An investigation of storm waves in the North Atlantic Ocean

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Since February 1953 the Ocean Weather Ship *Weather Explorer*, using a shipborne wave recorder, has taken records of waves at the positions 'India' (61° 00′ N, 15° 20′ W) and 'Juliet' (52° 20′ N, 20° 00′ W). An investigation of the records shows that conditions of wave generation in the deep ocean are different from those on the continental shelf.

In both deep and shallow water, the steepness of the highest waves decreases as the fetch increases until an equilibrium value is reached after about 100 miles. In deep water this equilibrium value increases with the wind speed, whereas in shallow water it tends to decrease with wind speed.

Different empirical rules are required for wave prediction in the deep ocean and the shallow sea. The difference can probably be attributed to a greater effect of turbulence near the coast, where the depth is of the same order as the wavelength and there are active tidal streams.

1. Introduction

A previous investigation of the generation of waves by wind (Darbyshire 1952) was based mainly on records taken at Perranporth, Cornwall. The only storms considered were those near the recording station, so that it would not be necessary to take account of the attenuation and velocity of the waves after they left the generating area. Most of the examples were taken from storms within 400 miles of the recording station and therefore on the continental shelf; this is a region where the tides and tidal streams are appreciable and the depth of water comparable with the length of the waves.

It is now possible to investigate wave generation in deep water, for since February 1953 waves have been recorded by the Ocean Weather Ship *Weather Explorer* using a shipborne wave recorder described by Tucker (1952). Records were taken eight times daily while the ship was at sea, most of them while she was stationary. The ship occupied in turn the positions 'India' (61° 00′ N, 15° 20′ W) and 'Juliet' (52° 20′ N, 20° 00′ W). The sensitivity of the recorder is nearly constant for wave periods from 8 to 24 s.

A statistical investigation of wave heights and periods from February 1953 to January 1954 (Darbyshire 1954) shows that high waves are much more frequent in the open sea than at Perranporth, but long-wave periods (mean period of the highest third of the waves) are more frequent at Perranporth.

Attempts to predict the wave characteristics in deep water using the formulae based on the observations made at Perranporth were not successful; at high wind speeds the predicted wave periods were always too large and the heights too small. The discrepancies are much larger than can be introduced by the application of the classical formula for attenuation of waves with depth to recordings made at the depth of 50 ft. near the coast as against 10 ft. in the ship, and they indicate that the processes of wave generation in deep water are significantly different from those in shallow water, and the present paper describes an extension of the previous methods to deep water.

2. Method of investigation

The records taken by the *Weather Explorer* are of 7 to 10 min duration, and not immediately suitable for conversion into the black and white silhouette form required by the Fourier wave analyzer, described by Barber, Ursell, Darbyshire & Tucker (1946). It was necessary to project them on to white paper $5\frac{1}{2}$ in. wide and trace the wave outline by hand, repeating four times to cover the circumference of the wheel of the analyzer. The black and white profile was obtained by painting the lower half of the record with black ink.

To avoid uncertainties about wave attenuation, the study was confined to waves generated by winds acting within about 100 miles of the ship's position, and a large number of records were analyzed to find forty-five which showed no evidence of extraneous swell from distant storms.

The wave spectra were examined as before. They consist of a number of discrete peaks, the height of each peak being a measure of the amplitude of a Fourier component corresponding to a submultiple of the length of the record. The energy per unit area of a simple sine wave of height h is $\frac{1}{8}g\rho h^2$, and if it is assumed (as in the previous paper) that all wave components act independently the total wave energy is $\frac{1}{8}g\rho\Sigma h_n^2 = \frac{1}{8}g\rho H^2$, where h_n is the height of the nth peak on the spectrum and H is the equivalent height of the waves, i.e. the height of a simple sine wave having the same energy as the complicated wave pattern. This argument is extended to a range of periods between $T-\frac{1}{2}$ and $T+\frac{1}{2}$ s, so that the energy contained in such an interval becomes

$$\tfrac{1}{8}g\rho \sum_{T-\frac{1}{2}}^{T+\frac{1}{2}} h_n^2 = \tfrac{1}{8}g\rho H_T^2,$$

where H_T is the equivalent height of waves of period between $T-\frac{1}{2}$ and $T+\frac{1}{2}$ s. Values of H_T were computed by taking the square root of the sum of the squares of the heights of the peaks on each wave spectrum between $T-\frac{1}{2}$ and $T+\frac{1}{2}$ s intervals.

