Global models of surface wave attenuation

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Received 15 August 2005; revised 18 January 2006; accepted 15 February 2006; published 31 May 2006.

[1] A large data set of fundamental mode Rayleigh wave amplitudes is analyzed to derive global models of surface wave attenuation (1/Q). The data set consists of measurements of Rayleigh wave amplitude anomalies in the period range 50-250 s for 347 earthquakes observed at 179 seismic stations. The amplitude anomalies are considered to depend on four factors: intrinsic attenuation along the ray path, elastic focusing effects along the ray path, a source factor accounting for uncertainties in the strength of excitation, and a receiver factor accounting for uncertainties in the response at the station. The amplitude data are inverted simultaneously for global maps of attenuation expanded in spherical harmonics up to degree 12, global maps of phase velocity expanded to degree 20, and source and receiver correction factors. All four variable types are shown to be important in explaining the amplitude anomalies. A data set of phase delay measurements provides additional constraints on velocity structure. The maps of attenuation obtained by simultaneous inversion for elastic and anelastic models contain important features that are not robustly imaged when the effect of focusing on wave amplitude is ignored. These include high attenuation along western North America and along the East Pacific Rise and other ridge systems and low attenuation associated with stable continental interiors. The global attenuation maps exhibit a strong correlation with phase velocity maps corrected for the effect of the crust, particularly for periods <200 s. The correlation suggests that the variability in both Q and velocity in the shallow upper mantle is primarily thermal in origin.

Citation: Dalton, C. A., and G. Ekström (2006), Global models of surface wave attenuation, J. Geophys. Res., 111, B05317, doi:10.1029/2005JB003997.

1. Introduction

[2] Over the past 20 years, the development of threedimensional models of the Earth's seismic velocity structure has been accelerated by several factors, among them an expanding inventory of earthquake data, rapid growth in computational power, and improvements in wave propagation theory and model parameterization schemes. As a result, today there is reasonable agreement between most of the current three-dimensional (3-D) global velocity models, at least at long wavelengths (e.g., S362D1 [Gu et al., 2001]; S20RTS [Ritsema and van Heijst, 2000], SAW16BV [Romanowicz and Gung, 2002], and SB4L18 [Masters et al., 2000]). Attenuation (1/Q) tomography has thus far been a less successful means of probing the Earth's interior. Four recent and relatively low-resolution models of shear attenuation in the upper mantle [Bhattacharvva et al., 1996; Reid et al., 2001; Selby and Woodhouse, 2002; Gung and Romanowicz, 2004] show some qualitative agreement but in general lack consistent features. The development of Q models has lagged behind that of velocity models because factors other than attenuation influence wave amplitude, which is the

datum in most attenuation studies. Principally, amplitudes are affected by focusing and defocusing due to lateral velocity variations [*Lay and Kanamori*, 1985; *Woodhouse and Wong*, 1986; *Selby and Woodhouse*, 2000], but uncertainties in the calculation of source excitation as well as inaccuracies and problems associated with the instrument response can also obscure the attenuation signal in the data.

[3] Despite these difficulties, advancing our knowledge about the Earth's anelastic structure is essential for several reasons. One, attenuation is thought to be highly sensitive to temperature variations: $Q^{-1} \propto \exp(-E/RT)$, where T is temperature and E and R are activation energy and the gas constant, respectively [e.g., Jackson et al., 2002]. Given the different sensitivities of velocity and Q to temperature and composition [e.g., Karato, 1993; Faul and Jackson, 2005], joint interpretation of attenuation and velocity models should aid in distinguishing the effects of thermal and chemical heterogeneity on these quantities. Second, an attenuating medium causes physical dispersion of seismic waves, with long-period waves traveling more slowly than high-frequency waves [Liu et al., 1976; Kanamori and Anderson, 1977]. This effect has been incorporated into the 1-D Earth model PREM [Dziewonski and Anderson, 1981], in which an abrupt decrease in shear wave velocity at the base of the lid and the resulting low-velocity zone for frequencies less than the reference value of 1 Hz is produced by dispersion. Lateral variations in Q cause lateral

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variations in dispersion [e.g., *Romanowicz*, 1990], affecting velocity by as much as 1-2% (at 200 s [*Dalton and Ekström*, 2005]), and should be considered in the construction and interpretation of 3-D velocity models derived from data over a wide range of frequencies. Finally, seismic wave amplitudes are sensitive to lateral velocity variations. Thus, when the effect of attenuation has been removed, the wave amplitude can help to constrain elastic structure, in particular smaller-scale features [*Laske and Masters*, 1996].

[4] Global studies of upper mantle attenuation structure have primarily considered amplitudes of long-period (50-300 s) Rayleigh waves. Much of the progress over the last 15 years has come from the development of techniques to isolate the attenuation signal in the amplitude data. Early studies combined amplitudes from four consecutive arrivals of long-period Rayleigh waves (e.g., R1, R2, R3, R4) for each source-receiver pair to eliminate source uncertainty and effectively cancel out the focusing signal [Romanowicz, 1990; Durek et al., 1993]. This technique has limited applicability since only even spherical harmonic degrees of heterogeneity (symmetric through the center of the Earth) can be retrieved. Romanowicz [1994] imaged both even and odd structure by selecting individual R1 and R2 amplitude measurements that did not appear strongly contaminated by focusing or errors in the source parameters. The selection criteria sought consistency between attenuation coefficients estimated using four consecutive wave trains and those measured using only the minor or major arc amplitudes. The resulting surface wave Q maps at 80–300 s formed the data set for the first 3-D global model of shear attenuation in the mantle [Romanowicz, 1995].

[5] Recently, Selby and Woodhouse [2000, 2002] and Billien et al. [2000] have treated the focusing effect more explicitly in the determination of their Q models, employing the linear approximation for ray theory developed by Woodhouse and Wong [1986]. Selby and Woodhouse [2000] observed that Rayleigh wave amplitudes in the period range 73-171 s contain a considerable amount of signal from elastic focusing, and they inferred that the attenuation maps obtained by inverting their amplitude data set were contaminated by focusing effects for wavelengths shorter than spherical harmonic degree 9. In constructing their 3-D shear attenuation model of the upper mantle, Selby and Woodhouse [2002] did not consider focusing out of concern that it could not be treated sufficiently accurately, but solved for a frequencydependent amplitude correction factor for each event to account for uncertainty in the source amplitude.

[6] Rather than predict focusing effects with existing phase velocity maps, *Billien et al.* [2000] inverted measurements of Rayleigh wave phase and attenuation jointly for spherical harmonic degree-20 maps of phase velocity and attenuation. Additional source or receiver factors were not included in their analysis. *Gung and Romanowicz* [2004] recently developed a 3-D model of shear attenuation in the upper mantle from three-component surface wave waveform data that included both overtones and fundamental modes. They did not explicitly correct for focusing, source, or instrument effects, and they inferred from tests with synthetic data that neglecting these factors did not significantly bias their degree-8 model.

