A Study of Microseisms in South Africa

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Summary

Microseisms have been recorded at Hermanus near Cape Town since 1961 October, using an N.I.O. two-component horizontal seismograph. Wave records have also been taken by means of a shipborne wave recorder aboard R.S. *Africana II*. The spectra of waves and microseisms due to a storm in 1962 April are compared and there is a two to one frequency relationship as would be expected from the wave interference theory of microseism generation. Estimates were made of the direction of approach of the microseisms, assuming various models and allowing for the effect of refraction. The evidence on the whole suggests that the microseisms consist of a mixture of Rayleigh and Love waves coming from the same range of directions.

1. Introduction

Since 1961 October, a two-component horizontal seismograph of the type described by Tucker (1958), and Darbyshire and Hinde (1961) has been installed at the Magnetic Observatory, Hermanus, Cape Province, about seventy-five miles south-east of Cape Town. Records have been taken regularly for fifteen minutes, eight times a day. Microseism activity was very low, less than 3μm until the onset of the winter in 1962 April. One storm in this month gave very interesting results and these form the subject of the present paper. It was fortunate that sea waves were also being systematically recorded at this time by means of an N.I.O. shipborne wave recorder on board the Division of Sea Fisheries research ship *Africana II*.

2. Description of the situation

Figure 1 shows sketches of the weather situation for April 9, 10, 11, 12, 13 and 14, at 12^h GMT, extracted from the weather charts issued by the South African Weather Bureau. A storm approaches from WSW of Cape Town and moves round the Cape, being approximately south of it at 1962 April 11 12^h and it then moves up along the coast. On the 13th, another storm appears and for a time two storms are effective. The positions of wave recording for 1962 April 10 22^h, 1962 April 11 22^h, 1962 April 12 11^h and 1962 April 15 00^h are shown by crosses on the most appropriate weather chart.

The wave records were frequency analysed by using the N.I.O. wave analyser (Barber & others, 1946). The microseism records for 1962 April 10 11^h, 1962 April 11 02^h, 1962 April 11 11^h, 1962 April 12 17^h and 1962 April 14 11^h, were digitized and analysed by the DEUCE computer. The power frequency spectra are shown in Figure 2, the frequency interval being 0·007 s^{-1} for the waves and also for the microseisms the frequencies of which have been halved to facilitate direct comparison. The ordinates on the microseism spectra represent the sum of the energies of the two components. As will be seen below, the positions of wave recording were outside the wave interference areas but the agreement between the two sets of spectra is very striking and gives good evidence for Longuet-Higgins' theory of wave interference (1950).

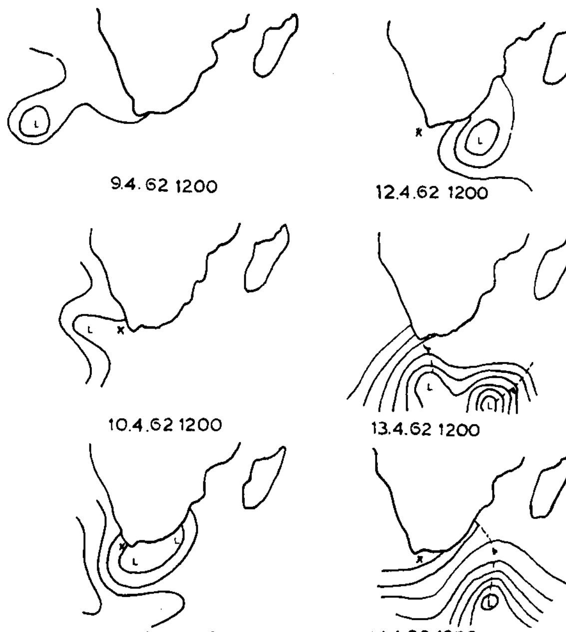

FIG. 1.—Position of storms, 1962 April 9 to 14.