The records lasting only 8 min are rather short to afford an adequate statistical sample of the wave conditions, but they can be made more useful by taking running means of H_T values along the spectrum, so that if H_{T1}, H_{T2} and H_{T3} correspond to three consecutive values then the modified value of H_{T2} is H'_{T2}, which is given by $\frac{1}{4}(H_{T1}+2H_{T2}+H_{T3})$. These modified values of H_T cannot be used to estimate $H = \Sigma H_T^2$, since $\Sigma H_T^2 \neq \Sigma H_T'^2$. To find H one has to go back to the original values.

The wind data were obtained as before from synoptic weather charts for every 6 h supplied by the Naval Weather Service, the gradient wind speeds blowing towards the ship's position being plotted at 100-mile intervals on a wind-data diagram.

3. Empirical results

The examples chosen covered a fairly wide range, in which the fetch (the distance the wind acts on the waves) varied from 50 to 400 miles. The previous paper showed that the effect of fetch was not very marked after 100 miles, and this appears to be true for deep-sea storms also.

The longest wave period (T_m) in each spectrum (standing significantly above the background level) was plotted against the gradient wind speed (U knots) in figure 1 *a*. Though the points are scattered, it is fairly clear that they do not fall on a straight line going through zero and the continuous curve represents $T_m = 2{\cdot}3U^{\frac{1}{2}}$ s.

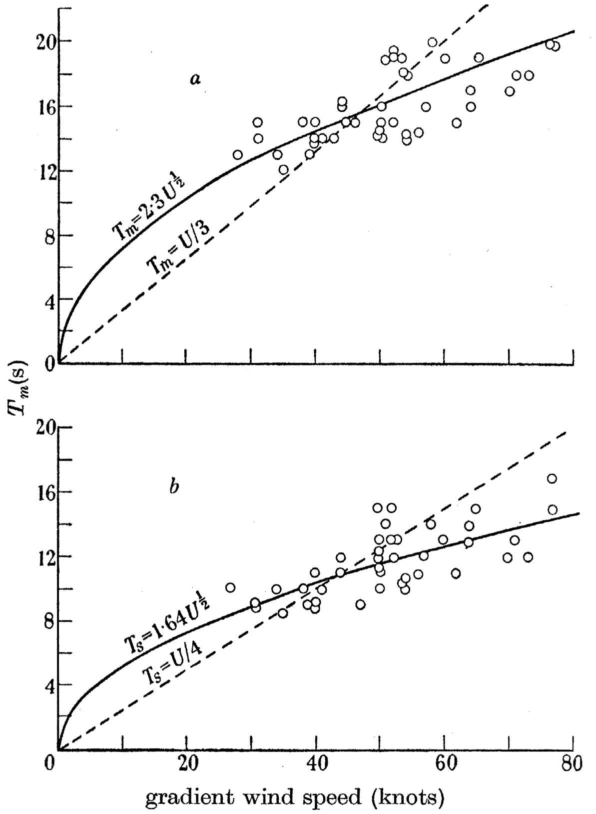

FIGURE 1. (*a*) Variation of maximum period, T_m, with gradient wind speed. (*b*) Variation of period of maximum amplitude, T_s, with gradient wind speed.

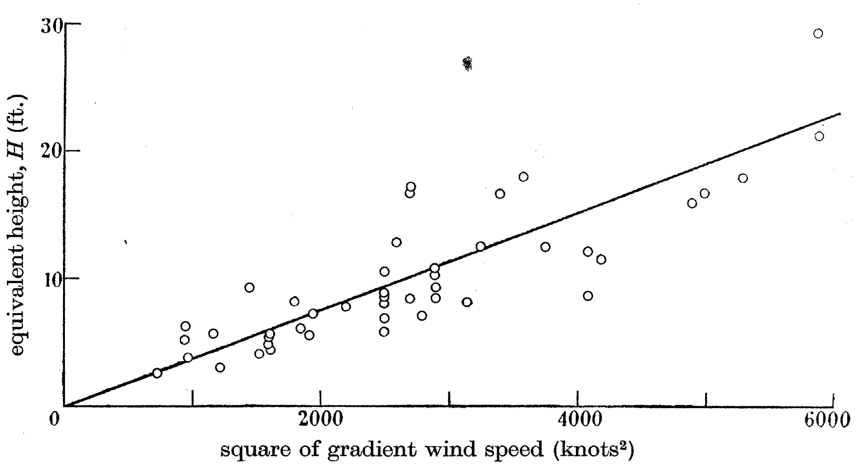

FIGURE 2. Variation of equivalent wave height with wind speed.

