Wave-Current Interaction Models for Rip Currents

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Two analytic models are developed to describe rip current cells on an open coastline with sloping planar foreshore and flat offshore bathymetry. Both models extend the work of LeBlond and Tang (1974) to include the refraction of the normally incident wave field by the nearshore circulation. The first model includes the effect of the currents on the incident wavelength, but no change in wave direction, and shows that no rip currents are possible. The second, more complete model considers the refraction of the waves as well and predicts rip current cells. The spacing of the rip currents and their associated variables are shown to be a function of one dimensionless parameter. Comparison of the predicted spacing with some published field data as well as with measurements obtained on the Isle of Sylt, West Germany, shows reasonable agreement with the second model.

INTRODUCTION

Numerous studies of nearshore circulation have been conducted to explain why rip currents, the narrow currents which flow rapidly seaward from the surf zone, often have very regular spacing along the beach. The first major mechanism was proposed by *Bowen* [1969] and *Bowen and Inman* [1969] and independently by *Harris* [1967]. This involved the concept of synchronous edge wave—incident wave interaction. The problem which arises with the proposed mechanism is the cause or existence of the synchronous edge wave, which travels parallel to the coast. Unless the beaches are flanked by headlands, as in the study of *Huntley and Bowen* [1975], where significant wave energy can be trapped by the headlands, the genesis of edge waves is difficult to explain. The field experiments of *Bowen and Inman* [1969] show that the field data on rip current spacing is best fit by mode 1 edge waves, yet a wave-wave interaction model developed by *Guza and Davis* [1974] only accounts for the generation of subharmonic edge waves, which do not cause rip currents. The model they proposed assumes total reflection from the beach, corresponding to 'surging' waves, and, in fact, *Guza and Inman* [1975] and *Guza and Bowen* [1976] have shown in the laboratory that waves breaking strongly damp out edge wave motions. Therefore the edge wave model for rip currents is apparently restricted to steep beaches and surging incident waves.

The regular spacing of rip currents can be induced by the presence of regularly varying bottom topography as shown by *Bowen* [1969], *Sonu* [1972], *Noda* [1973], *Birkemeier and Dalrymple* [1975], and *Mei and Liu* [1977]. Coastlines characterized by large offshore bars (often occurring after major storms) present an extreme example of this case. *Hite* [1925] noted the apparent setup of water behind the bars which then flowed seaward through rip channels. *Bruun* [1963] and *Dalrymple et al.* [1976] have presented different mechanisms to explain this case. Also, *Dalrymple* [1975] has shown that synchronous trains of incident waves can interact to create rip currents which are spaced according to the deep water wavelength and directions of the waves.

The possible importance of the wave-current interaction in nearshore circulation has been discussed for many years. As early as 1950, *Arthur* [1950] discussed the refraction effects of rip currents on the incident wave field, and later he speculated that this interaction may be important for the dynamics of rip currents [*Arthur,* 1962]. In model studies on rip currents caused by normally incident waves, *Harris* [1967] noted the slowing of the incident waves by the outflowing rip currents and also the refraction of the wave fronts that ensues.

The first theoretical attempt to show that rip currents may be supported by wave-current interaction was made by *LeBlond and Tang* [1974]; later, *Iwata* [1976] used a similar approach. Unfortunately, both analyses need revision, as we will point out later. Finally, *Mizuguchi* [1976] introduced a variable bottom friction coefficient whose functional dependence with depth was designed to ensure the existence of rip current cells.

The present study shows that steady nearshore circulation cells exist for normal wave incidence on a prismatic beach and for any dissipation mechanism in the surf zone for which the relationship between the wave height $\hat{H}$ and the total depth $\hat{h} + \hat{\eta}$ is well approximated by the relationship $\hat{H} = \kappa(\hat{h} + \hat{\eta})$, when κ is a breaking index of the order of unity.

For convenience in the presentation and in order to relate this work more easily to previous efforts, we discuss two models. In the first model the incident wave energy is affected by the waves, as in the paper by *LeBlond and Tang* [1974], and the local wavelength is changed by the presence of the currents. This model extends previous studies, yet no rip currents occur.

In a second model an additional wave-current interaction effect, the refraction of the waves on the current, is also included, and steady longshore periodic nearshore circulation cells are generated. The refraction of the waves by the outgoing rip currents causes them to impinge on the beach obliquely, thus generating longshore current flowing toward the location of the rip.

It is of importance to note that in these models the dissipation is introduced indirectly by its effects. More precisely, in the surf zone the energy equation is substituted by the relationship between the wave height $\hat{H}$ and the total depth $\hat{h} + \hat{\eta}$ given above. Since this condition is homogeneous in the unknowns, we may obtain the rip current cells but not the intensity of the dynamic variables and mean water level.

We assume that shallow water wave conditions (small depth to wavelength ratio) hold over the surf zone and a flat bottom

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offshore region. These are not unduly strong restrictions, as the nearshore circulation is confined to a very small region near the shore, extending only several surf zone widths offshore.

BASIC FORMULATION

The time- and depth-averaged equations of continuity of mass and motion have been developed elsewhere [e.g., *Phillips*, 1966]. The steady state continuity equation is

$$[\partial \hat{U}(\hat{h} + \hat{\eta})]/\partial \hat{x} + [\partial \hat{V}(\hat{h} + \hat{\eta})]/\partial \hat{y} = 0 \qquad (1)$$

where $\hat{h}$ is the still water depth, $(\hat{U}, \hat{V})$ are the horizontal vertically averaged velocities in the $(\hat{x}, \hat{y})$ directions, and $\hat{\eta}$ is the time-mean change in the water level due to the presence of the waves and currents. $\hat{\eta}$ is referred to as setup or setdown, depending on its sign. The transport stream function $\hat{\psi}$, defined by the relations

$$\hat{U}(\hat{h} + \hat{\eta}) = -\partial\hat{\psi}/\partial\hat{y} \qquad \hat{V}(\hat{h} + \hat{\eta}) = \partial\hat{\psi}/\partial\hat{x}$$

is now introduced to satisfy this equation exactly. See Figure 1 for the coordinate system.

