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# THE ORIGINS OF WATER WAVE THEORY

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■ **Abstract** After early work by Newton, the eighteenth and early nineteenth century French mathematicians Laplace, Lagrange, Poisson, and Cauchy made real theoretical advances in the linear theory of water waves; in Germany, Gerstner considered nonlinear waves, and the brothers Weber performed fine experiments. Later in Britain during 1837–1847, Russell, Green, Kelland, Airy, and Earnshaw all made substantial contributions, setting the scene for subsequent work by Stokes and others.



## 1. WATER WAVES BEFORE 1800: NEWTON, LAPLACE, LAGRANGE

Isaac Newton was the first to attempt a theory of water waves. In *Book II, Prop. XLV* of *Principia* (1687), he proposed a dubious analogy with oscillations in a U-tube, correctly deducing that the frequency of deep-water waves must be proportional to the inverse of the square root of the "breadth of the wave." Newton's arguments were repeated by many later authors, among them the Dutch Wilhelm-Jacob s'Gravesande (1721) and the French Charles Bossut (1786). But Newton was aware that his result was approximate, shrewdly observing that, "These things are true upon the supposition that the parts of water ascend or descend in a right line; but in truth, that ascent and descent is rather performed in a circle" (Newton transl. Motte, 1729).

Much later, after Leonhard Euler's (1757a,b; 1761) derivation of the equations of hydrodynamics, Pierre-Simon Laplace (1776) reexamined wave motion; but despite making considerable progress (see below), his work was disregarded. Joseph-Louis Lagrange (1781, 1786), perhaps independently, derived the linearized governing equations for small-amplitude waves, and obtained the solution in the limiting case of long plane waves in shallow water. This is repeated almost verbatim in *Méchanique Analitique* (1788). For shallow-water waves, he found that "the speed of propagation of waves will be that which a heavy body would acquire in falling from . . . half the height of the water in the canal" (Lagrange 1786); that is, $(gh)^{1/2}$ where $g$ is gravitational acceleration and $h$ the liquid depth. This wave speed is independent of wavelength, provided the latter is long compared with $h$.

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This shallow-water approximation employed the "method of parallel sections," whereby all liquid at each value of the horizontal coordinate $x$ is assumed to have the same horizontal velocity, but this velocity and the disturbed depth are allowed to vary slowly in $x$ and time $t$. Such an approximation was not new. Daniel Bernoulli had applied it much earlier in his *Hydrodynamica* (1738) to flow through pipes of variable cross section; and a similar approximation occurs in Laplace's theory of tides. Lagrange made a cursory comparison with experiments of de la Hire, and wrongly claimed that his shallow-water results should remain a good approximation for deep-water waves because most of the motion is confined near the surface.

Before 1800, few other works mentioned wave motion. Exceptions are works by M. Flaugergues (1793) and François de la Coudraye (1796), later summarized by Weber & Weber (1825). I have not seen the first of these works, but the Webers's summary convinces me that it added nothing of theoretical value. Coudraye's work, which won prizes from the Royal Academies of Dijon and Copenhagen, comprises rather vague essays on wind and waves. Perhaps the most interesting item in his book is a table reproduced from an earlier work on ship construction by L.E.G. du Maitz de Goimpy (1776). This shows expected values of water surface velocity, wave height, wavelength, and wave speed for various wind speeds, and increases in water-level at coasts. But this table is not based on observations; instead, it gives numbers that closely agree with theoretical hypotheses that surface velocity and wave speed are directly proportional to wind speed, and that wave height and wavelength are proportional to the square of the wind speed. Conveniently, these hypotheses are consistent with Newton's U-tube demonstration that wave frequency is inversely proportional to the square root of wavelength. Coudraye observes that this table does not present definite results, but may serve as a basis for future studies.

Just how near Laplace (1776) came to giving a satisfactory account of linear water waves is not widely known. It was he who first posed the general initial-value problem: Given any localized initial disturbance of the liquid surface, what is the subsequent motion? Cauchy and Poisson later addressed this problem at great length (see below). Because of the subject's difficulty, Laplace restricted attention to particular initial disturbances of wavelike form, but confined to a finite domain connected to undisturbed liquid at either end.

From first principles, he shows that small displacements starting from rest are governed by (what we now call) Laplace's equation. Rather than using Eulerian velocity components as is now usual, he describes the motion in a "Lagrangian" manner. With $x$ and $z$ denoting the small horizontal and vertical displacements of individual fluid particles with initial positions ($X$, $Z$), he arrives at the periodic solutions

$$x = A \cdot \sin \frac{X}{c} \cdot \left\{ e^{\frac{Z}{c}} + e^{-\frac{Z}{c}} \right\}, \quad z = -A \cdot \cos \frac{X}{c} \cdot \left\{ e^{\frac{Z}{c}} - e^{-\frac{Z}{c}} \right\}, \quad (1.1)$$

where $Z$ and $z$ are zero at the channel bottom and $A$ is a function of time $t$. [The first statement of solutions of Laplace's equation as products of exponentials and sines or cosines is probably that of Euler (1757a,b), but he did not have waves in mind.] Laplace aims to solve the initial-value problem with the vertical free-surface coordinate at time $t = 0$ given by $Z = l + u$: $l$ is the undisturbed depth, and $u$ equals $a[\cos(X/c) - \cos(h/c)]$ for $-h < X < h$, and 0 otherwise, for chosen constants $h$ and $c$. Accordingly, he supposes that Equation 1.1 holds only for $-h < X < h$, and dubiously deduces that

$$0 = -\frac{ga}{c} + \frac{gA}{c} \cdot \left\{e^{\frac{l}{c}} - e^{-\frac{l}{c}}\right\} + \frac{\partial\partial A}{\partial t^2} \cdot \left\{e^{\frac{l}{c}} + e^{-\frac{l}{c}}\right\} \tag{1.2}$$

from his linear free-surface condition for constant pressure,

$$0 = g\left(\frac{\partial u}{\partial X}\right) + g\left(\frac{\partial z}{\partial X}\right) + \left(\frac{\partial\partial x}{\partial t^2}\right).$$

He imposes the initial conditions $A = \partial A/\partial t = 0$ at $t = 0$ and integrates to find

$$A = \frac{a}{e^{\frac{l}{c}} - e^{-\frac{l}{c}}} \cdot \{1 - \cos \cdot nt\},$$

with the frequency

$$n = \sqrt{\left\{\frac{\frac{g}{c}\left(e^{\frac{l}{c}} - e^{-\frac{l}{c}}\right)}{e^{\frac{l}{c}} + e^{-\frac{l}{c}}}\right\}} \tag{1.3}$$

(see Figure 1). This, of course, is the correct frequency of small-amplitude plane waves in water of depth $l$ and wavenumber $1/c$. Laplace further observed that his product of cosines in $X/c$ and $nt$ of Equation 1.1 can be decomposed into oppositely traveling waves with forms $\cos(X/c \pm nt)$.

If Laplace had taken $u = a = 0$ at the outset and left an arbitrary constant in $A$, his work would have been totally correct. Instead, it was a near miss, almost giving the solution for linear plane waves with fixed depth, 42 years before Poisson and 65 years before Airy. Lagrange did not appreciate the significance of Laplace's work, although he mentions it briefly, and he restricted his own treatment to long waves in shallow water.

## 2. 1800–1830: GERSTNER, CAUCHY, POISSON, AND THE WEBERS

A remarkable early paper by Franz Joseph von Gerstner (1802) gave the first exact nonlinear solution for waves of finite amplitude on deep water. The Gerstner wave solution was long overlooked; even today it is usually regarded more as a curiosity than a result of practical importance because the wave is not irrotational.