The broken line represents $T_m = \frac{1}{3}U$ s, which was the corresponding relation given by the Perranporth data. Both give approximately the same result for wind speeds of 40 to 50 knots.

T_s, the period for which H_T is a maximum on each analysis, is plotted against the gradient wind speed in figure 1*b*. The curve shown corresponds to $T_s = 1{\cdot}64U^{\frac{1}{2}}$ s. The Perranporth relation $T_s = \frac{1}{4}U$ is shown by the broken line, and again the two give almost the same result at wind speeds between 40 and 50 knots. Values of the equivalent wave heights (H) of the wave spectra are plotted against the square of the gradient wind speeds in figure 2. The straight line corresponds to $H = 0{\cdot}0038U^2$ ft., whereas the measurements made in the coastal region agreed better with $H = 0{\cdot}027U^{\frac{3}{2}}$. The square law gives lower wave heights when the wind speed is less than 50 knots and greater heights at higher wind speeds.

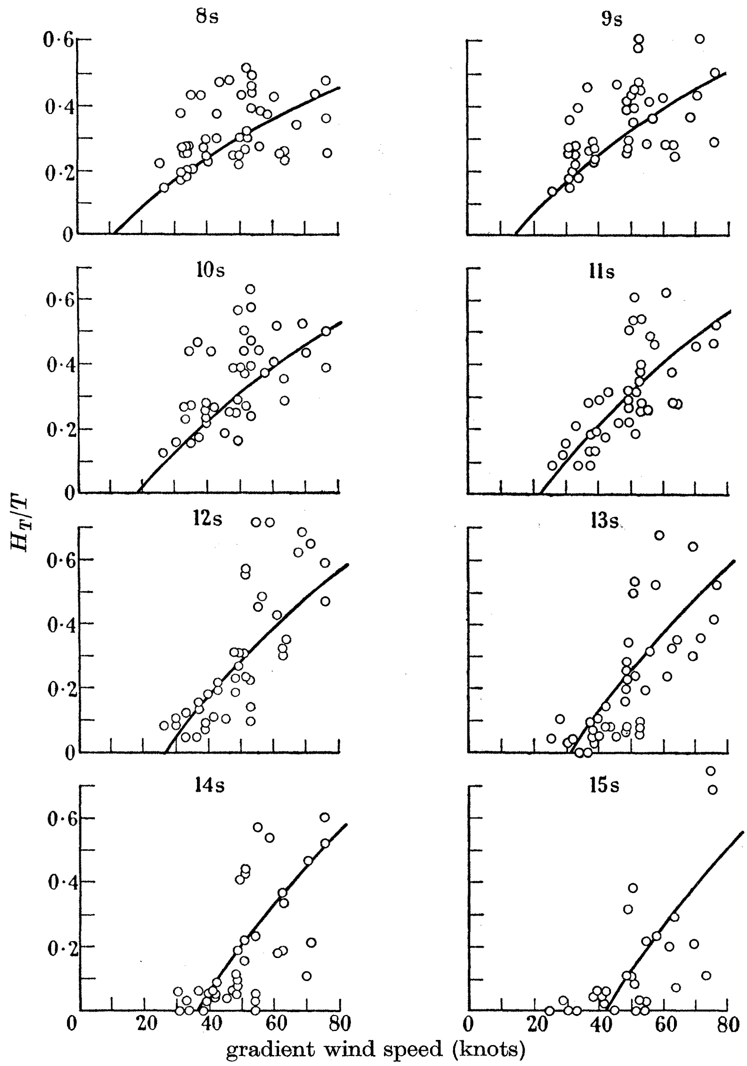

FIGURE 3. Variation of H_T/T with wind speed: open sea.