[7] In this study, four quantities are simultaneously determined from a large data set of Rayleigh wave amplitude

and phase delay measurements: maps of attenuation and phase velocity, and amplitude correction factors for each source and receiver included in the data set. Elastic focusing is treated using the path integral approximation of *Woodhouse* and Wong [1986]. Our Q maps contain features not seen in previous global attenuation models, including, at intermediate periods, continuous, linear zones of high attenuation along the East Pacific Rise and western North America and low attenuation beneath stable continental interiors that are also associated with fast Rayleigh wave phase velocity. We show how the retrieved attenuation maps are significantly dependent on the application of each of the corrections, and how focusing effects, when neglected, will map into inaccurate O structure. The method is outlined in section 2, and the data set of fundamental mode amplitude and phase measurements in the period range 50-250 s is described in section 3. Our preferred attenuation and phase velocity maps as well as source and receiver factors are presented in section 4. In section 5, we illustrate and discuss the importance of focusing, source, and receiver corrections on the retrieved Q maps. A comparison with the results of earlier studies in section 6 highlights the differences between our maps and previous attempts at modeling Q structure and shows a strong correlation between attenuation and velocities not observed before.

2. Method

[8] We describe the propagation of fundamental mode Rayleigh waves using ray theory. A surface wave seismogram, $u(\omega)$, can be written

$$u(\omega) = A(\omega) \exp[i\Phi(\omega)], \qquad (1)$$

where $A(\omega)$ and $\Phi(\omega)$ are the amplitude and phase of the wave as a function of the angular frequency ω . The wave amplitude is the quantity of primary interest for this paper and it can be expressed as

$$A(\omega) = A_{\rm S}(\omega)A_{\rm I}(\omega)A_{\rm F}(\omega)A_{\rm Q}(\omega), \qquad (2)$$

where $A_{\rm S}$ is due to excitation at the source, $A_{\rm I}$ is the receiver amplitude, $A_{\rm F}$ is the geometrical spreading factor, and $A_{\rm O}$ describes the decay due to attenuation along the ray path. Observations of wave amplitude are made with respect to a reference seismogram calculated using the appropriate moment tensor and centroid location from the Harvard centroid moment tensor (CMT) catalog [Dziewonski et al., 1981], the reported instrument response, and 1-D Earth structure from PREM. Therefore values of A_S, A_I, A_F, and A_O not equal to unity represent deviations away from the assumed source, receiver, geometrical spreading, and Qparameters. High or low values of $A_{\rm S}$ could be related to errors in the focal mechanism, scalar moment, or depth of the earthquake, or potentially to local Earth structure near the source [Selby and Woodhouse, 2002]. The factor A_I is most likely related to problems with the instrument response but could also contain effects of structure at the receiver site.

[9] The effect of focusing and defocusing of rays due to lateral heterogeneities in elastic velocity has been shown to cause significant modification of the wave amplitude [*Woodhouse and Wong*, 1986; *Wang and Dahlen*, 1994; Selby and Woodhouse, 2000]. Here, we treat this effect on amplitude using an expression from linearized ray theory,

$$\ln A_{\rm F}(\omega) = \frac{\delta c|_0}{2c_0}(\omega) + \frac{\delta c|_{\Delta}}{2c_0}(\omega) + \frac{1}{2}\operatorname{cosec}\Delta$$
$$\cdot \int_0^{\Delta} \left[\sin(\Delta - \phi)\sin\phi\partial_{\theta}^2 - \cos(\Delta - 2\phi)\right] \frac{\delta c}{c_0}(\omega)d\phi, (3)$$

where Δ is the epicentral distance, ϕ is the along-path coordinate, θ is the path-perpendicular coordinate, $\delta c/c_0$ is the relative perturbation in surface wave phase velocity, and $\delta c|_0$ and $\delta c|_{\Lambda}$ indicate the phase velocity perturbation at the source and receiver, respectively [Dahlen and Tromp, 1998]. This expression is slightly modified from the original one provided by Woodhouse and Wong [1986], as it includes a term with sensitivity to the phase velocity at the receiver. The wave amplitude due to focusing depends primarily on the second derivative of velocity perpendicular to the ray path. Waves traveling through a low-velocity trough are focused and amplified, and the opposite is true for propagation along a channel of fast velocity. Implicit in equation (3) is the assumption of an infinite frequency wave that does not deviate from the great circle path connecting the source and receiver. We argue in the Appendix that integrating along the great circle path instead of the true ray path is valid for the length scales in which we are interested.

[10] The perturbation in phase velocity, $\delta c/c_0(\omega)$, is expanded in spherical harmonics,

$$\frac{\delta c}{c_0}(\omega, \theta, \phi) = \sum_{l=0}^{L_c} \sum_{m=-l}^{l} C_{lm}(\omega) Y_{lm}(\theta, \phi), \tag{4}$$

where $Y_{lm}(\theta, \phi)$ are the fully normalized surface spherical harmonics of degree *l* and order *m*, L_c is the maximum degree of the phase velocity expansion, and $C_{lm}(\omega)$ are the coefficients to be determined. The focusing depends linearly on the phase velocity, and we write

$$\ln A_{\rm F}^{i,j}(\omega) = \sum_{l=0}^{L_c} \sum_{m=-l}^{l} C_{lm}(\omega) F_{lm}^{i,j},$$
(5)

where $F_{lm}^{i,j}$ represents the implementation of equation (3) in spherical harmonics for the path connecting the *i*th earth-quake and the *j*th receiver.

[11] The effect of attenuation on the wave amplitude, A_Q , is expressed as

$$A_{\rm Q}(\omega) = \exp\left[-\frac{\omega}{2U(\omega)} \int_{\rm path} \delta Q^{-1}(\omega, \theta, \phi) ds(\theta, \phi)\right], \quad (6)$$

where θ and ϕ are latitude and longitude, respectively, $U(\omega)$ is group velocity, and $\delta Q^{-1}(\omega, \theta, \phi)$ is the perturbation in surface wave attenuation away from the value predicted by PREM. Surface wave attenuation $Q^{-1}(\omega, \theta, \phi)$ is related to the Earth's intrinsic shear and bulk attenuation, $Q_{\mu}^{-1}(r, \theta, \phi)$ and $Q_{\kappa}^{-1}(r, \theta, \phi)$, by

$$Q^{-1}(\omega,\theta,\phi) = \int_0^a M(\omega,r)\mu(r)Q_{\mu}^{-1}(r,\theta,\phi)r^2dr + \int_0^a K(\omega,r)\kappa(r)Q_{\kappa}^{-1}(r,\theta,\phi)r^2dr, \qquad (7)$$



Figure 1. Kernels that describe the radial sensitivity of fundamental mode Rayleigh waves to the Earth's intrinsic shear attenuation for the range of periods examined in this study. The reference model is PREM [*Dziewonski and Anderson*, 1981].

where integration is over the radius of the Earth, and $K(\omega, r)$ $\kappa(r)$ and $M(\omega, r)\mu(r)$ are the kernels that describe the radial sensitivity of Rayleigh waves to bulk and shear attenuation (Figure 1). In this paper, we focus on the method for obtaining maps of surface wave attenuation at discrete frequencies and delay a discussion of the implications of our results for the Earth's intrinsic attenuation until a later paper.