Suppofons $u = a \cdot (\text{cof.}\,\frac{X}{c} - \text{cof.}\,\frac{h}{c})$, de manière que l'on ait $u = 0$, tant que $X$ n'eft pas compris entre les limites $+h$ & $-h$, ce qui revient à faire au-delà de ces limites $\text{cof.}\,\frac{X}{c}$ conftamment égal à $\text{cof.}\,\frac{h}{c}$; l'équation $(T)$ devient alors

$$0 = -\frac{ga}{c} \cdot \text{fin.}\,\frac{X}{c} + g\left(\frac{\partial z}{\partial X}\right) + \left(\frac{\partial\partial x}{\partial t^2}\right); \quad (T')$$

On peut y fatisfaire & remplir toutes les conditions du mouvement, en fuppofant

$$x = A \cdot \text{fin.}\,\frac{X}{c} \cdot \left\{e^{\frac{z}{c}} + e^{\frac{-z}{c}}\right\},$$

$$z = -A \cdot \text{cof.}\,\frac{X}{c} \cdot \left\{e^{\frac{z}{c}} - e^{\frac{-z}{c}}\right\},$$

$e$ étant le nombre dont le logarithme hyperbolique eft l'unité, & $A$ étant fonction de $t$ feul; car il eft aifé de voir, que ces valeurs de $x$ & de $z$, fatisfont aux équations

$$\left(\frac{\partial x}{\partial X}\right) + \left(\frac{\partial z}{\partial Z}\right) = 0, \quad \left(\frac{\partial x}{\partial Z}\right) = \left(\frac{\partial z}{\partial X}\right);$$

& à la condition de $z = 0$ lorfque $Z = 0$. Si l'on change dans ces valeurs, $Z$ en $l$, & qu'enfuite, on les fubftitue dans l'équation $(T')$, on aura

$$0 = -\frac{ga}{c} + \frac{gA}{c} \cdot \left\{e^{\frac{l}{c}} - e^{\frac{-l}{c}}\right\} + \frac{\partial\partial A}{\partial t^2} \cdot \left\{e^{\frac{l}{c}} + e^{\frac{-l}{c}}\right\}.$$

Si l'on intègre cette équation, en ayant foin de déterminer les conftantes arbitraires, de manière qu'à l'origine du mouvement, on ait $A = 0$, & $\frac{\partial A}{\partial t} = 0$, on trouvera facilement





$$A = \frac{a}{e^{\frac{l}{c}} - e^{\frac{-l}{c}}} \cdot \{1 - \text{cof.}\,nt\},$$

$n$ étant égale à $\sqrt{\left\{\frac{\frac{g}{c}\left(e^{\frac{l}{c}} - e^{\frac{-l}{c}}\right)}{e^{\frac{l}{c}} + e^{\frac{-l}{c}}}\right\}}$; partant on a généralement

$$x = a \cdot \frac{\left\{e^{\frac{z}{c}} + e^{\frac{-z}{c}}\right\}}{e^{\frac{l}{c}} - e^{\frac{-l}{c}}} \cdot \text{fin.}\,\frac{X}{c} \cdot \{1 - \text{cof.}\,nt\},$$

$$z = -a\,\frac{\left\{e^{\frac{z}{c}} - e^{\frac{-z}{c}}\right\}}{e^{\frac{l}{c}} - e^{\frac{-l}{c}}} \cdot \text{cof.}\,\frac{X}{c} \cdot \{1 - \text{cof.}\,nt\}.$$

Il réfulte de ces expreffions, que les molécules intérieures du fluide ofcillent d'une manière femblable à celles de la furface, avec cette feule différence, que leur mouvement dans le fens vertical, eft moindre dans la raifon de $e^{\frac{z}{c}} - e^{\frac{-z}{c}}$, à $e^{\frac{l}{c}} - e^{\frac{-l}{c}}$, & que leur mouvement dans le fens horizontal, eft moindre dans la raifon de $e^{\frac{z}{c}} + e^{\frac{-z}{c}}$, à $e^{\frac{l}{c}} + e^{\frac{-l}{c}}$; d'où il fuit, que fi $c$ eft peu confidérable, le mouvement du fluide fera prefque infenfible à une médiocre profondeur: il ne s'agit donc plus que de bien déterminer, pour tous les points fitués à la furface du fluide, la fignification des valeurs précédentes de $x$ & de $z$, qui, ayant été données par l'intégration d'équations aux différences partielles, doivent être plutôt regardées comme des fymboles, que comme de véritables expreffions analytiques. Si l'on confidéroit en effet fous ce dernier

Its independent rediscovery by W.J. Macquorn Rankine (1863) revived interest in it.

Thomas Young wrote extensively on tides, but only briefly on waves. On waves, he added nothing new but gave perhaps the first account in English of waves in shallow water (Young 1821, pp. 318–27).

In December 1813, the French Académie des Sciences announced a mathematical prize competition on surface wave propagation on liquid of indefinite depth. In July 1815, 25-year-old Augustin-Louis Cauchy submitted his entry, and, in August, Siméon D. Poisson, one of the judges, deposited a memoir of his own to record his independent work (Dalmedico 1988). Cauchy was awarded the prize in 1816, Poisson's memoir was published in 1818, and Cauchy's work eventually appeared in 1827, with an astonishing 188 pages of additional notes.

This work, which confronted the general initial-value problem for linearized water waves, displayed a mathematical sophistication far beyond anything attempted before, and for long afterwards. Both axisymmetric and two-dimensional cases were considered. The analysis employed de facto Fourier transforms, comprising superpositions of a continuum of standing-wave modes, each with their own frequency of oscillation; and then asymptotic approximations were employed to evaluate the integrals. Not only did the methods of analysis repel most readers; the results, too, seemed baffling and contrary to intuition. For instance, wave crests, propagating out from a localized two-dimensional disturbance, did so with uniform acceleration. Though correct (physically, as a result of interference of dispersive Fourier components), the negative reaction to this impressive work was hardly surprising. An accessible account of the Cauchy-Poisson analysis, but restricted to two-dimensional disturbances, was given by Horace Lamb in *Hydrodynamic*s (1895): the account in his 1895 first edition is brief, incomplete and relegated to small print, but he treats the subject more fully in the fourth edition of 1916. The Cauchy-Poisson analysis is now acknowledged as an important milestone in the mathematical theory of initial-value problems.

It is worth pointing out an error in Cauchy's derivation of the fundamental equations, which seems not to have been noticed previously, although Dalmedico (1988) and Grattan-Guinness (1990) highlighted this derivation.

Where $Q$ denotes the velocity potential, $x$, $z$ the horizontal coordinates, and $y$ the vertical coordinate, one has, uncontroversially (Cauchy 1827, pp. 56–57),

$$\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} + \frac{\partial^2 Q}{\partial z^2} = 0 \tag{2.1}$$

within the fluid, together with the linearized free-surface condition

**Figure 1** Extract from Laplace (1776) showing the first statement of the dispersion relation for linear gravity waves.

$$g\frac{\partial Q}{\partial y} + \frac{\partial^2 Q}{\partial t^2} = 0. \tag{2.2}$$

Cauchy then takes the second time-derivative of Equation 2.2, obtaining

$$\frac{\partial^4 Q}{\partial t^4} = -g\frac{\partial^3 Q}{\partial t^2 \partial y} = -g\frac{\partial^3 Q}{\partial y \partial t^2} = g^2\frac{\partial^2 Q}{\partial y^2} \tag{2.3}$$

on using Equation 2.2 again. Then Equation 2.1 gives

$$\frac{\partial^4 Q}{\partial t^4} + g^2\left(\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial z^2}\right) = 0. \tag{2.4}$$

If we assume periodic waves of form $exp[i(kx + lz - \omega t)]$, we recover the correct dispersion relation for deep-water waves, namely

$$\omega^2 = g(k^2 + l^2)^{1/2}. \tag{2.5}$$

Though all seems well, it is not. The argument, if valid, should apply equally to waves in liquid of finite depth because the bottom boundary condition has not been employed; but this obviously gives the wrong answer. The flaw lies in interchanging the order of differentiation of $y$ and $t$ in Equation 2.3. Equation 2.2 is valid only at the liquid surface, but such interchange is equivalent to taking its $y$-derivative, which is not permissible. Cauchy eventually realized that his equation applied only to infinite depth, citing Poisson's finite-depth result in his Note XV (Cauchy 1827, pp. 173–74).

Later objections to the impenetrability of the Cauchy-Poisson analysis had some substance: One will search in vain through the 300 pages of Cauchy's paper to find a clear statement of the dependence on wavelength of the frequency of plane deep-water gravity waves! Nevertheless, this fundamental result is implicit throughout his analysis. Poisson is much clearer: Before embarking on the general initial-value problem, he gives a brief and correct derivation of the frequency of linear sinusoidal standing waves in finite depth (Poisson 1818, pp. 79–85). Figure 2 shows his result.