The steady state equations of horizontal motion are

$$\hat{U}\frac{\partial \hat{U}}{\partial \hat{x}} + \hat{V}\frac{\partial \hat{U}}{\partial \hat{y}} = -\hat{g}\frac{\partial \hat{\eta}}{\partial \hat{x}} - \frac{1}{\hat{\rho}(\hat{h} + \hat{\eta})}\left(\frac{\partial \hat{S}_{\hat{x}\hat{x}}}{\partial \hat{x}} + \frac{\partial \hat{S}_{\hat{x}\hat{y}}}{\partial \hat{y}} + \hat{R}_x\right) \qquad (2a)$$

$$\hat{U}\frac{\partial \hat{V}}{\partial \hat{x}} + \hat{V}\frac{\partial \hat{V}}{\partial \hat{y}} = -\hat{g}\frac{\partial \hat{\eta}}{\partial \hat{y}} - \frac{1}{\hat{\rho}(\hat{h} + \hat{\eta})}\left(\frac{\partial \hat{S}_{\hat{y}\hat{x}}}{\partial \hat{x}} + \frac{\partial \hat{S}_{\hat{y}\hat{y}}}{\partial \hat{y}} + \hat{R}_y\right) \qquad (2b)$$

where $\hat{S}$ is the symmetric radiation stress tensor introduced by *Longuet-Higgins and Stewart* [e.g., 1964], $\hat{R}_x$ and $\hat{R}_y$ are the anisotropic resistance terms due to bottom friction, $\hat{g}$ is the gravitational acceleration, and $\hat{\rho}$ is the fluid density. Following *Longuet-Higgins* [1970] and *Iwata* [1976],

$$\hat{R}_x = \hat{\rho}F\kappa[\hat{g}(\hat{h} + \hat{\eta})]^{1/2}\pi^{-1}[(1 + \cos^2\theta)\hat{U} + [(\sin 2\theta)/2]\hat{V}] \qquad (3a)$$

$$\hat{R}_y = \hat{\rho}F\kappa[\hat{g}(\hat{h} + \hat{\eta})]^{1/2}\pi^{-1}[(1 + \sin^2\theta)\hat{V} + [(\sin 2\theta)/2]\hat{U}] \qquad (3b)$$

where θ is the angle of incidence of the waves (cf. Figure 1),

$$F = 1.41[\kappa/2\hat{k}\hat{k}_e]^{-2/3} \approx 1.41[(\hat{g}\kappa\hat{H}_b)^{1/2}/2\hat{\sigma}\hat{k}_e]^{-2/3} \qquad (3c)$$

[from *Kaijura*, 1958], $\hat{k}_e$ is the equivalent roughness of the bottom material, κ is the breaking index [*Galvin*, 1969], and the subscript b denotes values taken at the breaking line. In our calculations we approximate F by its value at the breaking line. Notice that for small angle of incidence θ, the onshore friction term is twice the longshore friction term $\hat{R}_y$. This is because of the influence of the wave-induced water particle motion, which is primarily in the onshore direction.

The steady state conservation of wave energy equation is

$$\frac{\partial[(\hat{U} + \hat{C}_g\hat{k}_x/\hat{k})E]}{\partial \hat{x}} + \frac{\partial(\hat{V} + \hat{C}_g\hat{k}_y/\hat{k})E}{\partial \hat{y}} = -\hat{S}_{\hat{x}\hat{x}}\frac{\partial \hat{U}}{\partial \hat{x}} - \hat{S}_{\hat{x}\hat{y}}\left[\frac{\partial \hat{U}}{\partial \hat{y}} + \frac{\partial \hat{V}}{\partial \hat{x}}\right] - \hat{S}_{\hat{y}\hat{y}}\frac{\partial \hat{V}}{\partial \hat{y}} - \hat{D} \qquad (4)$$

where $\hat{C}_g$ is the group velocity of the incident waves, which for shallow water is equal to $\hat{C} = [\hat{g}(\hat{h} + \hat{\eta})]^{1/2}$, the wave phase speed, and $\hat{D}$ is the energy dissipation due to wave breaking in the surf zone. The $(\hat{k}_x, \hat{k}_y)$ are the components of the wave number vector $\hat{\mathbf{k}}$ onto the $(\hat{x}, \hat{y})$ directions, and $\hat{k}$ is the magnitude of $\hat{\mathbf{k}}$.

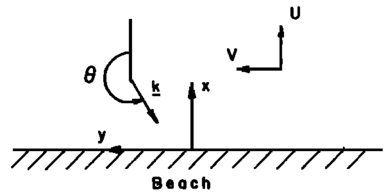

Fig. 1. Planform definition sketch for wave number and horizontal velocities.

The conservation of wave number in the steady state is given by

$$(\partial\hat{k}_x/\partial\hat{y}) - (\partial\hat{k}_y/\partial\hat{x}) = 0 \qquad (5)$$

where the wave number vector, $\hat{k} = \hat{k}_x\mathbf{i} + \hat{k}_y\mathbf{j}$ (where i, j are unit vectors in the $\hat{x}$, $\hat{y}$ directions, respectively), is related to the (constant) angular frequency of the waves $\hat{\sigma}$ and the water depth through the local dispersion relation,

$$\hat{\sigma} = \hat{k}[\hat{g}(\hat{h} + \hat{\eta})]^{-1/2} + \hat{\mathbf{k}}\cdot(\hat{U}\mathbf{i} + \hat{V}\mathbf{j}) \qquad (6)$$

SCALING AND PERTURBATION SCHEME

The problem is facilitated by making use of nondimensional variables. We use $\hat{\lambda}(= 2\pi/\hat{L}_r$, where $\hat{L}_r$ is the rip current spacing), the longshore wave number for the nearshore circulation, as the horizontal length scale and $\hat{\lambda}/m$, m, the beach slope, as the vertical length scale. Then the nondimensional quantities identified by the absence of circumflexes are

$$x = \hat{\lambda}\hat{x} \quad y = \hat{\lambda}\hat{y} \quad h = \hat{\lambda}m^{-1}\hat{h} \quad \hat{\eta} = \hat{\lambda}m^{-1}\hat{\eta}$$

$$k = (\hat{g}m\hat{\lambda}^{-1})^{1/2}\hat{\sigma}^{-1}\hat{k} \quad a = (\kappa/2)\hat{\lambda}m^{-1}\hat{a}$$

$$e = [\tfrac{1}{8}\hat{\rho}\hat{g}\kappa^2(m\hat{\lambda}^{-1})^2]^{-1}\hat{E} \quad \psi = (\hat{g}m\hat{\lambda}^{-1})^{-1/2}\hat{\lambda}^2m^{-1}\hat{\psi} \qquad (7)$$

$$S = [\tfrac{1}{8}\hat{\rho}\hat{g}\kappa^2(m\hat{\lambda}^{-1})^2]^{-1}\hat{S} \quad (C_g, U, V) = (\hat{g}m\hat{\lambda}^{-1})^{-1/2}(\hat{C}_g, \hat{U}, \hat{V})$$

$$D = [\tfrac{1}{8}\hat{\rho}\hat{g}\kappa^2(m\hat{\lambda}^{-1})^2]^{-1}[\hat{g}m\hat{\lambda}^{-1}]^{-1/2}\hat{\lambda}^{-1}\hat{D}$$

$$f = \kappa F/m\pi \quad A_D = \tilde{m}\kappa^2/8f$$

In the above relations the amplitude of the wave $\hat{a}$ relates to the energy by the equation $\hat{E} = \tfrac{1}{2}\hat{\rho}\hat{g}\hat{a}^2$, and in shallow water, disregarding the Reynolds stresses, the nondimensional radiation stresses are [*Phillips*, 1966, p. 51]

$$S_{xx} = \tfrac{1}{2}e(1 + 2\cos^2\theta)$$
$$S_{xy} = S_{yx} = e\sin\theta\cos\theta \qquad (8)$$
$$S_{yy} = \tfrac{1}{2}e(1 + 2\sin^2\theta)$$