As no vertical seismograph was available, it was not possible to determine from the horizontal seismographs whether the microseisms consisted of Rayleigh waves only or a mixture of Rayleigh and Love waves but a knowledge of the positions of wave interference would greatly help in this regard. Accordingly wave frequency and directional spectra were predicted from the weather charts by a method developed by J. Darbyshire (1961) for the North Atlantic and adapted by M. Darbyshire (1962) for South African waters. This method predicts the wave energies contained in wave period bands 5–7, 8–10, 11–13, 14–16, 17–19 s and direction classes 0°–45° and so on over a set of 300-mile side squares shown in Figure 3. This method indicated that there were bands of wave energy moving in opposite directions within the squares shown shaded. The square 23 applies to 1962 April 10 12^h, 44 to 1962 April 11 12^h, 36 to 1962 April 12 12^h and 57 to 1962 April 14 12^h. The energy values are shown in Table 1 and are compared with the sum of the variances of the two microseism records at that time.

There is some resemblance between the trends of the two sets of figures, both showing a marked decrease at 1962 April 12, but the waves show a marked rise between the 10th and 11th whereas the microseisms only show a slight increase. The great increase in wave activity on the 14th is not shown by the microseisms

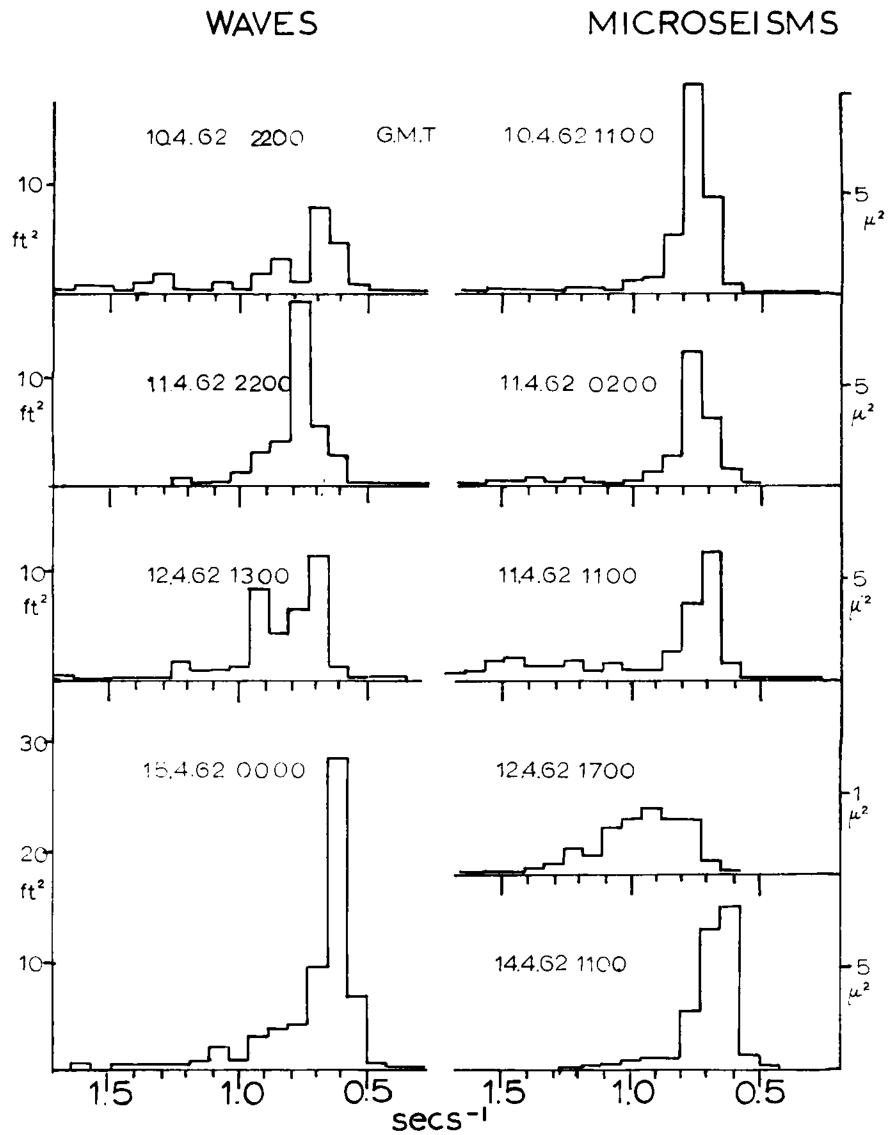

FIG. 2.—Frequency spectra of waves and microseisms, 1962 April 10 to 15.

but this can be explained to some extent by the wave interference area being further away from the recording station. The predicted wave amplitudes are not unreasonable but the periods are too short. It is possible that a given wind speed generates waves of higher period in these waters where there is a permanent swell than it would in the North Atlantic (see Phillips 1961). As the time for the waves to travel is only about twenty-four hours, however, errors in period leading to errors in group velocity will not greatly affect the placing of the wave interference areas. In this work no allowance has been made for coastal reflection but the waves were only coming head-on towards the coast in one of the examples, that of 1962 April 11 and the effect was probably slight apart from this.