Shortly before the delayed appearance of Cauchy's memoir, a very different work on waves was published in Leipzig: *Wellenlehre auf Experimente gegründet* (1825) by the brothers Ernst Heinrich Weber & Wilhelm Eduard Weber. The former was professor of human anatomy at the University of Leipzig; the latter, better known for his work on electricity, became professor of physics at the Universities of Halle, Göttingen, and Leipzig. This book describes the Webers's careful laboratory experiments on plane periodic wavetrains in a channel, together with extracts from, and comments on, such theoretical work as was then available. These include

**Figure 2** Extract from Poisson (1818) showing the first satisfactory derivation of the dispersion relation for linear standing waves.

satisfaire par une valeur de $\varphi$ composée d'exponentielles, de sinus et cosinus, et, sous cette forme, la solution la plus générale est celle-ci :

$$\varphi = \Sigma\left(\mathrm{A}e^{-az} + \mathrm{A}'e^{az}\right)\cos.(ax + a');$$

$e$ représentant la base des logarithmes dont le module est l'unité ; A, A′, $a$, $a'$, étant des quantités indépendantes de $x$ et $z$ ; et la caractéristique $\Sigma$ indiquant une somme qui s'étend à toutes les valeurs possibles, réelles ou imaginaires, de ces quatre quantités.

Différenciant cette valeur de $\varphi$ par rapport a $z$, et faisant ensuite $z = h$, on aura, en vertu de l'équation (5),

$$\Sigma\left(\mathrm{A}e^{-ah} - \mathrm{A}'e^{ah}\right)a.\cos.(ax + a') = 0;$$

or, cette équation ayant lieu pour toutes les valeurs de $x$, elle doit subsister séparément pour chaque terme de la somme $\Sigma$; on aura donc généralement

$$\mathrm{A}e^{-ah} - \mathrm{A}'e^{ah} = 0;$$

d'où l'on tire

$$\mathrm{A} = \mathrm{T}e^{ah}, \quad \mathrm{A}' = \mathrm{T}e^{-ah};$$

T étant une nouvelle indéterminée.

La valeur de $\varphi$ devient alors

$$\varphi = \Sigma\mathrm{T}\left(e^{a(h-z)} + e^{-a(h-z)}\right)\cos.(ax + a');$$

les trois indéterminées T, $a$ et $a'$ qu'elle renferme, pourraient être regardées comme des fonctions de $t$; mais pour sa-





tisfaire à l'équation (4), quelle que soit la valeur de $x$, il est aisé de voir qu'il faut supposer $a$ et $a'$ constante, et T seule dépendante de $t$. Prenant, dans cette hypothèse, les valeurs de $\frac{d\varphi}{dz}$ et $\frac{d^2\varphi}{dt^2}$, et y faisant $z = 0$, on aura, en vertu de cette équation (4),

$$\Sigma\left[g\mathrm{T}a\left(e^{-ah} - e^{ah}\right) - \frac{d^2\mathrm{T}}{dt^2}\left(e^{-ah} + e^{ah}\right)\right].\cos.(ax + a') = 0;$$

et à cause qu'elle doit subsister pour toutes les valeurs de $x$, on en conclura

$$\frac{d^2\mathrm{T}}{dt^2} + c^2\mathrm{T} = 0,$$

en faisant, pour abréger,

$$ga\left(e^{ah} - e^{-ah}\right) = c^2\left(e^{ah} + e^{-ah}\right). \qquad (7)$$

On tire de-là, en intégrant

$$\mathrm{T} = \mathrm{B}.\sin.\,ct + \mathrm{B}'.\cos.\,ct;$$

B et B′ étant les deux constantes arbitraires. Substituant cette valeur de T, dans celle de $\varphi$, il vient

$$\left.\begin{aligned}\varphi = \Sigma\mathrm{B}\left(e^{a(h-z)} + e^{-a(h-z)}\right)\cos.(ax + a').\sin.\,ct,\\ + \Sigma\mathrm{B}'\left(e^{a(h-z)} + e^{-a(h-z)}\right)\cos.(ax + a').\cos.\,ct;\end{aligned}\right\} \qquad (8)$$

expression dans laquelle les sommes $\Sigma$ s'étendent à toutes les valeurs possibles des constantes B, B′, $a$ et $a'$.

Nous pouvons regarder cette valeur de $\varphi$ en série d'expo-

Laplace (1776), Gerstner (1802), Flaugergues (1793), la Coudraye (1796), and Nicolas T. Brémontier (1809). Brémontier's book contains practical observations on wave interference and reflection, sediment transport, and harbor design, but no mathematics. Though the Webers's book made little impact when it appeared, its value was later recognized by Airy and Russell (see below).

## 3. BRITISH FLUID MECHANICS BEFORE 1838



Before 1835, the state of knowledge of fluid mechanics in Britain was unimpressive: few read and fewer understood the work of Johann and Daniel Bernoulli, Jean d'Alembert, and Leonhard Euler from a century earlier. Lagrange's *Méchanique Analitique* (1788), Laplace's *Mécanique Céleste* (1799), Poisson's *Mécanique* (1833), and Fourier's *Théorie Analytique de la Chaleur* (1822) were better known; but before 1835, few British scientists had mastered these analytical works sufficiently to make their own original contributions (see Guicciardini 1989 and Craik 1998, 1999, 2000a,b). Nevertheless, John Toplis (1814) and Henry Harte (1822, 1827) published English translations, with explanatory notes, of Book 1 of Laplace's *Mécanique Céleste*; Thomas Young anonymously wrote *Elementary Illustrations of the Celestial Mechanics of Laplace* (Book 1) (1821); and Nathaniel Bowditch's great American translation and commentary on Laplace's volumes 1–4 appeared during 1829–1839. All of these included descriptions of the equations of inviscid fluid mechanics and helped to raise awareness of this field.

Of works by British authors, Samuel Vince's old-fashioned *The Principles of Hydrostatics*, first published in 1798, was reprinted in a sixth edition in 1829. Henry Moseley's unimpressive 1830 *A Treatise on Hydrostatics and Hydrodynamics* included a chapter by James Challis "on the general equations of motion," presumably because Moseley was unfamiliar with the equations derived long before by Euler. Thomas Webster's 1836 *The Theory of the Equilibrium and Motion of Fluids* was "compiled principally from the writings of Poisson and Challis;" John Henry Pratt's 1836 *The Mathematical Principles of Mechanical Philosophy* had a brief section on the equations of inviscid flow; William Walton's 1847 *A Collection of Problems in Illustration of the Principles of Theoretical Hydrostatics and Hydrodynamics* and William H. Miller's *The Elements of Hydrostatics and Hydrodynamics* (1831) are uninspired, but the 1850 fourth edition of the latter at least contained a seven-page Appendix on the "Theory of Long Waves," showing some awareness of recent work.

James Challis (1803–1882) published no fewer than 14 papers on the equations of hydrodynamics between 1829 and 1845, and they are mostly worthless (e.g., Challis 1830, 1842). He convinced himself that Euler's equations for incompressible flow were incomplete; he claimed that he was first to derive the equations for compressible flow, given long before by Euler (1757a,b), and he argued that the condition that $udx + vdy + wdz$ be an exact differential is satisfied only by rectilinear flows!

Every one of the above-named authors was based, or had received his education, at Cambridge University, and the textbooks were mainly aimed at the captive audience of Cambridge undergraduates. [More on the Cambridge milieu of that time may be found in Harman (1985). Of particular note are the articles by D.B. Wilson contrasting Cambridge with the Scottish universities of Edinburgh and Glasgow, and by I. Grattan-Guinness on French influences on Cambridge before 1840.]

Typical of the few works on fluid mechanics published in Britain between 1800 and 1840 by non-Cambridge graduates are the reissues of James Ferguson's 1760 *Lectures on Select Subjects in Mechanics, Hydrostatics, Pneumatics and Optics*; Thomas Young's *Lectures on Natural Philosophy* (1807); John Playfair's *Outlines of Natural Philosophy* (1814); John Leslie's *Elements of Natural Philosophy* (1823) (with nearly 150 pages on hydrostatics and hydrodynamics but little mathematics); and expository articles on hydrodynamics and hydraulics by James Brewster (1808–1830), A. Thomson (1835), Dionysius Lardner (1831), and Henry Brougham (1829). These works, all by Scottish authors except Young's and Lardner's, were primarily aimed at a general audience, and were mainly concerned with practical hydraulic and pneumatical devices rather than mathematical theory.