We insert (7) and (8) into (1)–(6) to obtain

$$(h + \bar{\eta})\frac{\partial\bar{\eta}}{\partial x} = -\frac{\kappa^2}{16}\frac{\partial}{\partial x}[e(1 + 2\cos^2\theta)] - \frac{\kappa^2}{8}\frac{\partial}{\partial y}(e\sin\theta\cos\theta] + f(h + \bar{\eta})^{-1/2} \cdot \left[\partial\psi/\partial y(1 + \cos^2\theta) - \frac{\partial\psi}{\partial x}\frac{\sin^2\theta}{2}\right] - \left\{\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\left[\frac{1}{(h + \bar{\eta})}\frac{\partial\psi}{\partial y}\right]\right.$$

$$- \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \left[\frac{1}{(h + \bar{\eta})} \frac{\partial \psi}{\partial y} \right] \Bigg\} \tag{9}$$

$$(h + \bar{\eta}) \frac{\partial \bar{\eta}}{\partial y} = -\frac{\kappa^2}{8} \frac{\partial}{\partial x} [e \sin \theta \cos \theta] - \frac{\kappa^2}{16} \frac{\partial}{\partial y} [e(1 + 2\sin^2\theta)] - f(h + \bar{\eta})^{-1/2} \cdot \left[\frac{\partial \psi}{\partial x} (1 + \sin^2\theta) - \frac{\partial \psi}{\partial y} \frac{\sin^2\theta}{2} \right] + \Bigg\{ \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} \left[\frac{1}{(h + \bar{\eta})} \frac{\partial \psi}{\partial x} \right] - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \left[\frac{1}{(h + \bar{\eta})} \frac{\partial \psi}{\partial x} \right] \Bigg\} \tag{10}$$

$$\frac{\partial}{\partial x} \left\{ e \left[-\frac{1}{(h + \bar{\eta})} \frac{\partial \psi}{\partial y} + \cos \theta (h + \bar{\eta})^{1/2} \right] \right\} + \frac{\partial}{\partial y} \left\{ e \left[\frac{1}{(h + \bar{\eta})} \frac{\partial \psi}{\partial x} + \sin \theta (h + \bar{\eta})^{1/2} \right] \right\}$$
$$= \left(\frac{1 + 2\cos^2\theta}{2} \right) e \frac{\partial}{\partial x} \left[(h + \bar{\eta})^{-1} \frac{\partial \psi}{\partial y} \right] + (\sin \theta \cos \theta) \cdot e \left\{ \frac{\partial}{\partial y} \left[(h + \bar{\eta})^{-1} \frac{\partial \psi}{\partial y} \right] - \frac{\partial}{\partial x} \left[(h + \bar{\eta})^{-1} \frac{\partial \psi}{\partial x} \right] \right\} - \frac{(1 + 2\sin^2\theta)}{2} e \frac{\partial}{\partial y} \left[(h + \bar{\eta})^{-1} \frac{\partial \psi}{\partial x} \right] - D \tag{11}$$

$$(\partial k_y / \partial x) - (\partial k_x / \partial y) = 0 \tag{12}$$

$$1 = k(h + \bar{\eta})^{1/2} + \frac{\mathbf{k}}{(h + \bar{\eta})} \cdot \left(-\frac{\partial \psi}{\partial y} \mathbf{i} + \frac{\partial \psi}{\partial x} \mathbf{j} \right) \tag{13}$$

Offshore, (9)–(13) provide five equations for the five unknowns, $\bar{\eta}$, e, ψ, θ, k, as in (11), $D = 0$, whereas in the surf zone the energy balance equation (11) is replaced by the relationship

$$e = (h + \bar{\eta})^2 \tag{14}$$

It is well known that waves incident perpendicular to the shore induce a mean water level setdown offshore and a mean water level setup in the surf zone with no longshore variations. In this study we will hypothesize such a basic state and use a formal perturbation procedure to explore the possibility of rip current cells generated by the nonlinear wave-current interaction.

Accordingly, we seek solutions of our equations (8)–(13) in the offshore zone and (8)–(10) and (12)–(14) in the surf zone of the form

$$\begin{aligned} e &= e_0 + \epsilon e_1 \\ \bar{\eta} &= \bar{\eta}_0 + \epsilon \bar{\eta}_1 \\ \psi &= \epsilon \psi_1 \\ \theta &= \pi + \epsilon \theta_1 \\ \mathbf{k} &= (-k_0 - \epsilon k_1)\mathbf{i} - \epsilon k_0 \theta_1 \mathbf{j} \\ k &= k_0 + \epsilon k_1 \\ D &= D_0 + \epsilon D_1 \end{aligned} \tag{15}$$

where ϵ is an ordering parameter.

ZERO-ORDER EQUATIONS

We insert (15) in (9)–(13) and collect the leading terms to obtain

$$(h + \bar{\eta}_0) \frac{\partial \bar{\eta}_0}{\partial x} = -\tfrac{3}{16}\kappa^2 \frac{\partial e_0}{\partial x} \tag{16a}$$

$$(h + \bar{\eta}_0) \frac{\partial \bar{\eta}_0}{\partial y} = -\tfrac{1}{16}\kappa^2 \frac{\partial e_0}{\partial y} \tag{16b}$$

$$(\partial / \partial x)[e_0(h + \bar{\eta}_0)^{1/2}] = D_0 \tag{16c}$$

$$\partial k_0 / \partial y = 0 \tag{16d}$$

$$1 = k_0(h + \bar{\eta}_0)^{1/2} \tag{16e}$$

Offshore, $D_0 \equiv 0$, and (16c) implies that $e_0(h + \bar{\eta}_0)^{1/2}$ is a function of y, but (16d) implies that $k_0 = k_0(x)$; using this information successively in (16e) and then in (16b) yields $\bar{\eta}_0 = \bar{\eta}_0(x)$, $e_0 = e_0(x)$. In the offshore, then, the system (16) reduces to the system of ordinary differential equations

$$(h + \bar{\eta}_0) \frac{d}{dx} \bar{\eta}_0 = -\tfrac{3}{16}\kappa^2 \frac{de_0}{dx} \tag{17a}$$