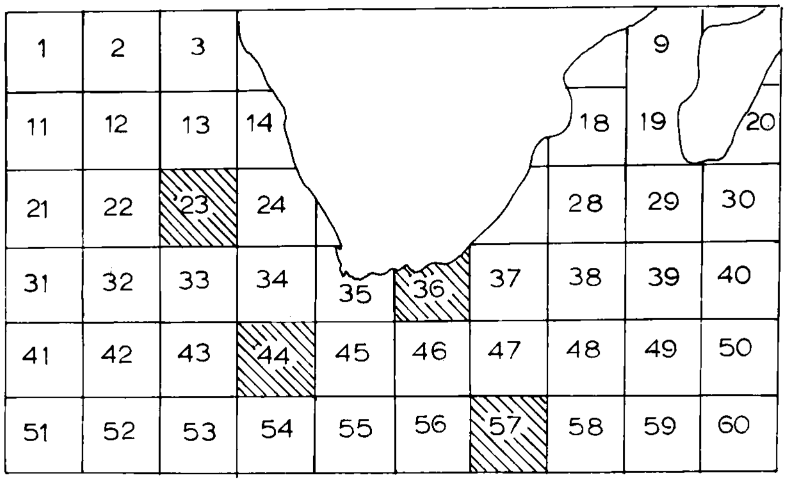

FIG. 3.—Diagram of 300-mile square-grid system showing areas of wave interference.

Table 1

Comparison of predicted wave energies and microseism variances

Date, time and location	Period band (s)	a_1^2 energy*	Direction	a_2^2 energy*	Direction	$a_1^2 a_2^2$	Total $a_1^2 a_2^2$	Total σ^2 for microseism (μm^2)
10.4.62	5–7	2·4	NE	1·1	SW	2·65		
1200	8–10	5·6	NE	2·2	SW	12·30	15·5	4·58
square 23	11–13	1·2	NE	0·44	SW	0·50		
11.4.62	5–7	1·5	SE	3·2	NW	4·8		
1200	8–10	3·5	SE	7·6	NW	26·5	34·8	5·07
square 44	11–13	0·8	SE	4·4	NW	3·5		
12.4.62	5–7	2·5	W	0·75	E	1·9		
1200	8–10	5·8	W	1·5	E	8·7	10·6	1·34
square 36	11–13	4·0	W	0				
14.4.62	5–7	2·5	SE	5·6	NW	14·0		
1200	8–10	5·8	SE	11·2	NW	64·5	122·5	5·05
square 57	11–13	4·0	SE	11·0	NW	44·0		
	14–16	0		1·6	NW	0		

* Unit of energy is $ft^{2/3}$ s.

3. Determination of direction of approach of microseisms

Various models have been suggested for the nature of microseisms approaching a given station. In the simplest case of all, the waves consist entirely of Rayleigh waves coming from a single direction. This would lead to a correlation coefficient of unity between the two horizontal component records which is seldom obtained. The next case is that of a mixture of Rayleigh and Love waves coming from a single direction. This was considered by Darbyshire (1954). A more complicated case was considered by Iyer (1959) who took the Rayleigh waves to come from one direction but the Love waves to be isotropic. These models are clearly idealistic as the Rayleigh waves will only rarely come from one direction and Longuet-Higgins has considered the general case of both Rayleigh and Love waves coming from a range of directions. The analysis however, in the case of two horizontal components, even when this spreading is taken into account, gives the same value of the mean angle of approach as that of the angle in the simple case, and values of R^2/L^2, the Rayleigh to Love wave activity ratio, are not very different. The various models will be considered in order. The analysis is based on the maximum value of the correlation between two records (or the coherence) and the variances. These were worked out by computer and are given in Table 2.