Plotting the values of H_T divided by T against the wind speed for each period from 8 to 15 s, as in figure 3, gives a good indication that the H_T values of waves longer than 10 s period increase with the wind speed. The evidence is not so conclusive for waves of 8 and 9 s period. This contrasts with the results found near the coast, in which the H_T/T values for waves of period 8 to 11 s decrease very sharply after an optimum wind speed was passed (see figure 4).

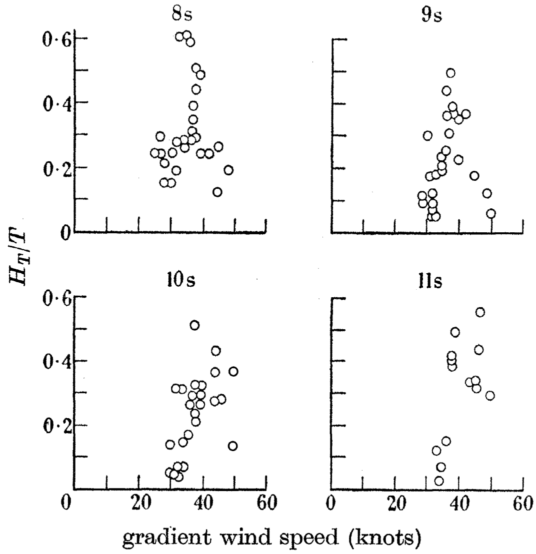

FIGURE 4. Variation of H_T/T with wind speed: west coast of the British Isles.

The results of figures 1 to 3 suggest an empirical relation between H_T, U and T such that H_T is zero when $T = 2{\cdot}3U^{\frac{1}{2}}$ and a maximum when $T = 1{\cdot}64U^{\frac{1}{2}}$, and $\int_0^\infty H_T^2 \mathrm{d}T = 0{\cdot}0038U^2$. These conditions are satisfied by the expression

$$H_T = 0{\cdot}0036\,(U^{\frac{1}{2}} - 0{\cdot}43T)\,T^{\frac{5}{2}},$$

and the curves in figure 3 correspond to this in the form

$$H_T/T = 0{\cdot}0036\,(U^{\frac{1}{2}} - 0{\cdot}43T)\,T^{\frac{3}{2}}.$$

The corresponding empirical formulae for the deep sea and Perranporth are:

	deep sea		Perranporth	
T_m	$2{\cdot}6U^{\frac{1}{2}}$	(1*a*)	$\frac{1}{3}U$	(1*b*)
T_s	$1{\cdot}64U^{\frac{1}{2}}$	(2*a*)	$\frac{1}{4}U$	(2*b*)
H	$0{\cdot}0038U^2$	(3*a*)	$0{\cdot}027U^{\frac{3}{2}}$	(3*b*)
H_T	$0{\cdot}0036\,(U^{\frac{1}{2}} - 0{\cdot}43T)\,T^{\frac{5}{2}}$	(4*a*)	$0{\cdot}44T \exp\{-(T/U - 0{\cdot}24)^2/0{\cdot}0027\}$	(4*b*)

T, T_m and T_s being expressed in seconds, H and H_T in feet and U in knots. It was shown in the previous paper that $H = \frac{1}{2}H_{\max.}$, where $H_{\max.}$ is the maximum height on the record.

Examples of wave spectra calculated from (4*a*) and (4*b*) are shown in figure 5. They agree with the observation that the spectral distribution in the open sea is wider and more uniform than near the coast.

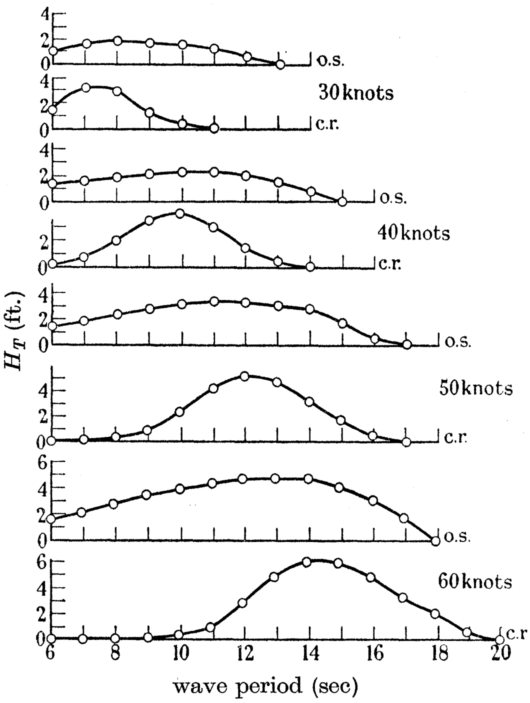

FIGURE 5. Predicted wave spectra for various wind speeds for open sea and western coastal region.