$$\frac{d}{dx} [e_0(h + \bar{\eta}_0)^{1/2}] = 0 \tag{17b}$$

$$k_0 = (h + \bar{\eta}_0)^{-1/2} \tag{17c}$$

On the other hand, in the surf zone, (15) and (14) together with (16a), (16b), (16d), and (16e) yield

$$(h + \bar{\eta}_0) \frac{d\bar{\eta}_0}{dx} = -\tfrac{3}{16}\kappa^2 \frac{de_0}{dx} \tag{18a}$$

$$k_0 = (h + \bar{\eta}_0)^{-1/2} \tag{18b}$$

$$e_0 = (h + \bar{\eta}_0)^2 \tag{18c}$$

We use (18c) in (18a) to obtain

$$\bar{\eta}_0 = -\tfrac{3}{8}\kappa^2(h + \bar{\eta}_0) + c_1 \tag{19a}$$

For a beach of constant slope m and shoreline at $x = 0$ when no waves are present, the bottom depth is $h = x$ ($\tilde{h} = m\tilde{x}$), and

$$\bar{\eta}_0 = -\frac{\tfrac{3}{8}\kappa^2 x}{1 + \tfrac{3}{8}\kappa^2} + c_1 \tag{19b}$$

We now translate our system of coordinates so that in the new coordinates, $h + \bar{\eta}_0 = 0$ at $x = 0$, and hence $c_1 = 0$. From here on we use the latter system of coordinates, and for convenience we refer to the new offshore distance simply by x. We cast (19b) in the form $h + \bar{\eta}_0 = \tilde{m}x$, with

$$\tilde{m} = 1/(1 + \tfrac{3}{8}\kappa^2) \tag{19c}$$

If we use (19c) in (16c) we find a posteriori that the dissipation in the surf zone is to first order

$$D_0 = \tilde{m}(h + \bar{\eta}_0)^{3/2} \tag{20}$$

Offshore, D_0 is assumed to be zero, and the bottom is assumed to be flat; therefore $e_0 = e_\infty$ and $\bar{\eta}_0 = \bar{\eta}(x_b)$, both constants, where x_b is the breaker line distance from shore. The mean depth offshore ($h + \bar{\eta}_0$) is a constant and coincides with the mean depth d_b at the breaker line. The position of the breaker line has been given by *Dalrymple et al.* [1977] in terms of wave period, beach slope, and breaker height.

FIRST-ORDER EQUATIONS

We insert

$$d = h + \bar{\eta}_0 \tag{21}$$

the mean zero-order total depth, into (9)–(14) and collect the first-order terms to obtain for the offshore region the following equations:

$$d\frac{\partial\bar{\eta}_1}{\partial x}+\bar{\eta}_1\frac{\partial\bar{\eta}_0}{\partial x}=-\frac{3\kappa^2}{16}\frac{\partial e_1}{\partial x}-\frac{\kappa^2}{8}\frac{\partial}{\partial y}(e_0\theta_1)+2fd^{-1/2}\frac{\partial\psi_1}{\partial y}\qquad(22a)$$

$$d\frac{\partial\bar{\eta}_1}{\partial y}=-\frac{\kappa^2}{8}\frac{\partial(e_0\theta_1)}{\partial x}-\frac{\kappa^2}{16}\frac{\partial e_1}{\partial y}-fd^{-1/2}\frac{\partial\psi_1}{\partial x}\qquad(22b)$$

$$-\frac{\partial}{\partial x}\left[e_0d^{-1}\frac{\partial\psi_1}{\partial y}\right]+\frac{\partial}{\partial y}\left[e_0d^{-1}\frac{\partial\psi_1}{\partial x}\right]$$
$$-\frac{\partial}{\partial x}[d^{1/2}e_1+\tfrac{1}{2}d^{-1/2}e_0\bar{\eta}_1]-\frac{\partial}{\partial y}[e_0\,d^{+1/2}\theta_1]$$
$$=\tfrac{3}{2}\,e_0\frac{\partial}{\partial x}\left[d^{-1}\frac{\partial\psi_1}{\partial y}\right]-\tfrac{1}{2}\,e_0\frac{\partial}{\partial y}\left[d^{-1}\frac{\partial\psi_1}{\partial x}\right]-D_1\qquad(22c)$$

$$-d^{1/2}\frac{\partial}{\partial x}[d^{-1/2}\theta_1]=\frac{\partial}{\partial y}\left[\tfrac{1}{2}d^{-1}\bar{\eta}_1+d^{-3/2}\frac{\partial\psi_1}{\partial y}\right]\qquad(22d)$$

The above equations are valid in the surf zone region once we substitute the energy balance equation (22*c*) by

$$e_1=2h\bar{\eta}_1\qquad(22e)$$

which follows from (14) and (15). The second term in the momentum equation in the x direction (22*a*) and the fourth term in the energy equation (22*c*), each containing $\bar{\eta}_1$, have been omitted in previous analyses. These arise due to the first-order perturbation of the phase and group velocity of the wave field given as

$$C_0+\epsilon C_1=(h+\bar{\eta}_0+\epsilon\eta_1)^{1/2}\simeq(d)^{1/2}+\frac{\epsilon}{2}d^{-1/2}\bar{\eta}_1$$

We now seek bounded, continuous, and periodic (in the longshore coordinate) solutions of these systems of equations; physically meaningful solutions must be bounded at the shore and at infinity, and from the system of equations themselves we show in the appendix that for periodic solutions in y the unknowns ψ_1, θ_1, e_1, and $\bar{\eta}_1$ must be continuous at the breaker line. A closer analysis reveals that the boundness condition implies that ψ_1, θ_1, e_1, and $\bar{\eta}_1$ tend to zero as x approaches zero (the shore) or infinity.

MODEL I: EXTREMELY SMALL REFRACTION ANGLE

If we further assume that $\theta_1=O(\epsilon)$, i.e., negligible angle of incidence, then (22) simplifies considerably. In particular,(22*d*) is now

$$\bar{\eta}_1=-2d^{-1/2}\,(\partial\psi_1/\,\partial y)\qquad(23)$$

Since $\partial k_x/\partial y=0$, there is no longshore variation in wave number to first order, and wavelength variations induced by the currents are balanced by corresponding variations of the first-order mean water level $\bar{\eta}_1$.

We introduce in (22) the periodic functions

$$\psi_1=\tilde{\psi}(x)\sin y\qquad e_1=\tilde{e}(x)\cos y\qquad\bar{\eta}_1=\tilde{\eta}(x)\cos y\qquad(24)$$

and cross differentiate (22*a*) and (22*b*) to obtain

$$\tilde{\psi}''+\tilde{\psi}'\left[\frac{4d}{3f}-\frac{d'}{2d}\right]+\tilde{\psi}\left[\frac{4d'}{3f}-\frac{2}{3f}\eta_0'-\frac{2}{3}\right]=0\qquad(25)$$

where the prime indicates differentiation with respect to x.