Table 2

Variance and correlation coefficients of microseism records

Date	Time	Direction	Variance σ^2	Total variance EW+NS	Correlation coefficient
10.4.62	1100	EW	1·435	4·580	−0·183
		NS	3·145		
11.4.62	0200	EW	1.995	4·020	−0·284
		NS	2·025		
11.4.62	1100	EW	1·726	5·069	0·03
		NS	3·343		
12.4.62	1700	EW	0·606	1·338	−0·148
		NS	0·732		
14.4.62	1100	EW	1·725	5·053	−0·246
		NS	3·328		

(1) *Mixture of Rayleigh and Love waves coming from the same direction.*

Let $R(t)$ represent the Rayleigh wave motion and $L(t)$ the Love wave motion. $R(t)^2$ is the Rayleigh wave activity and $L(t)^2$, the Love wave activity, x movement taken to be positive in W to E direction, $\bar{x}^2$ variance of EW component, y movement taken to be positive in S to N direction a $\bar{y}^2$ variance of NS component, θ measured from the positive x axis,

$$x = R(t)\cos\theta - L(t)\sin\theta, \qquad y = R(t)\sin\theta + L(t)\cos\theta$$

$$\bar{x}^2 = R(t)^2\cos^2\theta + L(t)^2\sin^2\theta, \qquad \bar{y}^2 = R(t)^2\sin^2\theta + L(t)^2\cos^2\theta$$

$$r_{xy} = \{R(t)^2 - L(t)^2\}\cos\theta\sin\theta = \tfrac{1}{2}\{R(t)^2 - L(t)^2\}\sin 2\theta$$

and

$$\bar{x}^2 - \bar{y}^2 = \{R(t)^2 - L(t)^2\}\cos 2\theta$$

so

$$\sin 2\theta = \frac{2r_{xy}}{R(t)^2 - L(t)^2}$$

$$\cos 2\theta = \frac{\bar{x}^2 - \bar{y}^2}{R(t)^2 - L(t^2)}$$

and so

$$\tan 2\theta = \frac{2r_{xy}\bar{x}\bar{y}}{\bar{x}^2-\bar{y}^2}.$$

The expressions for both $\sin 2\theta$ and $\cos 2\theta$ involve $\{R(t)^2-L(t)^2\}$ which can be positive or negative and so there is no restriction on the signs of these and

$$2\theta = n\pi + A$$

where

$$A = \tan^{-1}\frac{2r_{xy}\bar{x}\bar{y}}{\bar{x}^2-\bar{y}^2}$$

$$R(t)^2+L(t)^2 = \bar{x}^2+\bar{y}^2 \text{ and } R(t)^2-L(t)^2 = 2r_{xy}/\sin 2\theta$$

whence $R(t)/L(t)$ can be determined.

(2) *Rayleigh waves from one direction, Love waves isotropic.*

The analysis is similar but we assume initially that the Love waves come at an angle ϕ,

then

$$r_{xy} = \tfrac{1}{2}\frac{R(t)^2\sin 2\theta - L(t)^2\sin 2\phi}{\bar{x}\bar{y}}$$

and $\bar{x}^2-\bar{y}^2 = R(t)^2\cos 2\theta - L(t)^2\cos 2\phi$.

If all values of ϕ are equally probable, we can replace $\frac{1}{2}\sin 2\phi$ by

$$\tfrac{1}{2}\int_0^{2\pi} k\sin 2\phi\, d\phi = 0.$$

Then

$$r_{xy} = \frac{\frac{1}{2}R(t)^2\sin 2\theta}{\bar{x}\bar{y}}$$

$$\text{and } \bar{x}^2-\bar{y}^2 = R(t)^2\cos 2\theta$$

and once again

$$\tan 2\theta = \frac{2r_{xy}\bar{x}\bar{y}}{\bar{x}^2-\bar{y}^2}.$$

There is a difference between this case and case (1), however, as now $R(t)^2$ is always positive and the sign of $\sin 2\theta$ is determined by that of r_{xy} and that of $\cos 2\theta$ by that of $\bar{x}^2-\bar{y}^2$ and so we can only accept the solutions of $2\theta = n\pi+A$ which give the right sign to $\cos 2\theta$ and $\sin 2\theta$. $R(t)/L(t)$ can be found in a similar manner to case (1).

