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CARTESIAN DIVER OBSERVATIONS OF DOUBLE FREQUENCY PRESSURE FLUCTUATIONS IN THE UPPER LEVELS OF THE OCEAN

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Abstract. Pressure oscillations having a narrow spectrum centered at 0.175 Hz have been measured in the deep sea at depths between 100 and 300 m. At these depths the non-linear interaction between oppositely directed sea surface gravity waves causes pressure fluctuations, the intensity of which varies inversely with the square of the depth, uninfluenced by microseismic Rayleigh waves propagating on the seafloor below. The observed pressure fluctuations are consistent with the interaction between waves having a unimodel beam pattern whose half beam width is slightly larger than 90 degrees or with a shoreward propagating pencil beam of swells interacting with a small component reflected from the shore.

Some recently analyzed data recorded by a free vehicle, the Cartesian Diver (Duda, *et al.* 1988), demonstrate the existence of pressure fluctuations in the upper ocean at double the frequency of the surface wind waves. The observations were made where the ocean is 3.7 km deep (Figure 1) over a 48 hour period beginning at 1600 UT on May 5, 1988. The Diver moves alternately up and down under control of adjustable buoyancy through a depth range from the mixed layer to 300m. It moves smoothly at all times except when changing direction, essentially at its terminal velocity, which is approximately 0.14 m/s relative to the surrounding water. It has a device that records the rate of change of pressure in order to measure the vertical velocity of the diver in an absolute frame of reference and thereby infer the vertical component of the orbital velocity of internal waves. Owing to the smoothness of motion it has proven possible to extract from the pressure records evidence of minute pressure fluctuations of much higher frequency than those caused by the internal waves. The spectrum of pressure rate-of-change (Figure 2) shows, at low frequencies, the fluctuations brought about by the motion of the diver in response to the vertical orbital motions of internal waves. In addition the spectrum has a sharp peak near f_0 = 0.175 Hz rising well above instrumental noise level. We believe this is the pressure signature of the wave-wave interaction process. There is a very sensitive angular accelerometer in the diver that would have given an indication of any irregular motions of the diver. None were detectable.

At all depths the Diver was within the near field of the source of non-linear interactions of opposed surface wind-waves (Longuet-Higgins, 1950, Hasselmann, 1963). In this depth range, the variation of the pressure intensity with depth due to this non-linear mechanism has a known form that allows one to measure the input of energy from wave-wave interactions directly above the diver.

The observed spectrum of pressure varies in intensity both with depth and with time (Figure 3). The variation in time follows in the same sense as the local wave heights, but the latter are only roughly known and no analysis is justified. The reduction of pressure oscillations towards the end of the experiment does tend to show that measurement noise conceivably caused by oscillating motions of the Diver was not the source of the signal.

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The depth variation follows the theoretical prediction for a near surface source having a nearly "white" intensity in wavenumber space. The simplest model, valid well below the sea surface, is that the source spectrum is constant and isotropic up to a maximum wavenumber, k_m, and zero above. According to the theory of non-linear interaction between opposed wave trains, the effective value of k_m is a function of the directional pattern of the waves and of the wavenumber of waves of frequency $f_0/2$. It will be of the order of the latter, .03/m or less. The frequency spectrum of pressure as observed, is found by integrating over all contributing wavenumbers. Below the surface one must allow for exponential decay of each wavenumber component. In the near-field region then, the pressure frequency spectrum may be approximated by

$$P(\omega) = P_0(\mathbf{k}, \omega)\int_{-\pi}^{\pi}\int_0^{k_m} \exp\,(-2kd)k\,dk\,d\vartheta$$

where d is the depth at which the pressure frequency spectrum is measured and P_0 ($\mathbf{k}$, ω) is the value of the pressure wavenumber-frequency spectrum (for $|\mathbf{k}| < k_m$) just below the sea surface. Since P_0 is independent of $\mathbf{k}$ for any frequency ω, we write it as $P_0(\omega)$ below. With this form, P can be evaluated as

$$P = \pi\, P_0(\omega)\, [1 - (1+ u) \exp(-u)]/(2d^2) \quad (1)$$

where $u = 2\, k_m\, d$. According to this expression the spectrum approaches $\pi\, P_0(\omega)k_m^2$ just below the sea surface and