At the shore $x=0$, (25) has a regular singularity, and the roots of the indicial equation are zero and $\tfrac{3}{2}$. The solution associated with the root zero yields an unbounded energy $\tilde{e}$ at $x=0$; hence $\psi\sim\Lambda x^{3/2}$ as $x\to 0$, and it is not difficult to show that $\bar{\eta}$ and $\tilde{e}$ are $O(x^{1/2})$ as $x\to 0$. At infinity if $h'\to 0$, then the bounded solutions $\psi\sim C\exp[-\lambda_0x]$, where $\lambda_0=\{B+[B^2+(\tfrac{8}{3})]^{1/2}\}/2$, $B=4d_\infty/3f$, and $d_\infty=\lim d(x)$ as $x\to\infty$.

Rip current cells will exist if (25) has bounded solutions at $x=0$ and at $x=\infty$ and if there exists x_b positive such that

$$\lim_{\substack{x\to x_b\\ x>x_b}}\frac{\tilde{\psi}'(x)}{\tilde{\psi}(x)}=\lim_{\substack{x\to x_b\\ x<x_b}}\frac{\tilde{\psi}'(x)}{\tilde{\psi}(x)}\qquad(26)$$

This last equation is equivalent to the continuity of $\tilde{\psi}$, $\tilde{e}$, and $\tilde{\eta}$ at the breaker line.

In particular, for a surf zone with bottom slope and flat bottom offshore the above boundary problem becomes after (19) as follows: $\tilde{\psi}$ solution of

$$\tilde{\psi}''+\left(MX-\frac{1}{2x}\right)\tilde{\psi}'+\left[M\left(1+\frac{3\kappa^2}{16}\right)-\frac{2}{3}\right]\tilde{\psi}=0$$

such that $\tilde{\psi}\sim x^{3/2}$ as $x\to 0$, and $[\tilde{\psi}'(x_b)/\tilde{\psi}(x_b)]=-\lambda_0x_b$ for some suitable positive number x_b. Here the eigenparameter $M=4\bar{m}/3f$. Numerical experiments reveal that for $0<M<100$ and all reasonable values of x_b there are no nontrivial solutions of the above boundary value problem. It is then fairly safe to say that for arbitrary monotonic decreasing bottom beach profiles $h(x)$ the present model will not yield a steady circulation of current cells.

Examination of previous works. Iwata correctly concluded that no eigenvalues existed for his problem, but then he attempted a solution by an asymptotic approach. However, the asymptotic solution is invalid, as can be seen by using his equation for ψ (prior to his equation (4.14)) in our notation

$$\tilde{\psi}''-\left(\frac{3}{2x}+m^2A_Dx\right)\tilde{\psi}'-2m^2\left(1-\frac{A_D}{4}\right)\tilde{\psi}=0$$
$$A_D=\frac{\kappa^2\bar{m}}{8f}\qquad(27)$$

The matching condition (26) states that the slope of the stream function is the opposite sign of the stream function itself inside the surf zone. Therefore one expects $\tilde{\psi}(x)$ to reach a maximum and then decrease at such a rate as to smoothly match magnitude and slope with the exponentially decreasing offshore solution. However, from Iwata's equation, at the location of $\tilde{\psi}'=0$,

$$\tilde{\psi}''=2m^2[1-(A_D/4)]\tilde{\psi}$$

i.e., the curvature of $\tilde{\psi}$ has the same sign as $\tilde{\psi}$ for $0<A_D<4$, which contains the region of physical interest. Thus there is no maximum, and the matching condition (26) cannot be satisfied.

Mizuguchi [1976] also concluded that no rip currents or eigenvalues exist, using an approach similar to that of LeBlond and Tang. (He goes further to express f as proportional to x^l, where l is some integer, and obtains eigenvalues; however, there is no a priori reason to justify this.) *LeBlond and Tang*'s [1974] work contains a significant numerical error (their $\Lambda_1(x)$ and $\Lambda_2(x)$ are not linearly independent), and their results are invalid.

MODEL II: FIRST-ORDER WAVE-CURRENT INTERACTION

We set

$$(\psi_1, \theta_1) = [\tilde{\psi}(x), \tilde{\theta}(x)] \sin y$$

$$(\bar{\eta}_1, e_1) = (\tilde{\eta}(x), \tilde{e}(x)] \cos y \qquad (28)$$

into the first-order equations (22) to obtain the system of equations

$$d\tilde{\eta}' + \eta_0'\tilde{\eta} = -\frac{3\kappa^2}{16}\tilde{e}' - \frac{\kappa^2}{8}e_0\tilde{\theta} + 2fd^{-1/2}\tilde{\psi} \qquad (29a)$$

$$-d\tilde{\eta} = -\frac{\kappa^2}{8}(e_0\tilde{\theta})' + \frac{\kappa^2}{16}\tilde{e} - fd^{-1/2}\tilde{\psi}' \qquad (29b)$$

$$d^{1/2}(d^{-1/2}\tilde{\theta})' = \tfrac{1}{2}d^{-1}\tilde{\eta} + d^{-3/2}\tilde{\psi} \qquad (29c)$$

and

$$-(e_0d^{-1})'\tilde{\psi} - e_0d'^2\tilde{\psi}' + \tfrac{3}{2}e_0d'd^{-2}\tilde{\psi}$$
$$= (d^{1/2}\tilde{e} + \tfrac{1}{2}d^{-1/2}e_0\tilde{\eta})' + d^{1/2}(e_0\tilde{\theta}) \qquad (29d)$$

In the surf zone we substitute (29d) with

$$\tilde{e} = 2d\tilde{\eta} \qquad (29e)$$

and after inserting (29e) into (29a), (29b), and (29c) we obtain a system of first-order equations in the unknown $\mathbf{q}_\text{I} = (\tilde{\eta}, \tilde{\psi}, \tilde{\theta})$. We write this system in the compact notation

$$\mathbf{q}_\text{I}' = \mathbf{M}_\text{I}\mathbf{q} \qquad (30)$$

where $\mathbf{M}$ is a 3 × 3 matrix. Offshore, we write (29a)-(29d) as the system

$$\mathbf{q}_\text{II}' = \mathbf{M}_\text{II}\mathbf{q}_\text{II} \qquad (31)$$

where $\mathbf{q}_\text{II} = (\hat{\eta}, \hat{\psi}, \hat{\theta}, \hat{e})$ and $\mathbf{M}_\text{II}$ is a 4 × 4 matrix. The position $x = x_b$ of the breaker line is not known a priori, and we seek numerically solutions $\tilde{\eta}, \tilde{\psi}, \tilde{\theta}, \tilde{e}$ bounded in the positive x axis satisfying (29e) and (30) in the interval $0 \leq x \leq x_b$ and (31) in the interval $x \geq x_b$ and continuous at $x = x_b$.