[12] For an amplitude observation corresponding to the ith earthquake and the jth receiver, equation (6) can be simplified to

$$\mathcal{A}_{\mathbf{Q}}^{ij}(\omega) = \exp\left[-\frac{\omega X_{ij}}{2U(\omega)}\overline{\delta Q_{ij}^{-1}}(\omega)\right],\tag{8}$$

where $X_{i,j}$ is the length of the path and $\overline{\delta Q_{i,j}^{-1}}(\omega)$ is the average perturbation in Q^{-1} along that path. The lateral variations in δQ^{-1} are expanded with spherical harmonics,

$$\delta Q^{-1}(\omega, \theta, \phi) = \sum_{l=0}^{L_Q} \sum_{m=-l}^{l} \delta Q_{lm}^{-1}(\omega) Y_{lm}(\theta, \phi), \qquad (9)$$

where L_Q is the maximum degree of the Q^{-1} expansion, and $\delta Q_{lm}^{-1}(\omega)$ are the coefficients to be determined.

[13] In the inversion for our preferred surface wave attenuation maps presented in section 4, we solve for four quantities: $\ln A_{\rm S}$, $\ln A_{\rm I}$, $C_{lm}(\omega)$, and $\delta Q_{lm}^{-1}(\omega)$. For observations of amplitude anomalies $\ln A_{i,i}$, we can then write

$$\frac{-2U}{\omega X_{i,j}} \left[\ln A_{\rm S}^i + \ln A_{\rm I}^j + \sum_{l=0}^{L_c} \sum_{m=-l}^l C_{lm}(\omega) F_{lm}^{i,j} \right] \\ + \sum_{l=0}^{L_{\mathcal{Q}}} \sum_{m=-l}^l \delta \mathcal{Q}_{lm}^{-1}(\omega) \overline{Y}_{lm}^{i,j} = \frac{-2U \ln A_{i,j}}{\omega X_{i,j}},$$
(10)

where \overline{Y}_{lm}^{ij} is the path average of the spherical harmonic function $Y_{lm}(\theta, \phi)$. We also include measurements of phase delay in the inversion to provide additional constraints on the spherical harmonic coefficients of phase velocity. An observed phase anomaly $\delta\Phi(\omega)$ is attributed to a perturbation in phase along the propagation path and is modeled as [*Ekström et al.*, 1997]

$$\delta\Phi(\omega) = -\frac{\omega}{c_0(\omega)} \int_{\text{path}} \frac{\delta c}{c_0}(\omega, \theta, \phi) ds(\theta, \phi), \qquad (11)$$

which we implement in spherical harmonics as

$$-\frac{\omega X_{i,j}}{c_0}\sum_{l=0}^{L_c}\sum_{m=-l}^l C_{lm}(\omega)\overline{Y}_{lm}^{i,j} = \delta\Phi_{i,j}(\omega).$$
(12)

Equations (10) and (12) can then be used to form the inverse problem

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{d} + \mathbf{e} \;, \tag{13}$$

where **d** is the data vector, consisting of amplitude and phase measurements, **A** is the matrix containing the coefficients from the left-hand side of equations (10) and (12), and **x** contains the four types of unknowns. We solve the problem by least squares minimization of the error vector **e**.

3. Data

[14] The surface wave amplitude and phase anomaly observations that constitute the data set of this study were measured using the algorithm described by Ekström et al. [1997], which utilizes a phase-matched filter to isolate the fundamental mode from interfering overtones and measure its phase and amplitude. Velocity models derived from the phase measurements of this data set have been described previously [e.g., Ekström et al., 1997; Ekström and Dziewonski, 1998; Ekström, 2000]; however, the amplitudes have not been analyzed before. The observations were derived from earthquakes with $M_W > 6.0$ that occurred between 1993 and 2002 and measured from vertical component seismograms recorded by the stations of the Global Seismographic Network (GSN) of the Incorporated Research Institutions for Seismology (IRIS) and the U.S. Geological Survey, the China Digital Seismograph Network (CDSN), the Global Telemetered Seismograph Network (GTSN), and the MEDNET and GEOSCOPE networks. Fundamental mode Rayleigh waves with periods between 50 and 250 s are the focus of this paper (Figure 1).

[15] Our data are ratios, at each period, of observed to synthetic wave amplitude. The logarithm of each amplitude datum, ln $A_{i,j}$, is transformed into the quantity we fit in the inversion by multiplication of the factor $-2U/\omega X_{i,j}$ (equation (10), right-hand side). We use minor arc measurements (R1) for periods shorter than 150 s, and minor and major arc measurements (R1 and R2) for periods between 150 and 200 s. For periods longer than 200 s, R1, R2, R3, and R4 are used. The R4 data, which are fewer in number, are given additional weight in the inversion so that they have the same relative importance as R1; the same principle

applies to our treatment of R2 and R3. While the assumption that rays travel along the great circle connecting the source and receiver becomes less valid with increasing path length, we have found that higher orbits such as R3 and R4 are particularly useful for constraining even-degree structure at the longest periods of this study.

[16] The automated measurement procedure leads to a large set of amplitude anomalies. To ensure the quality of the data that are included in our inversion, we apply four selection criteria:

[17] 1. During the measurement process, a misfit value was assigned to each observation based on the level of fit between the observed and reference seismograms [*Ekström et al.*, 1997]. Paths with lower misfit are of higher quality, and we discard all observations with misfit values >0.15 for periods shorter than 150 s and >0.3 for longer periods, which corresponds to discarding >45% of the amplitude data at each period.

[18] 2. To avoid problems near the source and antipode, we discard paths for which the receiver lies within 20° of the source or its antipode.

[19] 3. For inversions from which source and receiver factors (A_S and A_I) are determined, we require that each earthquake and station has at least 30 paths associated with it to ensure that these factors are determined from a broad and even distribution of azimuths and path lengths.

[20] 4. We further reduce the amplitude data set that remains after the first three criteria have been applied by rejecting outliers, which we define as measurements that, at each period, deviate more than two standard deviations from the mean.

[21] The data set of phase anomalies is included in the inversion for additional constraint on the phase velocity maps. We apply the same criteria for misfit and epicentral distance to the phase data as we do to the amplitudes. Table 1 reports the original number of amplitude observations at each period, the number remaining after the four criteria listed above have been met, the number of events and stations that meet requirement 3, and the number of phase delay measurements. Figure 2 shows the path coverage achieved for Rayleigh waves at 75, 150, and 250 s.

4. Results

[22] We use the amplitude and phase data sets described in section 3 in a joint inversion for maps of attenuation, maps of phase velocity, and source and receiver correction factors. In our preferred results, the maps of attenuation are expanded to spherical harmonic degree 12, and the phase velocity maps are expanded to degree 20. While the phase velocity maps and correction factors do not require regularization, we find that in order to obtain physically reasonable values of attenuation, slight damping of the Q models is required. We choose to minimize a measure of the roughness of the attenuation maps, defined here as the squared RMS gradient of the model

$$\mathcal{R} \propto \left[\int_{\mathcal{S}} \left(\nabla \delta Q^{-1} \right) \left(\nabla \delta Q^{-1} \right) d\Omega \right]^{1/2}.$$
 (14)

Period, s	Total of Amplitudes	Number of Amplitudes Used	Number of Events	Number of Stations	Number of Phase	Arrivals Included
50	232,464	16,225	347	137	91,496	R1
75	232,509	16,066	343	136	90,980	R1
100	232,464	16,059	344	134	90,977	R1
125	232,464	16,076	343	134	90,977	R1
150	64,198	19,610	198	157	21,788	R1,R2
175	64,198	19,649	198	157	21,788	R1,R2
200	64,198	19,656	198	156	21,788	R1,R2
225	64,198	32,663	210	179	21,788	R1,R2,R3,R4
250	64,198	32,673	210	179	21,788	R1,R2,R3,R4

Table 1. Summary of Data Used in This Study

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[23] Selecting an appropriate level of damping requires some consideration, since there is still large uncertainty regarding the magnitude of lateral variation in Q^{-1} in the upper mantle; previous studies have noted a range of 50-100% [Romanowicz, 1994; Selby and Woodhouse, 2000; Reid et al., 2001]. Our choice is informed by two experiments. First, we inverted our data set of R1-R4 amplitudes for even degrees of structure, up to spherical harmonic degree 8, following the approach of Durek et al. [1993]. These inversions do not require regularization and result in Q maps that show a maximum variation of $\pm 40\%$ at 150-250 s. Second, we use the values of $A_{\rm S}$, $A_{\rm I}$, and $A_{\rm F}$ obtained from the joint inversion, which are mostly insensitive to how much the attenuation model is damped, together with the amplitude data to predict the average δQ^{-1} for each path. The range in $\delta Q^{-1}/Q^{-1}$ calculated in this way for similar path lengths is $\pm 60-80\%$. We choose damping factors that ensure that the variations in our maps lie in a similar range.