The British Association for the Advancement of Science was founded in 1831, largely through the initiative of the Scot David Brewster, and supported by others including Sir John Robison, W. Vernon Harcourt, and John Phillips (Howarth 1931, Ch. 1). Though the Cambridge professoriate were at first suspicious of the new national body, and none attended its inaugural meeting in York, the influential William Whewell of Trinity College and Cambridge's Professor of Mineralogy, wrote urging the Association to commission "reports . . . concerning the present state of science, drawn up by competent persons" (Howarth 1931, p. 25). Over the next few years, several reports on "Hydrodynamics" and on "Waves" were duly commissioned. The first two, by James Challis (1833, 1836), are mainly noteworthy for their pompous style and lack of substance, but they at least drew attention to the possibilities for advancement:

> The foregoing review of the theory of fluid motion . . . may suffice to show that this department of science is in an extremely imperfect state. Possibly it may on that account be the more likely to receive improvements; and I am disposed to think that such will be the case. (Challis 1833, p. 151)

## 4. 1838–1844: RUSSELL, GREEN, KELLAND, AIRY, AND EARNSHAW

Challis, Whewell, and the famous private tutor William Hopkins influenced others in Cambridge to take up the theoretical study of hydrodynamics. But perhaps as great an influence was the British Association's decision, in 1837, to set up a "Committee on Waves" to conduct observations and experiments. This committee consisted of John Scott Russell and Sir John Robison, and received a grant of £300 12s 0d plus £26 17s 6d for chronometers (Howarth 1931, pp. 266, 271). They were a good choice. Robison, the son of a former professor of Natural Philosophy at

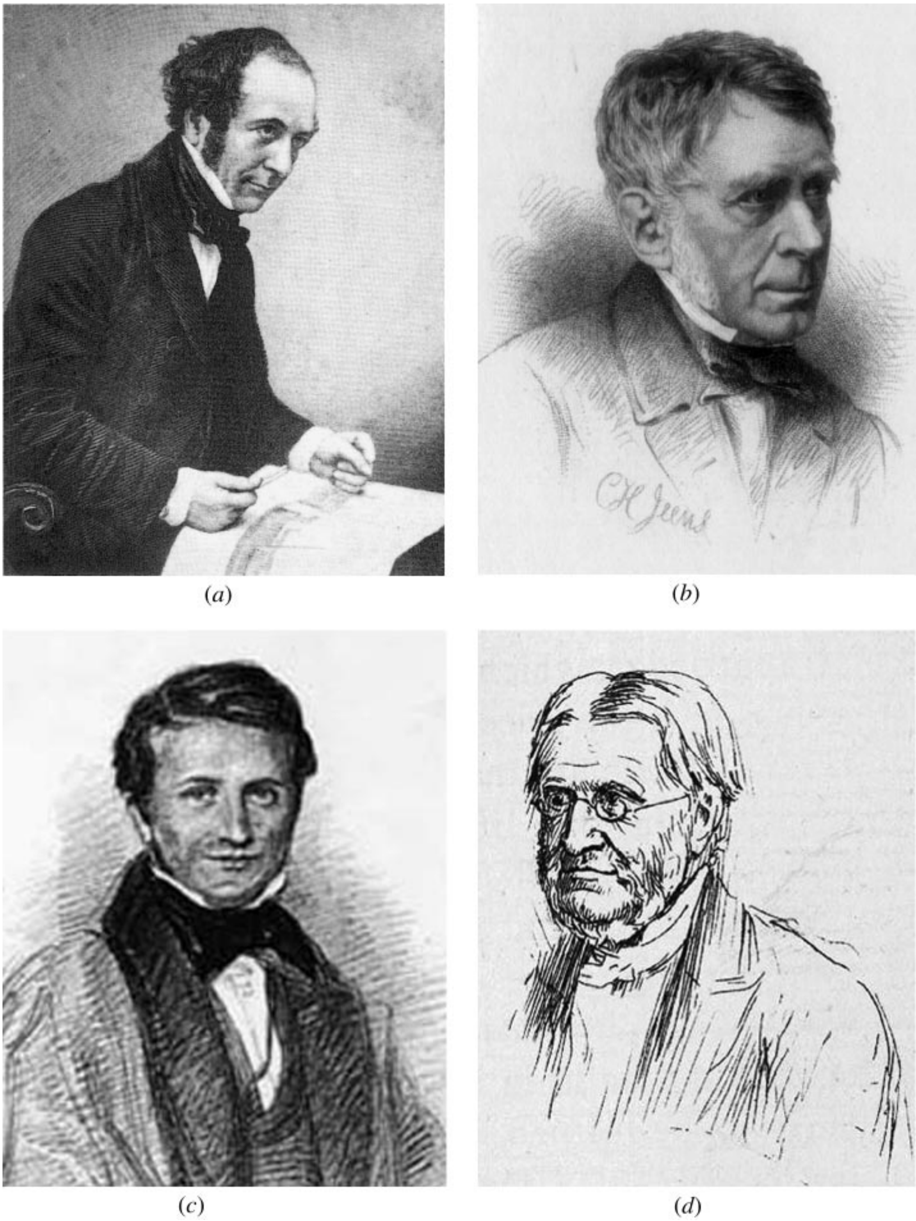


**Figure 3** Portraits of (*a*) Russell, (*b*) Airy, and (*c*, *d*) Kelland. No portrait of Green exists. Reproduced with permission: (*a*) from Emmerson (1977); (*b*) from Airy (1896); (*c*) from a pencil and watercolour drawing attributed to J.W. Slater, courtesy of Queens' College, Cambridge; (*d*) from an engraving by W. Hole in *Quasi Cursores*, Edinburgh University Press 1884.

Edinburgh, was a well-to-do amateur scientist and Secretary of the Royal Society of Edinburgh during 1828–1839 (Campbell & Smellie 1983). Russell had briefly taught at Edinburgh University, developed a steam carriage, and was then working on the resistance of ships. [The current *Encyclopaedia Britannica* wrongly states that Russell was appointed Professor of Natural Philosophy in 1832, but the 11th edition of 1911 is more reliable. He substituted for the professor only in 1832–1833, between the death of John Leslie and the appointment of James D. Forbes. In 1838, he was an unsuccessful applicant for the chair of Mathematics, which Philip Kelland secured. In 1844 he left Scotland for London to pursue a notable and turbulent career as an engineer and naval architect (Emmerson 1977).]

A substantial report by Russell & Robison (1837) was followed by a brief one (1840). Robison died in 1843 and Russell alone wrote a brief supplementary report (1842), then his major "Report on Waves" (1844). Russell paid tribute to Robison as "a kind friend," but made a point of claiming all the experimental work as his own:

> In all these researches the responsible duties were mine, and I alone accountable for them; but in forwarding the objects of the investigation I always found him a valuable counselor and a respected and cordial cooperator. (Russell 1844, p. 311)

These reports constitute a remarkable series of observations, at sea, in rivers and canals, and in Russell's own wave tank constructed for the purpose (see also Bullough 1988). The mathematicians had much to think about because in 1837, not even the linear theory of small-amplitude plane waves was well known. Though Russell's experiments are now famous for his discovery of the nonlinear solitary wave, it is not surprising that, in his own day, this aspect of his work proved contentious and misunderstood.

Those who published on water waves in the next few years were George Green, Philip Kelland, George Biddell Airy, and Samuel Earnshaw. After publishing a fundamental, but long-neglected work on electricity and magnetism in 1828, the self-taught Green became an undergraduate at Gonville & Caius College, Cambridge in 1833, at the mature age of 39. In 1837, he was appointed to a fellowship at Caius, but poor health caused him to return to Nottingham, where he died in 1841 (Cannell 2001). Earnshaw, too, was a Cambridge fellow, at St. Johns College. Kelland had studied at Cambridge, being senior wrangler and Smith's prizeman in 1834 and then a fellow of Queens' College. From 1838 until 1879, he was Professor of Mathematics at Edinburgh University, the first English-born and English-educated person to hold that office.