Solution technique. Offshore, because of the flat bottom assumption, the solution for the four variables takes the form of a set of exponential terms, $\{\mathbf{q}e^{\lambda x}\}$. Associated with the matrix equation (31), there are four eigenvalues ($\lambda(i), i = 1, 4$) and associated eigenvectors ($\mathbf{q}_\text{II}^i$, $i = 1, 4$) which span the space of all possible solutions offshore. For values of $A_D = \kappa^2\bar{m}/8f$ ranging from 1 to 6 the matrix $\mathbf{M}_\text{II}$ has one positive real eigenvalue (a growing solution offshore) and three eigenvalues with negative real part (decaying solutions), of which two were complex conjugates for $0 < A_D < 2.5$. The boundness condition for large x requires that the solution must be composed of the three decaying eigenvectors. Therefore the solution technique was to march in space the unique bounded solution of (29e) and (30) from $x = 0$ toward the offshore, checking at each x position whether or not the solution vector was perpendicular to the offshore eigenvector with the positive eigenvalue denoted, $\mathbf{q}_\text{II}^1$. The value of x at which this condition is met is then x_b, the breaker line location.

Mathematically, the condition used was [*Hildebrand*, 1965]

$$(\mathbf{q}\cdot\mathbf{Q}^1)/|\mathbf{q}|\ |\mathbf{Q}^1| = 0 \qquad (33)$$

where $\mathbf{Q}^i$ is the eigenvector associated with the transpose of $\mathbf{M}_\text{II}$ with an eigenvalue equal to that of $\mathbf{q}_\text{II}^i$, $i = 1, \cdots, 4$, and $\mathbf{q} = [\tilde{\eta}(x), \tilde{\psi}(x), \tilde{\theta}(x), \tilde{e}(x)], 0 \leq x \leq x_b$. This condition ensures that the solution contains modes that will satisfy the offshore boundary condition at $x = \infty$. Offshore the solution $\mathbf{q}$ is

$$\mathbf{q} = C_2\mathbf{q}_\text{II}^2 + C_3\mathbf{q}_\text{II}^3 + C_4\mathbf{q}_\text{II}^4 \qquad (34)$$

where $\{\mathbf{q}_\text{II}^i, i = 2, 3, 4\}$ are the decaying eigenvectors which span the remaining solution space and

$$C_i = \frac{\mathbf{q}(x_b)\cdot\mathbf{Q}^i}{|\mathbf{q}(x_b)|\ |\mathbf{Q}^i|} \qquad i = 2, 3, 4 \qquad (35)$$

In Table 1 and Figures 2 and 3, two cases for $A_D = 1.0$ and $A_D = 3.0$ are shown. The dimensionless breaker line locations, $x_b(= \hat{\lambda}\hat{x}_b)$, are also shown. For $A_D = 1.0$ the offshore solution is composed of two oscillatory components due to the presence of the complex eigenvalues. Their magnitude is quite small, and the offshore oscillations of $\tilde{\psi}$ are negligible. Also for this case, $\hat{L}_r = 2\pi\hat{x}_b/3.124$, or $\hat{L}_r \approx 1.6\hat{x}_b$. As A_D becomes greater, $\hat{L}_r$ becomes larger in proportion to $\hat{x}_b$, while for smaller A_D, $\hat{L}_r$ decreases. This trend can be seen in Figure 4. The nearshore circulations associated with Figures 2 and 3 are also confined to the nearshore region. After approximately three surf zone widths offshore, the circulation, or $\tilde{\psi}$, is negligible.

The influence of $\tilde{\theta}$ has caused the solution for ψ to have a discontinuous derivative at $x = x_b$, which causes the longshore velocity to be discontinuous at the breaker line. This behavior is not intrinsic to the model, for it is caused by the abrupt change of bottom slope, thus making $d\tilde{\theta}/dx$ discontinuous at the breaker line. Another manifestation of this discontinuity is the presence of two circulation cells. As there is a zero value for $\tilde{\psi}$ within the surf zone, there is one circulation system

TABLE 1. Eigenparameters for Offshore Variables $\mathbf{q}_\text{II} = (\tilde{\eta}, \tilde{\psi}, \tilde{\theta}, \tilde{e})$

	q_II^1	q_II^2	q_II^3	q_II^4
		$A_D = 1.0,\ x_b = 3.796$		
Eigenvalues	10.108	−5.215	−0.314 + 10.211i	$q_\text{II}^4 = (q_\text{II}^3)$
Eigenvectors*	−0.0638	0.000378	0.000244 + 0.00365i	
	−0.219	−0.174	0.0694 + 0.0118i	
	−0.00731	0.00969	0.0436 + 0.00316i	
	1.000	1.000	1.000 + 0i	
C_i	0	6.06 × 10⁹	−47.3 − 81.56i	
		$A_D = 3.0,\ x_b = 3.124$		
Eigenvalues	22.375	−12.844	−1.230	−0.149
Eigenvectors	−0.0802	0.000092	0.000067	0.001037
	−0.0889	0.0736	−0.031	0.0316
	−0.00534	0.00666	−0.0318	−2.477
	1.0	1.000	1.0	1.0
C_i	0	−8.996 × 10^{17}	48.368	3.155

*Note that the eigenvectors are known only to within an arbitrary multiplicative constant.

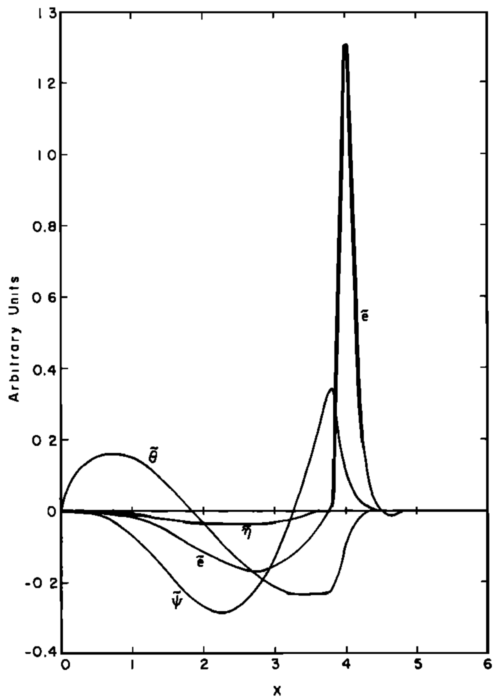

Fig. 2. Variation in the dimensionless offshore direction of the variables $\tilde{\eta}$, $\tilde{\psi}$, $\tilde{e}$, and $\tilde{\theta}$ for A_D = 1.0. The breaker line occurs at x = 3.79.

wholly contained within the surf zone and a counter circulating cell which crosses the breaker line. It is expected that for a uniformly sloping offshore bottom profile this discontinuity will be eliminated.