4. WAVE STEEPNESS

An expression for the steepness (height to length ratio), δ, of the highest waves in the open sea can be deduced from (2*a*) and (3*a*):

$$\delta = 0{\cdot}0038U^2/(5{\cdot}15 \times 1{\cdot}64^2U) = 0{\cdot}00028U, \tag{5}$$

assuming the relation λ (wavelength in feet) $= 5{\cdot}15 \times$ (period in seconds)^2. This implies that the steepness continues to increase linearly with the wind speed, whereas the equivalent formula for the coastal region

$$\delta = 0{\cdot}091/U^{\frac{1}{2}} \tag{6}$$

meant that it decreased. The values of wave steepness obtained from recordings in the Weather Ship are plotted against the wind speed in figure 6*a*, and those near the coast in figure 6*b*. The curves drawn are based on (5) and (6).

Sverdrup & Munk (1947) found that δ depended only on the ratio of the significant wave speed to the wind speed, c/U, a parameter which they called the 'wave age'. Using the same notation, (5) becomes

$$\delta c/U = 0{\cdot}00138U^{\frac{1}{2}}, \tag{7}$$

and (6) becomes

$$\delta c/U = 0{\cdot}068/U^{\frac{1}{2}}. \tag{8}$$

Thus (7) implies that $\delta c/U$ varies as $U^{\frac{1}{2}}$, whereas Sverdrup & Munk found that it should be independent of the wind speed and (8) implies that it varies inversely as $U^{\frac{1}{2}}$. The conditions of Sverdrup & Munk's observations would appear therefore to be an average of those found by these new wave recordings in deep water and those obtained in shallow water; this is very reasonable, since their observations were made in deep as well as shallow water.