[24] We quantify the fit of our results to the amplitude data set by calculating the variance reduction,

$$vr_{\rm amp}(\omega) = 1 - \frac{\sum_{n=1}^{N} \left[q_n^{\rm obs}(\omega) - q_n^{\rm pred}(\omega) \right]^2}{\sum_{n=1}^{N} \left[q_n^{\rm obs}(\omega) \right]^2}, \qquad (15)$$

where

$$q_n^{\rm obs}(\omega) = \frac{-2U(\omega)\ln A_n}{\omega X_n} \tag{16}$$

and

$$q_n^{\text{pred}}(\omega) = \sum_{l=0}^{L_Q} \sum_{m=-l}^{l} \delta Q_{lm}^{-1}(\omega) \overline{Y}_{lm}^n + \frac{-2U}{\omega X_n} \left[\ln A_{\text{S}}^n + \ln A_{\text{I}}^n + \sum_{l=0}^{L_c} \sum_{m=-l}^{l} C_{lm}(\omega) F_{lm}^n \right].$$
(17)

In (15)–(17), N is the total number of amplitude observations, and $A_{\rm S}^n$ and $A_{\rm I}^n$ are the source and receiver factors corresponding to the *n*th observation. The variance reduction for the phase data set is calculated as

$$\nu r_{\text{phase}}(\omega) = 1 - \frac{\sum_{k=1}^{K} \left\{ \left[\frac{\delta c}{c}(\omega) \right]_{k}^{\text{obs}} - \left[\frac{\delta c}{c}(\omega) \right]_{k}^{\text{pred}} \right\}^{2}}{\sum_{k=1}^{K} \left\{ \left[\frac{\delta c}{c}(\omega) \right]_{k}^{\text{obs}} \right\}^{2}}, \qquad (18)$$

where

$$\left[\frac{\delta c}{c}(\omega)\right]_{k}^{\text{obs}} = \frac{-c_{0}(\omega)\delta\Phi_{k}}{\omega X_{k}}$$
(19)

and

$$\left[\frac{\delta c}{c}(\omega)\right]_{k}^{\text{pred}} = \sum_{l=0}^{L_{c}} \sum_{m=-l}^{l} C_{lm}(\omega) \overline{Y}_{lm}^{k}, \qquad (20)$$

and K is the total number of phase measurements.



Figure 2. Hit count, in $5^{\circ} \times 5^{\circ}$ cells, for Rayleigh waves at three periods. Value in each cell is normalized by area and by the maximum value at that period: 935, 1429, and 3229 at 75, 150, and 250 s, respectively. Boxes shaded white indicate cells in which the hit count is <5% of the maximum value, and boxes shaded black show cells in which the hit count is >50% of the maximum.



Figure 3. Dependence of the variance reduction of the amplitude (left axis; solid black line) and phase delay (right axis; dashed gray line) data sets on the relative weight of the two data sets. This example shows the results for 150-s Rayleigh waves. The combined data set included 19,610 amplitudes and 21,788 phase measurements; a weighting factor of 0.90 gives both data sets equal importance in determining the coefficients of phase velocity. For values of the weighting factor ≥ 0.1 , the variance reduction of either data set is not acutely sensitive to the value of the weighting factor. Note the different scales on the vertical axes.

[25] Phase measurements are more abundant than amplitudes at most periods in this study (Table 1). So that the two data sets contribute equally to the determination of the coefficients of phase velocity, the amplitude data are assigned additional weight in the inversion. We implement this weighting scheme by multiplying each phase datum and the corresponding coefficients in the A matrix (equation (13)) by a weighting factor. Typically, the weighting factor is chosen to equal the ratio of the number of amplitude measurements to the number of phase measurements and is ~ 0.2 at short periods and between 0.9-1.0 for longer periods. Figure 3 illustrates how, as the relative importance of the phase delay measurements increases, the fit of the results to the amplitude data is slightly diminished while the ability of the retrieved phase velocity maps to predict the phase data improves.

4.1. Attenuation Maps

[26] Our preferred maps of Rayleigh wave attenuation expanded to spherical harmonic degree 12 are shown in Figure 4 at six periods. Between 50 and 150 s, the attenuation maps show a strong correlation with surface tectonic features that has not been demonstrated before. In particular, spreading ridges such as the East Pacific Rise, the Pacific-Antarctic Ridge, and portions of the Mid-Indian and Mid-Atlantic ridges appear as linear features of high attenuation. Much of the western coast of North America is also a region of distinctly high attenuation. At these periods, the areas of low attenuation are generally located beneath stable continental interiors, such as in the Baltic region, western Australia, Canada, Antarctica, and the cratons of Africa and Brazil. The northwestern Pacific also shows a pronounced high-Q anomaly at short periods. Surprisingly, the Red Sea rift zone, a region of very slow phase velocities at these periods, does not appear as a prominent zone of high attenuation for periods shorter than 150 s. Its absence may indicate that the thermal and compositional properties of that area influence Q and velocity in a complex way. Alternatively, its absence could be the result of limited resolution.

[27] At the longest periods of this study (≥ 200 s), a different pattern, which may reflect structure in the transition zone, emerges from the attenuation maps. *Gung and Romanowicz* [2004] also observe a change in the pattern of attenuation in the mid-upper mantle; it occurs between 200 and 300 km in their global 3-D Q_{μ} model. In our maps, this deeper pattern is dominated by a wide zone of high attenuation in the southeastern Pacific and a localized region of high attenuation in the Red Sea area is present but weaker than at 150 s. Areas of low attenuation are located along several subduction zones in the Pacific, including the Kuril, Japanese, Ryukyu, Java, New Hebrides, and Kermedec trenches. Pronounced low attenuation is also seen beneath India.

4.2. Phase Velocity Maps

[28] In section 4.1, we demonstrated that when source and receiver uncertainty and elastic focusing are properly accounted for, improved maps of surface wave attenuation can be retrieved from measurements of amplitude. The sensitivity of the amplitudes to lateral variations in phase velocity (equation (3)) also makes them a valuable data set for studying elastic structure. Laske and Masters [1996] used polarization and amplitude data in addition to phase measurements to construct phase velocity maps for Love and Rayleigh waves and concluded that these additional data sets were essential for resolving short-wavelength velocity structure. In Figure 5 (top), we show phase velocity maps obtained by inverting observations of phase delay that were measured by applying the technique of Ekström et al. [1997] to earthquakes that occurred between 1993 and 2002. The maps have not been damped. Figure 5 (bottom) shows the preferred velocity maps that result from our joint inversion, which includes amplitude measurements in addition to the phase data to constrain velocity variations; these maps are also not damped.