George Biddell Airy's long and influential article "Tides and Waves" was published in the *Encyclopaedia Metropolitana* in 1841. When it appeared, Airy's scientific reputation as mathematician and astronomer was already high: He was then Astronomer Royal at Greenwich Observatory, following a period at Cambridge as Lucasian Professor of Mathematics and then as Plumian Professor of Astronomy (between Vince and Challis). Airy's article (discussed below) has long been recognized as a major contribution to water wave theory.

In some respects, both Green and Kelland have legitimate claims to priority over Airy, and both were much influenced by Russell's experiments. Green's short paper "On the motion of waves in a variable canal of small depth and width" (1838), though restricted to long linear waves in shallow water, gives an exemplary analysis of the effects of slow variations. He explicitly introduces "a very small quantity" $\omega$, such that depth and width depend on $\omega x$, where $x$ is distance along the canal and neglects "quantities of the order $\omega^2$." This leads him to his final results that:

"... if $\beta$ represent the variable breadth of the canal and $\gamma$ its depth,

$\zeta =$ height of the wave $\propto \beta^{-1/2}\gamma^{-1/4}$,

$u =$ actual velocity of the fluid particles $\propto \beta^{-1/2}\gamma^{-3/4}$,

$dx =$ length of the wave $\propto \gamma^{1/2}$,

and $dx/dt =$ velocity of the wave's motion $= \sqrt{(g\gamma)}$."

Soon after, Green published a "Note on the motion of waves in canals" (1839). Interested by Russell's "Great Primary Wave," he calculates the horizontal displacement of particles by the passage of a localized wave of elevation to be $V/\gamma$, where $V$ is the volume of fluid raised above the undisturbed depth (with a similar result, of opposite sign, for a wave of depression). He then considers waves in a triangular channel with one side vertical and the other at an arbitrary angle, and finds that long waves of small amplitude have a velocity of propagation given by $\sqrt{(gc/2)}$, where $c$ is the maximum liquid depth. This he compares with Russell's measured data, and finds good agreement in those cases where "the elevation above the surface of equilibrium is very small compared with the depth $c$." He gives a table of theoretical and experimental values for all Russell's waves with $\zeta < c/20$, showing better agreement than obtained from Russell's suggested formula of $\sqrt{(2gc/3)}$.

Green concludes his note with a demonstration of the now familiar, but until then unproven, result that particle paths in the presence of deep-water traveling waves are circles, with radii that decrease exponentially with depth. [But the result easily follows from Laplace (1776); see Equation 1.1.] With arbitrary fixed depth, Airy (1841) deduced the corresponding elliptical paths soon afterwards.

The clarity of Green's exposition is reminiscent of that of George Gabriel Stokes, who admired Green's work. The young Stokes became an undergraduate at Cambridge in 1837 and was coached by William Hopkins. In autobiographical notes, Stokes recalled that, after graduation,

> I thought I would try my hand at original research; and, following a suggestion made to me by Mr. Hopkins while reading for my degree, I took up the subject of Hydrodynamics, then at rather a low ebb in the general reading of the place, notwithstanding that George Green, who had done admirable work in this and other departments, was resident in the University till he died. (Larmor 1907, p. 8)

The influence of the earlier continental work on these British authors was not so great as one might imagine. The Webers's book (1825) was known to Airy, who devoted seven pages of his article (1841b, pp. 344–50) to descriptions of, and comments on, their observations and those of Russell & Robison (1837). Russell only learned of the Webers's work, presumably from Airy (1841), while preparing his 1844 report: Referring to their "valuable series of experiments," he is "disposed to regret that this excellent book did not reach me till long after my own researches had advanced far towards completion." However, "it so happens that their labours and mine do not in the least degree supersede or interfere with each other," but "may be rather reckoned as supplementary the one to the other" (Russell 1844, p. 332 footnote).

The theoretical work of Lagrange on long waves was well known, but that of Laplace (1776) was not. Though aware of the long papers of Cauchy and Poisson, the new generation of British mathematicians was unimpressed. Kelland (1840a,b) tried to understand, and commented on, their work; but, with regard to recent advances in experimental knowledge (presumably he had in mind mainly those of Russell), he did not have

> . . . any reason to hope, that such men as Poisson and Cauchy will quit the delectable atmosphere in which they are involved, of abstruse analysis, for the more humble, but not less important task of endeavouring to treat the simpler problems in a manner not made general arbitrarily to lead to the most elegant formulae, but general to that extent, and in that mode, in which the problem in nature is so. It may seem strange, but I confess that it appears to me, that this problem has hitherto fallen into the hands of men too distinguished . . . . Hence their investigations have been such as apply only to the more abstruse and captivating branches of the science, while the simple extension of its fundamental conditions has been almost, if not altogether, overlooked. (Kelland 1840b, pp. 497–98).

Airy was even more dismissive. In his 1841 article "Tides and Waves" he asserted that his own theory of waves

> embraces . . . every case of general interest to which mathematics are at present applicable, but it does not comprehend those special cases which have been treated at so great length by Poisson . . . and Cauchy . . . . With respect to these we may express here an opinion, borrowed from other writers, but in which we join, that as regards their physical results these elaborate treatises are entirely uninteresting; although they rank among the leading works of the present century in regard to the improvement of pure mathematics. (Airy 1841, p. 344)

Stokes (1846) later expressed his own reservations of the Cauchy-Poisson work more mildly: "The mathematical treatment of such cases is extremely difficult; and after all, motions of this kind are not those which it is most interesting to investigate."

## 5. KELLAND'S 1840 ARTICLE

On arrival in Edinburgh, Philip Kelland met John Scott Russell and soon became interested in water waves, though previously more concerned with the propagation of heat. In 1840 Kelland contributed a short paper to the British Association for the Advancement of Science. In the same year, he published the first part of a long two-part paper "On the Theory of Waves" in the *Transactions of the Royal Society of Edinburgh*; the second part following in 1844. Kelland's work on waves is difficult to assess. These papers are needlessly long-winded and contain an infuriating mixture of good and bad. Correct analysis is intermingled with serious lapses of understanding. Kelland felt it necessary to publish lengthy and readily reproducible details of his algebraic calculations; at times he forgot that he had made assumptions and wrongly extrapolated his results to situations where they did not hold. Sometimes his assumptions were ill founded or, at least, without given justification. But, despite these limitations, he did obtain original results, and displayed some sound intuition. One can only wish that the Royal Society of Edinburgh had then had an editor able to give the sort of informed, firm, and impartial advice that Stokes later gave as Secretary to the Royal Society of London.

Kelland begins his analysis by considering "Wave-motion in a fluid of finite depth, on the hypothesis of parallel sections" (1840b, pp. 501–7); that is, he considers long waves in shallow water as Laplace had done 60 years before. With a free-surface displacement

$$z = h + a \sin\theta, \qquad \theta = (2\pi/\lambda)(ct - x)$$

where $h$ is depth, $a$ is wave amplitude, $c$ is wave speed, and $\lambda$ is wavelength, he correctly finds the associated horizontal and vertical velocities, and he eventually establishes that $c^2 = gh$ after some algebraic rambling. He also discusses, rather unconvincingly, the more general displacement $z = h + a\sin\theta + e\sin 2\theta$, which led to inconsistencies.

With these results as a guide, he next tackled waves in fluid of arbitrary depth directly from Euler's equations, without assuming irrotationality, but postulating velocity components $[u, v]$ of the form

$$[f(y)\sin\theta, F(y)\cos\theta]$$

for unknown functions $f$ and $F$. Neglecting products of sines and cosines, he found $f$ and $F$ to be the now-familiar exponential functions for irrotational waves. Unaware that his assumed form of solution is incompatible with the nonlinear equations, he obtains the surface displacement, implicitly, as

$$z = h + (e^{\alpha z} - e^{-\alpha z})a\sin\theta, \qquad \alpha = 2\pi/\lambda. \tag{5.1}$$

He then differentiates this to obtain $dz/dx$ (1840b, p. 512). From the nonlinear free-surface boundary conditions, he deduces a lengthy and inconsistent expression involving $z$ and $\theta$ (1840b, p. 512, equation 9). Kelland rescues this situation in dubious fashion by setting $\theta = 0$ and $z = h$ in his expression, obtaining the wave speed $c$ as (1840, p. 513)

$$\frac{2\pi}{\lambda}c^2 = g \cdot \frac{e^{\alpha h} - e^{-\alpha h}}{e^{\alpha h} + e^{-\alpha h}} \cdot \frac{1}{1 - (2\pi a/\lambda)^2 (e^{\alpha h} - e^{-\alpha h})^2}. \tag{5.2}$$

Setting $a = 0$ recovers the correct linear dispersion relation for infinitesimal waves; and it is not hard to confirm that Kelland's analysis can be justified (though it remains cumbersome) in this limit. This appears to be the first statement of the result, apart from Poisson's, because it anticipates Airy's article by a year.