(3), (4) and (5). *Cases taking into account a finite range of directions.*

This has been discussed by Longuet-Higgins (1962). In general we have:

$$x = \textstyle\sum_n R_n\cos\theta_n\cos(6nt+\alpha n) - \sum_m L_m\sin\theta m\cos(6mt+\alpha m)$$

$$y = \textstyle\sum_n R_n\sin\theta_n\cos(6nt+\alpha n) + \sum_m L_m\cos\theta m\cos(6mt+\alpha m);$$

if the αs are uniformly distributed, it can be shown that for a small interval $d\sigma d\theta$

$$\sum_{d\sigma d\theta}\tfrac{1}{2}R_n{}^2 = F(\sigma,\theta)d\sigma d\theta$$

$$\sum_{d\sigma d\theta}\tfrac{1}{2}L_n{}^2 = G(\sigma,\theta)d\sigma d\theta.$$

Then

$$\bar{x}^2 = \int_0^{2\pi} \cos^2\theta F d\theta + \int_0^{2\pi} \sin^2\theta G d\theta$$

$$\bar{y}^2 = \int_0^{2\pi} \sin^2\theta F d\theta + \int_0^{2\pi} \cos^2\theta G d\theta$$

$$\bar{x}\bar{y} = \Gamma xy \bar{x}\bar{y} = \int_0^{2\pi} \cos\theta \sin\theta F d\theta - \int_0^{2\pi} \cos\theta \sin\theta G d\theta.$$

It is more tractable analytically to assume a rectangular distribution of directions, and assuming the Rayleigh and Love waves have the same mean direction, thus

$$F = R^2 \text{ for } \alpha-\beta < \theta < \alpha+\beta$$

and is o outside these limits.

$$G = L^2 \text{ for } \alpha-\gamma < \theta < \alpha+\gamma.$$

Thus

$$\bar{x}^2 = \tfrac{1}{2}\beta \int_{\alpha-\beta}^{\alpha+\beta} R^2\cos^2\theta d\theta + \tfrac{1}{2}\gamma \int_{\alpha-\gamma}^{\alpha+\gamma} L^2\sin^2\theta d\theta$$

$$\bar{y}^2 = \tfrac{1}{2}\beta \int_{\alpha-\beta}^{\alpha+\beta} R^2\sin^2\theta d\theta + \tfrac{1}{2}\gamma \int_{\alpha-\gamma}^{\alpha+\gamma} L^2\cos^2\theta d\theta$$

$$\bar{x}\bar{y} = \tfrac{1}{2}\beta \int_{\alpha-\beta}^{\alpha+\beta} R^2\cos\theta \sin\theta d\theta - \tfrac{1}{2}\gamma \int_{\alpha-\beta}^{\alpha+\beta} L^2\cos\theta \sin\theta d\theta;$$

on integrating

$$\bar{x}^2 = \tfrac{1}{2}(R^2+L^2) + \frac{R^2}{4\beta}\sin 2\beta \cos 2\alpha - \frac{L^2}{4\gamma}\sin 2\gamma \cos 2\alpha$$

$$\bar{y}^2 = \tfrac{1}{2}(R^2+L^2) - \frac{R^2}{4\beta}\sin 2\beta \cos 2\alpha + \frac{L^2}{4\gamma}\sin 2\gamma \cos 2\alpha$$

$$\bar{x}\bar{y} = \frac{R^2}{4\beta}\sin 2\beta \sin 2\alpha - \frac{L^2}{4\gamma}\sin 2\gamma \sin 2\alpha$$

$$= \sin 2\alpha\left(\frac{R^2}{4\beta}\sin 2\beta - \frac{L^2}{4\gamma}\sin 2\gamma\right).$$

Then $\bar{x}^2+\bar{y}^2 = R^2+L^2$

$$\sin 2\alpha = \frac{2\bar{x}\bar{y}}{\left(\frac{R^2}{2\beta}\sin 2\beta - \frac{L^2}{2\gamma}\sin 2\gamma\right)}$$

$$\cos 2\alpha = \frac{\bar{x}^2-\bar{y}^2}{\left(\frac{R^2}{2\beta}\sin 2\beta - \frac{L^2}{2\gamma}\sin 2\gamma\right)}$$

and
$$\tan 2\alpha = \frac{2\bar{x}\bar{y}}{\bar{x}^2-\bar{y}^2}.rxy$$

which is the same equation as in the simple case. Three particular cases are of interest.