[29] Most of the large-scale features, such as the slow velocities associated with mid-ocean ridges and western North America and the fast velocities beneath cratons, are present in both sets of maps. However, many of the shortwavelength features in the maps derived from phase data only are not present in the maps constructed from both phase and amplitude data. Because amplitudes depend primarily on the size of the lateral gradients in elastic heterogeneity and not on the magnitude of the heterogeneity itself, they are more sensitive than phase anomalies to shortwavelength structure. That the inclusion of amplitude data in the determination of phase velocity maps has the effect of reducing the power in the maps at short wavelengths (Figure 6) suggests that the small-scale structures in



Figure 4. Our preferred Rayleigh wave attenuation maps. The six periods span the entire range examined in this study, and the maps are expanded to spherical harmonic degree 12. The color scale indicates percent attenuation anomaly relative to the global average value at each frequency (Figure 14); cold colors show low attenuation, and hot colors show regions of high attenuation.

Figure 5 (top) are not real features of the Earth. The maps that included amplitude measurements increase the residual variance of the phase data set by only 3-4%. Surprisingly, maps very similar to Figure 5 (bottom) can be obtained from the amplitude measurements alone (C. A. Dalton and G. Ekström, Constraints on global maps of phase velocity from surface wave amplitudes, submitted to *Geophysical Journal International*, 2006).

4.3. Source and Receiver Factors

[30] A by-product of the analysis described in sections 4.1 and 4.2 is a frequency-dependent amplitude factor for each event and station that provided data for the inversion. To ensure that these factors are determined from a broad and even distribution of azimuths and path lengths, we require that each source and receiver has at least 30 observations associated with it. Because the mean values of A_S and A_I can trade off (equation (10)), the receiver factors are constrained

by our requirement that deviations of their value from unity have zero mean,

$$\sum_{j=1}^{N_I} \ln(A_I^j) = 0, \qquad (21)$$

where N_I is the total number of stations used in the inversion (Table 1).

[31] Figure 7 shows the distribution of the source factors, which are plotted with respect to the mean value at each period. At 75 and 150 s, 57.1% and 93.9% of the events require a correction smaller than 15%, i.e., their correction factor falls between 0.85 and 1.15. We believe that the larger spread in source factors observed at shorter periods reflects a greater sensitivity to uncertainty in earthquake depth and local elastic structure. In addition, our data set at periods <150 s is dominated by smaller earthquakes, for which uncertainties in the source parameters can be larger.



Figure 5. Rayleigh wave phase velocity maps at 75 and 150 s, expanded to spherical harmonic degree 20. (top) Derived from the phase measurements only and not damped. (bottom) Derived from both the phase and amplitude measurements and not damped. The average has been removed from each map.



Figure 6. Power per spherical harmonic degree for the velocity maps derived from the phase data only (Figure 5, top) and for the maps derived from both the phase and amplitude data (Figure 5, bottom). Although neither set of maps was damped, the inclusion of amplitudes reduces the power in the maps for degrees greater than 10 at (left) 75 s and (right) 150 s. Note the different vertical scales.



Figure 7. Histogram of source factors determined from the joint inversion for (left) 343 events used at 75 s and (right) 198 events used at 150 s. The source factors are plotted with respect to the mean value at each period. Note the different scales.

[32] At both periods, several events require a large amplitude correction. To determine if uncertainty in the centroid depth and its effect on scalar moment can explain the large corrections, we recalculate CMT solutions for these earthquakes using surface wave waveforms in addition to the data set of mantle waves and body waves that was used in the original analysis [Ekström et al., 2005]. The inclusion of surface waves improves the estimate of centroid depth for the shallow earthquakes of this study (h < 50 km). In Table 2, we compare the resulting depths and scalar moments to the values from the CMT catalog for the nine events analyzed at 150 s. In every case, the change in scalar moment has the same sign (<1 or >1) as the source factor obtained from our joint inversion; this is also true for 37 of the 46 events analyzed at 75 s. While it appears from this analysis that many of the source factors can be explained by uncertainties in the earthquake depth, there may be other contributions, such as Earth structure near the earthquake, to source amplitude.

[33] Figure 8 shows histograms of the receiver factors for 75 and 150 s; 91.4% and 92.4% of the stations require a

correction smaller than 15%, respectively. At both periods, several stations require a large correction, which may be the result of errors in the reported gain of the instrument during part or all of the 10-year time period studied. *Ekström et al.* [2006] have recently examined the amplitude calibration of the stations of the GSN, MEDNET, GEOSCOPE, and other global networks with a particular interest in identifying any systematic problems with instrument gain. Their method determines the scaling factor required to achieve an optimal fit between long-period observed and synthetic seismograms that are correlated in phase.

[34] The scaling factors of *Ekström et al.* [2006] are compared with the receiver factors obtained from our joint inversion in Figure 9. There is very good agreement between the two sets of factors, particularly at 250 s, and both techniques identify station channels with significant gain problems.

[35] When we perform the joint inversion without data from stations for which the 10-year-averaged scaling factor (Figure 9) is <0.85 or >1.15, the resulting attenuation maps are correlated with our preferred *Q* maps at 0.95 and 0.98 at

Table 2.	Comparison	of	Source	Parameters	Determined	With	and	Without	Surface	Waves a	t 150	0 s
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		CMT Catalog		CMT With Sur			
Event	Source Factor	Depth, km	M_o	Depth, km	M_o	Fraction Change	
E092793C	1.20	17.3	9.1E25 ^a	16.9	1.0E26	1.10	
E042195D	1.20	23.0	6.6E26	32.5	6.9E26	1.05	
E061495B	1.30	15.0	7.5E25	12.0	8.0E25	1.07	
E100395B	0.85	25.0	3.9E26	25.0	2.9E26	0.74	
E120295A	0.85	16.0	8.8E25	21.5	7.7E25	0.88	
E030899C	0.80	15.0	2.6E26	32.5	1.9E26	0.73	
E100400G	0.81	15.0	3.0E26	29.1	1.9E26	0.63	
E040901A	1.23	15.0	1.2E26	12.0	1.2E26	1.00	
E111502B	1.20	15.0	1.1E26	12.0	1.2E26	1.10	

^aRead 9.1E25 as 9.1×10^{25} .



Figure 8. Distribution of receiver factors determined from the joint inversion for (left) 136 stations used at 75 s and (right) 157 stations used at 150 s.

75 and 150 s. This restriction eliminates 26 stations and 5332 amplitudes at 75 s and 11 stations and 1266 amplitudes at 150 s. The source and receiver factors are also quite similar to those corresponding to our preferred maps, with

correlation coefficients for the source factors of \sim 99% and for the receiver factors greater than 97%. While we are satisfied that our results are robust in spite of errors in the instrument gain, an understanding of and remedy for these



Figure 9. Station-by-station comparison of the instrument gain scaling factors determined by *Ekström et al.* [2006] and the receiver factors obtained from the joint inversion of this study. We have averaged Ekström et al.'s vertical component annual median scaling factors for each station over the time period 1993–2002, excluding values <0.5 and >2.0, in order to maximize overlap with the data set of this study. (left) Comparison of surface wave scaling factors with the 75-s receiver factors of this study for 103 stations. Correlation coefficient is 0.69. (right) Comparison of mantle wave scaling factors with the 250-s receiver factors of this study for 111 stations. Correlation coefficient is 0.94. The dashed gray line indicates where the data plot if the two values are equal (y = x). The solid black line is the best fitting line.