More surprising is the fact that the above result, if expanded in powers of the amplitude, yields the correct modification at $O(a^2)$ to the wave speed and frequency of deep-water waves with small but finite amplitude. That is to say, Kelland's formula correctly gives what is now commonly called the "Stokes frequency correction," about eight years before it was derived by Stokes (1847). In this, Kelland was fortunate—or would have been, had he received any credit for the discovery! Just why his dubious analysis, which excludes all higher harmonics from the velocity components, should lead to a correct result may be understood from a later observation by Rayleigh (1876), now largely forgotten. Equation 5.1 for the surface elevation involves only a single term in $\sin\theta$, with $z$ defined implicitly. Rayleigh showed that, when $z$ is calculated by successive approximations from Equation 5.1, the result contains higher harmonic terms that are correct up to third order in wave amplitude. It turns out that only these nonlinear terms affect the frequency shift up to this order, though this is not obvious.

Kelland then made a bad error, concluding that in shallow water the height of waves must be $a = \lambda/2\pi$, where $\lambda$ is the wavelength. Suspecting the validity of his result, he wrote that this, "if it have any truth at all, appears to shew the tendency of waves in shallow water to become semicircular, measuring from the mean points to the crests."

Kelland next adopts a more ambitious plan, representing velocity components and vertical surface displacement as a sum of harmonics, such as

$$z = h + a_1 e^{\alpha z}\sin\theta + a_2 e^{2\alpha z}\sin 2\theta + \text{\&c.} + f_1 e^{-\alpha z}\sin\theta + f_2 e^{-2\alpha z}\sin 2\theta + \text{\&c.}$$

He substitutes these expressions into the surface boundary conditions, obtaining complicated expressions representing quadratic and cubic interactions, because he allows the amplitudes of each harmonic to be of the same order of magnitude. Not surprisingly, the analysis gets bogged down and reaches no firm physical conclusions (Kelland 1840b, pp. 514–23). Though the details bear a superficial resemblance to modern wave interaction studies of the 1960s, Kelland assumed that each harmonic travels with the same wave velocity $c$. Despite the failure of this part of Kelland's analysis, he deserves some credit for attempting what I believe to be the first study of finite-amplitude waves to employ an amplitude-expansion technique involving a sum of harmonics. Airy's later attempt was restricted to waves in shallow water, and the definitive use of amplitude expansions appeared in Stokes (1847).

Kelland's work is mainly remembered for his study of waves in canals with nonrectangular cross sections. He investigates long waves of small amplitude in canals with triangular cross sections, and correctly derives the result that the wave speed is $\sqrt{(gh/2)}$, where $h$ is the maximum depth (Kelland 1840b, pp. 524–27). Though this result is also given in Green's 1839 paper, Kelland's work was independent. He, like Green, provided favorable numerical comparisons with Russell's experimental data, but used a different data set. Kelland's paper was read to the Royal Society of Edinburgh on April 1, 1839; Green's was read to the Cambridge Philosophical Society less than two months earlier on February 18, 1839.

Kelland goes further than Green in considering long waves in canals "of any shape whatever to the vertical section" **(**Kelland 1840b, pp. 527–31), deducing that the phase speed $c$ satisfies

$$c^2 = g \cdot \frac{\text{area of vertical section}}{\text{breadth at surface}}. \tag{5.3}$$

But Kelland's attempt to study canals with slowly varying breadth is wrong, and Green's is correct. Kelland finds that if the wavelength remains constant, "an hypothesis which must be considered as merely approximative," then the wave height varies inversely with the breadth. In contrast, Green's correct results, stated above, consistently include changes in both wavelength and wave height.

Kelland next addresses solitary wave motion. Apart from Green's brief note of 1839, this is the first theoretical attempt to model Russell's remarkable new observations. Unlike Green, who wisely confined attention to long waves in shallow water, Kelland attempts to describe wave motion in arbitrary depth $h$. He begins by postulating velocity components [$u$, $v$], which are zero outside the finite interval $-\pi/2 < \theta \equiv \alpha(x - ct) < 3\pi/2$, and within it satisfy

$$u = b(e^{\alpha z} + e^{-\alpha z})(1 + \sin\theta), \quad v = -b(e^{\alpha z} - e^{-\alpha z})\cos\theta.$$

The corresponding surface elevation $z$ is likewise assumed to be zero outside this interval, and after some pages of tedious and unconvincing calculations, he arrives at its form within the interval as

$$z = h + (b/\alpha c)(e^{\alpha z} - e^{-\alpha z})(1 + \sin\theta).$$

Kelland's analysis purports to be nonlinear, for any wave amplitude, despite his assumed form of solution. Some more ill-founded working leads him to a formula for the wave velocity (Kelland 1840b, p. 541):

$$c^2 = \frac{g}{\alpha} \cdot \frac{e^{\alpha h} - e^{-\alpha h}}{e^{\alpha h} + e^{-\alpha h}} \div \left(1 - e\alpha \frac{e^{\alpha h} - e^{-\alpha h}}{e^{\alpha h} + e^{-\alpha h}}\right), \tag{5.4}$$

where $e \equiv (b/c\alpha)(e^{\alpha h} - e^{-\alpha h})$ is half the maximum wave height. Kelland warns that the associated motion is discontinuous at the ends of the interval, and attempts to compare his results with some of Russell's observations. Russell (1844) reproduced the above formula, and unfavorably compared Airy's work with Kelland's (see below).

Kelland (1844) revisits many of the same topics of his 1840 paper, with minor variations and much the same mix of right and wrong. His only significant new solution is that for linear waves in a channel of arbitrary depth and triangular cross section, with one or both walls inclined at 45° to the vertical. He also addresses the Cauchy-Poisson initial-value problem, but without obtaining any new correct results.



## 6. EARNSHAW'S SOLITARY WAVES

The representation of a solitary wave by discontinuous expressions was taken up again by Earnshaw (1847). He, too, attempted a nonlinear theory, and it fails for similar reasons, which Stokes later clearly expressed in correspondence with William Thomson:

> Earnshaw's investigation gives a rigorous result for the portion of fluid comprised within planes at the beginning and end of the wave but the condition, at the connection of the wave and still water are violated as I pointed out to him when the paper was referred to me and as he admitted and altered his original MS accordingly. In fact the result as a whole would be true only on the supposition of the existence of impulsive forces acting for a moment on an infinitely thin layer of particles and changing at the next moment to another layer and so on. (Wilson 1990, Vol. 1, p. 305: Stokes to Thomson, Dec. 9, 1862)

Stokes's refereeing of Earnshaw's paper must have occasioned some delay. Stokes's own great paper on waves appeared later in the same volume of 1847, but Earnshaw's paper was read to the Society on December 1845 and Stokes's was not read until March 1847.

Earnshaw begins with an interesting and well-written introduction. He points out the conspicuous lack of success in obtaining general solutions to the equations of hydrodynamics, and he emphasizes the need to limit theoretical studies to restricted classes of motions, based on hypotheses suggested by experiments. Citing Russell's recent work, he then embarks on his attempted analysis of the "great solitary wave," supposing that "the horizontal velocity of every particle, in a transversal section of the canal, is the same." This restriction was suggested by Russell's observations and is the same as that in the "method of parallel sections" previously employed for waves in shallow water. But Earnshaw, like Kelland, does not suppose the depth to be small, nor the wave to have infinitesimal amplitude. Instead, he deduces, from the full equations of motion together with his restriction, that the horizontal velocity $u(x - ct)$ of a traveling wave must satisfy the equations

$$0 = d_t u + c d_x u, \tag{6.1}$$

$$\text{constant} = d_t d_x u + u d_x^2 u - (d_x u)^2 \tag{6.2}$$

where $d_x$, $d_t$ denote differentiation with respect to $x$ and $t$.