(3) *No Love waves present.*

Then
$$\cos 2\alpha = \frac{\bar{x}^2-\bar{y}^2}{R^2}\left(\frac{2\beta}{\sin 2\beta}\right)$$

$$\sin 2\alpha = \frac{2\bar{x}\bar{y}}{R^2}\left(\frac{2\beta}{\sin 2\beta}\right)$$

and again if $2\beta < 90°$ the signs of $\cos 2\alpha$ and $\sin 2\alpha$ are determined by that of $(\bar{x}^2-\bar{y}^2)$ and $\bar{x}\bar{y}$ and we have the same restrictions as in case (2). 2β can be calculated.

(4) *Rayleigh and Love waves from the same range of directions.*

Then
$$\cos 2\alpha = \frac{\bar{x}^2-\bar{y}^2}{(R^2-L^2)}\cdot\frac{2\beta}{\sin 2\beta}, \qquad \sin 2\alpha = \frac{2\bar{x}\bar{y}}{(R^2-L^2)}\cdot\frac{2\beta}{\sin 2\beta}$$

as the denominator can be positive or negative, there is no restriction on the sign of $\cos 2\alpha$ and $\sin 2\alpha$ and all the roots of $2\alpha = n\pi+A$ are admissible.

Now
$$R^2+L^2 = \bar{x}^2+\bar{y}^2, \qquad R^2-L^2 = \frac{2\bar{x}\bar{y}}{\sin 2\alpha}\cdot\frac{2\beta}{\sin 2\beta}$$

so the only difference between these equations and the corresponding ones for case (1) is the factor $2\beta/\sin 2\beta$ which is near unity for moderate values of β and so the value of R/L will not be greatly changed.

(5) *Love waves isotropic*, $\gamma = \pi$.

$$\cos 2\alpha = \frac{(\bar{x}^2-\bar{y}^2)}{R^2}\cdot\frac{2\beta}{\sin 2\beta}, \qquad \sin 2\alpha = \frac{\bar{x}\bar{y}}{R^2}\cdot\frac{2\beta}{\sin 2\beta}$$

which gives the same result as in case (3); again for $\beta < 90°$ as R^2 is always positive, there is the same restriction as in cases (2) and (3). The expression for R/L will again only be affected by the factor $2\beta/\sin 2\beta$.

Table 3 gives values of α (or θ) and the maximum possible value of β observed from Figure 4 which allows for the effect of refraction and those calculated according to these models, values of β being calculated for case (4) and in cases (5) and (6), given the observed value.

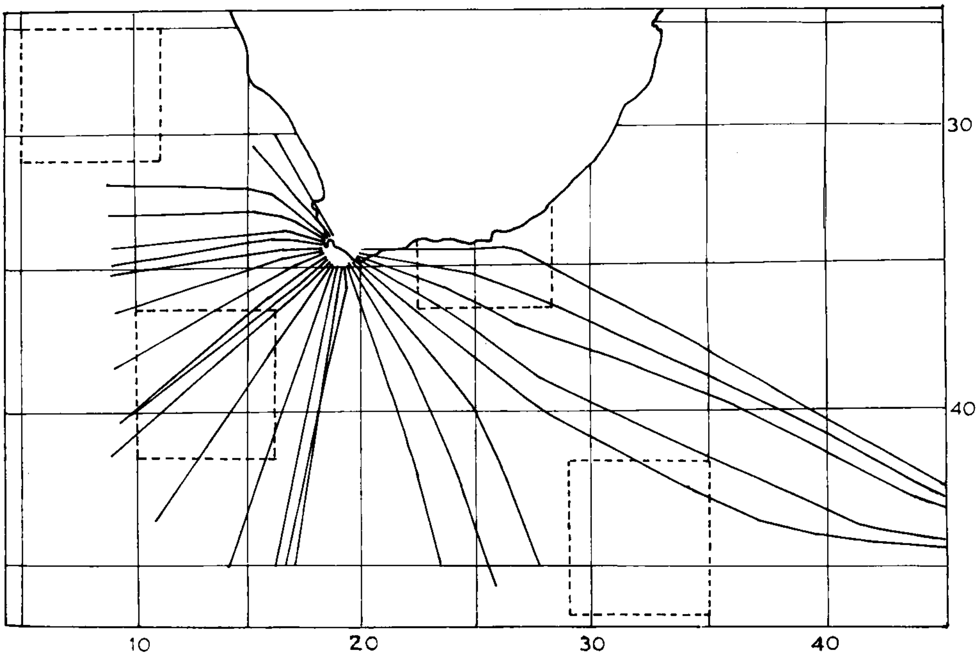

FIG. 4.—Refraction diagram for 6-sec microseisms.