Figure 10. (a) Amplitude variance reduction as a function of period for three separate inversions: our preferred results, which include degree-12 Q maps; an inversion that solves only for degree-12 Q maps; and an inversion that solves only for degree-30 Q maps. The number of unknowns for the joint inversion is approximately 1000. The number of unknowns for the degree-30 maps is 961. (b) The root-mean-square value of each of the four terms on the right-hand side of equation (17) as a function of period.

problems in addition to the implementation of routine instrument calibrations is clearly desirable.

5. Importance of the Attenuation, Focusing, Source, and Receiver Terms

[36] Our preferred degree-12 attenuation maps, degree-20 phase velocity maps, source factors, and receiver factors explain approximately 50% of the variance in the amplitude data. Here, we investigate the relative contribution of each of the four factors. Figure 10a shows the variance reduction of the amplitude data set achieved by our preferred results. For comparison, we also plot the variance reduction achieved by an inversion that solves only for degree-12 attenuation maps (i.e., it is assumed that the amplitudes can be explained entirely by attenuation). The inclusion of focusing effects and source and receiver factors, which increases the number of free parameters from 169 to as many as 1094, reduces variance by 30–40% more than the degree-12 maps alone.

[37] The number of unknown parameters in an inversion that solves only for degree-30 attenuation maps (961) is approximately the same as for our preferred results. We plot the variance reduction achieved by the degree-30 maps in Figure 10a. Our preferred results, which account for focusing effects and source and receiver uncertainty, reduce variance by nearly 20% more than the degree-30 maps. Since merely increasing the number of free parameters does not alone reduce variance by the amount observed, the terms that specifically consider elastic focusing and source and receiver amplitude must be responsible for the improved fit. [38] In order to quantify the relative importance of the attenuation, focusing, source, and receiver factors, we calculate the root mean square of each of the four terms on the right-hand side of equation (17) at each period (Figure 10b). Because the path coverage of our data set varies considerably with frequency (e.g., Figure 2), we use the same selection of minor arc paths at each period for the calculation of RMS. The RMS of the focusing term is slightly larger than that for attenuation at periods longer than 50 s. The source term is roughly constant at all frequencies, while the relative contribution of the receiver term increases with period.

[39] The impact of the terms that account for source and receiver uncertainty and focusing effects is observable not only in the calculation of variance reduction and RMS, but also in the features of the attenuation maps. We focus on two regions, the East Pacific Rise and North America, to illustrate the influence of these three factors on the *Q* maps. Figures 11a-11c and 12a-12c show three sets of degree-12 maps: the maps that result when elastic focusing and source and receiver uncertainty are ignored (i.e., it is assumed that the amplitude measurements can be explained only by attenuation) (Figures 11a and 12a); the maps obtained when the amplitude data are inverted for attenuation, and source and receiver factors (i.e., focusing effects are not removed) (Figures 11b and 12b); our preferred attenuation maps (Figures 11c and 12c), which result when source and receiver factors and focusing effects are included in the inversion of amplitude data. The differences between the three maps are pronounced. For example, when elastic focusing and source and receiver uncertainty are ignored, the linear zones of high attenuation along the East Pacific



Figure 11. Maps of attenuation for 75-s Rayleigh waves, centered on the East Pacific Rise, illustrating the effects of source and receiver factors and focusing. The path coverage and damping parameters are identical in all four maps. To facilitate a simple comparison, the color scale shows δQ^{-1} the perturbation in surface wave attenuation; Q^{-1} is 0.00819 in PREM. (a) The O map that results when the amplitude data are inverted for attenuation maps only. No source, receiver, or focusing terms were included. (b) Map that results when the amplitude data are inverted for three quantities: attenuation maps, source factors, and receiver factors. (c) Our preferred Q map, which includes corrections for source and receiver uncertainty and elastic focusing. (d) The focusing Q map that results when a synthetic data set of focusing-predicted amplitudes is inverted for attenuation only.

Rise and western North America in the preferred Q maps are replaced, in some spots, by areas of low attenuation (Figures 11a and 12a).

[40] Figures 11b and 12b show the Q maps that include corrections for source and receiver uncertainties. Differences between these maps and the Q maps not corrected for source and receiver uncertainty are small but important. At 75 s, amplitude correction factors for earthquakes located near the ridges have caused the patches of high attenuation to align along the East Pacific Rise and the Chile Rise. At 150 s, attenuation has been increased in southern California

and Mexico as a result of amplitude corrections for events and stations located in that area. Failure to account for uncertainty in the source and receiver amplitudes can result in inaccurate attenuation structure. For example, if the scalar moment used to predict the synthetic seismograms with respect to which the amplitude observations are made is too high for an event, the ratio of observed to synthetic amplitude for that event will generally be <1 at all stations, regardless of propagation effects such as attenuation and focusing. This reduced amplitude will map into a region of falsely high attenuation located near the earthquake when source uncertainty is not considered.

[41] The inclusion of focusing effects in addition to source and receiver factors in the inversion (Figures 11c and 12c) causes the patches of high attenuation along the East Pacific Rise to merge into one smooth and continuous feature, particularly between -45° S and -15° S. At 150 s, the zone of low attenuation beneath western North America has been replaced by high attenuation, and the low attenuation beneath the Canadian Shield has been further reduced as a result of the focusing correction.

[42] On a global scale, the Q maps determined from our joint inversion are mostly insensitive to the maximum spherical harmonic degree of the phase velocity maps that are simultaneously determined. At 75 s, the correlation between our preferred Q maps, which are determined simultaneously with degree-20 phase velocity maps, and Q maps determined simultaneously with degree-12 phase velocity maps is 0.96; the correlation of our preferred Q maps with Q maps determined simultaneously with degree-28 phase velocity maps is 0.97. At 150 s, these numbers are 0.94 and 0.98 for degree-12 and degree-28 phase velocity maps, respectively.

[43] To investigate the patterns of attenuation expected if our preferred Q maps were contaminated by elastic focusing, we invert a synthetic data set of focusing-predicted amplitudes (A_F) for degree-12 maps of attenuation. The synthetic data were generated using the phase velocity maps obtained from the joint inversion, and the resulting Q maps, hereinafter referred to as focusing Q maps, are shown in Figures 11d and 12d. The slow Rayleigh wave phase velocities characteristic of the East Pacific Rise at 75 s and the western United States at 150 s cause significant focusing of the many minor arc waves that travel along these features. The resulting enhancement of wave amplitude maps into a region of pronounced low attenuation when focusing effects are ignored.