For positive solitary waves, Earnshaw eventually arrived at results for $u$ and the total water depth, as functions of $x - ct$, and for the wave speed $c$. In these, the wavelength and maximum wave height are independent parameters, and the solution is supposed to terminate at those values of $x - ct$ on either side of the single crest, where the water depth reaches that of undisturbed liquid. Because, for his solution, the vertical acceleration of individual fluid particles must remain constant, the free surface has finite slope at these discontinuities.

Earnshaw's "solution" was no advance on Kelland's but merely used a different invalid hypothesis. As Stokes pointed out, forces must continuously be exerted in front of and behind the wave to maintain this solution. Eventually, Rayleigh (1876) derived the correct approximate solution, retaining both dispersion and nonlinearity, and he further observed that Earnshaw's solution is not irrotational. Rayleigh learned of Boussinesq's (1871) earlier version after obtaining his own. The now-famous paper by Korteweg & de Vries (1895) appeared much later (see Miles 1981). Bullough (1988) and Bullough & Caudrey (1995) give an account of the later history of solitary waves, with a detailed description of Russell's experiments on them. Earnshaw (1860) later worked more successfully on sound waves in gases.

## 7. AIRY'S *TIDES & WAVES* (1841)

Airy's long article for the *Encyclopaedia Metropolitana* can be securely dated to 1841. The Encyclopaedia bears no date, and first appeared in 59 parts between 1817 and 1845; Airy's article is in "Mixed Sciences" volume 3. But Airy's autobiography (1896) supplies the precise chronology: "In 1838 the Rev. H.J. Rose (Editor of the *Encyclopaedia Metropolitana*) had proposed my writing a Paper on Tides; in Oct. 1840 I gave him notice that I must connect Tides with Waves, and in that way I will take up the subject" (p. 142). "On May 26th [1841] the manuscript of my article 'Tides and Waves,' for the *Encyclopaedia Metropolitana* was sent to the printer" (p. 152). Airy's work (1841) was also issued separately, along with two other articles on *Trigonometry* and *On the Figure of the Earth* (with his name given incorrectly on the title-page as George "Biddle" Airy). (The page references cited below are to this undated printing.)

Although Airy's main focus of interest was tidal phenomena, he also wrote a substantial section on the "Theory of Waves in Canals" (1841, pp. 281–344) and an "Account of Experiments on Waves" (1841, pp. 344–51). Much of the work is original and is markedly concerned with modeling observed or observable phenomena. At times, he is cavalier in his assumptions and approximations, seemingly eager to press on towards a result that he can claim to agree with observation. I describe below only those parts that seem most valuable and well founded and relevant to our theme.

Airy gives the now-standard linear theory for plane waves in his section IV subsection 2, pages 289–96 (1841). This is cumbersome but correct, yielding the dispersion relation for the wave velocity $n/m$:

$$\left(\frac{n}{m}\right)^2 = \frac{g}{m}\cdot\frac{e^{mk} - e^{-mk}}{e^{mk} + e^{-mk}}, \tag{7.1}$$

where $n$ is the frequency, $m$ the wavenumber, and $k$ the liquid depth. He also shows that particle paths are ellipses (or circles for infinite depth). Airy gives four tables of results showing the period and velocity of plane waves as functions of depth and wavelength (tables 1 and 2), the relative horizontal and vertical displacements of fluid particles at intervals below the free surface (table 4), and the wavelength and wave velocity of the "free tide-wave" in water of various depths, this being a long wave with the period equal to the semidiurnal period of nearly 12 hours 24 minutes (table 3). These were doubtless useful because a published table of the function tanh$x$ did not exist at that time.

Subsection 3 of section IV is entitled the "Theory of Long Waves in which the Elevation of the Water Bears a Sensible Proportion to the Depth of the Canal." On making the now-standard approximations for long nonlinear but nondispersive waves, Airy arrives at the equation

$$\frac{d^2X}{dt^2} = gk\frac{\frac{d^2X}{dx^2}}{\left(1 + \frac{dX}{dx}\right)^3}, \tag{2}$$

where $X(x, t)$ denotes the horizontal displacement of a fluid particle. Here, $dX/dx$ must be small, but Airy does not say so. The corresponding height $V$ of the surface above the bottom is recovered from the continuity equation as $V = k/(1 + dX/dx)$.

This looks unlike the now-standard long-wave equations usually expressed as coupled first-order equations for the surface elevation and horizontal velocity. The latter theory developed from analogous equations first derived independently in acoustics by Earnshaw (1860) and Riemann (1858). Airy's equation is equivalent to the latter, for waves of small but finite amplitude, and Horace Lamb describes both versions in *Hydrodynamics* (1895). But Lamb characterized Airy's version as "less convenient," and I am unaware of any subsequent work on Equation 7.2.

Airy uses his equation to deduce that the surface elevation is

$$V = k\left\{1 - am\sin(mvt - mx') + \tfrac{3}{4}a^2m^3 \cdot x' \cdot \sin(2mvt - 2mx')\right\}, \tag{7.3}$$

where $v$ is the linear wave speed $(gk)^{1/2}$ and $x'$ is "measured from the point where the canal communicates with the open sea." In fact, Airy had developed an approximation for small $a$, deriving the second term of a power series expansion in $a$, but without making this clear. We would now say that the linear factor in $x'$ arises from a secularity at second order, because an $O(a^2)$ quadratic term resonates with the free wave with wavenumber $2m$. Thus, "When the wave leaves the open sea, its front slope and its rear slope are equal in length, and similar in form. But as it advances in the canal, its front slope becomes short and steep, and its rear slope becomes long and gentle..." (Airy 1841, p. 300).

By considering the time intervals between successive maxima of $V$ when the secular term in $x'$ is small, Airy deduces that "the phase of high water has travelled along the canal with the velocity... $\sqrt{\{gk(1+3b)\}}$," where $b$ is the maximum elevation divided by the mean depth $k$. Correspondingly, the minima at low water travel with the lesser velocity with $-b$ replacing $b$. He next continues his approximation to $O(a^3)$, finding terms that grow as $x'^2$ and showing his results in a figure, reproduced in Figure 4. Then, in a rather contorted manner, he considers the added effect of a mean current in the canal.

Airy then turns to small-amplitude (linear) waves in canals with slowly varying depth and cross-section. Green (1838, 1839) and Kelland (1840b) developed similar theories, as described above. Airy's results for long waves agree with this earlier work, which he does not cite, but his approximations seem ad hoc and unconvincing, particularly for waves that are not long.

Airy also attempted to construct a "Theory of Waves on Canals when friction is taken into account" (1841, pp. 329–39). Restricting attention to long, small-amplitude (linear) waves, he reasonably models the frictional force as proportional to the horizontal velocity; and he studies various cases, including waves in canals subject to both friction and an external temporally periodic tidal force. Depending on the problem studied, solutions display either spatial or temporal exponential decay. Surprisingly, he leaves the simplest problem, the temporal damping of a uniform wavetrain, until last.

In his section V. "Account of Experiments on Waves," Airy (1841, pp. 344–51) describes the Webers's experiments and the more recent ones of Russell. But his description of the latter is based on Russell & Robison's report of 1837, not Russell's fuller one of 1844, which had yet to appear. Though Airy describes Russell's experiments as "upon the whole, the most important body of experimental information in regard to the motion of Waves which we possess," he is critical of "Mr. Russell's references to theory" as giving "a most erroneous notion of the extent of the Theory of Waves at the date of these experiments" (p. 345).