$$P = \pi\, P_0(\omega)/(2d^2) \quad (2)$$

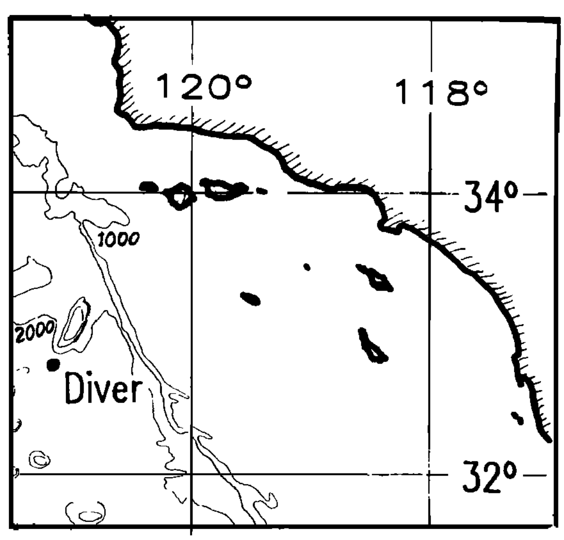

Fig. 1. Location where the observations were made is shown by the large dot. The isobaths are in fathoms (1 fathom ≅ 1.8 m).

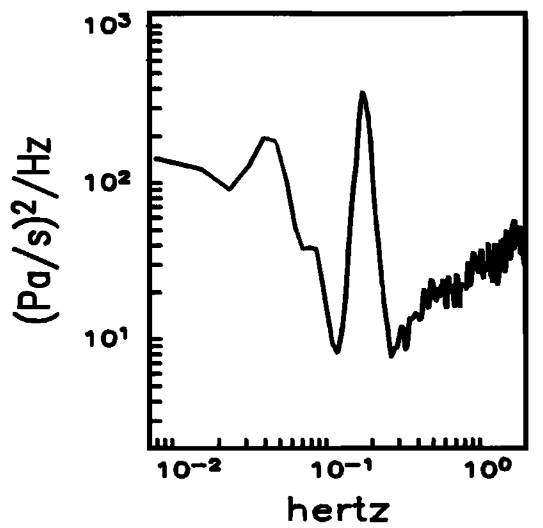

Fig. 2. A typical spectrum of the rate of change of pressure at the diver during an eight hour period as it cycled between 100 and 300 m.

at depths such that u >> 1 (but still in the near field of the sources, that is down to many hundreds of meters for .18 Hz oscillations). The reduction of the spectrum with increasing depth is a consequence of the gradual loss of shorter wavelength components of the pressure with increasing depth.

According to Hasselmann (1963) an adequate approximation for the intensity of the pressure can be represented in terms of the directional frequency spectrum for sea surface wave elevations, $Z(\omega,\vartheta)$, by the expression

$$P_0(\omega) = \frac{1}{2}\,\omega(\rho g)^2 \int_{-\pi}^{\pi} Z(\omega/2,\vartheta) Z(\omega/2,\vartheta+\pi) d\vartheta \qquad (3)$$

Below the near field region the pressure fluctuations have an acoustic character and their variation with depth is expected to show interference effects from bottom reflection of the pressure waves as well as pressures induced by microseismic motions of the seafloor generated by distant sources. These non-local effects vanish toward the sea surface because the surface is a pressure release for bottom generated motions. In related studies, Bradner and co-workers (1970) report on midwater seismic measurements made from a neutral buoyant float at various depths ranging from 430 to 1210 m. Also Cotaras, *et al.* (1988) have measured the spectrum of acoustic energy near the sea surface in the frequency band from 1 to 20 Hz. In neither of these studies have the investigators discussed the near field region of the acoustic sources.

Our observations fit reasonably well to the theoretical curve (Figure 4). The plotted curves differ slightly from equation (1) because they take into account the compressibility of the water. Here the data points represent mean square dp/dt found by integrating over the "microseism" peak after subtracting a noise background and averaging over all data measured while the Cartesian diver was within 20m depth windows centered at 110m, 130m,...,290m depths. In short, we have a measure of the local wave-wave interaction intensity in a depth region (the near field) where the unknown parameters of bottom reflection and remote sources are unimportant. The observationally determined value for $P_0(\omega)$ provides this information.

The observed spectra of pressures are narrow, centered near radian frequency $\omega_0 = 1.1$/s, with effective frequency bandwidth $\Delta\omega = .24$/s, defined in terms of the mean square rate of change of pressure by

$$\langle \dot{p}^2 \rangle = \omega_0^2\, P(\omega_0)\, \Delta\omega \qquad (4)$$

At depths such that $2\,k_m\,d >> 1$, the mean square rate of change of pressure, found by combining (2) and (4) is

$$\langle \dot{p}^2 \rangle = \frac{\pi\omega_0^2\Delta\omega}{2d^2}\, P_0(\omega_0) \qquad (5)$$