[44] Figure 13 shows the global correlation between the focusing Q maps and the attenuation maps corrected only for source and receiver factors, and between the focusing Q maps and our preferred Q maps. At 75 s, the two sets of correlation factors are nearly identical through degree 5. For degrees 6 and greater, the correlation between the focusing Q map and the Q map corrected for source and receiver factors increases steadily while the correlation between the focusing Q map and the preferred Q map remains fairly constant at a value near -0.5. This pattern suggests that the Q map corrected only for source and receiver factors is contaminated by focusing effects that have not yet been removed, and, as was first noted by *Selby and Woodhouse* [2000], this is particularly true of the short-wavelength features in the map. The anticorrelation between the focus-



Figure 12. Same as Figure 11, but for 150-s Rayleigh waves and centered on North America. Q^{-1} is 0.00757 in PREM at 150 s.

ing Q map and the preferred Q map is not surprising, given the strong focusing potential of certain regions with strong velocity gradients, such as the western United States, the East Pacific Rise, and the cratons of Africa. The conclusions are essentially the same at 150 s (Figure 13, right).

[45] To investigate the possibility of trade-offs between the four terms that are simultaneously solved for (i.e., source factors, receiver factors, Q maps, and velocity maps), we have run numerous inversions with and without each of these terms. The degree-20 phase velocity maps are extremely robust regardless of which of the other three terms are included in the inversion. This is not surprising, given that our data set of phase delay measurements also constrains the velocity variations. The source and receiver factors are also quite robust. Even when we assume that the amplitude data depend only on the source factors (and do not solve for receiver factors, Q maps, or phase velocity maps), the resulting source factors are highly correlated with those corresponding to our preferred Q maps (correlation coefficient >0.94). When only receiver factors are determined from the inversion (source, attenuation, and focusing effects on amplitude are ignored), correlation with the receiver factors corresponding to our preferred Q maps is 0.95 at 150 s and 0.77 at 75 s. While there may be a small amount of trade-off between the receiver factors and the

other terms, particularly at short periods, the strong agreement between our receiver factors and the instrument scaling factors of *Ekström et al.* [2006] (i.e., Figure 9) supports the case for well-determined receiver factors. The Q maps are the least robust of the four quantities for which we invert; they depend on whether the other three terms have been included in the inversion. This underscores the importance of including corrections for focusing, source, and receiver effects in the inversion for attenuation, which is one of the main conclusions of this paper.

6. Discussion

6.1. Spherically Symmetric Term

[46] In Figure 14, we compare the global average Rayleigh wave Q values obtained from spherical harmonic degree zero of our preferred maps to predictions from 1-D Q models PREM, QM1 [*Widmer et al.*, 1991], and QL6 [*Durek and Ekström*, 1996]. Predictions from the radial Q_{μ} model of *Resovsky et al.* [2005], hereinafter RTV05, also are shown; we used the mean values of the probability density functions determined from that study for the predictions shown here. The global average values from the surface wave Q maps of *Selby and Woodhouse* [2000] are plotted as well. Between 50 and 125 s, the values from our



Figure 13. Grey lines show the global correlation coefficient between the focusing Q maps (e.g., Figures 11d and 12d) and the Q maps that result when source and receiver factors but not focusing effects are included in the inversion of amplitude data (e.g., Figures 11b and 12b). Black lines show the correlation between the focusing Q maps and our preferred Q maps (Figure 4). Bold line plots the correlation at each spherical harmonic degree; thin dashed line shows the cumulative correlation. (left) 75 s. (right) 150 s.

study are more attenuating then PREM and less so than QL6 and RTV05. Agreement with the results of Selby and Woodhouse [2000] is quite good. Our global average Q values agree well with both QL6 and RTV05 at 150 and 175 s and require less attenuation than QL6 and PREM at the longest periods. The similarity of QL6 and this study at 150 and 175 s suggests that the differences at longer periods must originate from the deepest structure sampled by the long-period surface waves, most likely near 400-500 km depth, where Q_{μ} is 165 in QL6 and 143 in PREM. Indeed, the agreement with RTV05, which contains a jump in Q_{μ} at 400 km from 170 to 185, supports the argument for a higher value of Q_{μ} in the transition zone. As has been noted by others [Durek and Ekström, 1997], QM1 consistently underpredicts attenuation throughout the period range considered here.

6.2. Comparison With Other Global Studies

[47] In Figure 15, we compare our preferred Q maps with the results of two previous surface wave attenuation studies, *Romanowicz* [1995] and *Selby and Woodhouse* [2000]. The attenuation maps constructed by *Romanowicz* [1995] were not explicitly corrected for uncertainty associated with source and receiver amplitudes or focusing effects, but amplitude data that exhibited indications of severe focusing or source problems were rejected during the data selection process. At 120 s (Figure 15, left), the two maps share certain large-scale features, such as high attenuation near the East Pacific Rise and low attenuation beneath Canada, parts of Eurasia, and northwestern Africa. The correlation coefficient is 0.44. The map of *Selby and Woodhouse* [2000] at 146 s, which also did not include any corrections, is



Figure 14. Spherically symmetric Rayleigh wave Q as a function of period from the degree-zero component of our preferred Q maps. For comparison, values from SW00 [Selby and Woodhouse, 2000], QL6 [Durek and Ekström, 1996], PREM, QM1 [Widmer et al., 1991], and RTV05 [Resovsky et al., 2005] are also plotted. The error bars on the Q values of this study represent the range of global average values determined from inversions that included different subsets of the data.



Figure 15. (left) Comparison of the 120-s Rayleigh wave Q maps of *Romanowicz* [1995] and this study. Both sets of maps are expanded to spherical harmonic degree 8, although the resolution of Romanowicz's map is closer to degree 6. (right) Comparison of the 146-s Q map of *Selby and Woodhouse* [2000] with the 150-s map of this study. Both maps are expanded to degree 12. The average value has been removed from all four maps.

compared to our preferred 150-s map in Figure 15 (right). Correlation coefficient is 0.24. The maps show reasonable agreement for parts of Africa, Australia, and Eurasia. The strong zone of low attenuation along the western United States in the map of Selby and Woodhouse we believe is an artefact related to focusing effects, which were not removed in that study (compare Figure 12).

[48] Joint interpretation of Q and velocity should aid in distinguishing the relative importance of temperature, composition, and the presence of melt/fluids on wave speed and amplitude in various regions. To date, comparisons between attenuation and velocity models have been qualitative in nature and inconclusive. Romanowicz [1990] observed a correlation between velocity and attenuation for great circle paths that was largest (correlation coefficient ~ -0.5) for 180-s Rayleigh waves. Billien et al. [2000] reported good correspondence between velocity and attenuation for Rayleigh waves at 40-50 s that diminished for longer periods. They also noted that the degree-2 pattern in attenuation at 160 s (150–300 km) was not clearly correlated with degree 2 in velocity. Gung and Romanowicz [2004] observed a correlation between Q and tectonics above 200-km depth and a degree-2 pattern in Q for depths >400 km that agreed well with the location of superplumes as inferred from elastic tomography.