FIELD DATA COMPARISON

There are very few published sources of field data to compare with the rip current spacing predicted by theory. Two published sources were used herein. The first was the field data taken at El Moreno Beach, Mexico, published by Bowen and Inman. Despite extreme values of breaker angles, ranging from 10° to 21°, the comparison with theory, Figure 4, is not bad; however, the comparison with the edge wave prediction for spacing gives an average error of 3.5%, while the present theory has an average error of 63.3%. The second data source was that of *Balsillie* [1975], which lists data taken at the Coastal Engineering Research Center's Prototype Experimental Groin at Point Mugu, California. The data consist of wave period, breaking wave height, and breaking angles as well as surf zone width and rip current spacing. The beach slope was estimated by dividing the approximate breaking depth by the surf zone width, or $m = \hat{H}_b/\kappa\hat{x}_b$, where κ = 0.8. Of all the data tabulated only those taken 1000 ft (~305 m) north of the pier were used in order to avoid undue interference from the pier. Further, any data for days when the wind speed exceeded 10 knots (5 m/s) or the breaking angle exceeded ±5° of the beach normal were neglected. Approximately 40 individual observations remained during the period May 1972 to April 1973. The observations have a significant range of error, as the measurements are made visually by observers who estimated wave characteristics and rip current spacing. It is anticipated, however, that owing to the large amount of data used the variability in A_D and $\hat{\lambda}\hat{x}_b$ will, in the mean, cancel out. As can readily be seen in Figure 4, the data tend to confirm the theory quite well, even following the predicted curve of $\hat{\lambda}\hat{x}_b$ versus A_D as A_D becomes small. The wave breaker type during the measurement period was also recorded. Fifty-nine percent of the rips occurred during types of spilling breakers corresponding to the assumption made in the derivation, while 36% occurred when the waves were classified as spilling-plunging. Only one observation of a rip current was made during surging waves, which would correspond to the edge wave model of rip current formation.

From November 10 to 16, 1976, observations were made at the North Sea at the Isle of Sylt, West Germany, by the first author in conjunction with a study being conducted by the Leichweiss Institut für Wasserbau, Technische Universität Braunschweig. During this time, detectable rip currents occurred twice; each time when the wave angle at the breaker line θ_b was less than or equal to ±5°. The data for the measurements are shown in Table 2, and their average values are plotted in Figure 3. For the other days the wave angle exceeded 15° to the beach normal, and no periodic rips occurred. In comparing the average spacing of the rip currents to their predicted spacing, the present theory yields an average error of 4.1%, while the edge wave model of Bowen errs by over 50%.

The observed rip currents strongly interacted with the beach profile during each occurrence. The profile was, in fact, a composite profile consisting of a steep foreshore where the waves broke and a flat berm over which the swash occurred. The swash divided at the limit of uprush and returned seaward through small swash channels in the berm and then further seaward past the breaker line as a rip current. Figure 5 shows these rip currents.

CONCLUSIONS

The two models for steady nearshore circulation induced by normally incident waves have been explored for a planar foreshore of constant slope and a flat offshore bathymetry. The flow pattern in the nearshore zone consists of two cells, one wholly contained within the surf zone and one extending offshore of the breaker line. The presence of two counter rotating cells instead of one is presumed to be introduced by the discontinuity in the beach slope which occurs at the breaker line, although the laboratory experiment of *Mizuguchi and Horikawa* [1976, case 15] shows the same general circulation for a

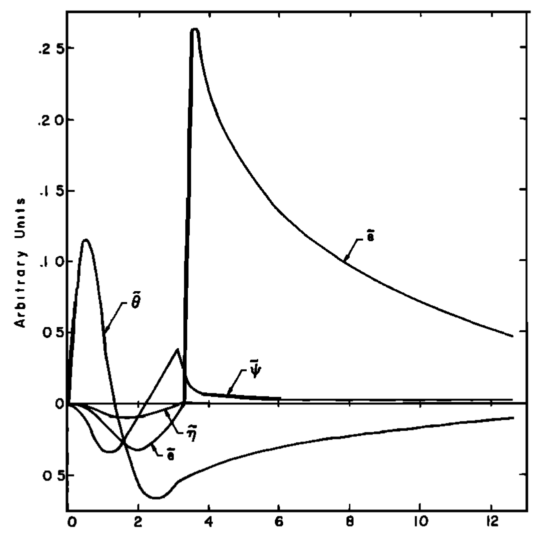

Fig. 3. Variation in the dimensionless offshore direction of the variables $\tilde{\eta}$, $\tilde{\psi}$, $\tilde{e}$, and $\tilde{\theta}$ for A_D = 3.0. The breaker line occurs at x = 3.12.

mild beach slope. The longshore dependencies of the variables and the computed results indicate that the offshore directed flows in both cells cause the waves to be refracted into these currents. However, because of the work done by the currents against the radiation stresses, the wave energy is reduced at the location of the rip currents. The longshore flow is therefore from regions of high wave energy and setup to regions of low energy, in the same manner as that of *Bowen* [1969].

This analysis, while showing that wave-current interaction can in fact support steady state rip current circulation, has not treated the initiation mechanism, that is, the original time-dependent instability which leads to this steady state.

Another important factor in the rip current system is the convective acceleration (last terms in (9) and (10)) which plays no role in our analysis but is most likely responsible for the strong offshore jetlike currents observed in the circulation cells.

Finally, a better understanding of the dissipation and friction mechanisms is necessary to improve our knowledge of the nearshore circulation.

APPENDIX

The matching conditions at the breaker line are derived from the equations of motion themselves. The procedure will be to integrate an equation over a small region containing a portion of the breaker line. For example, consider the integral of the continuity equation (1) which holds in the inshore and offshore regions

$$\int_{l_1}^{l_2}\int_{x_b-\delta}^{x_b+\delta}\left(\frac{\partial \hat{U}(\hat{h}+\hat{\eta})}{\partial x}+\frac{\partial \hat{V}(\hat{h}+\hat{\eta})}{\partial y}\right)dx\,dy=0 \qquad \text{(A1)}$$

where l_1 and l_2 are arbitrary. Integrating the first term with respect to x we find

$$\int_{l_1}^{l_2}[\hat{U}(\hat{h}+\hat{\eta})|_{x=x_b+\delta}-\hat{U}(h(\hat{h}+\hat{\eta})|_{x=x_b-\delta}]\,dy$$

$$+\int_{l_1}^{l_b}\int_{x_b-\delta}^{x_b+\delta}\frac{\partial \hat{V}(\hat{h}+\hat{\eta})}{\partial y}\,dx\,dy=0$$

and integrating the second term with respect to y we find

$$\int_{l_1}^{l_2}[\hat{U}(\hat{h}+\hat{\eta})|_{x=x_b+\delta}-\hat{U}(\hat{h}+\hat{\eta})|_{x=x_b-\delta}]\,dy$$

$$+\int_{x_b-\delta}^{x_b+\delta}[(\hat{V}(\hat{h}+\hat{\eta})|_{y=l_2}-\hat{V}(\hat{h}+\hat{\eta})|_{y=l_1}]\,dx=0$$