From the data shown in Figure 4 we find

$$P_0(\omega_0) = 8.8\mathrm{x}10^5 Pa^2 m^2 s \qquad (6)$$

and from eq (3) the "interaction integral",

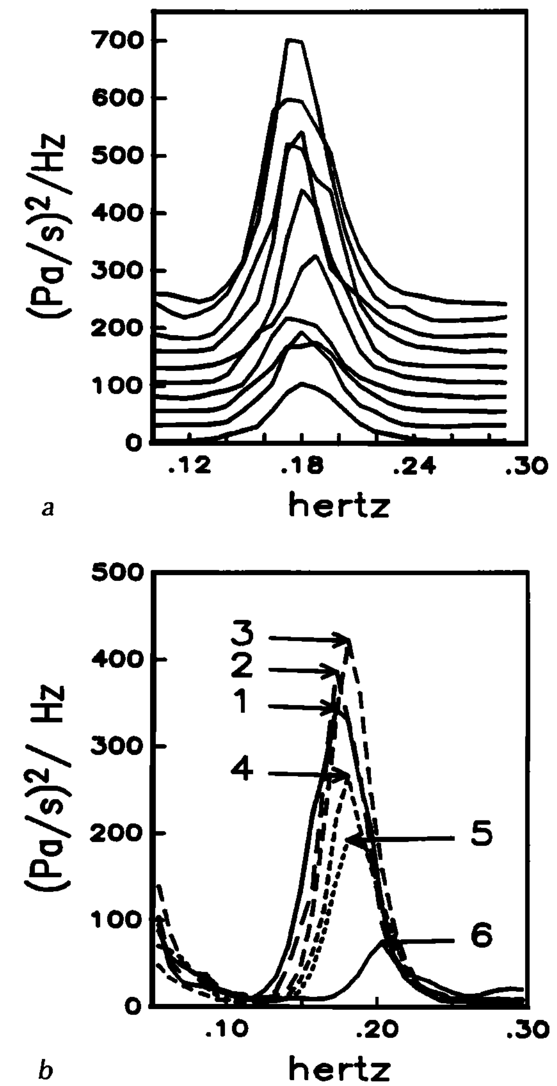

Fig. 3. Variations of spectral intensity in space and time. a) Spectra made by averaging all data measured in 20 m windows centered at depths of 110, 130, ..., 290 m. The deepest spectrum is shown lowest in the figure; shallower ones are spaced upward by 25 $(Pa/s)^2$/Hz. b) Spectra made by averaging all data measured in the successive eight hour interval indicated by the numerals.

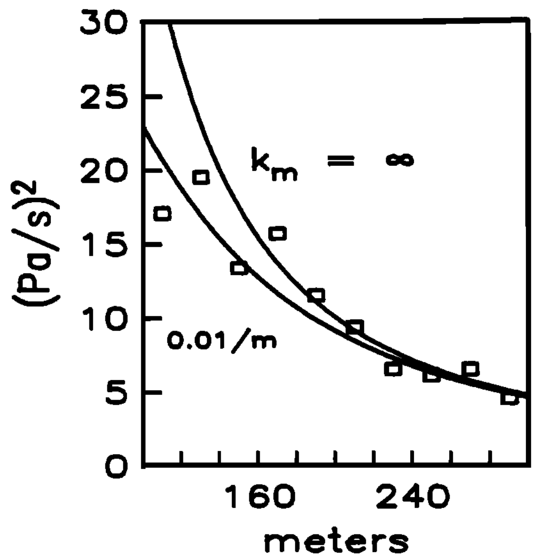

Fig. 4. The mean square rate of change of pressure as a function of depth. The continuous curves show the expected variation for two values of the maximum wavenumber as noted.

$$I(\omega_0) = \int_{-\pi}^{\pi} Z(\omega_0/2,\vartheta)Z(\omega_0/2,\vartheta+\pi)d\vartheta = .016\ m^4s^2 \quad (7)$$

we have some information on the wave field over the diver but little directional information. According to shipboard observers, the significant wave height decreased from 2.1 to 1.5 m during the 48 hours of observation, and the wind speed decreased from 10 to 6 m/s. It is commonly assumed that the rms wave elevation is equal to the significant wave height divided by 4.0. Thus the mean square elevation averaged over the entire observing period was

$$\int_{-\pi}^{\pi}\int_{0}^{\infty} Z(\omega,\vartheta)\,d\omega d\vartheta = .21\ m^2 \quad (8)$$