[49] In Figure 16, we compare the preferred Q maps of this study with the phase velocity maps that are simulta-

neously determined from our joint inversion. The effect of the crust on the velocity maps has been removed by subtracting phase velocity maps predicted for a 3-D Earth model consisting of crustal structure described by CRUST2.0 [Bassin et al., 2000] and the laterally homogeneous PREM mantle. At both periods, the similarity between the two sets of maps is remarkable; they are correlated at -0.78 through degree 12 and at -0.83 through degree 8 at 75 s. At 150 s, the agreement is still quite good, with correlation coefficients of -0.73 through degree 12 and -0.79 through degree 8. If our preferred Q maps are instead compared to the phase velocity maps of Ekström et al. [1997], with the effect of the crust removed, the correlation through degree 12 is -0.77 and -0.73 at 75 and 150 s, respectively. The correlations are much stronger than have previously been observed and suggest a common cause, most likely thermal in origin, for the observed variability in both quantities. The improvement is possible because of the measures taken to isolate the signal of attenuation in the amplitude data set. The exact value of the correlation coefficient depends slightly on the relative weight of the amplitude data and the phase data (i.e., Figure 3). A weighting factor that assigns a small relative weight to the phase measurements decreases the magnitude of the correlation by 0.03-0.05, which does not alter our conclusion of a strong correlation between phase velocity and attenuation. We find that a 1% increase in

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Figure 16. Comparison of (bottom) the preferred Q maps of this study with (top) the phase velocity maps that are determined simultaneously from the joint inversion. Both sets of maps are expanded to spherical harmonic degree 12. The effect of the crust has been removed from the velocity maps; see text for more detail. Velocity variations are plotted in units of $\delta c/c(\%)$ and attenuation maps are plotted in units of $\delta Q^{-1}/Q^{-1}(\%)$. (left) 75 s. (right) 150 s. The average value has been removed from all maps.

velocity roughly corresponds to a 12.4% decrease in attenuation at 75 s and a 14.5% decrease at 150 s (Figure 17).

7. Conclusions

[50] We have presented the results of a simultaneous inversion of Rayleigh wave amplitude and phase delay

measurements for four quantities: global degree-12 maps of attenuation, global degree-20 maps of phase velocity, and amplitude correction factors for each source and receiver included in the data set. The results of the current study can be summarized as follows:

[51] 1. The global attenuation maps that result from this analysis contain features not imaged in previous attenuation



Figure 17. Comparison of the phase velocity and attenuation maps of Figure 16 sampled at 1442 evenly spaced points. (left) 75 s. Correlation is -0.78. Best fitting line $\delta Q^{-1}/Q^{-1}(\%) = -12.4(\delta c/c)(\%)$. (right) 150 s. Correlation = -0.73. Best fitting line $\delta Q^{-1}/Q^{-1}(\%) = -14.5(\delta c/c)(\%)$.



Figure A1. (a,b) Comparison of focusing-predicted amplitude anomalies calculated with ray perturbation theory (horizontal axis) and exact ray theory (vertical axis) for 75-s and 150-s Rayleigh waves. The gray boxes in the top figures define the area shown in the bottom figures. (c,d) The points in the top figures have been contoured according to the number of points that fall into each 0.05×0.05 cell. The contours are labeled by the percentage of the total number of points they enclose. The innermost contours, which are not labeled, contain 32% of the 29,125 points at 75 s and 47% of the 13,074 points at 150 s. Note the different scales.

studies, including continuous and linear zones of high attenuation along the East Pacific Rise and western North America.

[52] 2. The maps of attenuation exhibit a strong correlation with maps of phase velocity corrected for the effect of the crust.

[53] 3. The retrieval of the attenuation maps and the high level of agreement between attenuation and velocity are

possible because the effects of source and receiver uncertainty and focusing have been explicitly accounted for. Failure to account for any of these factors, in particular focusing, can result in inaccurate Q structure.

[54] 4. The phase velocity maps, source factors, and receiver factors also determined from the simultaneous inversion are robust, and there is minimal trade-off between these three terms.



Figure A2. (top) Q map that results when a synthetic data set of focusing-predicted amplitudes, calculated using exact ray theory, is inverted for degree-12 maps of attenuation and degree-20 phase velocity maps. This example is for 75-s Rayleigh waves. (bottom) As above, but here phase measurements have also been included in the inversion, as is the case for our preferred Q maps, to help constrain velocity structure. The maps are damped by the same amount as our 75-s preferred Q map.

[55] The attenuation models we retrieve in this study are 2-D maps of Rayleigh wave attenuation at discrete frequencies. We are currently developing a global 3-D model of shear attenuation in the upper mantle using the method described in this paper. A thorough analysis of this 3-D model as well as comparison to other 3-D Q_{μ} models will be presented in a future paper.

Appendix A: Ray Tracing

[56] In the calculation of attenuation and focusing, we assume that each surface wave ray propagates along the great circle connecting the source and receiver (i.e., ray perturbation theory). It has been shown previously, however, that rays may deviate from the great circle path in the presence of lateral variations in phase velocity [*Lay and Kanamori*, 1985; *Woodhouse and Wong*, 1986]. As a result of their dependence on off-path velocity structure (equation (3)), amplitudes are more sensitive to errors in the assumed propagation path than phase anomalies are [e.g., *Wang and Dahlen*, 1994; *Larson et al.*, 1998]. In this appendix, we discuss how our results are affected by integrating attenuation and focusing effects over the great circle path instead of over the true ray path (i.e., exact ray theory).

[57] In Figures A1a and A1b, we compare focusingpredicted amplitude anomalies calculated with ray perturbation theory and with exact ray theory. The degree-20 phase velocity maps obtained from our joint inversion were the input model for the predictions. In Figures A1c and A1d, the same data have been contoured according to their density. At both 75 and 150 s, 85% of the points fall very near the 1:1 line and exhibit absolute values of $\ln A_F < 0.4$ and <0.25, respectively. For most of the paths in our data set, predictions of focusing made with ray perturbation theory and with exact ray theory differ only slightly, an observation that is not readily apparent from Figures A1a and A1b.

[58] We investigate potential trade-offs between our attenuation and phase velocity maps by inverting the focusing-predicted amplitude anomalies for two quantities: maps of attenuation and maps of phase velocity. Since the input attenuation model for this test is spherically symmetric PREM, any lateral Q variations retrieved must result from leakage of elastic velocity signal into the attenuation maps. The results (Figure A2, top) show small but nonzero variations in attenuation, which are largest along western North America, the eastern coast of Eurasia, and in the northeastern Pacific. The magnitude of these variations is, at its maximum, 20% of the variations in the 75-s preferred Omap, and in most areas is much smaller. When the phase delay measurements are included to further constrain the velocity maps, the amount of leakage into the Q maps is reduced (Figure A2, bottom). The errors introduced by integrating along the great circle path do not change our conclusions; in fact, the results indicate that in certain areas, such as the western United States, Red Sea region, and East Pacific Rise, attenuation may be even higher than suggested by our preferred maps (Figure 4).

[59] We also use the two sets of focusing-predicted amplitudes to investigate more directly how the preferred Q maps are affected by our use of ray perturbation theory instead of exact ray theory. The prediction of focusing is subtracted from the amplitude observations prior to inversion, and the remaining amplitude signal is then inverted for three quantities simultaneously: maps of attenuation, source factors, and receiver factors. The resulting Q map for 75-s Rayleigh waves is nearly identical to our preferred Q map (correlation coefficient is 0.99). The relative values of the retrieved source and receiver factors are also very similar.

[60] Acknowledgments. We thank A. M. Dziewonski, M. Nettles, G. A. Abers, and G. Hirth for helpful discussions. We are grateful to N. D. Selby and B. Romanowicz for making their models available to us. J. Trampert and B. Romanowicz provided thoughtful and constructive comments during the review process. This work was supported by an NSF Graduate Research Fellowship and NSF grant EAR-02-07608. All of the map-based figures were produced with the GMT mapping software [*Wessel and Smith*, 1991].

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