Russell had distinguished four types of waves: "Waves of translation," including his newly found solitary waves, tides, and bores; "Oscillatory waves," or periodic wavetrains; "Capillary waves;" and "Corpuscular waves," which are compressive sound waves propagating through water. But Airy was unconvinced by the importance Russell attached to "The Great Primary Wave, or solitary wave, writing that

> We are not disposed to recognize this wave as deserving the epithets "great" or "primary" (the wave being the solitary wave whose theory is discussed in (226.) &c.) and we conceive that, ever since it was known that the theory of

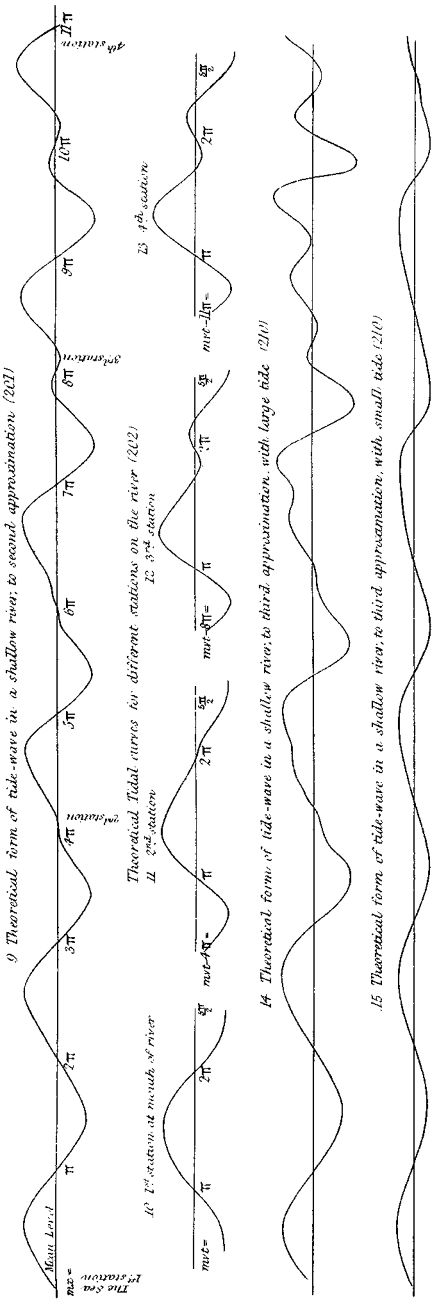


**Figure 4** Diagram from Airy (1841) showing the progressive steepening of waves in shallow water.

> shallow waves of great length was contained in the equation $\frac{d^2X}{dt^2} = gk\frac{d^2X}{dx^2}$, . . . the theory of the solitary wave has been perfectly well known. (Airy 1841, p. 46)

In his "(226.) &c." (pp. 307–9), Airy had argued that, in this linear shallow-water approximation, one may find many wave profiles of finite extent and form $X(z)$, where $X$ is the horizontal particle displacement and $z = vt - x$ is a coordinate that moves with the wave speed $v = (gk)^{1/2}$. Such profiles connect smoothly with a flat surface at, say, $z < 0$ and $z > a$, if $X(0) = 0$, $X(a) = b$, and at least the first three derivatives of $X$ vanish at 0 and $a$. Airy gives several examples, including $X = c \cdot z^4 \cdot (a - z)^4$ and $c \cdot z^5 \cdot (a - z)^5$ with $b = 0$ and $c$ constant, and

$$X = \frac{2b}{3\pi}\left\{\frac{3}{2}\cdot\frac{\pi z}{a} - \sin\frac{2\pi z}{a} + \frac{1}{8}\cdot\sin\frac{4\pi z}{a}\right\}.$$

Airy understands that such solutions do not exist for superpositions of short sinusoidal waves because "each of these would tend to travel on with its own peculiar velocity," but "when the waves are long, the peculiar velocities are very nearly the same for the different waves" (1841, p. 309).

As we have seen, Kelland (1840b, 1844) and later Earnshaw (1847) also believed that it was acceptable to use functions defined on finite regions, and joined to a flat surface at points where discontinuities occur. At that time, this was a reasonable supposition because a similar one was known to apply successfully to waves on strings. However, unlike Airy, both Kelland and Earnshaw considered the wave to be nonlinear, with a wave speed that differed somewhat from linear theory. In contrast, Airy categorically stated that: "provided it be long in proportion to the depth of the fluid. . . [the wave] can, when moving freely, have no other velocity than $\sqrt{(gk)}$. . . [but] Mr. Russell was not aware of the influence of the length of the wave in any case and therefore has not given it. . . ."

Russell was rightly displeased by these remarks, and took the chance to reply in his 1844 "Report on Waves." He expresses disappointment with Airy's recent "elaborate paper on waves." Airy's formula (Equation 7.1) for the velocity of small-amplitude waves has a form "closely resembling that which Mr. Kelland had previously obtained," so Airy had "advanced in this direction little beyond his predecessor." He also takes issue with Airy's claim of good agreement with his [Russell's] experiments, and he unfavorably contrasts Airy's boldness with Kelland's modesty, the latter having "not yielded to the temptation of twisting his theory to exhibit some apparent approximation to the facts, nor distorted the facts to make them appear to serve the theory, a proceeding not without precedent" (Russell 1844, p. 334).

Airy's article covers many topics of wave motion with originality and inventiveness. Much, though not all, is correct, at least as an approximation, and it seems unfair to criticize his shortcomings. However, one is left with the overall impression that Airy was too eager to rush on to the next matter of interest for tidal motion, and took insufficient care to present a clear mathematical analysis.

But he (and no doubt his editor) must have been conscious that the article was exceedingly long for a general encyclopaedia, and so space for any one topic was restricted. His copious references to Laplace on tides contrast with an almost total lack of acknowledgment of the theoretical work of his own contemporaries. Yet both Green and Kelland had published their work before Airy's article appeared in 1841.

It is hard to believe that Airy both researched and wrote all of this long and original article between October 1840 and his delivery of the manuscript in May 1841. More probably, he already had the project well in hand before then. If this were so, then his work and that of Kelland may well have been carried out independently.

Airy continued to work on tides after his encyclopedia article appeared, making observations and commissioning others at various English and Irish seaports.[1] These led to several publications, culminating in Airy (1845).

Much later, in 1876, Airy asked Stokes:

> Can you answer me the following question:
>
> In my Tides and Waves, written, I think, in 1837, I produced the theories of "Free Waves" and "Forced Waves", then subjectively new and original to me. Do you know any place in which they had been exhibited before that time? (Larmor 1907, vol. 2, p. 174: Airy to Stokes, Jan. 3, 1876)

In reply, Stokes referred Airy to his 1846 British Association report, and correctly dates the publication of Airy's article to not before 1841. He reminds Airy that Cauchy's general formulation incorporated the correct relation between period and wavelength for sinusoidal standing waves in deep water, and that Poisson's version did so for waves in arbitrary constant depth. But Cauchy, like Poisson, "did not stop to discuss the nature of the motion in the case of one of these "simple harmonic" disturbances, but put out his force in the difficult problem of determining the result of a single splash." (Larmor 1907, vol. 2, p. 176, Stokes to Airy, Jan. 4, 1876). Stokes also mentioned that, with regard to the early work of Laplace (1776), "I have not been able to find this paper, but I think I once saw it. I think it refers

[1]A letter to his wife on February 27, 1842 (Airy 1896, pp. 155–56) vividly describes his encounter with a stormy sea at Weymouth: ". . . there was the surf in all its glory. I cannot give you an idea of its majestic appearance. . . . My impression is that the height of the surf was from 10 to 20 feet. But the striking part was the clouds of spray. . . . A great swell is seen coming, growing steeper and steeper; then it all turns over and you see a face just like the pictures of falls of Niagara; but in a little more than one second this is totally lost and there is nothing before you but an impenetrable cloud of white spray. In about another second there comes from the bottom of this cloud the foaming current of water up the bank, and it returns grating the pebbles together till their jar penetrates the very brain. I stood in the face of the wind and rain watching this a good while, and should have stood longer but that I was so miserably wet. . . . I have now borrowed somebody else's trowsers while mine are drying. . . ."

to deep water only." (In fact, Laplace considered arbitrary depth, as discussed in Section 1.)

Surprisingly, Stokes then dismissed Gerstner's earlier work as "in some respects radically erroneous. . . . As the theory is not rigorous I don't count it;" but he modified this view soon after. He went on to assure Airy that:

> I am not aware that anyone before you showed so simply, from first principles, the equality of the horizontal motion of the particles in a vertical line in the case of long waves, or carried the investigation beyond the first order of small quantities, or considered the case of forced waves except in so far as it is virtually involved in the problem of the tides. . . (Larmor 1907, vol. 2, p. 177, Stokes to Airy, Jan 5, 1876)

Airy's article was published at just the time Stokes was starting to work on water waves. This, together with Russell's remarkable experiments and Green's and Kelland's papers, provided just the stimulus Stokes needed for his own great work on waves, to be discussed in the *Annual Review of Fluid Mechanics* volume 37.



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