Defining the saltus of a function f at x_b $[[f]]=\lim_{\delta\to 0}[f(x_b+\delta)-f(x_b-\delta)]$, we find from the last equation

$$\int_{l_1}^{l_2}[[\hat{U}(\hat{h}+\hat{\eta})]]\,dy=-\lim_{\delta\to 0}$$

$$\cdot\int_{x_b-\delta}^{x_b+\delta}[\hat{V}(\hat{h}+\hat{\eta})\,|_{y=l_2}-\hat{V}(\hat{h}+\hat{\eta})|_{y=l_1}]dx$$

If $\hat{V}(\hat{h}+\hat{\eta})$ is continuous across x_b or has a finite discontinuity, the right-hand side of this equation is null, i.e.,

$$\int_{l_1}^{l_2}[[\hat{U}(\hat{h}+\hat{\eta})]]\,dy=0$$

but since the values l_1 and l_2 are arbitrary, the integrand is zero:

$$[[\hat{U}(\hat{h}+\hat{\eta})]]=0 \qquad \text{(A2)}$$

That is, the onshore-offshore flow of water is continuous

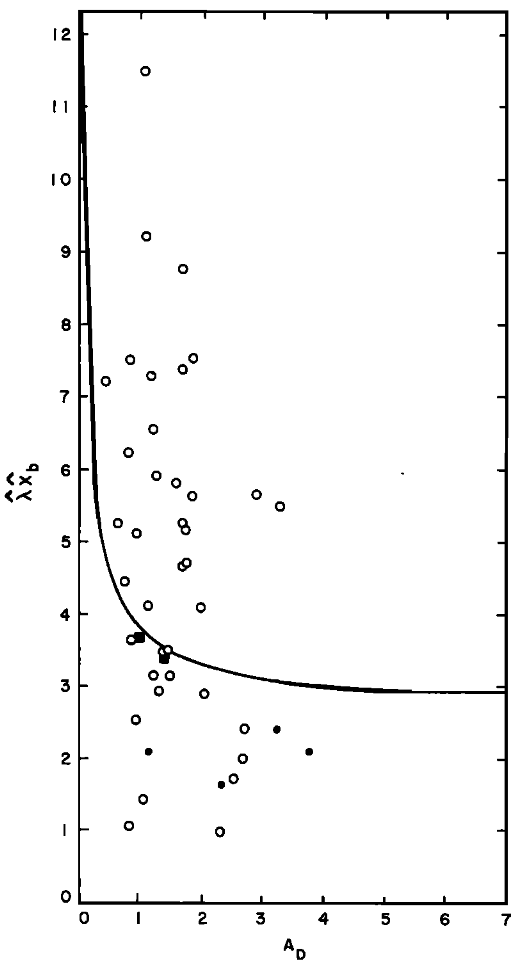

Fig. 4. Dimensionless rip current spacing $\hat{\lambda}\hat{x}_b$, where $\hat{\lambda}=2\pi/\hat{L}_r$ and $\hat{L}_r$ is the rip current spacing, versus A_D, the bottom slope to friction ratio, defined after (30). Field data sources are from *Balsillie* [1975] (open circles), *Bowen and Inman* [1969] (solid circles), and the Isle of Sylt (solid squares). For all data points the effective beach slope mm^* was taken as $(\hat{H}_b/\kappa\hat{x}_b)$ and $\hat{k}_e=4\times10^{-4}$ m. One data point ($\hat{\lambda}\hat{x}_b=14.6$, $A_D=2.05$) was not plotted.

TABLE 2. Rip Current Observations at Isle of Sylt, November 1976

	November 10, 1976	November 13, 1976
Wave period T, s	5.3	9.1
Breaking wave height $\hat{H}_b$, cm	30–40	30–40
Breaking angle θ_b, deg	+5	−5
Surf zone width $\hat{x}_b$, m	15	15
Beach slope m, in surf zone	0.07	0.1
Rip current spacing $\hat{L}_r$, m	35.1	44.6
	34.8	46.2
	24.1	30.8
	19.6	20.1
	16.6	22.0
	24.1	20.3
	17.8	25.2
	34.8	27.8
		24.3
		22.9
		26.3
		29.0
		27.0
		17.7
Average spacing	25.8	27.4

Fig. 5. Photograph of rip currents observed at the Isle of Sylt on November 13, 1976. The Technical University of Braunschweig installed the instrumented profile located at the center of picture.

across the breaker line. Using the same procedure for the other equations of motion we find

$$[[\theta_1]] = 0 \quad (A3)$$

from the y momentum equation (19b)

$$[[\bar{\eta}_1]] = -\frac{3\kappa^2}{16d_b}[[e_1]] \quad (A4)$$

from the x momentum equation (22a) and assuming D_1 has a finite discontinuity,

$$[[\bar{\eta}_1]] = -2d_b^{-1}[[e_1]] \quad (A5)$$

from the energy equation (22c). In matrix form, (A4) and (A5) are

$$\begin{bmatrix} 1 & \frac{3\kappa^2}{16d_b} \\ 1 & \frac{2}{d_b} \end{bmatrix} \begin{bmatrix} [[\eta_1]] \\ [[e_1]] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This matrix equation has nontrivial solutions only if the determinant of the coefficient matrix is zero, which requires $\kappa = (32/3)^{1/2}$. Since this value of κ is far too large than the observed values, η_1 and e_1 are also continuous across the breaker line.

Because of variations in the first-order mean water level and wave energy the breaker line is perturbed from its zero-order position; x_b and offshore and inshore variables must be matched at the actual first-order breaker line position. However, any first-order quantity, say, $G(x, y)$, at the perturbed breaker line x_{b_1} can be expanded in a first-order Taylor series about the zero-order breaker line x_{b_0}.

$$G(x_{b_1}, y) = G(x_{b_0}, y) + (x_{b_1} - x_{b_0})(\partial G/\partial X)(x_{b_0}, y) \quad (A6)$$

Since $(x_{b_1} - x_{b_0})$ is also a first-order quantity, the second term on the right-hand side is $O(\epsilon^2)$ and can be neglected [*Mei and Liu*, 1977]. Therefore, to first order, all matching conditions can be applied at the zero-order breaker line position x_{b_0}, which is precisely the distance x_b used elsewhere in this study.

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