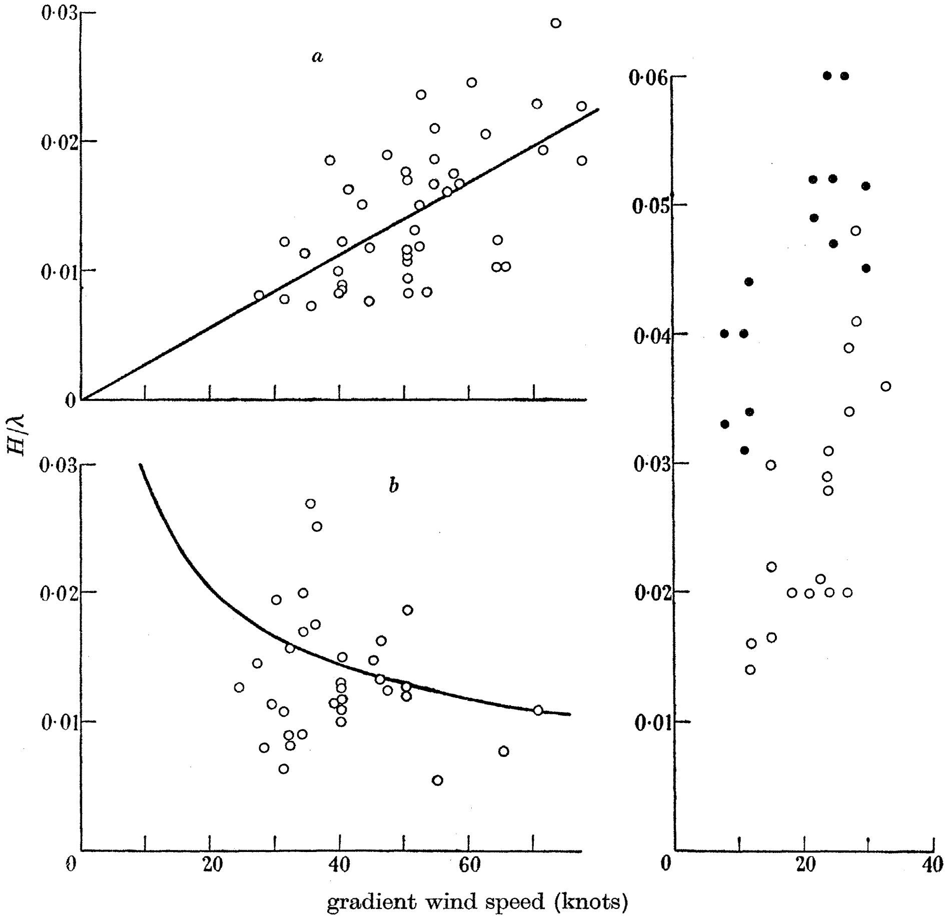

FIGURE 6. Variation of wave steepness with wind speed: (a) open sea; (b) west coast of the British Isles.

FIGURE 7. Variation of wave steepness with wind speed for Lough Neagh (○) and Staines reservoir (●).

5. Variation of wave steepness with fetch

Figure 7 shows the steepness of the highest waves on Lough Neagh and Staines reservoir plotted against $\frac{4}{3}$ times the surface wind speed (to compare with gradient winds over the sea). The fetch in Lough Neagh is 5 to 16 miles and on Staines reservoir about $\frac{1}{2}$ mile. The waves have periods of 1 to 4 s, and the conditions approximate to those in the open sea, since there are no appreciable currents and the depth is large compared with the wavelength. There appears to be an approximately linear relation between wave steepness and wind speed, but for a given wind speed, over these small fetches the waves are much steeper than in the open sea, those at Lough Neagh being four times and those at Staines reservoir about six times the values given by (5). These differences indicate that the steepness of the dominant waves diminishes as the fetch increases, and taken in conjunction with the data given in the previous paper (which showed that there was no marked change after 100 miles) suggest that the steepness falls to a steady value after about 100 miles. The steepness increases with wind speed at all fetches. It is reasonable to suppose that waves of a given period absorb less energy as they grow steeper and approach the limiting value for stability. They are then surpassed in height by longer waves which are less steep; these are in turn surpassed as the fetch increases, till after 100 to 200 miles the energy intake is limited primarily by the wave speed becoming comparable with that of the wind.

6. Factors governing wave generation

Existing knowledge about the nature of wave generation is hardly sufficient to account for the differences under open-sea and coastal conditions. It appears that waves gain their energy in two ways (Sverdrup & Munk 1947). The normal stress of the wave surface may do work on the waves because of an eddy being formed on the leeward side when the waves move more slowly than the wind, so that more work is done on the wave when the water particles are moving downwards on the windward side than is done against it when they move up, against the normal pressure, on the leeward side. This theory, developed by Jeffreys (1925, 1926), implies that the energizing factor becomes proportional to $(U-c)^2$, where c is the wave velocity and U the wind speed. Neumann (1950) and others have suggested that this theory applies only to the slower moving wavelets superimposed on the longer waves. The growth of such wavelets would make the longer waves present a rougher surface to the wind, and the effect of tangential stress may become important. The tangential stress appears to do work on the waves because of the small net movement of the water particles during each cycle. This movement has been confirmed experimentally (Beach Erosion Board 1941) and was shown by Stokes (1847) to be necessary if the waves are irrotational. The tangential stress τ_{xz} is usually taken to be proportional to the square of the wind speed, although there is much uncertainty about this.