The Fleet Numerical Oceanographic Center, United States Navy, has provided us with additional information (based on hindcasting) that the significant waves, swells of 10 s period, were propagating toward the south east. We consider that the frequency of these hindcast waves, 0.1 Hz, is within expectable error of $f_0/2 = 0.087$ Hz. In the light of these data we consider two models. In the first the sea surface waves are assumed to have a unimodal directional distribution symmetrical about the wind direction and follow the form used by Longuet-Higgins, *et al.* (1963) to discuss wave observations from a pitch and roll buoy:

$$Z(\omega,\vartheta) = Z_1(\omega)\cos^q(\vartheta/2)/H(q) \quad (9)$$

where ϑ is measured from the downwind direction, q is a beamwidth parameter, and H(q) is a normalization factor such that the integral of $Z_1(\omega)$ over frequency is the mean square wave elevation. Both radio backscatter observations of wind waves (Tyler, *et al.* 1974) and direct measurements of microseismic pressures at the sea bed (Webb and Cox, 1986) suggest that $q \geq 4$ for the longer period waves generated by a steady wind. In the second model a bimodal directional distribution is assumed. A well collimated incident swell from a distant source interacts with a broadly scattered component reflected from the steep shores of the islands and mainland of Southern California. The incident swell is expected to have very little interaction with itself, but the reflected component although small will have a strong effect because it is oppositely directed. In analogy with equation (9) we assume that the spectra of the incident and reflected waves can be represented by

$$Z_i(\omega,\vartheta) = (1-a)Z_1(\omega)\theta_1(\vartheta)$$

$$Z_r(\omega,\vartheta) = a\,Z_1(\omega)\theta_2(\vartheta)$$

respectively, where a is a small term representing the proportion of total wave energy in the reflected beam. We can assume that the beam width of the reflected waves is much larger than that of the incident waves because the shore lines are irregular. For a rough calculation it is enough to assume that θ_1 is a delta function around the wind direction and that θ_2 has the value $1/\Delta\vartheta$ in the opposite direction.

According to either model, the integral in equation (7) is $I(\omega_0) = J\,[Z_1(\frac{\omega_0}{2})]^2$, where for the first model (Webb and Cox, eq. (15))

$$J = [H(q)]^{-2}\int_{-\pi}^{\pi}\cos^q(\tfrac{\vartheta}{2})\,|\sin^q(\tfrac{\vartheta}{2})|\,d\vartheta$$
$$= \Gamma(\tfrac{1}{2}q+1)/[2^{1+q}\pi^{\frac{1}{2}}\Gamma(\tfrac{1}{2}q+\tfrac{1}{2})] \quad (10)$$

in terms of the gamma function, Γ, and for the second model

$$J = a(1-a)/\Delta\vartheta$$

we now estimate $Z_1(\omega_0/2)$ by dividing the mean square wave elevation by the bandwidth of the wave spectrum. For an estimate of the latter we take $\Delta\omega/2 = .12$/s from figure (3). This makes the mean value of $Z_1(\omega_0/2)$ equal

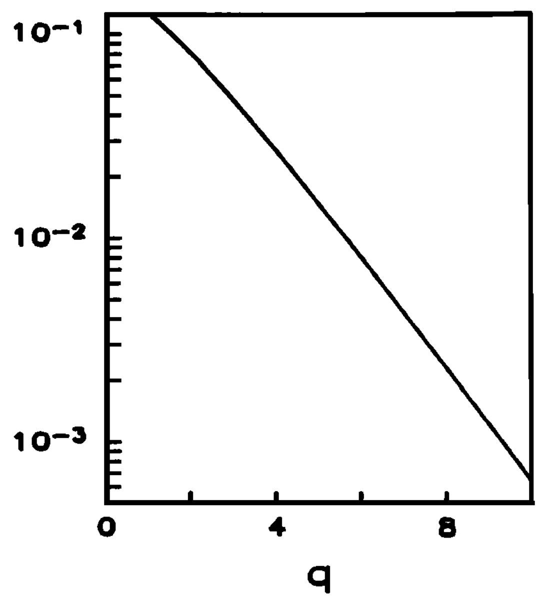

Fig. 5. The interaction integral according to equation (10).

to 1.7 m^2 s during the observation time. The uncertainty in this figure derives both from the inaccuracy of visual estimates of wave heights and from inaccurate knowledge of the bandwidth, but is probably not more than a factor of 2. With this value and the observed value for the interaction integral, equation (7), we can now solve for

$$J = I(\omega_0)/[Z_1(\omega_0/2)]^2 = 0.005$$

again within a factor of two. From Figure (5) we find that this result can be understood in terms of model 1 if the beam parameter q is between 5 and 8, or model 2 if the reflected energy a is a few tenths of a percent. (This assumes that the scattering beamwidth $\Delta\vartheta$ is of the order of one radian.) Thus either model presents a reasonable picture and we suspect it may only prove possible to distinguish the two mechanisms when new measurements are made at sufficient distance from shorelines that the scattered wave components have a distinctly different frequency character than the primary incident waves.

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