Table 3

Observed and computed values of α and β

Example	Date Time	α observed	β observed	Model 1 α	Model 1 R/L	Model 2 α	Model 2 R/L	Model 3 α	Model 3 β	Model 4 α	Model 4 R/L	Model 5 α	Model 5 R/L
a	10.4.62 1100	135°	5°	102°	1·50	102°	0·82	102°	61½°	102°	1·50	102°	0·82
b	11.4.62 1100	220°	30°	178°	0·71	267°	0·70	267°	65°	178°	0·70	267°	0·71
c	12.4.62 1700	340°	20°	299°	1·20	299°	0·46	299°	76°	299°	1·21	299°	0·48
d	14.4.62 1100	320°	10°	288°	1·41	288°	0·78	288°	61°	288°	1·41	288°	0·78

The analysis was repeated by using a narrow band of frequencies, which included about the ten highest harmonics on the spectrum. The correlation coefficients and the variances could be worked out from the Fourier coefficients, for if:

$$x = \sum_{n=i}^{n=j} A_{xn} \cos nwt + \sum_{n=i}^{n=j} B_{xn} \sin nwt$$

$$y = \sum_{n=i}^{n=j} A_{yn} \cos nwt + \sum_{n=i}^{n=j} B_{yn} \sin nwt$$

$$2\bar{x}\bar{y} = \sum_{i}^{j} A_{xn}A_{yn} + \sum_{i}^{j} B_{xn}B_{yn}$$

$$2\bar{x}^2 = \sum_{i}^{j} (A_{xn}^2 + B_{xn}^2)$$

$$2\bar{y}^2 = \sum_{i}^{j} (A_{yn}^2 + B_{yn}^2),$$

iw and jw being the limits of the frequency band. The results are shown in Table 4.

Table 4
Observed and computed values of α and β

Example	Period range secs	α	β	Model 1 α	Model 1 R/L	Model 2 α	Model 2 R/L	Model 3 α	Model 3 β	Model 4 α	Model 4 R/L	Model 5 α	Model 5 R/L
a	6·4–7·2	135°	5°	102°	2·50	102°	1·64	102°	40°	102°	2·50	102°	1·64
b	6·7–7·7	220°	30°	188°	0·67	278°	0·80	278°	57°	188°	0·60	278°	0·93
c	5·4–6·5	340°	20°	327°	1·41	327°	0·70	327°	66°	327°	1·48	327°	0·77
d	7·1–8·2	320°	10°	300°	1·83	300°	1·08	300°	54°	300°	1·84	300°	1·09

The computed angles α are too small by 30° to 40° in the case of Table 3. The agreement, using a narrow frequency band is better, the computed angles now being between 13° and 32° too small. Of the various models used, (1) and (2) can be discarded as being unrealistic. Model (3) gives much too high values of β and it seems therefore unlikely that the waves consist only of Rayleigh waves. Model (4) on the whole gives better values of α than (5) and so gives a more accurate representation than any other model.

4. Refraction of microseisms

Figure 4 shows a refraction diagram for 6 s microseisms. It is based on the same assumptions as the diagrams prepared for Bermuda, (Darbyshire 1955), the British Isles (Darbyshire & Darbyshire 1957), and the western North Atlantic (Iyer, Lambert & Hinde 1958). The same procedure is followed as in the last named reference, the waves being assumed to travel outwards from the recording station and using the reciprocity principle to find divergence and convergence areas. The velocity of distortional waves in the sea bed was assumed as before to be 2·8 km/s. The depths east of Cape Agulhas are based on recent soundings taken by the S.A.S. *Natal* and R.S. *Africana II*, in connexion with the International Indian Océan Expedition. Fewer soundings are available for the portion to the west and this part may not be so reliable.

5. Conclusions

Microseisms and waves recorded near Cape Town show a two-to-one frequency relationship which supports Longuet-Higgins's theory of microseism generation. Estimates of the direction of approach, taking into account the effect of refraction give some indication that the microseisms consist of a mixture of Love and Rayleigh waves coming from the same range of directions.

6. Acknowledgments

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