Since waves do not grow longer and higher indefinitely, there must be some dissipation of energy. One possible cause is that the waves moving faster than the wind would leave an eddy on the windward side so that the waves lose energy to the wind, the process used in Jeffreys's wave generation theory being reversed. If, however, this effect applies only to the wavelets superimposed on the longer waves, the attenuation cannot be explained since the wavelets will still be travelling more slowly than the wind. Another possible cause of the dissipation is the effect of turbulent friction. This effect is usually represented by a parameter called the 'eddy viscosity', ν_e, which has the same dimensions as ordinary kinematical viscosity and enters the hydrodynamical equations in the same way. Various expressions have been given relating this to the wind speed, but they have all been based on the measurements of currents produced by the wind and there is little doubt that they give much too high a value for the rate of dissipation when applied to waves. The turbulence set up in the water may be regarded as being caused mainly by the waves, and formulae have been suggested relating the eddy viscosity to wave characteristics. Groen & Dorrenstein (1950) suggested that it was proportional to the four-thirds power of the wavelength, while Bowden (1950) suggested from dimensional considerations that it was proportional to the product of the amplitude and the wave velocity. It is necessary, however, that the eddy-viscosity term should approach infinity as the waves get longer to explain the fact that waves moving appreciably faster than the wind are not generated. It is therefore suggested that $\nu_\epsilon = k_2 aUf(c^2/U^2)$, where $f(c^2/U^2) \to \infty$ as c becomes greater than U, k_2 being a non-dimensional constant. Thus energy lost per unit area per second due to eddy viscosity $= 2\nu_\epsilon g^3 \rho a^2 c^{-4} = 2k_2 a^3 \rho U g^3 c^{-4} f(c^2/U^2)$, and therefore the maximum height allowed by the dissipation factor is

$$a = k_1 \rho' Uc/2k_2 g\rho f(c^2/U^2),$$

where k_1 is the stress coefficient. The spectral distribution would then be governed by $c/f(c^2/U^2)$.

Another limitation, already mentioned, is that the waves tend to break as their steepness approaches the limiting value, and this must govern the heights of the shorter-period waves. It has been assumed so far that wave components all act independently, but this hypothesis may not be satisfactory for a storm area because waves of one period tend to be shielded from the wind by other waves so that the effect of wave generation on individual wave components is governed by the total spectral distribution. This may explain the importance of the $c/U^{\frac{1}{2}}$ term rather than the c/U term in the generation of open-sea waves, where there is a wider spectrum than in that of coastal waves.

These arguments apply to coastal waves as well as deep-sea waves, but the effect of turbulence is likely to depend on the depth and to be different in the case of coastal storms, which were examined in the previous paper where the waves were generated mainly in sea areas in which the depth is comparable with the wavelength. In coastal regions the tidal streams will affect the waves in many ways. It has been shown by Mosby (1948), Bowden & Fairbairn (1952) and others that there are minute-by-minute fluctuations in the strength and direction of tidal streams, and this must increase the turbulence and act on the waves directly when they are in very shallow water as the change in the overall wave velocity will alter the effect of refraction on the wave direction. Apart from this, the effect of tidal streams will be considerable if the waves are generated because of the roughness due to the superimposed wavelets. It has been shown by Unna (1947) that waves steepen when they are travelling against the tidal stream and become less steep when they are travelling with it. The waves measured at Perranporth have travelled on the continental shelf for at least one tidal cycle, and so for a considerable proportion of the time the superimposed wavelets will be steeper and tending to break. This would increase the roughness of the waves and increase the energizing factor. It is probable that the effect of the increase in roughness when moving against the stream is more than the effect of the decrease when moving with the stream. The net effect of the tidal streams would then be to increase the height of the waves for moderate wind speeds as indicated by (3*a*) and (3*b*). The same effect would steepen waves which are longer than wavelets and there would be a greater tendency for them to break; this may explain in part the greater loss of energy by shorter-period waves near the coast. It is probable, however, that the turbulence near the coast would also tend to attenuate shorter waves relatively more than in the open sea. A combination of the two effects could explain how at wind speeds under 50 knots the wave height is greater near the coast than in deep water, with a sharp falling off in the height of waves of shorter periods than the optimum. The fact that wave heights become greater in deep water than near the coast, and the mean period shorter, can be explained by the short-period waves not losing so much energy at wind speeds greater than 50 knots.

7. Conclusions

Conditions governing wave generation in the open sea are different from those acting on the continental shelf area.

In both deep and shallow water there is evidence that the steepness of the highest waves decreases as the fetch increases until an equilibrium value is reached at about 100 miles. In deep water the equilibrium value appears to increase with the wind speed, whereas in the coastal region west of Cornwall it tended to decrease with the wind speed.

Different empirical rules are required for wave prediction in the deep ocean and shallow sea. The difference can be attributed to the effect of turbulence being different near the coast where the depth is of the same order as the wavelength, and also to the effect of tidal streams.

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