 m^2/s** with a southern wind **(green feature)** and **2.3 m^2/s** with a northen wind **(blue feature)**
- **h > -5 m: " the wave sees the bottom"** and the transports deacrease to **0 m^2/s**

3) bottom currents: mainly southtward crosshore currents (longshore currents mean intensity: 0.02 m/s and crosshore currents mean intensity: 0.3 m/s)

4) surface currents: mainly northwards crosshore currents (longshore currents mean intensity : 0.05 m/s and crosshore currents mean intensity: 0.3 m/s)

This two currents have the same behaviour and intensities and compensate themselves:

- **h < - 65 m**: crosshore currents decrease for all the simulations because of the offshore boundary condition
- **- 65 m < h < - 5 m**: crosshore currents remains relatively constant to **0.28 m/s** for the southern wave simulation **(red feature)**, **0.3 m/s** with a southern wind **(green feature)** and **0.25 m/s** with a northen wind **(blue feature)**
- **h > -5 m:** surface and bottom currents increase respectively up to **0.6 m/s** and to **1.3 m/s**

5) z- profiles: highlight the **current direction shear**

- **offshore**: - 2000 m and - 800 m depth

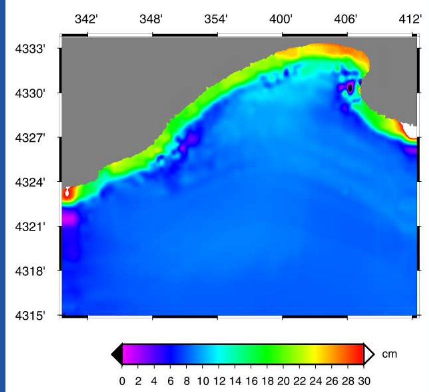
1- The inversion occurs on a **300 m thickness**.
2- Contrary to other simulations, the surface current with a northern wind is directed northward.

- **on the small domain**: - 70 m and - 25 m depth

1- The inversion occurs between the **surface** and the **bottom**.
2- Bottom currents **increase** when the wave "sees the bottom".

Application to the gulf of Aigues-Mortes

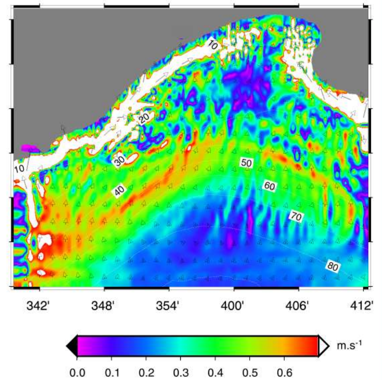
Southern wave condition: 2 m amplitude and 10 s period



Sea surface elevation:

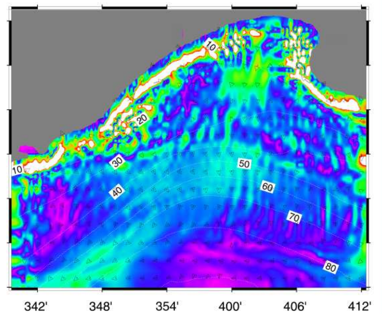
it is similar to the academic case.

- **Offshore**: the elevation remains constant and is about +10 cm
- **-20 m < h < -15 m**, it decreases up to 4 cm
- **h > -5 m**, it strongly increases and reaches +20 cm.



Surface currents:

- **offshore**: the velocities are about 0.15 m/s and directed northward.
- **-60 m < h < -30 m** : they increase strongly and reach 0.6 m/s.
- **h > -5 m:** they strongly increase : > 0.6 m/s and they become longshore velocities.



Bottom currents:

- **offshore**: the velocities are about 0.05 m/s and directed southward.
- **-60 m < h < -30 m**: they increase and reach 0.3 m/s.
- **h > -5 m:** they strongly increase: > 0.6 m/s and they become longshore velocities. In this area there are two nearshore drifts.

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